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Slightly ν -Closed Mappings

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Abstract

The aim of this paper is to introduce and study the concept of slightly ν -closed mappings and the interrelationship between other closed maps.

Keywords: *ν -closed set, slightly ν -closed map.*

1 Introduction:

Mappings play an important role in the study of modern mathematics, especially in Topology and Functional analysis. Closed mappings are one such mappings which are studied for different types of closed sets by various mathematicians for

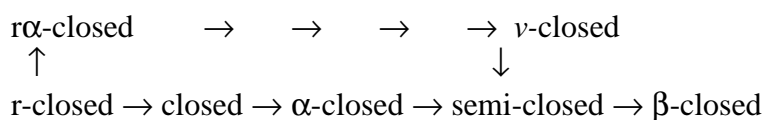
the past many years. N.Biswas, discussed about semiopen mappings in the year 1970, A.S. Mashhour, M.E. Abd El-Monsef and S.N. El-Deeb studied preopen mappings in the year 1982 and S.N.El-Deeb, and I.A.Hasanien defined and studied about preclosed mappings in the year 1983. Further Asit kumar sen and P. Bhattacharya discussed about pre-closed mappings in the year 1993. A.S. Mashhour, I.A. Hasanein and S.N. El-Deeb introduced α -open and α -closed mappings in the year in 1983, F. Cammaroto and T. Noiri discussed about semipre-open and semipre-closed mappings in the year 1989 and G.B. Navalagi further verified few results about semipreclosed mappings. M.E. Abd El-Monsef, S.N. El-Deeb and R.A. Mahmoud introduced β -open mappings in the year 1983 and Saeid Jafari and T.Noiri, studied about β -closed mappings in the year 2000. In the year 2010, S. Balasubramanian and P.A.S. Vjayanathi introduced ν -open mappings and in the year 2011, further defined almost ν -open mappings and also they introduced ν -closed and almost ν -closed mappings. C.W. Baker studied slightly-open and slightly-closed mappings in the year 2011. Inspired with these concepts and its interesting properties we in this paper tried to study a new variety of closed maps called slightly ν -closed and almost slightly ν -closed maps. Throughout the paper X, Y means topological spaces (X, τ) and (Y, σ) on which no separation axioms are assured.

2 Preliminaries:

Definition 2.1: $A \subseteq X$ is said to be

- Regular open[pre-open; semi-open; α -open; β -open] if $A = \text{int}(\text{cl}(A))$ [$A \subseteq \text{int}(\text{cl}(A))$; $A \subseteq \text{cl}(\text{int}(A))$; $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$; $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$] and regular closed[pre-closed; semi-closed; α -closed; β -closed] if $A = \text{cl}(\text{int}(A))$ [$\text{cl}(\text{int}(A)) \subseteq A$; $\text{int}(\text{cl}(A)) \subseteq A$; $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$; $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$]
- ν -open if there exists a r -open set U such that $U \subseteq A \subseteq \text{cl}(U)$.
- g -closed[rg -closed] if $\text{cl}(A) \subset U$ [$r\text{cl}(A) \subset U$] whenever $A \subset U$ and U is open[r -open] in X .
- g -open[rg -open] if its complement $X - A$ is g -closed[rg -closed].

Remark 1: We have the following implication diagrams for closed sets.



Definition 2.2: A function $f: X \rightarrow Y$ is said to be

- Continuous [resp: semi-continuous, r -continuous, ν -continuous] if the inverse image of every open set is open [resp: semi open, regular open, ν -open].

- b) *Irresolute* [resp: *r-irresolute*, *v-irresolute*] if the inverse image of every semi open [resp: *regular open*, *v-open*] set is semi open [resp: *regular open*, *v-open*].
- c) *Open* [resp: *semi-open*, *r-open*, *v-open*] if the image of every open set is open [resp: *semi-open*, *regular-open*, *v-open*].
- d) *Closed* [resp: *semi-closed*, *r-closed*, *v-closed*] if the image of every closed set is closed [resp: *semi- closed*, *regular- closed*, *v-closed*].
- e) *Contra-open* [resp: *contra-semi-open*, *contra-r-open*, *contra- v-open*] if the image of every open set is closed [resp: *semi-closed*, *regular-closed*, *v-closed*].
- f) *Contra-closed* [resp: *contra-semi-closed*, *contra-r-closed*, *contra-v-closed*] if the image of every closed set is open [resp: *semi-open*, *regular-open*, *v-open*].
- g) *Slightly-open* [resp: *slightly-semi-open*, *slightly-r-open*, *slightly-v-open*] if the image of every clopen set is open [resp: *semi-open*, *regular-open*, *v-open*].
- h) *Slightly-closed* [resp: *slightly-semi-closed*, *slightly-r-closed*] if the image of every clopen set is closed [resp: *semi- closed*, *regular- closed*].
- i) *g-continuous* [resp: *rg-continuous*] if the inverse image of every closed set is *g-closed* [resp: *rg-closed*].

Definition 2.3: X is said to be v -regular space (or $v-T_3$ space) if for a open set F and a point $x \notin F$, there exists disjoint v -open sets G and H such that $F \subseteq G$ and $x \in H$.

Definition 2.4: X is said to be $T_{1/2}[r-T_{1/2}]$ if every (regular) generalized closed set is (regular) closed.

3 Slightly v -Closed Mappings:

Definition 3.1: A function $f: X \rightarrow Y$ is said to be

- (i) *Slightly v -closed* if the image of every clopen set in X is v -closed in Y .
- (ii) *Almost slightly v -closed* if the image of every r -clopen set in X is v -closed in Y .

Example 1: Let $X = Y = \{a, b, c\}$; $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$; $\sigma = \{\emptyset, \{a, c\}, Y\}$. Let $f: X \rightarrow Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is slightly v -closed and almost slightly v -closed.

Example 2: Let $X = Y = \{a, b, c\}$; $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$; $\sigma = \{\emptyset, \{a, c\}, Y\}$. Let $f: X \rightarrow Y$ be defined $f(a) = b$, $f(b) = c$ and $f(c) = a$. Then f is not slightly v -closed and not almost slightly v -closed.

Theorem 3.1: Every slightly v -closed map is almost slightly v -closed but not conversely.

Proof: Follows directly from the definitions.

Note 1: We have the following implication diagrams for slightly closed mappings.

$$\begin{array}{ccccccc} \text{i) sl.r}\alpha\text{-closed.map} & \rightarrow & & \rightarrow & & \rightarrow & \text{sl.v.closed.map} \\ & \uparrow & & & & & \downarrow \\ \text{sl.r.closed.map} & \rightarrow & \text{sl.closed.map} & \rightarrow & \text{sl.}\alpha\text{-closed.map} & \rightarrow & \text{sl.s.closed.map} \rightarrow \\ \text{sl.}\beta\text{-closed.map} & & & & & & \end{array}$$

$$\begin{array}{ccccccc} \text{ii) al.sl.r}\alpha\text{-closed.map} & \rightarrow & & \rightarrow & & \rightarrow & \text{al.sl.v.closed.map} \\ & \uparrow & & & & & \downarrow \\ \text{Al.sl.r.closed.map} & \rightarrow & \text{al.sl.closed.map} & \rightarrow & \text{al.sl.}\alpha\text{-closed.map} & \rightarrow & \text{al.sl.s.closed.map} \rightarrow \\ \text{sl.}\beta\text{-closed.map} & & & & & & \end{array}$$

None is reversible.

Note 2: We have the following implication diagram.

$$\begin{array}{ccccccc} \text{i) Contra } \nu\text{-open} & \rightarrow & \text{Slightly } \nu\text{-closed} & \rightarrow & \nu\text{-closed} \\ & \downarrow & & & \downarrow \\ \text{Almost Contra } \nu\text{-open} & \rightarrow & \text{Almost Slightly } \nu\text{-closed} & \rightarrow & \text{Almost-}\nu\text{-closed} \end{array}$$

$$\begin{array}{ccccccc} \text{ii) Contra } \nu\text{-closed} & \rightarrow & \text{Slightly } \nu\text{-open} & \rightarrow & \nu\text{-open} \\ & \downarrow & & & \downarrow \\ \text{Almost Contra } \nu\text{-closed} & \rightarrow & \text{Almost Slightly } \nu\text{-open} & \rightarrow & \text{Almost-}\nu\text{-open} \end{array}$$

None is reversible.

Example 3: Let $X = Y = \{a, b, c\}$; $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$; $\sigma = \{\emptyset, \{a, c\}, Y\}$. Let $f: X \rightarrow Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is slightly ν -closed, almost slightly ν -closed, slightly ν -open, almost slightly ν -open but not ν -closed, ν -open, contra- ν -open and contra- ν -closed.

Example 4: Let $X = Y = \{a, b, c\}$; $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$; $\sigma = \{\emptyset, \{a\}, \{b, c\}, Y\}$. Let $f: X \rightarrow Y$ be defined $f(a) = b$, $f(b) = c$ and $f(c) = a$. Then f is slightly ν -closed, almost slightly ν -closed, slightly ν -open, almost slightly ν -open but not almost ν -closed, almost ν -open, almost contra- ν -open and almost contra- ν -closed.

Theorem 3.2:

- (i) If $R\alpha C(Y) = \nu C(Y)$ then f is [almost-]slightly α -closed iff f is [almost-]slightly ν -closed.
- (ii) If $\nu C(Y) = RC(Y)$ then f is [almost-]slightly r -closed iff f is [almost-]slightly ν -closed.
- (iii) If $\nu C(Y) = \alpha C(Y)$ then f is [almost-]slightly α -closed iff f is [almost-]slightly ν -closed.

Example 5: Let $X = Y = \{a, b, c\}$; $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$; $\sigma = \{\emptyset, \{a, c\}, Y\}$. Let $f: X \rightarrow Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is slightly pre-closed, slightly v -closed, slightly β -closed, slightly open, slightly α -closed, slightly $r\alpha$ -closed, slightly semi-closed and slightly r -closed.

Example 6: Let $X = Y = \{a, b, c\}$; $\tau = \{\emptyset, \{a\}, X\}$; $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$. Let $f: X \rightarrow Y$ be defined $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then f is slightly open, slightly pre-closed, slightly β -closed, slightly α -closed, slightly $r\alpha$ -closed, slightly semi-closed, slightly v -closed and slightly r -closed.

Example 7: Let $X = Y = \{a, b, c\}$; $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$; $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Y\}$. Assume $f: X \rightarrow Y$ be the identity map. Then f is slightly open, slightly pre-closed, slightly β -closed, slightly α -closed, slightly semi-closed, slightly $r\alpha$ -closed, slightly v -closed and slightly r -closed.

Theorem 3.3: *If f is [almost-] slightly closed and g is v -closed then $g \circ f$ is [almost-] slightly v -closed.*

Proof: Let $A \subseteq X$ be r -clopen $\Rightarrow f(A)$ is closed in $Y \Rightarrow g(f(A)) = g \circ f(A)$ is v -closed in Z . Hence $g \circ f$ is slightly v -closed.

Theorem 3.4: *If f is [almost-] slightly closed and g is r -closed then $g \circ f$ is [almost-] slightly v -closed.*

Proof: Follows from Remark 1 and Theorem 3.3.

Theorem 3.5: *If f and g are [almost-] slightly r -closed then $g \circ f$ is [almost-] slightly v -closed.*

Proof: Let $A \subseteq X$ be r -clopen $\Rightarrow f(A)$ is r -closed in $Y \Rightarrow f(A)$ is closed in $Y \Rightarrow g(f(A))$ is r -closed in $Z \Rightarrow g \circ f(A)$ is v -closed in Z . Hence $g \circ f$ is [almost-] slightly v -closed.

Theorem 3.6: *If f is [almost-] slightly r -closed and g is v -closed then $g \circ f$ is [almost-] slightly v -closed.*

Proof: Let $A \subseteq X$ be r -clopen $\Rightarrow f(A)$ is r -closed in $Y \Rightarrow f(A)$ is closed in $Y \Rightarrow g(f(A)) = g \circ f(A)$ is v -closed in Z . Hence $g \circ f$ is [almost-] slightly v -closed.

Theorem 3.7: *If f is [almost-] slightly closed and g is $r\alpha$ -closed then $g \circ f$ is [almost-] slightly v -closed.*

Proof: Let $A \subseteq X$ be r -clopen $\Rightarrow f(A)$ is closed in $Y \Rightarrow g(f(A))$ is $r\alpha$ -closed in $Z \Rightarrow g \circ f(A)$ is v -closed in Z . Hence $g \circ f$ is [almost-] slightly v -closed.

Corollary 3.1:

- a) If f is [almost-] slightly open [[almost-] slightly r -closed] and g is [almost-] v -closed then $g \circ f$ is [almost-] slightly semi-closed and hence [almost-] slightly β -closed.
- b) If f is [almost-] slightly r -closed and g is [almost-] $r\alpha$ -closed then $g \circ f$ is [almost-] slightly semi-closed and hence [almost-] slightly β -closed.

Theorem 3.8: If $f: X \rightarrow Y$ is slightly v -closed, then $v(\text{cl}\{f(A)\}) \subset f(\text{cl}\{A\})$

Proof: Let $A \subset X$ and $f: X \rightarrow Y$ is slightly v -closed gives $f(\text{cl}\{A\})$ is v -closed in Y

and $f(A) \subset f(\text{cl}\{A\})$ which in turn gives $v(\text{cl}\{f(A)\}) \subset v\text{cl}\{(f(\text{cl}\{A\}))\}$ - - - - - (1)

Since $f(\text{cl}\{A\})$ is v -closed in Y , $v\text{cl}\{(f(\text{cl}\{A\}))\} = f(\text{cl}\{A\})$ - - - - - (2)

From (1) and (2) we have $v(\text{cl}\{f(A)\}) \subset (f(\text{cl}\{A\}))$ for every subset A of X .

Remark 2: converse is not true in general.

Corollary 3.2: If $f: X \rightarrow Y$ is slightly r -closed, then $v(\text{cl}\{f(A)\}) \subset f(\text{cl}\{A\})$

Theorem 3.9: If $f: X \rightarrow Y$ is slightly v -closed and $A \subset X$ is r -closed, then $f(A)$ is τ_v -closed in Y .

Proof: Let $A \subset X$ and $f: X \rightarrow Y$ is slightly v -closed implies $v(\text{cl}\{f(A)\}) \subset f(\text{cl}\{A\})$ which in turn implies $v(\text{cl}\{f(A)\}) \subset f(A)$, since $f(A) = f(\text{cl}\{A\})$. But $f(A) \subset v(\text{cl}\{f(A)\})$. Combaining we get $f(A) = v(\text{cl}\{f(A)\})$. Hence $f(A)$ is τ_v -closed in Y .

Corollary 3.3: If $f: X \rightarrow Y$ is slightly r -closed, then $f(A)$ is τ_v -closed in Y if A is r -closed set in X .

Theorem 3.10: If $v(\text{cl}\{A\}) = r(\text{cl}\{A\})$ for every $A \subset Y$, then the following are equivalent:

- a) $f: X \rightarrow Y$ is slightly v -closed map
- b) $v(\text{cl}\{f(A)\}) \subset f(\text{cl}\{A\})$

Proof: (a) \Rightarrow (b) follows from theorem 3.8

(b) \Rightarrow (a) Let A be any r -closed set in X , then $f(A) = f(\text{cl}\{A\}) \supset v(\text{cl}\{f(A)\})$ by hypothesis. We have $f(A) \subset v(\text{cl}\{f(A)\})$. Combaining we get $f(A) = v(\text{cl}\{f(A)\}) = r(\text{cl}\{f(A)\})$ [by given condition] which implies $f(A)$ is r -closed and hence v -closed. Thus f is v -closed.

Theorem 3.11: $f: X \rightarrow Y$ is slightly v -closed iff for each subset S of Y and each r -clopen set U containing $f^{-1}(S)$, there is a v -closed set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Proof: Assume f is slightly v -closed, $S \subset Y$ and U an r -closed set of X containing $f^{-1}(S)$, then $f(X - U)$ is v -closed in Y and $V = Y - f(X - U)$ is v -closed in Y . $f^{-1}(S) \subset U$ implies $S \subset V$ and $f^{-1}(V) = X - f^{-1}(f(X - U)) \subset X - (X - U) = U$.

Conversely let F be r -clopen in X , then $f^{-1}(f(F^c)) \subset F^c$. By hypothesis, exists $V \in VC(Y)$ such that $f(F^c) \subset V$ and $f^{-1}(V) \subset F^c$ and so $f \subset (f^{-1}(V))^c$. Hence $V^c \subset f(F) \subset f[(f^{-1}(V))^c] \subset V^c$ implies $f(F) \subset V^c$, which implies $f(F) = V^c$. Thus $f(F)$ is v -closed in Y and therefore f is v -closed.

Remark 3: Composition of two [almost-] slightly v -closed maps is not [almost-] slightly v -closed in general.

Theorem 3.12: Let X, Y, Z be topological spaces and every v -closed set is closed [r -closed] in Y . Then the composition of two [almost-] slightly v -closed [[almost-] slightly r -closed] maps is [almost-] slightly v -closed.

Proof: (a) Let f and g be slightly v -closed maps. Let A be any clopen set in $X \Rightarrow f(A)$ is closed in Y (by assumption) $\Rightarrow g(f(A)) = g \circ f(A)$ is v -closed in Z . Therefore $g \circ f$ is slightly v -closed.

Example 8: Let $X = Y = Z = \{a, b, c\}$; $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$; $\sigma = \{\emptyset, \{a, c\}, Y\}$ and $\eta = \{\emptyset, \{a\}, \{b, c\}, Z\}$. $f: X \rightarrow Y$ be defined $f(a) = c, f(b) = b$ and $f(c) = a$ and $g: Y \rightarrow Z$ be defined $g(a) = b, g(b) = a$ and $g(c) = c$, then g and f are slightly v -closed and $g \circ f$ is also slightly v -closed.

Theorem 3.13: If $f: X \rightarrow Y$ is [almost-] slightly g -closed, $g: Y \rightarrow Z$ is v -closed [r -closed] and Y is $T_{1/2}$ [r - $T_{1/2}$] then $g \circ f$ is [almost-] slightly v -closed.

Proof: (a) Let A be a clopen set in X . Then $f(A)$ is g -closed set in $Y \Rightarrow f(A)$ is closed in Y as Y is $T_{1/2} \Rightarrow g(f(A)) = g \circ f(A)$ is v -closed in Z since g is v -closed. Hence $g \circ f$ is slightly v -closed.

Corollary 3.4: If $f: X \rightarrow Y$ is [almost-] slightly g -closed, $g: Y \rightarrow Z$ is v -closed [r -closed] and Y is $T_{1/2}$ [r - $T_{1/2}$] then $g \circ f$ is [almost-] slightly semi-closed and hence [almost-] slightly β -closed.

Theorem 3.14: If $f: X \rightarrow Y$ is [almost-] slightly rg -closed, $g: Y \rightarrow Z$ is v -closed [r -closed] and Y is r - $T_{1/2}$, then $g \circ f$ is [almost-] slightly v -closed.

Corollary 3.5: If $f: X \rightarrow Y$ is [almost-] slightly rg -closed, $g: Y \rightarrow Z$ is [almost-] v -closed [[almost-] r -closed] and Y is r - $T_{1/2}$, then $g \circ f$ is [almost-] slightly semi-closed and hence [almost-] slightly β -closed.

Theorem 3.15: *If $f:X \rightarrow Y$, $g:Y \rightarrow Z$ be two mappings such that $g \circ f$ is [almost-] slightly v -closed [[almost-] slightly r -closed] then the following statements are true.*

- a) *If f is continuous [r -continuous] and surjective then g is [almost-] slightly v -closed.*
- b) *If f is g -continuous, surjective and X is $T_{1/2}$ then g is [almost-] slightly v -closed.*
- c) *If f is rg -continuous, surjective and X is r - $T_{1/2}$ then g is [almost-] slightly v -closed.*

Proof: (a) Let A be a clopen set in $Y \Rightarrow f^{-1}(A)$ is closed in $X \Rightarrow (g \circ f)(f^{-1}(A))$ is v -closed in $Z \Rightarrow g(A)$ is v -closed in Z . Hence g is slightly v -closed.

Similarly one can prove the remaining parts and hence omitted.

Corollary 3.6: *If $f:X \rightarrow Y$, $g:Y \rightarrow Z$ be two mappings such that $g \circ f$ is v -closed [r -closed] then the following statements are true.*

- a) *If f is continuous [r -continuous] and surjective then g is [almost-] slightly semi-closed and hence [almost-] slightly β -closed.*
- b) *If f is g continuous, surjective and X is $T_{1/2}$ then g is [almost-] slightly semi-closed and hence [almost-] slightly β -closed.*
- c) *If f is rg -continuous, surjective and X is r - $T_{1/2}$ then g is [almost-] slightly semi-closed and hence [almost-] slightly β -closed.*

To prove the following theorem we use the following Definition.

Theorem 3.16: *If X is v -regular, $f: X \rightarrow Y$ is r -closed, nearly-continuous, v -closed surjection and $\bar{A} = A$ for every v -closed set in Y , then Y is v -regular.*

Proof: Let $p \in U \in VO(Y)$. Then there exists a point $x \in X$ such that $f(x) = p$ as f is surjective. Since X is v -regular and f is r -continuous there exists $V \in RO(X)$ such that $x \in V \subseteq \bar{V} \subseteq f^{-1}(U)$ which implies $p \in f(V) \subseteq f(\bar{V}) \subseteq f(f^{-1}(U)) = U \rightarrow (1)$

Since f is v -closed, $f(\bar{V}) \subseteq U$, By hypothesis $\overline{f(\bar{V})} = f(\bar{V})$ and $\overline{f(\bar{V})} = \overline{f(V)} \rightarrow (2)$

By (1) & (2) we have $p \in f(V) \subseteq f(\bar{V}) \subseteq U$ and $f(V)$ is v -open. Hence Y is v -regular.

Corollary 3.7: *If X is v -regular, $f: X \rightarrow Y$ is r -closed, nearly-continuous, v -closed surjection and $\bar{A} = A$ for every r -closed set in Y , then Y is v -regular.*

Theorem 3.17: *If $f:X \rightarrow Y$ is [almost-] slightly v -closed [[almost-] slightly r -closed] and A is a closed set of X then $f_A:(X, \tau(A)) \rightarrow (Y, \sigma)$ is [almost-] slightly v -closed.*

Proof: Let F be a clopen set in A . Then $F = A \cap E$ for some closed set E of X and so F is closed in $X \Rightarrow f(A)$ is v -closed in Y . But $f(F) = f_A(F)$. Therefore f_A is slightly v -closed.

Corollary 3.8: *If $f: X \rightarrow Y$ is [almost-] slightly v -closed [[almost-] slightly r -closed] and A is a closed set of X then $f_A: (X, \tau(A)) \rightarrow (Y, \sigma)$ is [almost-] slightly semi-closed and hence [almost-] slightly β -closed.*

Theorem 3.18: *If $f: X \rightarrow Y$ is [almost-] slightly v -closed [[almost-] slightly r -closed], X is $T_{1/2}$ and A is g -closed set of X then $f_A: (X, \tau(A)) \rightarrow (Y, \sigma)$ is [almost-] slightly v -closed.*

Proof: Let F be a clopen set in A . Then $F = A \cap E$ for some closed set E of X and so F is closed in $X \Rightarrow f(A)$ is v -closed in Y . But $f(F) = f_A(F)$. Therefore f_A is slightly v -closed.

Corollary 3.9: *If $f: X \rightarrow Y$ is [almost-] slightly v -closed [[almost-] slightly r -closed], X is $T_{1/2}$, A is g -closed set of X then $f_A: (X, \tau(A)) \rightarrow (Y, \sigma)$ is [almost-] slightly semi-closed and hence [almost-] slightly β -closed.*

Theorem 3.19: *If $f_i: X_i \rightarrow Y_i$ be [almost-] slightly v -closed [[almost-] slightly r -closed] for $i = 1, 2$. Let $f: X_1 \times X_2 \rightarrow Y_1 \times Y_2$ be defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then $f: X_1 \times X_2 \rightarrow Y_1 \times Y_2$ is [almost-] slightly v -closed.*

Proof: Let $U_1 \times U_2 \subseteq X_1 \times X_2$ where U_i is clopen in X_i for $i = 1, 2$. Then $f(U_1 \times U_2) = f_1(U_1) \times f_2(U_2)$ is v -closed set in $Y_1 \times Y_2$. Hence f is slightly v -closed.

Corollary 3.10: *If $f_i: X_i \rightarrow Y_i$ be [almost-] slightly v -closed [[almost-] slightly r -closed] for $i = 1, 2$. Let $f: X_1 \times X_2 \rightarrow Y_1 \times Y_2$ be defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$, then $f: X_1 \times X_2 \rightarrow Y_1 \times Y_2$ is [almost-] slightly semi-closed and hence [almost-] slightly β -closed.*

Theorem 3.20: *Let $h: X \rightarrow X_1 \times X_2$ be [almost-] slightly v -closed. Let $f_i: X \rightarrow X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then $f_i: X \rightarrow X_i$ is [almost-] slightly v -closed for $i = 1, 2$.*

Proof: Let U_1 be r -clopen in X_1 , then $U_1 \times X_2$ is r -clopen in $X_1 \times X_2$, and $h(U_1 \times X_2)$ is v -closed in X . But $f_1(U_1) = h(U_1 \times X_2)$, therefore f_1 is slightly v -closed. Similarly we can show that f_2 is also slightly v -closed and thus $f_i: X \rightarrow X_i$ is slightly v -closed for $i = 1, 2$.

Corollary 3.11: *Let $h: X \rightarrow X_1 \times X_2$ be [almost-] slightly v -closed. Let $f_i: X \rightarrow X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then $f_i: X \rightarrow X_i$ is [almost-] slightly semi-closed and hence [almost-] slightly β -closed for $i = 1, 2$.*

Conclusion

In this paper we introduced the concept of slightly ν -closed and almost slightly ν -closed mappings, studied their basic properties and the interrelationship between other slightly closed maps.

References

- [1] D. Andrijevic, Semi pre-open sets, *Mat. Vesnik*, 38(1986), 24-32.
- [2] A.K. Sen and P. Bhattacharya, On preclosed mappings, *Bull. Cal. Math. Soc.*, 85(1993), 409-412.
- [3] C.W. Baker, On weak forms of contra-open and contra-closed mappings, *International Journal of Pure and Applied Mathematics*, 73(3) (2011), 281-287.
- [4] S. Balasubramanian and P.A.S. Vyjayanthi, ν -open mappings, *Scientia Magna*, 6(4) (2010), 118-124.
- [5] S. Balasubramanian, C. Sandhya and P.A.S. Vyjayanthi, Almost ν -open mappings, *Inter. J. Math. Archive*, 2(10) (2011), 1943-1948.
- [6] S. Balasubramanian, P.A.S. Vyjayanthi and C. Sandhya, Contra ν -closed map, *International Journal of Engineering Sciences and Research Technology*, 2(3) (2013), 544-549.
- [7] S. Balasubramanian, C. Sandhya and P.A.S. Vyjayanthi, Contra ν -open map, *International Journal of Engineering Sciences and Research Technology*, 2(3) (2013), 561-566.
- [8] S. Balasubramanian, C. Sandhya and P.A.S. Vyjayanthi, Slightly ν -open map (Communicated).
- [9] G. Di Maio and T. Noiri, On s-closed spaces, *I.J.P.A.M.*, 18(3) (1987), 226-233.
- [10] J. Dontchev, On generating semi-pre open sets, *Mem. Fac. Sci. Kochi Univ. Ser.A., Math.*, 16(1995), 35-48.
- [11] W. Dunham, $T_{1/2}$ spaces, *Kyungpook Math. J.*, 17(1977), 161-169.
- [12] P.E. Long and L.L. Herington, Basic properties of regular closed functions, *Rend. Cir. Mat. Palermo*, 27(1978), 20-28.
- [13] A.S. Mashour, I.A. Hasanein and S.N. El. Deep, α -continuous and α -open mappings, *Acta Math. Hungar.*, 41(1983), 213-218.
- [14] N. Levine, Semi open sets and semi continuity in topological spaces, *Amer. Math. Monthly*, 70(1963), 36-41.
- [15] N. Levine, Generalized closed sets in topological spaces, *Rend. Civr. Mat. Palermo*, 19(1970), 89-96.
- [16] N. Palaniappan, Studies on regular-generalized closed sets and maps in topological spaces, *Ph. D Thesis*, (1995), Alagappa University, Karikudi.
- [17] M.E.A. El. Monsef, S.N. El. Deep and R.A. Mohmoud, β -open sets and β -continuous mappings, *Bull. Fac. Sci. Assiut Univ.*, A12(1) (1983), 77-90.
- [18] K. Balachandran, P. Sundaram and H. Maki, On generalized continuous maps in topological spaces, *Mem. Fac. Sci. Kochi Univ. Ser. A. Math*, 12(1991), 5-13.

- [19] A.S. Mashhour, M.E.A. El-Monsef and S.N. El. Deep, On pre continuous and weak pre continuous mappings, *Proc. Math. Phy. Soc.*, Egypt, 53(1982), 47-53.
- [20] O. Njastad, On some classes of nearly open sets, *Pacific J. Math.*, 109(2) (1984), 118-126.