Gen. Math. Notes, Vol. 7, No. 2, December 2011, pp. 1-14
ISSN 2219-7184; Copyright © ICSRS Publication, 2011
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Generalized Intuitionistic Fuzzy Soft Sets and Its Applications

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(Received: 12-5-11 / Accepted: 29-11-11)

Abstract

In this paper we define generalized intuitionistic fuzzy soft sets and set theoretical operations with illustrating examples. It also proposes a similarity measure for generalized intuitionistic fuzzy soft set and used it to find out the similarity between synthetic texture and natural texture. A congenial method for solving multi criteria decision making problem in generalized intuitionistic fuzzy soft set frame work is presented.

Keywords: *Fuzzy soft sets, Generalized Intuitionistic fuzzy soft sets, Similarity measure, Multi criteria decision making problem.*

1 Introduction

We are living in a real world where we have to handle situations involving uncertainty, imprecision and vagueness. Moreover the great deal of data involved in economics, engineering, medical science and other fields are not always vivid and includes all kinds of uncertainty. But in classical mathematics all the mathematical tools for modeling, reasoning and calculation are certain or precise which deals with certain problems. So that they can't solve those complex problems in real life situations. In recent years researchers have become interested to deal with the complexity of uncertain data. There are a wide range of theories such as probability theory, fuzzy set theory, vague set theory which are considered as mathematical approaches to modeling vagueness. But each of these theories has its own inherent difficulties, which are pointed out in [1].The reason why there exist such difficulties is lack of the theory of expressing parameters. The tools for making sure parameters are so poor that uncertainty of parameters becomes the bottleneck of using these theories. To solve this problem, in 1999 D. Molodtsov set up the basic theory of soft sets which can well deal with uncertain, fuzzy, unclear information. This theory has proven useful in many different fields such as the Smoothness of functions, Game theory, Operations research, Riemann integration, Perron integration, Probability theory, and Measurement theory.

At present study on soft set is still discovering. Maji et al. introduced several algebraic operations in soft set theory and published a detailed theoretical study on soft sets [2]. The same authors [3] also extended crisp soft sets to fuzzy soft sets. The algebraic nature of soft set has been studied by several researchers. Aktas and Cagman [4] initiated soft groups, F. Feng [5] defined soft semirings. Q.M Sun [6] introduced a basic version of soft module theory, which extends the notion of a module by including some algebraic structures in soft sets.

In the meantime soft set theory has been applied practically in many domains. Maji et al.[7] used soft set in decision making problem. D Chen [8] proposed a reasonable definition of parameter reduction of soft sets and improved the application of a soft set in a decision making problem. Milind [9] presented a novel method for classification of natural textures using the notions of soft set theory. An attempt to assess sound quality based on soft set approach has been made by Bozena kostek [10].

In this paper we concentrate on intuitionistic fuzzy soft set. Intuitionistic fuzzy set was introduced by K.T.Atanassov [11,12] as an extensions of the standard fuzzy sets. Later Maji et al. [13,14] introduced the concept of intuitionistic fuzzy soft set. This paper generalizes intuitionistic fuzzy soft set.

The paper is organized as follows: Section 2 reviews the notions of soft sets, Intutionistic fuzzy soft set and relevant definitions used in the proposed work. In section 3 we introduce the concept of generalised intuitionistic fuzzy soft sets and define some operations such as subset, union, intersection, compliment all explained with examples. We also give some results based on it. In section 4 we propose a similarity measure of generalized intuitionistic fuzzy soft sets in a way similar to that of P Majumdar [15] and used it to find the similarity of natural texture and its synthetic copy. In section 5 we present a novel method for solving multi-criteria decision-making problem in generalized intuitionistic fuzzy soft set environment. At last we conclude the paper with a summary and outlook for further research in section 6.

2 Preliminaries

Definition 2.1 *[1] Let U be an initial universal set and let E be set of parameters. Let P(U) denote the power set of U. A pair (F, E) is called a soft set over U if F is a mapping given by F : E → P(U)*

Definition 2.2 *[13] Let U be an initial universal set and let E be set of parameters. Let P(U) denotes the set of all intuitionistic fuzzy sets of U. A pair (F, A) is called an intuitionistic fuzzy soft set over U if F is a mapping given by F : A → P(U)*
We write an Intuitionistic fuzzy soft set shortly as IF soft set.

Example 2.3 We give an example of an IF soft set. Suppose that there are five people in the universe given by, $U = \{p_1, p_2, p_3, p_4, p_5\}$ and $E = \{e_1, e_2, e_3\}$ where e_1 stands for young , e_2 stands for smart, e_3 stands for middle-aged. Suppose that

$$F(e_1) = \left\{\frac{p_1}{(0.5,0.2)}, \frac{p_2}{(0.9,0.1)}, \frac{p_3}{(0.4,0.3)}, \frac{p_4}{(0,0.56)}, \frac{p_5}{(0.2,0.5)}\right\}$$
$$F(e_2) = \left\{\frac{p_1}{(0.3,0.2)}, \frac{p_2}{(0.9,0.1)}, \frac{p_3}{(.11,.77)}, \frac{p_4}{(0.8,0.13)}, \frac{p_5}{(0.5,0.5)}\right\}$$
$$F(e_3) = \left\{\frac{p_1}{(0.5,0.1)}, \frac{p_2}{(0.3,0.75)}, \frac{p_3}{(0.7,0.27)}, \frac{p_4}{0(.7,0.13)}, \frac{p_5}{(0.7,0.31)}\right\}$$

Thus IF soft set is a parameterized family of all Intuitionistic fuzzy set of *U* and gives us a approximate description of the object.

Definition 2.4 *[13] For two intutionistic fuzzy soft set (F, A) and (G, B) over the common universe U, we say that (F, A) is an intutionistic fuzzy soft subset of (G, B) if*

1. A ⊂ B,
2. F(e) is an intuitionistic fuzzy subset of G(e).

We write (F, A) ⊆ (G, B). (F, A) is said to be a intutionistic fuzzy soft super set of (G, B) if (G, B) is intutionistic fuzzy soft sub set of (F, A).We denote this as (F, A)⊇(G, B) .

Definition 2.5 *[13] Two intutionistic fuzzy soft set (F, A) and (G, B) over the common universe U are said to be intutionistic fuzzy soft equal if (F, A) is an intutionistic fuzzy soft subset set of (G, B) and (G, B) is intutionistic fuzzy soft sub set of (F, A).*

Definition 2.6 *[13] The compliment of an intuitionistic fuzzy soft set (F, A), denoted by $(F, A)^c$ is defined by $(F, A)^c = (F^c, \rceil A)$ where $F^c : \rceil A \to P(U)$ is the mapping given by $F^c(\rceil e) =$ intuitionistic fuzzy compliment of F(e) for every 'e' in A.*

Definition 2.7 *[13] A soft set (F; A) over U is said to be null intuitionistic fuzzy* *soft set denoted by* Φ *if* $\forall\, e \in A$ $F(e)$ *= intuitionistic fuzzy set 0 of U where 0 =* $\{(x, 0, 1) : x \in U\}$.

Definition 2.8 *[13] A soft set (F; A) over U is said to be absolute intuitionistic fuzzy soft set denoted by* $\bar{A}$ *, if* $\forall\, e \in A$,

$F(e)$ *= intutionistic fuzzy set 1 of U where* $1 = \{ (x, 1, 0) : x \in U\}$.

Definition 2.9 *[16] (Lattice*$(L_*, \leq_*)$*)*
Consider the set L and the operation defined by

$$L = \{(x_1; x_2): x_1, x_2 \in [0; 1]; x_1 + x_2 \leq 1\},$$

$(x_1, x_2) \leq_* (y_1, y_2) \Leftrightarrow x_1 \leq y_1$ *and* $x_2 \geq y_2$. *Then* $(L_*, \leq_*))$ *is a complete lattice.*

Definition 2.10 *[16] The operators* $\wedge$ *and* $\vee$ *on* $(L_*, \leq_*))$ *are defined as follows:*

$(x_1, x_2) \wedge (y_1, y_2) = (\min(x_1, y_1), \max(x_2, y_2))$,
$(x_1, x_2) \vee (y_1, y_2) = (\max(x_1, y_1), \min(x_2, y_2))$, *for* $(x_1, x_2); (y_1, y_2) \in L_*$.

3 Generalized Intuitionistic Fuzzy Soft Sets

In this section we define intuitoinistic fuzzy soft set in a more generalized way and discuss some related properties.

Definition 3.1 *Let* $U = \{x_1, x_2 \ldots\ldots\ldots x_n\}$ *be universal set and E be set of parameters. The pair (U, E) is a soft universe. Let* $F : E \to \mathcal{P}(U)$ *and* $< \mu, \upsilon >$ *be intutionistic fuzzy subset of E , i.e,* $\mu, \upsilon : E \to [0; 1]$, *where* $\mathcal{P}(U)$ *denotes the set of all IF sub sets of U. Let* $F_{\mu\upsilon}$ *be the mapping* $F_{\mu\upsilon} : E \to \mathcal{P}(U) \times I^2$ *defined as follows :*
$F_{\mu\upsilon}(e) = (F(e), \mu(e), \nu(e))$ *where* $F(e) \in \mathcal{P}(U)$ *Then* $F_{\mu\upsilon}$ *is called generalised intuitionistic fuzzy soft set (GIFSS in short) over the soft set (U, E).*
Obviously, every intuitionistic fuzzy set has the form $(F(e), \mu(e), \nu(e))$ *where* $\mu(e) = 1, \forall\, e \in E$ *and* $\upsilon(e) = 0, \forall\, e \in E$.
In short, for each parameter e, $F_{\mu\upsilon}(e)$ *gives not only the extent to which each element in U belongs or not to F(e) but also indicates how much such belonging possible or not.*

Example 3.2 Let U is the set of medicines under consideration given by
$U = \{m_1, m_2, m_3\}$ and $E = \{e_1, e_2, e_3\}$ where e_1 stands for malaria , e_2 stands for typhoid and e_3 stands for head ache. Let $<\mu, \upsilon>$ be IF subset of E defined as follows :

$\mu(e_1) = 0.1$, $\mu(e_2) = 0.6$, $\mu(e_3) = 0.8$
$\upsilon(e_1) = 0.8$, $\upsilon(e_2) = 0.3$, $\upsilon(e_3) = 0.2$

We define a function $F_{\mu\upsilon}$ *: E → 𝒫(U) × I*^{2} as follows :

$F_{\mu\upsilon}$ (e$_1$) = $\left\{\frac{m_1}{(0.7,0.2)}, \frac{m_2}{(0.4,0.3)}, \frac{m_3}{(0.3,0.5)}, (0.1,0.8)\right\}$
$F_{\mu\upsilon}$ (e$_2$) = $\left\{\frac{m_1}{(0.1,0.8)}, \frac{m_2}{(0.2,0.7)}, \frac{m_3}{(.9,0.1)}, (0.6,0.3)\right\}$
$F_{\mu\upsilon}$ (e$_3$) = $\left\{\frac{m_1}{(0.8,0.1)}, \frac{m_2}{(0.5,0.5)}, \frac{m}{(0.2,0.7)}, (0.8,0.2)\right\}$

then $F_{\mu\upsilon}$ is GIFSS over *(U,E).* Here $F_{\mu\upsilon}$ point out how much each medicine m$_i$ is effective or not for the disease e , but it also gives the approximation about the degree of membership and degree of non-membership of such combination F(e). Now the GIFSS discussed above can be represented in tabular form as follows :

	e$_1$	e$_2$	e$_3$
m$_1$	(0.7,0.2)	(0.1,0.8)	(0.8,0.1)
m$_2$	(0.4,0.3)	(0.2,0.7)	(0.5,0.5)
m$_3$	(0.3,0.5)	(0.9,0.1)	(0.2,0.7)
$<\mu,\nu>$	(0.1,0.8)	(0.6,0.3)	(0.8,0.2)

Table 1 $F_{\mu\upsilon}$

Definition 3.3 *Let* $F_{\mu\upsilon}$ *and* $G_{\alpha\beta}$ *be two GIFSS over (U, E). Then* $F_{\mu\upsilon}$ *is said to be generalised IF soft subset of* $G_{\alpha\beta}$ *if*

1. $< \mu,\upsilon >$ *is IF subset of* $< \alpha,\beta >$
2. *F(e) is also IF subset of G(e) for each parameter e.*

We denote this as $F_{\mu\upsilon} \;\tilde{\subseteq}\; G_{\alpha\beta}$ *.*

Example 3.4 Consider the GIFSS $F_{\mu\upsilon}$ over (U,E) as given in the example 3.2. Let $G_{\alpha\beta}$ another GIFSS over (U,E) defined as follows:

$G_{\alpha\beta}$ (e$_1$) = $\left\{\frac{m_1}{(0.2,0.7)}, \frac{m}{(0.3,0.6)}, \frac{m_3}{(0.1,0.8)}, (0.1,0.8)\right\}$
$G_{\alpha\beta}$ (e$_2$) = $\left\{\frac{m}{(0,0.8)}, \frac{m_2}{(0.1,0.9)}, \frac{m_3}{(.72,0.13)}, (0.3,0.65)\right\}$
$G_{\alpha\beta}$ (e$_3$) = $\left\{\frac{m_1}{(0.71,0.3)}, \frac{m_2}{(0.33,0.65)}, \frac{m}{(0.1,0.5)}, (0.5,0.5)\right\}$

Clearly we have $G_{\alpha\beta} \;\tilde{\subseteq}\; F_{\mu\upsilon}$

Definition 3.5 *[17] Given a fuzzy t-norm t and t-conorm s satisfying* *t(a,b) ≤ 1− s(1-a,1-b) for all a,b ∈ [0,1], the mapping T and S defined by*

$$T(x, y) = (t(x_1, y_1), s(x_2, y_2))$$

$$S(x, y) = \big(s(x_1, y_1), t(x_2, y_2)\big) \; for\ every\ \ x = (x_1, x_2) and\ y = (y_1, y_2)$$

in [0,1]$\times$ [0,1] *is IF t norm and IF t-conorm respectively.*

Definition 3.6 *[17] If n is involutive fuzzy negator then the mapping N defined by N(x) = (n(1-x_2),1−n(x_1)) for all* $x = (x_1, x_2) \in$ *[0,1]*$\times$ [0,1] *is involutive IF negator.*
In the rest of this paper we take IF t-norm T and IF t- conorm S satisfying Archimedean property and involutive IF negator N.

Definition 3.7 *Let $F_{\mu\upsilon}$ be GIFSS over (U,E). Then compliment of $F_{\mu\upsilon}$ denoted by $(F_{\mu\upsilon})^c$ is defined by $(F_{\mu\upsilon})^c = G_{\alpha\beta}$ where $(\alpha(e), \beta(e)) = N(\mu(e), \nu(e))$ and G(e) = N(F(e)) for every e ∈ E .*
Note that $(F_{\mu\upsilon}{}^c)^c = F_{\mu\upsilon}$ as IF compliment is involutive.

Definition 3.8 *Union of two GIFSS $F_{\mu\upsilon}$ and $G_{\alpha\beta}$ denoted by $F_{\mu\upsilon} \tilde{\cup} G_{\alpha\beta}$ is GIFSS $H_{\gamma\delta}$ defined as $H_{\gamma\delta}$: E → $\mathcal{P}$(U) × I^2 such that*

$H_{\gamma\delta}$ (e) = (H(e),γ(e),δ(e)) where H(e) = S (F(e), G(e)),
γ(e) = S (μ(e),α(e)), δ(e) = S (ν(e), β(e))

and S is IF t conorm.

Definition 3.9 *Intersection of two GIFSS $F_{\mu\upsilon}$ and $G_{\alpha\beta}$ denoted by $F_{\mu\upsilon} \tilde{\cap} G_{\alpha\beta}$ is GIFSS H defined as H : E → $\mathcal{P}$(U) × I^2 such that $H_{\gamma\delta}$ (e) = (H(e),γ(e),δ(e)) where H(e) = T(F(e), G(e)), γ(e) = T (μ(e),α(e)), δ(e) = T (ν(e), β(e)) and T is IF t-norm.*

Example 3.10 Let us consider the two GIFSS $F_{\mu\upsilon}$ and $G_{\alpha\beta}$ given in examples 3.2 and 3.4. Let us define a fuzzy t-norm on [0,1] as follows : t(a, b) = ab and the t-conorm on [0,1] as s(a, b) = a + b −ab. Consider the fuzzy compliment n defined by n(a) = 1− a. Then IF t-norm, IF t-co-norm and IF compliment is given by

$$T(x, y) = (x_1 y_1;\ x_2 + y_2 - x_2 y_2)$$
$$S(x, y) = (x_1 + y_1 - x_1 y_1,\ x_2 y_2)$$
$$N(x) = (x_2,\ 1 - x_1)$$

for every $x = (x_1, x_2), y = (y_1, y_2) \in [0,1] \times [0,1]$

	e_1	e_2	e_3
m_1	(0.76,0.14)	(0.1,0.64)	(0.942,0.03)
m_2	(0.57,0.18)	(0.28,0.63)	(0.66,0.32)
m_3	(0.07,0.4)	(0.972,0.013)	(0.28,0.49)
$<\mu,\nu>$	(0.19,0.64)	(0.42,0.195)	(0.9,0.1)

Table 2: $F_{\mu\upsilon}\ \tilde{\cup}\ G_{\alpha\beta}$

	e_1	e_2	e_3
m_1	(0.14,0.72)	(0.0,0.96)	(0.568,0.37)
m_2	(0.12,0.72)	(0.02,0.97)	(0.165,0.825)
m_3	(0.03,0.9)	(0.648,0.217)	(0.02,0.91)
$<\mu,\nu>$	(0.01,0.96)	(0.18,0.755)	(0.4,0.6)

Table 3: $F_{\mu\upsilon}\ \tilde{\cap}\ G_{\alpha\beta}$

Definition 3.11 *A GIFSS is said to be a generalized absolute IF soft set denoted by* $A_{\alpha_1\alpha_2}$ *if* $A_{\alpha_1\alpha_2}$ *is the mapping defined by* $A_{\alpha_1\alpha_2} : E \rightarrow \mathcal{P}(U) \times I^2$ *such that*

$A_{\alpha_1\alpha_2}(e) = \big(F(e), \alpha_1(e), \alpha_2(e)\big)$ *such that* $F(e) = \tilde{1}$,
$\alpha_1(e) = 1, \alpha_2(e) = 0$ *for every e in E.*

Definition 3.12 *A GIFSS is said to be a generalized null IF soft set denoted by* $\phi_{\theta_1\theta_2}$ *if* $\phi_{\theta_1\theta_2}$ *is the mapping defined by* $\phi_{\theta_1\theta_2} : E \rightarrow \mathcal{P}(U) \times I^2$ *such that*

$\phi_{\theta_1\theta_2}(e) = \big(F(e), \theta_1(e), \theta_2(e)\big)$ *such that* $F(e) = \tilde{0}$,
$\theta_1(e) = \theta_2(e) = 0$ *for every e in E.*

Proposition 3.13 *Let* $F_{\mu\upsilon}$ *be a GIFSS over (U, E) then the following holds:*

(i) $F_{\mu\upsilon}\ \tilde{\cup}\ F_{\mu\upsilon}\ \tilde{\supseteq}\ F_{\mu\upsilon}$
(ii) $F_{\mu\upsilon}\ \tilde{\supseteq}\ F_{\mu\upsilon}\ \tilde{\cap}\ F_{\mu\upsilon}$
(iii) $F_{\mu\upsilon}\ \tilde{\cup}\ \phi_{\theta_1\theta_2} = F_{\mu\upsilon}$
(iv) $F_{\mu\upsilon}\ \tilde{\cap}\ A_{\alpha_1\alpha_2} = F_{\mu\upsilon}$

Proof. Result follows trivially from definitions 3.7, 3.8, 3.9, 3.11 and 3.12.

Proposition 3.14 *Let* $F_{\mu\upsilon}$, $G_{\alpha\beta}$ *and* $H_{\gamma\delta}$ *be three GIFSS over (U, E). Then the following holds:*

(i) $F_{\mu\upsilon} \tilde{\cup} G_{\alpha\beta} = G_{\alpha\beta} \tilde{\cup} F_{\mu\upsilon}$

(ii) $F_{\mu\upsilon} \tilde{\cap} G_{\alpha\beta} = G_{\alpha\beta} \tilde{\cap} F_{\mu\upsilon}$

(iii) $F_{\mu\upsilon} \tilde{\cup} (G_{\alpha\beta} \tilde{\cup} H_{\gamma\delta}) = (F_{\mu\upsilon} \tilde{\cup} G_{\alpha\beta}) \tilde{\cup} H_{\gamma\delta}$

(iv) $F_{\mu\upsilon} \tilde{\cap} (G_{\alpha\beta} \tilde{\cap} H_{\gamma\delta}) = (F_{\mu\upsilon} \tilde{\cap} G_{\alpha\beta}) \tilde{\cap} H_{\gamma\delta}$

Proof. Result follows trivially from definition 3.7 and 3.8

Remark 3.15 *The following do not hold.*

(i) $(F_{\mu\upsilon} \tilde{\cup} G_{\alpha\beta})^c = (F_{\mu\upsilon})^c \tilde{\cap} (G_{\alpha\beta})^c$

(ii) $(F_{\mu\upsilon} \tilde{\cap} G_{\alpha\beta})^c = (F_{\mu\upsilon})^c \tilde{\cup} (G_{\alpha\beta})^c$

(iii) $F_{\mu\upsilon} \tilde{\cup} (F_{\mu\upsilon})^c = A_{\alpha_1\alpha_2}$

(iv) $F_{\mu\upsilon} \tilde{\cap} (F_{\mu\upsilon})^c = \phi_{\theta_1\theta_2}$

This can be illustrated by considering the example 3.10
Here

$$\left(F_{\mu\upsilon} \tilde{\cup} G_{\alpha\beta}\right)^c(e_1) = \left\{\frac{m_1}{(0.14,0.24)}, \frac{m}{(0.18,0.43)}, \frac{m_3}{(0.4,0.93)}, (0.64,0.81)\right\}$$

$$(F_{\mu\upsilon})^c(e_1) \tilde{\cap} (G_{\alpha\beta})^c(e_1)$$

$$= \left\{\frac{m_1}{(0.14,0.86)}, \frac{m}{(0.18,0.88)}, \frac{m_3}{(0.4,0.97)}, (0.64,0.99)\right\}$$

$$\left(F_{\mu\upsilon} \tilde{\cup} G_{\alpha\beta}\right)^c \neq (F_{\mu\upsilon})^c \tilde{\cap} (G_{\alpha\beta})^c$$

Similarly we can show that equalities (ii),(iii),(iv) also does not hold.

Remark 3.16 *If we take standard IF intersection, union and compliment then above (i) and (ii) in remark 3.15 holds.*

4 Similarity Measure of GIFSS

In several situations we are interested to know whether two sets or patterns are identical or approximately same or to what extent they are identical. One of the basic mathematical tool we often use in this context is the measure of similarity. Pinaki Majumdar[15] gave the definition of similarity measure of two soft sets. In this section we define the similarity measure of two GIFSS and study some of its results. Here we define the similarity measure based on the matching function.

Definition 4.1 *[14] For any two intuitionistic fuzzy sets A and B the similarity measure S (A* **,** *B) between A and B is defined by*

$S(A, B) = \frac{\sum_y A_y B_y}{max(\sum_y {A_y}^2, \sum {B_y}^2)}$

Where A_x *is the vector*$(\mu_A(x), \nu_A(x), \pi_A(x))$, B_x *is the vector*

$(\mu_B(x), \nu_B(x), \pi_B(x))$ $\forall\, x \in E$ *and* $\pi_A(x) = 1 - \nu_A(x) - \mu_A(x)$

Definition 4.2 *Let U = {* $x_1, x_2, \ldots x_n$ *} be universal set and let E = {*$e_1, e_2, \ldots e_m$ *} be set of parameters. Let* $F_{\gamma\delta}$ *and* $G_{\alpha\beta}$ *be two GIFSS over (U, E). Let* $\hat{F}$ *= {F(*e_i*); i = 1, 2,….m} and* $\hat{G}$ *= {G(*e_i*); i = 1, 2, ……m} be two families of intuitionistic fuzzy soft sets. Let* $S_i(\hat{F}, \hat{G})$ *denotes the similarity measure between the intuitionistic fuzzy sets F(*e_i*) and G(*e_i*) and* $S(\gamma\delta, \alpha\beta)$ *denotes the similarity measure between the intuitionistic fuzzy sets* $< \gamma\delta >$ *and* $< \alpha\beta >$ *. Then the similarity measure between the two GIFSS* $F_{\gamma\delta}$ *, and* $G_{\alpha\beta}$ *is given by* $S(F_{\gamma\delta}, G_{\alpha\beta}) = S(\hat{F}, \hat{G}) \,.\, S(\gamma\delta, \alpha\beta)$ *where*

$S(\hat{F}, \hat{G}) = max_i\, S_i(\hat{F}, \hat{G})$

Proposition 4.3 *Let* $F_{\gamma\delta}$ *and* $G_{\alpha\beta}$ *be two GIFSS over (U, E). Then the following holds.*

(i) $S(F_{\gamma\delta}, G_{\alpha\beta}) = S(G_{\alpha\beta}, F_{\gamma\delta})$

(ii) $0 \leq S(F_{\gamma\delta}, G_{\alpha\beta}) \leq 1$

(iii) $S(F_{\gamma\delta}, F_{\gamma\delta}) = 1$

Proof. Trivially follows from definition 4.2

Definition 4.4 *Let us denote the set of all GIFSS over (U, E) by GIFSS(U).WE define a relation* $\approx^{\alpha}$ *on GIFSS(U) called* α *similar as follows.*
Two GIFSS $F_{\gamma\delta}$ *and* $G_{\alpha\beta}$ *is said to be* α *similar denoted as* $F_{\gamma\delta} \approx^{\alpha} G_{\alpha\beta}$ *iff* $S(F_{\gamma\delta}, G_{\alpha\beta}) \geq \alpha$ *for all* $\alpha \in [0, 1]$.

Proposition 4.5 *The relationn* $\approx^{\alpha}$ *is reflexive and symmetric, but not transitive.*

Proof. Reflexive and symmetric properties follows from proposition 3.
In the following example we will show that the relationn $\approx^{\alpha}$ is not transitive.

Example 4.6 Let *U = {*x_1, x_2*}* be the universe and *E = {*e_1, e_2*}* be the set of parameters.
Let $\alpha = \frac{1}{3}$. We define three GIFSS $F_{\gamma\delta}, G_{\alpha\beta}, H_{\phi\omega}$ over *(U, E)* and is given in the tabulated form as :

	e_1	e_2
x_1	(0.1,0)	(0,0)
x_2	(1,0)	(0, 0.3)
(γ, δ)	(0.1,0)	(0.2,0.7)

Table 4: $F_{\gamma\delta}$

	e_1	e_2
x_1	(0.1,0.1)	(0.2,0.8)
x_2	(0,0.3)	(0.1,0.9)
(α, β)	(0.2,0.6)	(0.5,0.5)

Table 5: $G_{\alpha\beta}$

	e_1	e_2
x_1	(0.1,0.1)	(0.2,0.8)
x_2	(0,0.3)	(0.1,0)
(ϕ, ω)	(0.2,0.6)	(0.5,0.5)

Table 6: $H_{\phi\omega}$

Then $S(F_{\gamma\delta}, G_{\alpha\beta}) = 0.422 \geq \frac{1}{3}$, $S(G_{\alpha\beta}, H_{\phi\omega}) = 0.477 \geq \frac{1}{3}$, $S(F_{\gamma\delta}, H_{\phi\omega}) = 0.191 \leq \frac{1}{3}$

Definition 4.7. *Let $F_{\gamma\delta}$ and $G_{\alpha\beta}$ be two GIFSS over (U, E).We call the two GIFSS significantly similar if $S(F_{\gamma\delta}, G_{\alpha\beta}) > \frac{1}{2}$.*

Example 4.8. Texture synthesis is a common method that adds realism to computer generated images. The ultimate goal in texture synthesis is to produce a synthetic copy of a given natural texture in a such way that both textures are identical. Suppose a natural texture is represented in the form of a GIFSS over (U, E). Here U = $\{x_1, x_2\}$ is set of experts and E = $\{e_1, e_2, e_3\}$ is the texture features. Let $G_{\alpha\beta}$ denotes representation of the synthetic copy of the natural texture. Both GIFSS are given tabulated form as :

	e_1	e_2	e_3
x_1	(0.5,0.5)	(0.4,0.6)	(0.6,0.2)
x_2	(0.9,0.1)	(0.1,0.8)	(0.5,0.3)
(γ,δ)	(0.8,0.1)	(0.4,0.3)	(0.8,0.1)

Table 7: $F_{\gamma\delta}$

	e_1	e_2	e_3
x_1	(0.6,0.3)	(0.2,0.5)	(0.5,0.2)
x_2	(0.8,0.1)	(0.2,0.6)	(0.4,0.3)
(α,β)	(0.7,0.2)	(0.5,0.2)	(0.7,0.1)

Table 8: $G_{\alpha\beta}$

We have to check whether synthetic copy is similar to natural texture or not. Now we have S($F_{\gamma\delta}$,$G_{\alpha\beta}$)= 0:822 $\geq \frac{1}{2}$ .Thus we conclude that both textures are significiantly similar.

5 Application of GIFSS in Multi Criterion Decision Making Problem

In this section we mainly focus on the application of GIFSS in multi criterion decision making problem. We define multi criteria decision making problem in GIFSS based on the work done by H W Liu. [18]

Definition 5.2 (Multi criteria decision making problem in GIFSS)

Let M be a set of alternatives and let C be a set of crirerian where M= $\{ M_1, M_2, M_3, \dots\dots . M_m, \}$ and C= $\{ c_1, c_2, \dots\dots . c_n \}$. Assume that the characteristics of M_i are expressed by GIFSS as follows: $\{(c_1, F(c_1), \mu(c_1), \nu(c_1)), \dots\dots (c_n, F(c_n), \mu(c_n), \nu(c_n))\}$ where $F(c_j) = (\alpha_{ij}, \beta_{ij}), i = 12 \dots\dots m$ and α_{ij} denotes the degree to which M_i satisfy the criteria c_j and β_{ij} denotes the degree to which M_i does not satisfy the criteria c_j. Also $\mu(c_j)$ denotes the degree of possibility of the belongingness $F(c_j)$ and $\nu(c_j)$ denotes the degree in which the belongings $F(c_j)$ is not possible. Here not that $(\alpha_{ij}, \beta_{ij})$, and $(\mu(c_j), \nu(c_j)) \in L^*$

Assume that there is a decision maker who wants to choose an alternative *which satisfy the criteria* $c_j, c_k \dots\dots, c_p$ *or* c_s *(A)*

Definition 5.2 *We define the evaluation value for the alternative* M_i *satisfying the decision makers requirement (A) as follows:*

$$E_v(M_i) = S(T_{j,k \dots\dots p}(\alpha_{iq}, \beta_{iq}), (\alpha_{sq}, \beta_{sq})) \qquad (B)$$

where T is IF t-norm and S is IF t-conorm on L_*.
We call the function E_v *define on M as evaluation function for GIFSS decision making problem. The evaluation value* $E_v(M_i)$ *is also expressed as*

$$E_v(M_i) = (\alpha_{M_i}, \beta_{M_i})$$

Remark 5.3 *The evaluation value for the IF set* $< \mu, \nu >$ *is expressed as follows*

$$E_v < \mu, \nu > = (\mu_{cd}, \nu_{cd})$$

Definition 5.4 *The degree of suitability to which the alternative satisfy the decision maker's requirement can be measured by the following score function* J_n *(for any integer) or* J_∞*:*

$$J_n E_V(M_i) = \alpha_{M_i} + \mu_{cd}\pi_{E_V(M_i)} + \mu_{cd}(1 - \mu_{cd} - \nu_{cd})\pi_{E_V(M_i)} + \dots\dots\dots. \mu_{cd}(1 - \mu_{cd} - \nu_{cd})^{n-1}\pi_{E_V(M_i)} \qquad (C)$$

$$J_\infty E_V(M_i) = \alpha_{M_i} + \frac{\mu_{cd}}{\mu_{cd} + \nu_{cd}} \pi_{E_V(M_i)} \qquad (D)$$

where $\pi_{E_V(M_i)} = 1 - \alpha_{M_i} - \beta_{M_i}$ *and* $\mu_{cd} + \nu_{cd} \neq 0$

Steps of multi criteria decision making problem in GIFSS

1. Calculate the evaluation value E_v for the alternative M_i and the Intuitionistic fuzzy set $< \mu, \nu >$
2. Seek the degree of suitability J_n to which the alternative M_i for i = 1, 2 ….m satisfy the decision maker's requirement.
3. If there exists $i_0 \in \{1, 2\dots\dots m\}$ such that $J_n(E_v(M_{i0}))$ is the largest value among the values $J_n(E_v(M_i))$ (i = 1, 2….m) then the alternative M_{i0} is the best value.

Remark 5.5 *If necessary, we can also use* J_∞ *(*$E_v(M_i)$*) to choose the best alternative.*

Example 5.6 Consider a plot selection problem. Suppose there are 3 plots $\{p_1, p_2, p_3\}$ which form the set of alternatives. Suppose there exists three criterion C_1(greenery), C_2(cheap), C_3(hill side) that are taken into account in this problem. Now decision maker want to choose a plot depending upon the criteria c_1, c_2 or c_2. Let the observations made are expressed as follows:

	c_1	c_2	c_3
p_1	(0.2,0.2)	(0.3,0.1)	(0.2,0.0)
p_2	(0.3,0.3)	(0.2,0.2)	(0.3,0.1)
P_3	(0.4,0.4)	(0.5,0.4)	(0.3,0.2)
(μ, ν)	(0.4,0.4)	(0.6,0.3)	(0.6,0.4)

Taking IF t- norm T = ∧ and IF t-conorm S = ∨ in (B) we get the evaluation values M_1, M_2, M_3 and $< \mu, \nu >$ as
$E_v(M_1)$ = (0.2,0), $E_v(M_2)$ = (0.3,0.1), $E_v(M_3)$ = (0.4,0.2) and
$E_v(\mu, \nu)$ = (0.6,0.4) = (μ_{cd}, ν_{cd})
substituting the values of $\alpha_{M_i}, \beta_{M_i}, \mu_{cd}$ *and* ν_{cd} in (C) and (D) we obtain

$J_1(E_v(M_1)) = 0.68$, $J_1(E_v(M_2)) = 0.66$, $J_1(E_v(M_3)) = 0.64$
$J_\infty(E_v(M_1)) = 0.68$, $J_\infty(E_v(M_2)) = 0.66$, $J_\infty(E_v(M_3)) = 0.64$

Now $J_1(E_v(M_1)) = \max \max_i J_1(Ev(Mi))$ i
Hence M_1 is the best choice. Also note that since

$J_\infty(E_v(M_1)) \geq J_n(E_v(M_i)) \geq J_{n-1}(E_v(M_i))$ for $n \geq 2$,
we have for n > 1
$J_n(E_v(M_1)) > J_1(E_v(M_1)) = 0:68$
$> 0:66 = J_1(E_v(M_2))$
$> J_n(E_v(M_2)) = 0:66 >$
$> 0:64 = J_1(E_v(M_3)) > J_n(E_v(M_3))$
So our best choice is always M_1, no matter what the positive integer n is.

6 Conclusion

At present studies on theory and applications of the exension of soft set is going on. Based on this, we introduced the concept of Generalized Intuitionistic fuzzy soft sets and studied some of the related results. We have shown that GIFSS generalize Intuitionistic fuzzy soft sets. We also presented a method to find out the similarity measure of two Generalized intuitionistic fuzzy soft sets and applied it to know whether two textures are similar or not. We also disscuss about the multi crtiteria decision making problem in Generalized intuitionistic fuzzy soft sets and tried to solve one decision making problem. In future one can think of the algebraic nature of Generalized intuitionistic fuzzy soft sets and thus still extend it.

Acknowledgements

The first author acknowledges the financial assistance given by the Council of Scientific and Industrial Research, Government of India throughout the preparation of this paper.

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