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Quasi-Open Sets in Bispaces

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Abstract

The notions of quasi-open sets, quasi-continuity, semi-open sets and quasi-Hausdorffness were studied in [4]. Here we study the same in more general structure of a bispace and investigate how far several results as valid in a bitopological space are affected in a bispace.

Keywords: *Bispaces, Quasi open sets, Quasi continuity, Semi open sets, Quasi Hausdorffness.*

1 Introduction

The notion of a topological space was generalized to a σ-space(or simply space) by A.D.Alexandroof[1] weakening the union requirements. J.C. Kelly[6] introduced the idea of a bitopological space. Several works on bitopological spaces have been done in [4]. The concept of σ-space was used by Lahiri and Das [8] to generalize the notion of a bitopological space to a bispace where several ideas like pairwise Hausdorffness, pairwise bicompactness etc. were also studied. The concept of quasi-open sets and quasi-continuity were studied by M.C.Datta [4] in a bitopological space. Indeed quasi-continuity is weaker than Pervin's continuity [10] and quasi-Hausdorffness is more general than a pairwise Hausdorffness in a bitopological space [4]. Here we have studied the ideas of quasi open sets, quasi-continuity and quasi-Hausdorffness in a bispace and investigate how far several results as valid in a bitopological space are affected in a bispace. J. Swart [11] introduced the idea of least upper bound topology in a bitopological space whose members are called semi-open set [4]. In fact this notion is not same with the notion of semi-open set introduced by Levine[9] and [3]. In this paper we have also studied the idea of semi-open set due to M.C. Datta [4] in a bispace.

Definition 1.1 *A set X is called an Alexandroff space or simply a space if in it is chosen a system of subsets $\mathcal{F}$ satisfying the following axioms:*

1. *The intersection of a countable number of sets from $\mathcal{F}$ is a set in $\mathcal{F}$.*
2. *The union of a finite number of sets from $\mathcal{F}$ is a set in $\mathcal{F}$.*
3. *The void set ϕ is a set in $\mathcal{F}$.*
4. *The whole set X is a set in $\mathcal{F}$.*

Sets of $\mathcal{F}$ are called closed sets. Their complementary sets are called open. It is clear that instead of closed sets in the definition of the space, one may put open sets with subject to the conditions of countable summability, finite intersectibility and the condition that X and void set ϕ should be open. The collection of all such open sets will sometimes be denoted by τ and the space by (X, τ). Note that, in general, τ is not a topology as can be easily seen by taking $X = R$, the set of real numbers and τ as the collection of all F_σ -sets in R.

Definition 1.2 *To every set M of (X, τ) we correlate its closure $\overline{M}$, the intersection of all closed sets containing M. Sometimes the closure of a set M will be denoted by $\tau - clM$ or simply clM when there is no confusion about τ.*

Generally the closure of a set in a space is not a closed set.

From the axioms, it easily follows that

1)$\overline{M \cup N} = \overline{M} \cup \overline{N}$; 2)$M \subset \overline{M}$; 3)$\overline{M} = \overline{\overline{M}}$; 4)$\overline{\phi} = \phi$.

Definition 1.3 *The interior of a set M in (X, τ) is defined as the union of all open sets contained in M and is denoted by $\tau - intM$ or $intM$ when there is no confusion.*

Definition 1.4 *Let X be a nonempty set. If τ_1 and τ_2 be two collections of subsets of X such that (X, τ_1) and (X, τ_2) are two spaces, then X is called a bispace and is denoted by (X, τ_1, τ_2).*

2 Quasi Open Sets

Throughout our discussion, (X, τ_1, τ_2) or simply X stands for a bispace, R stands for the set of real numbers , Q stands for the set of rational numbers and sets are always subsets of X unless otherwise stated.

Definition 2.1 *A subset A in a bispace (X,τ_1,τ_2) is said to be quasi-open if for every $x \in A$ there exists a τ_1-open neighbourhood $U_x \subset A$ or a τ_2-open neighbourhood $V_x \subset A$.*

In bitopological space quasi open sets are precisely the unions of τ_1-open and τ_2-open sets (proposition 2.2 [4]). But in bispace quasi open sets may not be the unions of τ_1-open and τ_2-open sets as shown in the following example:

Example 2.2 *Let $X = [0,2] - Q$, where Q is the set of rational numbers. Let $\{F_i\}$ be the collection of all countable subsets in $[0,1] - Q$ and $\{G_i\}$ be the collection of all countable subsets in $[1,2] - Q$.*
Let $\tau_1 = \{X, \phi, F_i\}$, $\tau_2 = \{X, \phi, G_i\}$ and $A = [\frac{1}{2}, \frac{3}{2}] - Q$. Then A is neither τ_1 open nor τ_2 open, but A is quasi open because $A = (\bigcup\limits_{r \in A_1} \{r\}) \cup (\bigcup\limits_{q \in A_2} \{q\})$ where $A_1 = [\frac{1}{2}, 1] - Q, A_2 = [1, \frac{3}{2}] - Q$ and each $\{r\}$ is τ_1-open and $\{q\}$ is τ_2-open.

However any quasi open set is the union of τ_1-open and τ_2-open sets in the form $(\bigcup\limits_i U_{x_i}) \cup (\bigcup\limits_j V_{x_j})$ where U_{x_i} and V_{x_j} are respectively the τ_1-open and τ_2-open neighbourhoods of x_i and x_j.

Note 1: Clearly every τ_1-open(τ_2-open) set is quasi-open and arbitrary union of quasi-open sets is quasi-open. But finite intersection of quasi-open sets need not be quasi-open as shown in the following example.

Example 2.3 *Let $X = [0,3]$. Let $\{G_i\}$ be the collection of all countable subsets in $[0,1] - Q$ and $\{F_i\}$ be the collection of all countable subsets in $[2,3] - Q$.*

Let $\tau_1 = \{X, \Phi, G_i \cup \{\sqrt{2}\}\}$ and $\tau_2 = \{X, \Phi, F_i \cup \{\sqrt{2}\}\}$. Then clearly each τ_1-open or τ_2-open set is quasi-open but the intersection of any τ_1-open and τ_2-open set other than X and ϕ is $\{\sqrt{2}\}$ which is not quasi-open.

Definition 2.4 *A subset A in a bispace (X,τ_1,τ_2) is said to be quasi-closed set if its complement is quasi-open.*

Note 2: Clearly every τ_1-closed (τ_2-closed) set is quasi-closed. Arbitrary intersection of quasi-closed sets is quasi-closed but finite union of quasi-closed sets need not be quasi-closed as shown in the above example 2.3 that union of any two τ_1-closed and τ_2-closed set is $X - \{\sqrt{2}\}$ which is not quasi-closed.

In bitopological space every quasi-closed set is the intersection of a τ_1-closed and τ_2-closed sets but in a bispace this is not true. Because in Example 2.2, $A^c = (\bigcap\limits_{r \in A_1} (X - \{r\})) \cap (\bigcap\limits_{q \in A_2} (X - \{q\}))$ which is not the intersection of τ_1-closed and τ_2-closed sets.

Definition 2.5 *The quasi-closure of a subset* A *in a bispace* (X, τ_1, τ_2) *is the set* $(\tau_1 - cl(A)) \cap (\tau_2 - cl(A))$*. The quasi-closure of* A *is denoted by* $\overline{A}$*.*

Theorem 2.6 *Quasi-closure of a set is quasi-closed.*

Proof: $\overline{A} = (\tau_1 - cl(A)) \cap (\tau_2 - cl(A)) = (\cap P_i) \cap (\cap Q_i)$, where $\{P_i\}$ and $\{Q_i\}$ are family of τ_1-closed and τ_2-closed set respectively satisfying the conditions $A \subset P_i$ and $A \subset Q_i$.
Therefore $\overline{A}^c = (\bigcap\limits_i P_i)^c \cup (\bigcap\limits_i Q_i)^c = (\bigcup\limits_i P_i^c) \cup (\bigcup\limits_i Q_i^c)$ which implies that $\overline{A}^c$ is quasi open and hence $\overline{A}$ is quasi closed.

Note 3: If $A \subset (X, \tau_1, \tau_2)$, then $\overline{A}$ is the smallest quasi closed set containing A.

Theorem 2.7 *If* $x \in \overline{A}$ *then every open set* U *containing* x *intersects* A*.*

Proof: If possible, let there exist an open set U containing x such that $A \cap U = \phi$ which implies that $A \subset X - U$ where $X - U$ is closed set. Since $x \notin X - U$ and $X - U$ is a closed set containing A , $x \notin \overline{A}$. So if $x \in \overline{A}$, then every open set U containing x intersects A.

Corollary 2.8 $\overline{A} \subset A \cup A'$.

Definition 2.9 *Let* (X, τ) *be a space. A family of open sets* B *is said to form a base (open) for* τ *if and only if every open set can be expressed as countable union of members of* B*.*

Theorem 2.10 *A collection of subsets* B *of a set* X *forms an open base of a suitable space structure* τ *of* X *if and only if*

1) the null set $\phi \in B$

2) X *is the countable union of some sets belonging to* B*.*

3) intersection of any two sets belonging to B *is expressible as countable union of some sets belonging to* B*.*

Definition 2.11 *Let* (X, τ) *be a space. A family of subsets* S *of* X *is said to form a subbase of a space structure* τ *if the collection of subsets obtained as the intersection of all finite sub-collections of* S *constitute a base of* τ*.*

Theorem 2.12 *A collection of subsets* S *of a given set* X *forms a subbase of a suitable space structure of* X *if and only if*

1) either $\phi \in S$ *or* ϕ *is the intersection of a finite number of subsets belonging to* S

2) X *is the countable union of subsets belonging to* S*.*

Proof: Let S form a subbase of a space structure τ of X and let B be the base generated by S. As $\phi \in B$ either $\phi \in S$ or S must contains some subsets, finite in number whose intersection is the null set ϕ . As X is the countable union of some sets belonging to B, so $X = \bigcup\limits_{i=1}^{\infty} V_i$, where $V_i \in B$. Again, each V_i is the intersection of finite numbers of subsets belonging to S i.e, $V_i = S_i^1 \cap S_i^2 \cap .... \cap S_i^k$, for some finite number k, where $S_i^j \in S$. So $V_i \subset S_i^1$ (taking first number of the intersection) for all $i = 1, 2.......\infty$. Therefore $X = \bigcup\limits_{i=1}^{\infty} V_i \subset \bigcup\limits_{i=1}^{\infty} S_i^1$, on the other hand $\bigcup\limits_{i=1}^{\infty} S_i^1 \subset X$. Therefore, $X = \bigcup\limits_{i=1}^{\infty} S_i^1$, i.e, X is the countable union of subsets belongs to S.

Conversely, let the condition hold. Let B be the set formed by the intersection of finite members of S. Clearly $\phi \in B$ by (1) . Since $S \subset B$ and X is the countable union of subsets belonging to S, therefore $X \in B$.

Let $V_1, V_2 \in B$, and let $V_1 = A_1 \cap A_2 \cap .... \cap A_k$, $V_2 = B_1 \cap B_2 \cap .... \cap B_m$ where $A_i, B_j \in S$, $i = 1, 2, ..., k$ and $j = 1, 2, ..., m$. So $V_1 \cap V_2 = A_1 \cap A_2 \cap ....\cap A_k \cap B_1 \cap ......\cap B_m = V_3(say) \in B$. So $V_1 \cap V_2 = \bigcup\limits_{i=1}^{\infty} V_i$, where $V_i = V_3$ for all i.
So B form a base and hence S form a subbase of space structure τ of X.

Definition 2.13 *Let* (X, τ_1, τ_2) *be a bispace . The space* (X, τ) *is called least upper bound space generated by* τ_1 *and* τ_2 *if* τ *is generated by subbase* $\tau_1 \cup \tau_2$.

Definition 2.14 *Let* (X, τ_1, τ_2) *be a bispace.* $A \subset X$ *is said to be semi-open if it is open in the least upper bound space structure* τ *generated by* τ_1 *and* τ_2.

Definition 2.15 *Complement of semi-open set is called semi-closed. Semi-closure of any set* $A \subset X$ *is intersection of all semi-closed sets containing* A.

In (X, τ_1, τ_2) every τ_1-closed or τ_2 closed is semi-closed. For let A be τ_1-closed. Then A^c is τ_1-open. So $A^c \in \tau_1 \cup \tau_2$ which implies that A^c is a subbasic open set of least upper bound space structure τ generated by τ_1 and τ_2. Therefore A^c is semi-open and hence A is semi-closed. Similarly every τ_2-closed set is semi-closed set.

Remark 2.16 *In a bitopological space every quasi-closed is semi-closed, but in bispace quasi-closed sets may not be semi-closed i.e, quasi-open set may not be semi-open as shown in the following example.*

Example 2.17 *Consider X, τ_1, τ_2 and $A \subset X$ as in example 2.2. Then A is a quasi-open and uncountable set. Clearly $\tau_1 \cup \tau_2$ form a subbase of least upper bound space structure τ. Other than X, ϕ every member of $\tau_1 \cup \tau_2$ is countable set. So finite intersection of member of $\tau_1 \cup \tau_2$ is also a countable set i.e, all basic open set of τ is countable. So every member of τ is countable, since open sets are formed by countable union of basic open sets. So A can not be a semi-open set (i.e, $A \notin \tau$) but A is quasi-open.*

Definition 2.18 *Let $f : (X_1, \tau_1, \tau_2) \to (X_2, \tau_1^{'}, \tau_2^{'})$ be a mapping. Then f is said to be quasi continuous if the inverse image of every quasi open set in $(X_2, \tau_1^{'}, \tau_2^{'})$ is quasi open in (X_1, τ_1, τ_2).*

Definition 2.19 *A function f maps a bispace (X, τ_1, τ_2) into a bispace $(X^{'}, \tau_1^{'}, \tau_2^{'})$ is said to be $(\tau_1\tau_1^{'}, \tau_2\tau_2^{'})$ continuous or simply continuous if and only if the induced mappings $f_1 : (X, \tau_1) \to (X^{'}, \tau_1^{'})$ and $f_2 : (X, \tau_2) \to (X^{'}, \tau_2^{'})$ are continuous.*
Clearly every continuous mapping is quasi continuous. But converse may not be true as shown in the following example.

Example 2.20 *Example of a function which is quasi continuous but not continuous:*

Let $X = [0, 2]$, $\tau_1 = \{X, \phi, G_{i,}$ where G_i are countable subsets in $[0, 1] - Q\}$ and $\tau_2 = \{X, \phi, F_{i,}$ where F_i are countable subsets in $[1, 2] - Q\}$ and $\tau_2^{'} = \tau_1^{'} = \{X, \phi, P_{i,}$ where P_i are countable subsets in $[0, 2] - Q\}$. Let $f : (X, \tau_1, \tau_2) \to (X, \tau_1^{'}, \tau_2^{'})$ be the identity mapping. Clearly $\{\sqrt{2}\} \in \tau_1^{'}$ but $f^{-1}(\{\sqrt{2}\}) = \{\sqrt{2}\}$ $\notin \tau_1$. Also $\{\frac{1}{\sqrt{2}}\} \in \tau_2^{'}$ but $f^{-1}(\frac{1}{\sqrt{2}}) = \{\frac{1}{\sqrt{2}}\} \notin \tau_2$. So the identity mapping f is not continuous.

Let G be any quasi open set in $(X, \tau_1^{'}, \tau_2^{'})$, let $x \in f^{-1}(G) = G$, where G is a countable set in $[0, 2] - Q$. So x is an irrational number in $[0, 2] - Q$. Now if $x \in [0, 1] - Q$ then $\{x\} \in \tau_1$ and if $x \in [1, 2] - Q$ then $\{x\} \in \tau_2$. Thus in any case, for any $x \in f^{-1}(G)$, there is τ_1-open or τ_2-open set $\{x\}$ such that $x \in \{x\} \subset G$. So $f^{-1}(G)$ is quasi open in (X, τ_1, τ_2) and hence f is quasi continuous.

Definition 2.21 *A space (or a set) is called bicompact if every open cover of it has a finite subcover.*

Definition 2.22 *A set $A \subset (X, \tau_1, \tau_2)$ is said to be semi-bicompact if it is compact in the least upper bound space structure generated by τ_1 and τ_2; in other words, A is semi-bicompact if and only if any given covering of A by semi-open subsets of X there exists a finite sub-covering.*

In bitopological space the quasi-continuous image of a semi-compact set is semi-compact. But in bispace it is not true.

Proposition 2.23 *Let* $f : (X, \tau_1, \tau_2) \to (X^{'}, \tau_1^{'}, \tau_2^{'})$ *be continuous and surjective and let* (X, τ_1, τ_2) *be semi-bicompact. Then* $(X^{'}, \tau_1^{'}, \tau_2^{'})$ *is semi-bicompact.*

Proof: Let $\{U_\alpha\}_{\alpha\in\Lambda}$ be a covering of $X^{'}$, where each U_α is semi-open in $(X^{'}, \tau_1^{'}, \tau_2^{'})$. Then U_α is of the form $\bigcup\limits_{i,j=1}^{\infty}(V_{\alpha i} \cap W_{\alpha j})$ where $V_{\alpha i}$ is $\tau_1^{'}$-open and $W_{\alpha j}$ is $\tau_2^{'}$-open.
Then $f^{-1}(U_\alpha) = \bigcup\limits_{i,j=1}^{\infty}(f^{-1}(V_{\alpha i}) \cap f^{-1}(W_{\alpha j}))$. Since $V_{\alpha i}$ is $\tau_1^{'}$-open, $f^{-1}(V_{\alpha i})$ is τ_1-open for each $i = i_1, i_2, ...i_k$ and similarly $f^{-1}(W_{\alpha j})$ is τ_2-open for each $j = j_1, j_{2,}......j_m$. Therefore, $\{f^{-1}(U_\alpha)\}$ is a covering of X where $f^{-1}(U_\alpha)$ is semi open in (X, τ_1, τ_2). Since X is semi-bicompact , there exists finite subset $\Lambda_1 \subset \Lambda$ such that $\{f^{-1}(U_\alpha)\}_{\alpha\in\Lambda_1}$ covers X. Therefore $\{U_\alpha\}_{\alpha\in\Lambda_1}$ is finite subcovers of $X^{'}$. Therefore $X^{'}$ is semi-bicompact.

3 Quasi-Hausdorff Space

Definition 3.1 *A bispace* (X, τ_1, τ_2) *is said to be quasi Hausdorff if given* $x_1 \neq x_2$*, there exist quasi open sets* U_1, U_2 *such that* $x_1 \in U_1, x_2 \in U_2$ *and* $U_1 \cap U_2 = \phi$*.*

Theorem 3.2 *Every quasi Hausdorff bispace* (X, τ_1, τ_2) *is Hausdorff with respect to least upper bound space structure* τ *generated by* τ_1 *and* τ_2*.*

Proof: Since (X, τ_1, τ_2) is quasi-Hausdorff, for every $x_1, x_2 \in X, x_1 \neq x_2$, there exist quasi-open sets U_1 and U_2 such that $x_1 \in U_1, x_2 \in U_2$, $U_1 \cap U_2 = \phi$. So U_1 and U_2 is of the form $U_1 = (\bigcup\limits_i U_{x_i}) \cup (\bigcup\limits_j V_{x_j})$ and $U_2 = (\bigcup\limits_i U_{y_i}) \cup (\bigcup\limits_j V_{y_j})$ where $U_{x_i}, U_{y_i} \in \tau_1$ and $V_{x_j}, V_{y_j} \in \tau_2$.
Since $U_1 \cap U_2 = \phi$, i.e, $(\bigcup\limits_i U_{x_i}) \cup (\bigcup\limits_j V_{x_j}) \cap$ $(\bigcup\limits_i U_{y_i}) \cup (\bigcup\limits_j V_{y_j}) = \phi$, and since $x_1 \in U_1$, there exists a $U_{x_i}($ or $V_{x_j})$ such that $x_1 \in U_{x_i}($ or $V_{x_j})$ and since $x_2 \in U_2$ implies there exists a U_{y_i} (or V_{y_j}) such that $x_2 \in U_{y_i}($ or $V_{y_j})$.

So in any case (U_{x_i}, U_{y_i}) [or (U_{x_i}, V_{y_j}) or (V_{x_j}, U_{y_i}) or (V_{x_j}, V_{y_j})] is a pair of τ-open sets which separates strongly x_1, x_2, since $U_{x_i} \cap U_{y_i} = \phi$, $U_{x_i} \cap V_{y_j} = \phi$, $V_{x_j} \cap U_{y_i} = \phi$,$V_{x_j} \cap V_{y_j} = \phi$. So (X, τ) is Hausdorff.

Lemma 3.3 *Let* (X, τ_1, τ_2) *be a quasi Hausdorff bispace and let* U_1 *and* U_2 *be two quasi open sets such that* $U_1 \cap U_2 = \phi$ *then* $\overline{U_1} \cap U_2 = \phi$ *where* $\overline{U_1}$ *is the quasi closure of* U_1*.*

Proof: Let U_1 and U_2 be quasi open sets such that $U_1 \cap U_2 = \phi$. Suppose $\overline{U_1} \cap U_2 \neq \phi$. Let $y \in \overline{U_1} \cap U_2$. Now $y \in \overline{U_1}$ implies that every τ_1-neighbourhood and τ_2-neighbourhood of y meet U_1 .

But U_2 is a quasi open set containing y. So there exists a τ_1-open neighbourhood or a τ_2-open neighbourhood W of y such that $W \subset U_2$. Since $U_1 \cap U_2 = \phi$ implies $U_1 \cap W = \phi$ which contradicts the above fact. So $\overline{U_1} \cap U_2 = \phi$.

Theorem 3.4 *A semi-bicompact subset in a quasi-Hausdorff bispace is quasi-closed if the following condition 'C' is satisfied.*

'C': Quasi-closure of every quasi-open set is quasi-open.

Proof: Let A be a semi-bicompact subset of a quasi-Hausdorff space (X, τ_1, τ_2) which satisfies condition 'C'. We show $X - A$ is quasi open.

Let $s \in X - A$. Then $s \neq a_i$ for each $a_i \in A$. Since (X, τ_1, τ_2) is quasi-Hausdorff there exist quasi-open sets U_{a_i} and V_{a_i} containing a and s respectively such that $U_{a_i} \cap V_{a_i} = \phi$. So by lemma 3.3, we get $U_{a_i} \cap \overline{V_{a_i}} = \phi$. The collection $\{U_{a_i}\}_{a_i \in A}$ is a quasi-open covering of A. Now U_{a_i} is of the form $U_{a_i} = (\cup_{x \in \Lambda_1} U^{a_i}_x) \cup (\cup_{y \in \Lambda_2} V^{a_i}_y)$ where $U^{a_i}_x$ and $V^{a_i}_y$ are τ_1-open and τ_2-open sets respectively. So $\cup\{(\cup U^{a_i}_x) \cup (\cup V^{a_i}_y) : a_i \in A\} \supset A$. Since $U^{a_i}_x$ and $V^{a_i}_y$ are also semi-open sets, the collection $\{U^{a_i}_x, V^{a_i}_y : a_i \in A, x \in \Lambda_1, y \in \Lambda_2\}$ form an semi-open cover for A.

So there exist finite number of sets $U^{a_1}_{x_1}, U^{a_2}_{x_2}, U^{a_3}_{x_3}, \ldots\ldots U^{a_n}_{x_n}, V^{a'_1}_{y1}, V^{a'_2}_{y2}, \ldots\ldots V^{a'_m}_{y_m}$ such that $A \subset U^{a_1}_{x_1} \cup U^{a_2}_{x_2} \cup \ldots\ldots \cup U^{a_n}_{x_n} \cup V^{a'_1}_{y1} \cup V^{a'_2}_{y2} \ldots\ldots \cup V^{a'_m}_{y_m}$. Clearly $U^{a_i}_{x_i} \cap \overline{V_{a_i}} = \phi, V^{a'_i}_{y_i} \cap \overline{V_{a_i}} = \phi$, since $U^{a_i}_{x_i} \subset U_{a_i}$, $V^{a'_i}_{y_i} \subset U_{a_i}$.
Let $W_s = \overline{V_{a_1}} \cap \overline{V_{a_2}} \cap \ldots\ldots \cap \overline{V_{a_n}} \cap \overline{V_{a'_1}} \cap \overline{V_{a'_2}} \cap \ldots\ldots \cap \overline{V_{a'_m}}$.
Then $\overline{V_{a_1} \cap V_{a_2} \cap \ldots\ldots \cap V_{a_n} \cap V_{a'_1} \cap \ldots\ldots \cap V_{a'_m}} \subset \overline{V_{a_1}} \cap \overline{V_{a_2}} \cap \ldots\ldots \cap \overline{V_{a_n}} \cap \overline{V_{a'_1}} \cap \overline{V_{a'_2}} \cap \ldots\ldots \cap \overline{V_{a'_m}} = W_s$.

But $\overline{V_{a_1} \cap V_{a_2} \cap \ldots\ldots \cap V_{a_n} \cap V_{a'_1} \cap \ldots\ldots \cap V_{a'_m}} = W'_s$ is quasi open, by condition 'C'.

So W_s contain a quasi open set W'_s containing s . Also $A \cap W_s \subset (U^{a_1}_{x_1} \cup U^{a_2}_{x_2} \cup \ldots\ldots \cup U^{a_n}_{x_n} \cup V^{a'_1}_{y1} \cup V^{a'_2}_{y2} \ldots\ldots \cup V^{a'_m}_{y_m}) \cap W_s = \phi$.
Thus for each $s \in X - A$, there exists a quasi open set $W'_s \subset W_s$ such that $W_s \cap A = \phi$ that is $s \in W'_s \subset W_s \subset X - A$. Hence $X - A$ is quasi open implies A is quasi closed.

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