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On Pairwise Fuzzy Baire Bitopological Spaces

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Abstract

In this paper, the concepts of pairwise fuzzy Baire bitopological spaces are introduced and characterizations of pairwise fuzzy Baire bitopological spaces are studied. Several examples are given to illustrate the concepts introduced in this paper.

Keywords: *Pairwise fuzzy dense, pairwise fuzzy nowhere dense, pairwise fuzzy first category, pairwise fuzzy second category and pairwise fuzzy Baire spaces.*

1 Introduction

The concepts of fuzzy sets and fuzzy set operations were first introduced by L.A.Zadeh in his classical paper [14] in the year 1965. Thereafter the paper of C.L.Chang [4] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. In recent years, fuzzy topology has been found to be very useful in solving many practical problems. Shihong Du et.al [8] are currently working to fuzzify the very successful 9 - intersection Egenhofer model for depicting topological relations in Geographic Information Systems (GIS) query. Tang [9] has used a slightly changed version of Chang's fuzzy topological spaces to model spatial objects for GIS databases and Structured Query Language (SQL) for GIS. In 1989, Kandil [7] introduced the concept of fuzzy bitopological spaces. The concepts of Baire spaces in fuzzy setting was introduced and studied by G.Thangaraj and S.Anjalmose in [12]. The concepts of Baire bitopological spaces have been studied extensively in classical topology in [1], [2], [5] and [6]. In this paper the concepts of pairwise fuzzy Baire bitopological spaces are introduced and several characterizations of pairwise fuzzy Baire bitopological spaces are studied. Several examples are given to illustrate the concepts introduced in this paper.

2 Preliminaries

Now we introduce some basic notions and results used in the sequel. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang, (1968). By a fuzzy bitopological space Kandil, (1989) we mean an ordered triple (X, T_1, T_2) where T_1 and T_2 are fuzzy topologies on the non-empty set X. The complement λ' of a fuzzy set λ is defined by $\lambda'(x) = 1-\lambda(x)$ for $x \in X$.

Definition 2.1 *Let λ and μ be any two fuzzy sets in (X, T). Then we define $\lambda \vee \mu : X \to [0, 1]$ as follows : $(\lambda \vee \mu)(x) = Max\{\lambda(x), \mu(x)\}$. Also we define $\lambda \wedge \mu : X \to [0, 1]$ as follows : $(\lambda \wedge \mu)(x) = Min\{\lambda(x), \mu(x)\}$.*

Definition 2.2 *Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). We define $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$ and $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$.*

For any fuzzy set λ in a fuzzy topological space (X, T), it is easy to see that $1 - cl(\lambda) = int(1 - \lambda)$ and $1 - int(\lambda) = cl(1 - \lambda)$[3].

Definition 2.3 *[10] A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy dense if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$.*

Definition 2.4 *[13] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy dense set if $cl_{T_1}cl_{T_2}(\lambda) = cl_{T_2}cl_{T_1}(\lambda) = 1$.*

Definition 2.5 *[10] A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy nowhere dense if there exists no non - zero fuzzy open set μ in (X, T) such that $\mu < cl(\lambda)$. That is, $intcl(\lambda) = 0$.*

Definition 2.6 *[11] A fuzzy topological space (X, T) is called a fuzzy open hereditarily irresolvable space if $intcl(\lambda) \neq 0$, then $int(\lambda) \neq 0$ for any non-zero fuzzy set λ in (X, T).*

Definition 2.7 *[13] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy open hereditarily irresolvable space if $int_{T_2} cl_{T_1}(\lambda) \neq 0$, for any T_1-fuzzy open set λ in (X, T_1, T_2) implies that $int_{T_2}(\lambda) \neq 0$ and $int_{T_1} cl_{T_2}(\mu) \neq 0$ for any T_2-fuzzy open set μ in (X, T_1, T_2) implies that $int_{T_1}(\mu) \neq 0$.*

3 Pairwise Fuzzy Nowhere Dense Sets

Motivated by the classical concept introduced in [6] we shall now define :

Definition 3.1 *A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy nowhere dense set if $int_{T_1} cl_{T_2}(\lambda) = int_{T_2} cl_{T_1}(\lambda) = 0$.*

Example 3.1 Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and ν are defined on X as follows :

$$\lambda : X \to [0,1] \text{ is defined as } \lambda(a) = 0.5; \quad \lambda(b) = 0.4; \quad \lambda(c) = 0.6.$$
$$\mu : X \to [0,1] \text{ is defined as } \mu(a) = 0.6; \quad \mu(b) = 0.2; \quad \mu(c) = 0.8.$$
$$\nu : X \to [0,1] \text{ is defined as } \nu(a) = 0.4; \quad \nu(b) = 0.6; \quad \nu(c) = 0.2.$$

Clearly $T_1 = \{0, \lambda, 1\}$ and $T_2 = \{0, \mu, 1\}$ are fuzzy topologies on X and by easy computations, $cl_{T_2}(\nu) = 1 - \mu$; $cl_{T_1}(\nu) = 1 - \lambda$; $int_{T_1}(1 - \mu) = 0$; $int_{T_2}(1 - \lambda) = 0$. Then $int_{T_1} cl_{T_2}(\nu) = int_{T_2} cl_{T_1}(\nu) = 0$ and hence ν is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).

Remarks: The complement of a pairwise fuzzy nowhere dense set in a fuzzy bitopological space **need not** be a pairwise fuzzy nowhere dense set. For, consider the above example 3.1. By easy computations, we have $int_{T_1} cl_{T_2}(1 - \nu) = 1 \neq 0$ and $int_{T_2} cl_{T_1}(1 - \nu) = 1 \neq 0$. Hence $1 - \nu$ is not a pairwise fuzzy nowhere dense set in (X, T_1, T_2), whereas ν is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).

Proposition 3.1 *If λ is a fuzzy T_i-closed set with $int_{T_i}(\lambda) = 0$, $(i = 1, 2)$, in a fuzzy bitopological space (X, T_1, T_2) , then λ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).*

Proof. Let λ be a fuzzy T_i-closed set in (X, T_1, T_2), $(i = 1, 2)$. Then $cl_{T_1}(\lambda) = \lambda$ and $cl_{T_2}(\lambda) = \lambda$. Also we have $int_{T_1}(\lambda) = 0$ and $int_{T_2}(\lambda) = 0$. Then $int_{T_1}(cl_{T_2}(\lambda)) = 0$ and $int_{T_2}(cl_{T_1}(\lambda)) = 0$ implies that $int_{T_1} cl_{T_2}(\lambda) = int_{T_2} cl_{T_1}(\lambda) = 0$. Therefore λ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).

Proposition 3.2 *If λ is a pairwise fuzzy nowhere dense set in a fuzzy bitopological space (X, T_1, T_2), then $int_{T_i}(\lambda) = 0, \quad (i = 1, 2)$.*

Proof. Let λ be a pairwise fuzzy nowhere dense set in (X, T_1, T_2). Then we have $int_{T_1}cl_{T_2}(\lambda) = int_{T_2}cl_{T_1}(\lambda) = 0$. Now $\lambda \leq cl_{T_2}(\lambda)$ implies that $int_{T_1}(\lambda) \leq int_{T_1}cl_{T_2}(\lambda)$. Then $int_{T_1}(\lambda) = 0$. Also $\lambda \leq cl_{T_1}(\lambda)$ implies that $int_{T_2}(\lambda) \leq int_{T_2}cl_{T_1}(\lambda)$. Then $int_{T_2}(\lambda) = 0$.

Proposition 3.3 *If λ is a pairwise fuzzy nowhere dense set in a fuzzy bitopological space (X, T_1, T_2), then $1 - \lambda$ is a pairwise fuzzy dense set in (X, T_1, T_2).*

Proof. Let λ be a pairwise fuzzy nowhere dense set in (X, T_1, T_2). Then we have $int_{T_1}cl_{T_2}(\lambda) = int_{T_2}cl_{T_1}(\lambda) = 0$. Now $1 - int_{T_1}cl_{T_2}(\lambda) = 1 - 0 = 1$. Then $cl_{T_1}(1 - cl_{T_2}(\lambda)) = 1$, which implies that $cl_{T_1}int_{T_2}(1-\lambda) = 1$. But $cl_{T_1}int_{T_2}(1-\lambda) \leq cl_{T_1}cl_{T_2}(1-\lambda)$. Hence $1 \leq cl_{T_1}cl_{T_2}(1-\lambda)$. That is, $cl_{T_1}cl_{T_2}(1-\lambda) = 1$. Also $1 - int_{T_2}cl_{T_1}(\lambda) = 1 - 0 = 1$. Then we have $cl_{T_2}(1 - cl_{T_1}(\lambda)) = 1$, which implies that $cl_{T_2}int_{T_1}(1-\lambda) = 1$. But $cl_{T_2}int_{T_1}(1-\lambda) \leq cl_{T_2}cl_{T_1}(1-\lambda)$. Hence $1 \leq cl_{T_2}cl_{T_1}(1-\lambda)$. That is, $cl_{T_2}cl_{T_1}(1-\lambda) = 1$. Hence we have $cl_{T_1}cl_{T_2}(1-\lambda) = cl_{T_2}cl_{T_1}(1-\lambda) = 1$. Therefore, $1-\lambda$ is a pairwise fuzzy dense set in (X, T_1, T_2).

Remarks: If λ is a pairwise fuzzy dense set in (X, T_1, T_2) , then $1 - \lambda$ **need not** be a pairwise fuzzy nowhere dense set in (X, T_1, T_2). For, consider the following example:

Example 3.2 Let $X = \{a, b, c\}$. The fuzzy sets $\lambda_i : X \to [0, 1], \quad (i = 1, 2, 3)$ and $\mu_j : X \to [0, 1], \quad (j = 1, 2, 3)$ are defined as follows :

$\lambda_1 : X \to [0, 1]$ is defined as $\lambda_1(a) = 0.2; \quad \lambda_1(b) = 0.3; \quad \lambda_1(c) = 0.4.$
$\lambda_2 : X \to [0, 1]$ is defined as $\lambda_2(a) = 0.5; \quad \lambda_2(b) = 0.4; \quad \lambda_2(c) = 0.7.$
$\lambda_3 : X \to [0, 1]$ is defined as $\lambda_3(a) = 0.4; \quad \lambda_3(b) = 0.5; \quad \lambda_3(c) = 0.6.$
$\mu_1 : X \to [0, 1]$ is defined as $\mu_1(a) = 0.4; \quad \mu_1(b) = 0; \quad \mu_1(c) = 0.$
$\mu_2 : X \to [0, 1]$ is defined as $\mu_2(a) = 0.7; \quad \mu_2(b) = 0.2; \quad \mu_2(c) = 0.$
$\mu_3 : X \to [0, 1]$ is defined as $\mu_3(a) = 0.5; \quad \mu_3(b) = 0; \quad \mu_3(c) = 0.3.$

Clearly $T_1 = \{0, \lambda_1, \lambda_2, 1\}$ and $T_2 = \{0, \mu_1, \mu_2, 1\}$ are fuzzy topologies on X. By easy computations, $cl_{T_1}cl_{T_2}(1 - \lambda_1) = cl_{T_1}(1) = 1$ and $cl_{T_2}cl_{T_1}(1 - \lambda_1) = cl_{T_2}(1-\lambda_1) = 1$ and therefore $1-\lambda_1$ is a pairwise fuzzy dense set in (X, T_1, T_2). Also we have $int_{T_1}cl_{T_2}(\lambda_1) = int_{T_1}(1 - \mu_2) = \lambda_1 \neq 0$ and $int_{T_2}cl_{T_1}(\lambda_1) = int_{T_2}(1 - \lambda_1) = \mu_2 \neq 0$. That is, $int_{T_1}cl_{T_2}(\lambda_1) \neq 0$ and $int_{T_2}cl_{T_1}(\lambda_1) \neq 0$. Hence λ_1 is not a pairwise fuzzy nowhere dense set in (X, T_1, T_2).

Proposition 3.4 *If λ is a T_2-fuzzy open, T_1-fuzzy dense and a T_1-fuzzy open, T_2-fuzzy dense set in a fuzzy bitopological space (X, T_1, T_2), then $1 - \lambda$ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).*

Proof. Let λ be a T_2-fuzzy open, T_1-fuzzy dense in (X, T_1, T_2). Then $1-\lambda$ is a T_2-fuzzy closed set and $cl_{T_1}(\lambda) = 1$. Hence $cl_{T_2}(1-\lambda) = 1-\lambda$ and $1 - cl_{T_1}(\lambda) = 0$ which implies that $cl_{T_2}(1-\lambda) = 1-\lambda$ and $int_{T_1}(1-\lambda) = 0$. Therefore $int_{T_1} cl_{T_2}(1-\lambda) = 0$. Also λ is a T_1-fuzzy open, T_2-fuzzy dense set in (X, T_1, T_2) implies that $1-\lambda$ is a T_1-fuzzy closed set and $cl_{T_2}(\lambda) = 1$. Hence we have $cl_{T_1}(1-\lambda) = 1-\lambda$ and $1 - cl_{T_2}(\lambda) = 0$. Then $cl_{T_1}(1-\lambda) = 1-\lambda$ and $int_{T_2}(1-\lambda) = 0$ implies that $int_{T_2} cl_{T_1}(1-\lambda) = 0$. Hence we have $int_{T_1} cl_{T_2}(1-\lambda) = int_{T_2} cl_{T_1}(1-\lambda) = 0$. Therefore $1-\lambda$ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).

Proposition 3.5 *If λ is a T_2-fuzzy open, T_1-fuzzy dense and a T_1-fuzzy open, T_2-fuzzy dense set in a fuzzy bitopological space (X, T_1, T_2) and if $\mu \leq (1-\lambda)$, then μ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).*

Proof. Since λ is a T_2-fuzzy open, T_1-fuzzy dense and a T_1-fuzzy open, T_2-fuzzy dense set in a fuzzy bitopological space (X, T_1, T_2), by Proposition 3.4, $1-\lambda$ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2). Then we have $int_{T_1} cl_{T_2}(1-\lambda) = 0$ and $int_{T_2} cl_{T_1}(1-\lambda) = 0$. Now $\mu \leq (1-\lambda)$implies that $int_{T_1} cl_{T_2}(\mu) \leq int_{T_1} cl_{T_2}(1-\lambda)$ and $int_{T_2} cl_{T_1}(\mu) \leq int_{T_2} cl_{T_1}(1-\lambda)$. Hence $int_{T_1} cl_{T_2}(\mu) \leq 0$ and $int_{T_2} cl_{T_1}(\mu) \leq 0$. That is, $int_{T_1} cl_{T_2}(\mu) = int_{T_2} cl_{T_1}(\mu) = 0$. Therefore μ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).

Theorem 3.1 *[13] If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy open hereditarily irresolvable bitopological space and if $int_{T_2}(\lambda) = 0$ for a T_1-fuzzy open set λ in (X, T_1, T_2) and $int_{T_1}(\mu) = 0$ for a T_2-fuzzy open set μ in (X, T_1, T_2), then $int_{T_2} cl_{T_1}(\lambda) = 0$ and $int_{T_1} cl_{T_2}(\mu) = 0$.*

Proposition 3.6 *If λ is a T_i $(i = 1, 2)$ fuzzy closed and a T_i $(i = 1, 2)$ fuzzy dense set in a fuzzy open hereditarily irresolvable bitopological space (X, T_1, T_2), then $1-\lambda$ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).*

Proof. Letλ be a T_i $(i = 1, 2)$ fuzzy closed and a T_i $(i = 1, 2)$ fuzzy dense set in (X, T_1, T_2). Then $1-\lambda$ is a T_i $(i = 1, 2)$ fuzzy open set in (X, T_1, T_2) and $cl_{T_1}(\lambda) = 1$, $cl_{T_2}(\lambda) = 1$. Now $1 - cl_{T_1}(\lambda) = 0$, $1 - cl_{T_2}(\lambda) = 0$ implies that $int_{T_1}(1-\lambda) = 0$, $int_{T_2}(1-\lambda) = 0$. Since (X, T_1, T_2) is an open hereditarily irresolvable space, $int_{T_2}(1-\lambda) = 0$, where $1-\lambda$ is a T_1 fuzzy open set in (X, T_1, T_2) and $int_{T_1}(1-\lambda) = 0$, where $1-\lambda$ is a T_2 fuzzy open set in (X, T_1, T_2), implies that $int_{T_2} cl_{T_1}(1-\lambda) = 0$ and $int_{T_1} cl_{T_2}(1-\lambda) = 0$. Hence $1-\lambda$ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).

Remarks: If λ and μ are pairwise fuzzy nowhere dense sets in (X, T_1, T_2), then $\lambda \vee \mu$ **need not** be a pairwise fuzzy nowhere dense set in (X, T_1, T_2). For, consider the following example :

Example 3.3 Let $X = \{a, b, c\}$. The fuzzy sets $\lambda_i : X \to [0,1]$, $(i = 1,2,3)$ and $\mu_j : X \to [0,1]$, $(j = 1,2,3)$ are defined as follows :

$\lambda_1 : X \to [0,1]$ is defined as $\lambda_1(a) = 0.5$; $\lambda_1(b) = 0.7$; $\lambda_1(c) = 0.6$.
$\lambda_2 : X \to [0,1]$ is defined as $\lambda_2(a) = 0.4$; $\lambda_2(b) = 0.6$; $\lambda_2(c) = 0.5$.
$\lambda_3 : X \to [0,1]$ is defined as $\lambda_3(a) = 0.6$; $\lambda_3(b) = 0.5$; $\lambda_3(c) = 0.4$.
$\mu_1 : X \to [0,1]$ is defined as $\mu_1(a) = 0.8$; $\mu_1(b) = 0.5$; $\mu_1(c) = 0.7$.
$\mu_2 : X \to [0,1]$ is defined as $\mu_2(a) = 0.6$; $\mu_2(b) = 0.9$; $\mu_2(c) = 0.4$.
$\mu_3 : X \to [0,1]$ is defined as $\mu_3(a) = 0.4$; $\mu_3(b) = 0.7$; $\mu_3(c) = 0.8$.

Clearly $T_1 = \{0, \lambda_1, \lambda_2, \lambda_3, \lambda_1 \vee \lambda_3, \lambda_2 \vee \lambda_3, \lambda_1 \wedge \lambda_3, \lambda_2 \wedge \lambda_3, \lambda_2 \vee (\lambda_1 \wedge \lambda_3), (\lambda_1 \vee \lambda_2 \vee \lambda_3), 1\}$ and $T_2 = \{0, \mu_1, \mu_2, \mu_3, \mu_1 \vee \mu_2, \mu_1 \vee \mu_3, \mu_2 \vee \mu_3, \mu_1 \wedge \mu_2, \mu_1 \wedge \mu_3, \mu_2 \wedge \mu_3, \mu_1 \vee (\mu_2 \wedge \mu_3), \mu_1 \wedge (\mu_2 \vee \mu_3), \mu_2 \vee (\mu_1 \wedge \mu_3), \mu_2 \wedge (\mu_1 \vee \mu_3), \mu_3 \vee (\mu_1 \wedge \mu_2), \mu_3 \wedge (\mu_1 \vee \mu_2), (\mu_1 \vee \mu_2 \vee \mu_3), 1\}$ are fuzzy topologies on X.
Now consider the following fuzzy sets $\alpha : X \to [0,1]$ and $\beta : X \to [0,1]$ defined on X.

$\alpha : X \to [0,1]$ is defined as $\alpha(a) = 0.6$; $\alpha(b) = 0.3$; $\alpha(c) = 0.4$.
$\beta : X \to [0,1]$ is defined as $\beta(a) = 0.4$; $\beta(b) = 0.3$; $\beta(c) = 0.6$.

By easy computations,$int_{T_1} cl_{T_2}(\alpha) = int_{T_1}[1-(\mu_2 \wedge \mu_3)] = 0$ and $int_{T_2} cl_{T_1}(\alpha) = int_{T_2}(1 - \lambda_2) = 0$. Also $int_{T_1} cl_{T_2}(\beta) = int_{T_1}[1 - (\mu_2 \wedge (\mu_1 \vee \mu_3))] = 0$ and $int_{T_2} cl_{T_1}(\beta) = int_{T_2}(1 - \lambda_3) = 0$. Hence α and β are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). But $int_{T_2} cl_{T_1}(\alpha \vee \beta) = int_{T_2}[1-(\lambda_2 \wedge \lambda_3)] = \mu_1 \wedge \mu_2 \neq 0$, eventhough $int_{T_1} cl_{T_2}(\alpha \vee \beta) = 0$. Therefore $\alpha \vee \beta$ is **not** a pairwise fuzzy nowhere dense set in (X, T_1, T_2).

Definition 3.2 *Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a pairwise fuzzy first category set if $\lambda = \vee_{n=1}^{\infty}(\lambda_n)$, where λ_n's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Any other fuzzy set in (X, T_1, T_2) is said to be a pairwise fuzzy second category set in (X, T_1, T_2).*

Definition 3.3 *If λ is a pairwise fuzzy first category set in a fuzzy bitopological space (X, T_1, T_2), then the fuzzy set $1 - \lambda$ is called a pairwise fuzzy residual set in (X, T_1, T_2).*

Definition 3.4 *A fuzzy bitopological space(X, T_1, T_2) is called pairwise fuzzy first category if the fuzzy set 1_X is a pairwise fuzzy first category set in (X, T_1, T_2). That is, $1_X = \vee_{n=1}^{\infty}(\lambda_n)$, where λ_n's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Otherwise, (X, T_1, T_2) will be called a pairwise fuzzy second category space.*

Proposition 3.7 *If λ is a pairwise fuzzy first category set in (X, T_1, T_2), then $1 - \lambda = \wedge_{n=1}^{\infty}(\mu_n)$, where $cl_{T_i}(\mu_n) = 1$, $(i = 1, 2)$.*

Proof. Let λ be a pairwise fuzzy first category set in (X, T_1, T_2). Then we have $\lambda = \vee_{n=1}^{\infty}(\lambda_n)$, where λ_n's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Now $1-\lambda = 1-\vee_{n=1}^{\infty}(\lambda_n) = \wedge_{n=1}^{\infty}(1-\lambda_n)$. Since λ_n is a pairwise fuzzy nowhere dense set in (X, T_1, T_2), by Proposition 3.3, we have $1-\lambda_n$ is a pairwise fuzzy dense set in (X, T_1, T_2). Let us put $\mu_n = 1-\lambda_n$. Then $1-\lambda = \wedge_{n=1}^{\infty}(\mu_n)$, where $cl_{T_i}(\mu_n) = 1, \quad (i = 1, 2)$.

4 Pairwise Fuzzy Baire Bitopological Spaces

Motivated by the classical concept introduced in [2] we shall now define :

Definition 4.1 *A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy Baire if $int_{T_i}(\vee_{n=1}^{\infty}(\lambda_n)) = 0$, $(i = 1, 2)$, where λ_n's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2).*

Example 4.1 Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and ν are defined on X as follows :

$$\lambda : X \to [0,1] \text{ is defined as } \lambda(a) = 0.5; \quad \lambda(b) = 0.7; \quad \lambda(c) = 0.6.$$
$$\mu : X \to [0,1] \text{ is defined as } \mu(a) = 0.4; \quad \mu(b) = 0.6; \quad \mu(c) = 0.5.$$
$$\nu : X \to [0,1] \text{ is defined as } \nu(a) = 0.6; \quad \nu(b) = 0.5; \quad \nu(c) = 0.4.$$

Clearly $T_1 = \{0, \lambda, \mu, \nu, \lambda \vee \nu, \mu \vee \nu, \lambda \wedge \nu, \mu \wedge \nu, \lambda \wedge (\mu \vee \nu), \lambda \vee \mu \vee \nu, 1\}$ and $T_2 = \{0, \lambda, \mu, 1\}$ are fuzzy topologies on X. By easy computations, we have $cl_{T_2}(1-\lambda) = 1-\lambda$, $cl_{T_2}(1-\mu) = 1-\mu$, $cl_{T_2}[1-(\lambda\vee\nu)] = 1-\lambda$, $cl_{T_2}[1-(\mu\vee\nu)] = 1-\mu$, $cl_{T_2}[1-(\lambda \wedge (\mu \vee \nu))] = 1-\mu$, $int_{T_2}(1-\lambda) = 0$, $int_{T_2}(1-\mu) = 0$, $int_{T_2}[1-(\lambda \vee \nu)] = 0$, $int_{T_2}[1-(\mu \vee \nu)] = 0$, $int_{T_2}[1-(\lambda \wedge (\mu \vee \nu))] = 0$. Also $int_{T_1}(1-\lambda) = 0$, $int_{T_1}(1-\mu) = 0$, $int_{T_1}[1-(\lambda \vee \nu)] = 0$, $int_{T_1}[1-(\mu \vee \nu)] = 0$, $int_{T_1}[1-(\lambda \wedge (\mu \vee \nu))] = 0$ and $cl_{T_1}(\nu) = 1-(\mu \wedge \nu)$, $cl_{T_1}(\mu \wedge \nu) = 1-\nu$, $cl_{T_1}(\lambda \wedge \nu) = 1-(\lambda \wedge \nu)$, $cl_{T_1}[1-(\lambda \vee \nu)] = 1-(\lambda \vee \nu)$, $cl_{T_1}[1-(\mu \vee \nu)] = 1-(\mu \vee \nu)$, $cl_{T_1}(1-\lambda) = 1-\lambda$, $cl_{T_1}(1-\mu) = 1-\mu$, $cl_{T_1}(1-\nu) = 1-\nu$, $cl_{T_1}[1-(\lambda\wedge(\mu\vee\nu))] = 1-(\lambda\wedge(\mu\vee\nu))$. Now $1-\lambda$, $1-\mu$, $1-(\lambda\vee\nu)$, $1-(\mu\vee\nu)$ and $1-[\lambda \wedge (\mu \vee \nu)]$ are pairwise fuzzy nowhere dense sets in (X, T_1, T_2) and $int_{T_1}\{(1-\lambda) \vee (1-\mu) \vee [1-(\lambda \vee \nu)] \vee [1-(\mu \vee \nu)] \vee [1-(\lambda \wedge (\mu \vee \nu))]\} = 0$ and $int_{T_2}\{(1-\lambda) \vee (1-\mu) \vee [1-(\lambda \vee \nu)] \vee [1-(\mu \vee \nu)] \vee [1-(\lambda \wedge (\mu \vee \nu))]\} = 0$. Hence the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy Baire space.

Example 4.2 Let $X = \{a, b, c\}$. The fuzzy sets $\lambda_i : X \to [0,1]$, $(i = 1, 2, 3)$ and $\mu_j : X \to [0,1]$, $(j = 1, 2, 3)$ are defined as follows :

$$\lambda_1 : X \to [0,1] \text{ is defined as } \lambda_1(a) = 0.5; \quad \lambda_1(b) = 0.7; \quad \lambda_1(c) = 0.6.$$
$$\lambda_2 : X \to [0,1] \text{ is defined as } \lambda_2(a) = 0.4; \quad \lambda_2(b) = 0.6; \quad \lambda_2(c) = 0.5.$$
$$\lambda_3 : X \to [0,1] \text{ is defined as } \lambda_3(a) = 0.6; \quad \lambda_3(b) = 0.5; \quad \lambda_3(c) = 0.4.$$

$\mu_1 : X \to [0,1]$ is defined as $\mu_1(a) = 0.8$; $\mu_1(b) = 0.5$; $\mu_1(c) = 0.7$.
$\mu_2 : X \to [0,1]$ is defined as $\mu_2(a) = 0.6$; $\mu_2(b) = 0.9$; $\mu_2(c) = 0.4$.
$\mu_3 : X \to [0,1]$ is defined as $\mu_3(a) = 0.4$; $\mu_3(b) = 0.7$; $\mu_3(c) = 0.8$.

Clearly $T_1 = \{0, \lambda_1, \lambda_2, \lambda_3, \lambda_1 \vee \lambda_3, \lambda_2 \vee \lambda_3, \lambda_1 \wedge \lambda_3, \lambda_2 \wedge \lambda_3, \lambda_2 \vee (\lambda_1 \wedge \lambda_3), \lambda_1 \vee \lambda_2 \vee \lambda_3, 1\}$ and $T_2 = \{0, \mu_1, \mu_2, \mu_3, \mu_1 \vee \mu_2, \mu_1 \vee \mu_3, \mu_2 \vee \mu_3, \mu_1 \wedge \mu_2, \mu_1 \wedge \mu_3, \mu_2 \wedge \mu_3, \mu_1 \vee (\mu_2 \wedge \mu_3), \mu_1 \wedge (\mu_2 \vee \mu_3), \mu_2 \vee (\mu_1 \wedge \mu_3), \mu_2 \wedge (\mu_1 \vee \mu_3), \mu_3 \vee (\mu_1 \wedge \mu_2), \mu_3 \wedge (\mu_1 \vee \mu_2), (\mu_1 \vee \mu_2 \vee \mu_3), 1\}$ are fuzzy topologies on X. Now consider the following fuzzy sets $\alpha : X \to [0,1]$ and $\beta : X \to [0,1]$ defined on X.

$\alpha : X \to [0,1]$ is defined as $\alpha(a) = 0.6$; $\alpha(b) = 0.3$; $\alpha(c) = 0.4$.
$\beta : X \to [0,1]$ is defined as $\beta(a) = 0.4$; $\beta(b) = 0.3$; $\beta(c) = 0.6$.

Now α, β, $1-\lambda_1$, $1-\mu_1$, $1-\mu_3$, $1-(\mu_1 \vee \mu_2)$ are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). But $int_{T_i}\{\alpha \vee \beta \vee (1-\lambda_1) \vee (1-\mu_1) \vee (1-\mu_3) \vee (1-(\mu_1 \vee \mu_2))\} \neq 0$, $(i = 1, 2)$, since $int_{T_1}\{\alpha \vee \beta \vee (1-\lambda_1) \vee (1-\mu_1) \vee (1-\mu_3) \vee (1-(\mu_1 \vee \mu_2))\} = \lambda_3 \neq 0$ and $int_{T_2}\{\alpha \vee \beta \vee (1-\lambda_1) \vee (1-\mu_1) \vee (1-\mu_3) \vee (1-(\mu_1 \vee \mu_2))\} = \mu_1 \wedge \mu_2 \neq 0$. Hence (X, T_1, T_2) is **not** a pairwise fuzzy Baire bitopological space.

Proposition 4.1 *Let (X, T_1, T_2) be a fuzzy bitopological space. Then the following are equivalent :*

(1). (X, T_1, T_2) is a pairwise fuzzy Baire space.

(2). $int_{T_i}(\lambda) = 0$, $(i = 1, 2)$, for every pairwise fuzzy first category set λ in (X, T_1, T_2).

(3). $cl_{T_i}(\mu) = 1$, $(i = 1, 2)$, for every pairwise fuzzy residual set μ in (X, T_1, T_2).

Proof. $(\mathbf{1}) \Rightarrow (\mathbf{2})$ Let λ be a pairwise fuzzy first category set in (X, T_1, T_2). Then $\lambda = \vee_{n=1}^{\infty}(\lambda_n)$, where λ_n's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Now $int_{T_i}(\lambda) = int_{T_i}(\vee_{n=1}^{\infty}(\lambda_n)) = 0$, $(i = 1, 2)$, [since (X, T_1, T_2) is a pairwise fuzzy Baire space]. Therefore $int_{T_i}(\lambda) = 0$, where λ_n's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2).
$(\mathbf{2}) \Rightarrow (\mathbf{3})$ Let μ be a pairwise fuzzy residual set in (X, T_1, T_2). Then $1 - \mu$ is a pairwise fuzzy first category set in (X, T_1, T_2). By hypothesis, $int_{T_i}(1 - \mu) = 0$, $(i = 1, 2)$, which implies that $1 - cl_{T_i}(\mu) = 0$. Hence $cl_{T_i}(\mu) = 1$, $(i = 1, 2)$.
$(\mathbf{3}) \Rightarrow (\mathbf{1})$ Let λ be a pairwise fuzzy first category set in (X, T_1, T_2). Then $\lambda = \vee_{n=1}^{\infty}(\lambda_n)$, where λ_n's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Now λ is a pairwise fuzzy first category set in (X, T_1, T_2) implies that $1 - \lambda$ is a pairwise fuzzy residual set in (X, T_1, T_2). By hypothesis, we have $cl_{T_i}(1-\lambda) = 1$, $(i = 1, 2)$, which implies that $1 - int_{T_i}(\lambda) = 0$, $(i = 1, 2)$. Then $int_{T_i}(\lambda) = 1$. That is, $int_{T_i}(\vee_{n=1}^{\infty}(\lambda_n)) = 0$, where λ_n's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Hence the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy Baire space.

Proposition 4.2 *If the fuzzy bitopological space* (X, T_1, T_2) *is a pairwise fuzzy Baire space, then* (X, T_1, T_2) *is a pairwise fuzzy second category space.*

Proof. Let (X, T_1, T_2) be a pairwise fuzzy Baire bitopological space. Then we have $int_{T_i}(\vee_{n=1}^{\infty}(\lambda_n)) = 0, \ (i = 1, 2)$, where λ_n's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Now we claim that $\vee_{n=1}^{\infty}(\lambda_n) \neq 1_X, \quad (i = 1, 2)$. Suppose that $\vee_{n=1}^{\infty}(\lambda_n) = 1_X$. Then $int_{T_i}(\vee_{n=1}^{\infty}(\lambda_n)) = int 1_X = 1_X, \ (i = 1, 2)$, which implies that $0 = 1$, a contradiction. Hence we must have $\vee_{n=1}^{\infty}(\lambda_n) \neq 1_X$. Therefore the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy second category space.

Remarks: The converse of the above Proposition **need not** be true. A pairwise fuzzy second category space **need not** be a pairwise fuzzy Baire space. For, in example 4.2, $\alpha, \beta, 1-\lambda_1, 1-\mu_1, 1-\mu_3$ and $1-(\mu_1 \vee \mu_2)$ are pairwise fuzzy nowhere dense sets in (X, T_1, T_2) and $\{\alpha \vee \beta \vee (1-\lambda_1) \vee (1-\mu_1) \vee (1-\mu_3) \vee (1-(\mu_1 \vee \mu_2))\} = \lambda_3 \neq 1_X$. Hence (X, T_1, T_2) is a pairwise fuzzy second category space. But (X, T_1, T_2) is **not** a pairwise fuzzy Baire bitopological space, since $int_{T_1}\{\alpha \vee \beta \vee (1-\lambda_1) \vee (1-\mu_1) \vee (1-\mu_3) \vee (1-(\mu_1 \vee \mu_2))\} = \lambda_3 \neq 0$ and also $int_{T_2}\{\alpha \vee \beta \vee (1-\lambda_1) \vee (1-\mu_1) \vee (1-\mu_3) \vee (1-(\mu_1 \vee \mu_2))\} = \mu_1 \wedge \mu_2 \neq 0$.

Proposition 4.3 *If* $int_{T_i}(\vee_{n=1}^{\infty}(\lambda_n)) = 0, \quad (i = 1, 2)$, *where* $int_{T_i}(\lambda_n) = 0$ *and* $1 - \lambda_n \in T_i, \quad (i = 1, 2)$, *then the fuzzy bitopological space* (X, T_1, T_2) *is a pairwise fuzzy Baire space.*

Proof. Now $1-\lambda_n \in T_i, \ (i = 1, 2 \ \ and \ \ n \geq 1)$, implies that $int_{T_i}(1-\lambda_n) = 1-\lambda_n$. Then $1-cl_{T_i}(\lambda_n) = 1-\lambda_n$ and hence $cl_{T_i}(\lambda_n) = \lambda_n, \ (i = 1, 2 \ \ and \ \ n \geq 1)$. Now $int_{T_i}(\lambda_n) = 0$ and $cl_{T_i}(\lambda_n) = \lambda_n$ implies that $int_{T_i}(cl_{T_i}(\lambda_n)) = 0, \ (i = 1, 2 \ \ and \ \ n \geq 1)$. In particular, $int_{T_1}(cl_{T_2}(\lambda_n)) = 0$ and $int_{T_2}(cl_{T_1}(\lambda_n)) = 0$, for $n \geq 1$. Hence λ_n's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Therefore we have $int_{T_i}(\vee_{n=1}^{\infty}(\lambda_n)) = 0, \quad n \geq 1$ where λ_n's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Hence (X, T_1, T_2) is a pairwise fuzzy Baire space.

Proposition 4.4 *If* $cl_{T_i}(\wedge_{n=1}^{\infty}(\lambda_n)) = 1, \ (i = 1, 2)$ *where* λ_n*'s,* $(i = 1, 2)$, *are* T_i-*fuzzy dense and* T_i-*fuzzy open sets in* (X, T_1, T_2), *then* (X, T_1, T_2) *is a pairwise fuzzy Baire space.*

Proof. Let λ_n's, $(i = 1, 2 \ \ and \ \ n \geq 1)$, be T_i-fuzzy dense and T_i-fuzzy open sets in (X, T_1, T_2). Now $cl_{T_i}(\wedge_{n=1}^{\infty}(\lambda_n)) = 1$ implies that $1-cl_{T_i}(\wedge_{n=1}^{\infty}(\lambda_n)) = 0$, for $(i = 1, 2)$. Then we have $int_{T_i}(1 - \wedge_{n=1}^{\infty}(\lambda_n)) = 0$, $(i = 1, 2)$ and hence $int_{T_i}(\vee_{n=1}^{\infty}(1 - \lambda_n)) = 0. \Longrightarrow (1)$. Since λ_n's are T_i-fuzzy dense sets in (X, T_1, T_2), $cl_{T_i}(\lambda_n) = 1, \ \ (i = 1, 2 \ \ and \ \ n \geq 1)$. Then $1 - cl_{T_i}(\lambda_n) = 0$, which implies that $int_{T_i}(1-\lambda_n) = 0$ and $\lambda_n = 1-(1-\lambda_n) \in T_i$, $(i = 1, 2 \ \ and \ \ n \geq 1)$. Hence from (1), by Proposition 4.3, we have (X, T_1, T_2) is a pairwise fuzzy Baire space.

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