Gen. Math. Notes, Vol. 10, No. 2, June 2012, pp.22-29
ISSN 2219-7184;
www.i-csrs.org
Available free online at http://www.geman.in

A New Type of Weak Continuity

G. Shanmugam[1] and N. Rajesh[2]

[1]Department of Mathematics, Jeppiaar Engineering College
Chennai- 600119, Tamilnadu, India
E-mail: gsm.maths@gmail.com
[2]Department of Mathematics, Rajah Serfoji Govt. College
Thanjavur- 613005, Tamilnadu, India
E-mail: nrajesh_topology@yahoo.co.in

(Received: 21-4-12 / Accepted: 19-6-12)

Abstract

In this paper, we introduce a new class of functions called weakly $sg\alpha$-continuous functions and investigate some of their fundamental properties.

Keywords: *$sg\alpha$-open set, weakly $sg\alpha$-continuous function, $sg\alpha$-connected space, $sg\alpha$-compact space, $sg\alpha$-closed graph.*

1 Introduction

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the variously modified forms of continuity, seperation axioms etc. by utiliaing generalized open sets. One of the most well known notions and also an inspiration source is the notion of α-open [3] sets introduced by Njastad in 1965. Quite recently, as generalization of closed sets called $sg\alpha$-closed sets were introduced and studied by the present author [4]. In [5] the authors, introduced the notion of $sg\alpha$-continuity and investigated its fundamental properties. In this paper, we introduce a new class of functions called weakly $sg\alpha$-continuous functions and investigate some of their fundamental properties.

2 Preliminaries

Throughout this paper (X, τ) and (Y, σ) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), $\mathrm{Cl}(A)$, $\mathrm{Int}(A)$ and A^c denote the closure of A, the interior of A and the complement of A in X, respectively.

Definition 2.1 *A subset A of a space X is called semi-open [2] (resp. α-open [3]) if $A \subset \mathrm{Cl}(\mathrm{Int}(A))$ (resp. $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$). The complement of α-open set is called α-closed.*

The α-closure of a subset A of X, denoted by $\alpha\,\mathrm{Cl}(A)$ is defined to be the intersection of all α-closed sets containing A in X.

Definition 2.2 *A subset A of a space X is called $sg\alpha$-closed [4] if $\alpha\,\mathrm{Cl}(A) \subset U$ whenever $A \subset U$ and U is semiopen in X. The complement of $sg\alpha$-closed set is called $sg\alpha$-open. The family of all $sg\alpha$-open subsets of (X, τ) is denoted by $sg\alpha O(\tau)$.*

The family of all $sg\alpha$-open (resp. $sg\alpha$-closed) sets of X is denoted by $sg\alpha(\tau)$ (resp. $sg\alpha C(X)$). We set $sg\alpha O(X, x) = \{U | U \in sg\alpha(\tau) \text{ and } x \in U\}$. In [4] shown that the set $sg\alpha(\tau)$ forms a topology, which is finer than τ.

Definition 2.3 *The intersection of all $sg\alpha$-closed sets containing A is called the $sg\alpha$-closure [4] of A and is denoted by $sg\alpha$-$\mathrm{Cl}(A)$. A set A is $sg\alpha$-closed if and only if $sg\alpha$-$\mathrm{Cl}(A) = A$ [4].*

Lemma 2.4 *[4] Let A be a subset of a topological space (X, τ). Then $x \in sg\alpha$-$\mathrm{Cl}(A)$ if and only if $U \cap A \neq \varnothing$ for every $U \in sg\alpha O(X, x)$.*

Definition 2.5 *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $sg\alpha$-continuous [5] (resp. $sg\alpha$-irresolute [6]) if $f^{-1}(V) \in sg\alpha(\tau)$ for every open set V of Y (resp. $V \in sg\alpha(\sigma)$).*

Definition 2.6 *A topological space (X, τ) is said to be $sg\alpha$-regular [7] if for each closed set F and each $x \notin F$, there exist disjoint $sg\alpha$-open sets U and V such that $x \in U$ and $F \subset V$.*

Lemma 2.7 *For a topological space (X, τ), the following are equivalent:*

(i) X is $sg\alpha$-regular;

(ii) for each open set U and each $x \in U$, there exists $V \in sg\alpha(\tau)$ such that $x \in V \subset sg\alpha$-$\mathrm{Cl}(V) \subset U$.

3 Weakly $sg\alpha$-Continuous Functions

Definition 3.1 *A function $f : (X, \tau) \to (Y, \sigma)$ is called weakly $sg\alpha$-continuous if for each $x \in X$ and each open set V containing $f(x)$ there exists $U \in sg\alpha o(X, x)$ such that $f(U) \subseteq sg\alpha\text{-Cl}(V)$.*

It is clear that every $sg\alpha$-continuous function is weakly $sg\alpha$-continuous but not converse.

Example 3.2 *Let $X = \{a, b, c\}$, $\tau = \{\varnothing, \{b\}, X\}$ and $\sigma = \{\varnothing,\ \{a\},\ X\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is weakly $sg\alpha$-continuous but not $sg\alpha$-continuous.*

Theorem 3.3 *Let (X, τ) be a $sg\alpha$-regular space. Then $f : (X, \tau) \to (Y, \sigma)$ is a $sg\alpha$-continuous if and only if it is weakly $sg\alpha$-continuous.*

Proof: The proof follows from Lemma 2.7.

Theorem 3.4 *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(i) f *is weakly $sg\alpha$-continuous;*

(ii) $f^{-1}(V) \subset sg\alpha\text{-Int}(f^{-1}(sg\alpha\text{-Cl}(V)))$ *for every open set V of Y;*

(iii) $sg\alpha\text{-Cl}(f^{-1}(sg\alpha\text{-Int}(F))) \subset f^{-1}(F)$ *for every closed set F of Y;*

(iv) $sg\alpha\text{-Cl}(f^{-1}(sg\alpha\text{-Int}(\text{Cl}(B)))) \subset f^{-1}(\text{Cl}(B))$ *for every subset B of Y;*

(v) $f^{-1}(\text{Int}(B)) \subset sg\alpha\text{-Int}(f^{-1}(sg\alpha\text{-Cl}(\text{Int}(B))))$ *for every subset B of Y;*

(vi) $sg\alpha\text{-Cl}(f^{-1}(V)) \subset f^{-1}(sg\alpha\text{-Cl}(V))$ *for every open set V of Y.*

Proof: (i)⇒(ii): Let V be an open subset of Y and $x \in f^{-1}(V)$. Then $f(x) \in V$. There exists $U \in sg\alpha O(X, x)$ such that $f(U) \subset sg\alpha\text{-Cl}(V)$. Thus, we obtain $x \in U \subset f^{-1}(sg\alpha\text{-Cl}(V))$. This implies that $x \in sg\alpha\text{-Int}(f^{-1}(sg\alpha\text{-Cl}(V)))$ and consequently $f^{-1}(V) \subset sg\alpha\text{-Int}(f^{-1}(sg\alpha\text{-Cl}(V)))$.
(ii)⇒(iii): Let F be any closed set of Y. Then $Y \backslash F$ is open in Y. By (ii), we have $sg\alpha\text{-Cl}(f^{-1}(sg\alpha\text{-Int}(F))) \subset f^{-1}(F)$.
(iii)⇒(iv): Let B be any subset of Y. Then $\text{Cl}(B)$ is closed in Y and by (iii), we obtain $sg\alpha\text{-Cl}(f^{-1}(sg\alpha\text{-Int}(\text{Cl}(B)))) \subset f^{-1}(\text{Cl}(B))$.
(iv)⇒(v): Let B be any subset of Y. Then we have $f^{-1}(\text{Int}(B)) = X \setminus f^{-1}(\text{Cl}(Y \backslash B)) \subset X \setminus sg\alpha\text{-Cl}(f^{-1}(sg\alpha\text{-Int}(\text{Cl}(Y \backslash B)))) = sg\alpha\text{-Int}(f^{-1}(sg\alpha\text{-Cl}(\text{Int}(B))))$.
(v)⇒(vi): Let V be any open subset of Y. Suppose that $x \notin f^{-1}(sg\alpha\text{-Cl}(V))$.

Then $f(x) \notin sg\alpha$-Cl(V) and there exists $U \in sg\alpha O(Y, f(x))$ such that $U \cap V = \varnothing$; hence $sg\alpha$-Cl$(U) \cap V = \varnothing$. By (v), we have $x \in f^{-1}(U) \subset sg\alpha$-Int$(f^{-1}(sg\alpha\text{-Cl}(U)))$ and hence there exists $W \in sg\alpha O(X, x)$ such that $W \subset f^{-1}(sg\alpha\text{-Cl}(U))$. Since $sg\alpha$-Cl$(U) \cap V = \varnothing$, $W \cap f^{-1}(V) = \varnothing$ and by Lemma 2.4 $x \notin sg\alpha$-Cl$(f^{-1}(V))$. Therefore, we obtain $sg\alpha$-Cl$(f^{-1}(V)) \subset f^{-1}(sg\alpha$-Cl$(V))$.
(vi)$\Rightarrow$(i): Let $x \in X$ and V any open subset of Y containing $f(x)$. By (vi), we have $x \in f^{-1}(V) \subset f^{-1}(\text{Int}(sg\alpha\text{-Cl}(V))) \subset f^{-1}(sg\alpha\text{-Int}(sg\alpha\text{-Cl}(V))) = X \setminus f^{-1}(sg\alpha\text{-Cl}(Y \backslash sg\alpha\text{-Cl}(V))) \subset X \setminus sg\alpha\text{-Cl}(f^{-1}(Y \backslash sg\alpha\text{-Cl}(V))) = sg\alpha\text{-Int}(f^{-1}(sg\alpha\text{-Cl}(V)))$. Therefore, there exists $U \in sg\alpha O(X, x)$ such that $U \subset sg\alpha$-Cl(V). This shows that f is weakly $sg\alpha$-continuous.

Definition 3.5 *A function $f : (X, \tau) \to (Y, \sigma)$ is said to have a strongly $sg\alpha$-closed graph [8] if for $(x, y) \in (X \times Y) \setminus G(f)$, there exists $U \in sg\alpha O(X, x)$ and an open set V of Y containing y such that $(U \times V) \cap G(f) = \varnothing$.*

Lemma 3.6 *[8] Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then its graph $G(f)$ is strongly $sg\alpha$-closed in $X \times Y$ if and only if for each point $(x, y) \in (X \times Y) \setminus G(f)$, there exist a $sg\alpha$-open set U of X and an open set V of Y, containing x and y, respectively, such that $f(U) \cap V = \varnothing$.*

Theorem 3.7 *If $f : (X, \tau) \to (Y, \sigma)$ is a weakly $sg\alpha$-continuous function and (Y, σ) is a Hausdorff space, then the graph $G(f)$ is a $sg\alpha$-closed set of $X \times Y$.*

Proof: Let $(x, y) \in (X \times Y) \backslash G(f)$. Then, we have $y \neq f(x)$. Since (Y, σ) is Hausdorff, there exist disjoint open sets W and V such that $f(x) \in W$ and $y \in V$. Since f is weakly $sg\alpha$-continuous, there exists a $sg\alpha$-open set U containing x such that $f(U) \subseteq sg\alpha$-Cl(W). Since W and V are disjoint subsets of Y, we have $V \cap sg\alpha$-Cl$(W) = \varnothing$. This shows that $(U \times V) \cap G(f) = \varnothing$ and hence by Lemma 3.6 $G(f)$ is $sg\alpha$-closed.

Definition 3.8 *By a weakly $sg\alpha$-continuous retraction, we mean a weakly $sg\alpha$-continuous function $f : X \to A$, where $A \subset X$ and $f|A$ is the identity function on A.*

Theorem 3.9 *Let A be a subset of X and $f : (X, \tau) \to (Y, \sigma)$ be a weakly $sg\alpha$-continuous restraction of X onto A. If (X, τ) is a Hausdorff space, then A is a $sg\alpha$-closed set in X.*

Proof: Supoose that A is not $sg\alpha$-closed in X. Then there exists a point $x \in sg\alpha$-Cl$(A) \backslash A$. Since f is weakly $sg\alpha$-continuous restraction, we have $f(x) \neq x$. Since X is Hausdorff, there exist disjoint open sets U and V of X such that $x \in U$ and $f(x) \in V$. Thus, we get $U \cap sg\alpha$-Cl$(V) = \varnothing$. Now, let $W \in$

$sg\alpha O(X,x)$. Then $U \cap W \in sg\alpha O(X,x)$ and hence $(U \cap W) \cap A \neq \varnothing$, because $x \in sg\alpha\text{-Cl}(A)$. Let $y \in (U \cap W) \cap A$. Since $y \in A$, $f(y) \in U$ and hence $f(y) \notin sg\alpha\text{-Cl}(V)$. This gives that $f(W) \nsubseteq sg\alpha\text{-Cl}(V)$. This contradicts that f is weakly $sg\alpha$-continuous. Hence A is $sg\alpha$-closed in X.

Definition 3.10 *A space (X,τ) is called $sg\alpha$-connected [5] if X cannot be written as the disjoint union of two nonempty $sg\alpha$-open sets.*

Theorem 3.11 *Let $f : (X,\tau) \to (Y,\sigma)$ be a weakly $sg\alpha$-continuous surjective function. If X is $sg\alpha$-connected, then Y is connected.*

Proof: Suppose that (Y,σ) is not connected. Then there exist nonempty disjoint open sets V_1 and V_2 in Y such that $V_1 \cup V_2 = Y$. Since f is surjective, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are nonempty disjoint subsets of X such that $f^{-1}(V_1) \cup f^{-1}(V_2) = X$. By Theorem 3.4, we have $f^{-1}(V_i) \subseteq sg\alpha\text{-Int}(f^{-1}(sg\alpha\text{-Cl}(V_i)))$, i = 1,2. Since V_i is open and closed and every closed set is $sg\alpha$-closed, we obtain $f^{-1}(V_i) \subseteq sg\alpha\text{-Int}(f^{-1}(V_i))$ and hence $f^{-1}(V_i)$ is $sg\alpha$-open for i = 1,2. This implies that (X,τ) is not $sg\alpha$-connected.

Definition 3.12 *A space (X,τ) is said to be ultra $sg\alpha$-Urysohn if for each pair of distinct points x and y in X, there exist open sets U, V containing x, y respectively such that $sg\alpha\text{-Cl}(U) \cap sg\alpha\text{-Cl}(V) = \varnothing$.*

Definition 3.13 *A space (X,τ) is said to be $sg\alpha$-T_2 [8] if for each pair of distinct points x and y in X, there exist $U \in sg\alpha O(X,x)$ and $V \in sg\alpha O(X,y)$ such that $U \cap V = \varnothing$.*

Theorem 3.14 *Let $f : (X,\tau) \to (Y,\sigma)$ be a weakly $sg\alpha$-continuous injective function. If Y is ultra $sg\alpha$-Urysohn, then X is $sg\alpha$-T_2.*

Proof: Let x_1 and x_2 be any two distinct points of X. Since f is injective, $f(x_1) \neq f(x_2)$. Since (Y,σ) is ultra $sg\alpha$-Urysohn, there exist $V_1, V_2 \in \sigma$ such that $f(x_1) \in V_1$, $f(x_2) \in V_2$ and $sg\alpha\,\text{Cl}\ (V_1) \cap sg\alpha\text{-Cl}(V_2) = \varnothing$. This gives $f^{-1}(sg\alpha\text{-Cl}(V_1)) \cap f^{-1}(sg\alpha\text{-Cl}(V_2)) = \varnothing$ and hence $sg\alpha\text{-Int}(f^{-1}(sg\alpha\text{-Cl}(V_1))) \cap sg\alpha\text{-Int}(f^{-1}(sg\alpha\,\text{Cl}(V_2))) = \varnothing$. Since f is weakly $sg\alpha$-continuous, $x_i \in f^{-1}(V_i) \subset sg\alpha\text{-Int}(f^{-1}(sg\alpha\text{-Cl}(V_i)))$, $i = 1,2$. By Theorem 3.4 and this indicates that the space (X,τ) is $sg\alpha$-T_2.

4 Additional Properties

Definition 4.1 *For a function $f : (X,\tau) \to (Y,\sigma)$, the graph $G(f)$ is said to be ultra $sg\alpha$-closed if for each $(x,y) \in (X \times Y)\backslash G(f)$, there exist $U \in sg\alpha O(X,x)$, $V \in sg\alpha O(Y,y)$ such that $(U \times sg\alpha\text{-Cl}(V)) \cap G(f) = \varnothing$.*

Lemma 4.2 *The function $f : (X, \tau) \to (Y, \sigma)$ has a ultra $sg\alpha$-closed graph if and only if for every $(x, y) \in (X \times Y)\backslash G(f)$ there exist $U \in sg\alpha O(X, x)$, $V \in sg\alpha O(Y, y)$ and $f(U) \cap sg\alpha\text{-Cl}(V) = \varnothing$.*

Proof: It is an immediate consequence of Definition 4.1.

Theorem 4.3 *Let $f : (X, \tau) \to (Y, \sigma)$ be a weakly $sg\alpha$-continuous function. If (Y, σ) is ultra $sg\alpha$-Urysohn, then the graph $G(f)$ is ultra $sg\alpha$-closed.*

Proof: Let $(x, y) \in (X \times Y)\backslash G(f)$. Then $y \neq f(x)$. Since Y is ultra $sg\alpha$-Urysohn, there exist open sets V and W containing x and y, repectively such that $sg\alpha\text{-Cl}(V) \cap sg\alpha\text{-Cl}(W) = \varnothing$. Since f is weakly $sg\alpha$-continuous, there exist $U \in sg\alpha O(X, x)$ such that $f(U) \subset sg\alpha\text{-Cl}(U)$. This implies that $f(U) \cap sg\alpha\text{-Cl}(W) = \varnothing$. So, by Lemma 4.2 $G(f)$ is ultra $sg\alpha$-closed.

Theorem 4.4 *If $f : (X, \tau) \to (Y, \sigma)$ is an injective weakly $sg\alpha$-continuous function with a ultra $sg\alpha$-closed graph, then the space (X, τ) is $sg\alpha$-T_2.*

Proof: Let x and y be any distinct points of X. Then, since f is injective, we have $f(x) \neq f(y)$. Then we have $(x, f(y)) \in (X \times Y) \setminus G(f)$. Since $G(f)$ is ultra $sg\alpha$-closed, by Lemma 4.2 there exist $U \in sg\alpha O(X, x)$ and an open set V of Y containing $f(y)$ such that $f(U) \cap sg\alpha\text{-Cl}(V) = \varnothing$. Since f is weakly $sg\alpha$-continuous, there exists $W \in sg\alpha O(X, y)$ such that $f(W) \subset sg\alpha\text{-Cl}(V)$. Therefore, we have $f(U) \cap G(f) = \varnothing$. Since f is injective, we obtain $U \cap W = \varnothing$. This shows that (X, τ) is a $sg\alpha$-T_2 space.

Theorem 4.5 *If $f : (X, \tau) \to (Y, \sigma)$ is a $sg\alpha$-continuous function and (Y, σ) is a T_2 space, then the graph $G(f)$ is ultra-$sg\alpha$-closed.*

Proof: Let $(x, y) \in (X \times Y)\backslash G(f)$. The T_2 ness of Y gives the existence of an open set V containing y such that $f(x) \notin \text{Cl}(V)$. Now $\text{Cl}(V)$ is a closed set in Y. So, $Y\backslash \text{Cl}(V)$ is an open set in Y containing $f(x)$. Therefore, by the $sg\alpha$-continuity of f there exist $U \in sg\alpha O(X, x)$ such that $f(U) \subseteq Y\backslash \text{Cl}(V)$, hence $f(U) \cap \text{Cl}(V) = \varnothing$. Since $sg\alpha\text{-Cl}(A) \subseteq \text{Cl}(A)$ for every subset A of X, once obtain $f(U) \cap sg\alpha\text{-Cl}(V) = \varnothing$. By Lemma 4.2, $G(f)$ is ultra $sg\alpha$-closed.

Theorem 4.6 *If $f : (X, \tau) \to (Y, \sigma)$ is a $sg\alpha$-irresolute function and (Y, σ) is a $sg\alpha$-T_2 space, then the graph $G(f)$ is ultra $sg\alpha$-closed.*

Proof: Similar proof of Theorem 4.5.

Definition 4.7 *A space (X, τ) is said to be*

(i) $sg\alpha$-compact [5] if every cover of X by $sg\alpha$-open sets has a finite subcover;

(ii) $sg\alpha$-closed [5] if every cover of X by $sg\alpha$-open sets has a finite subcover whose $sg\alpha$-closure cover X.

Definition 4.8 *A subset A of a space X is said to be $sg\alpha$-closed relative to X [5] if for every cover $\{V_\alpha\colon \alpha \in \Lambda\}$ of A by $sg\alpha$-open sets of X, there exists a finite subset Λ_0 of Λ such that $A \subset \bigcup \{sg\alpha\text{-Cl}(V_\alpha) \mid \alpha \in \Lambda_0\}$.*

Theorem 4.9 *If $f : (X, \tau) \to (Y, \sigma)$ is a weakly $sg\alpha$-continuous function and A is a $sg\alpha$-compact subset of (X, τ), then $f(A)$ is $sg\alpha$-closed relative to (Y, σ).*

Proof: Let $\{V_i | i \in \Lambda\}$ be any cover of $f(K)$ by open sets of (Y, σ). For each $x \in X$, there exists $\alpha(x) \in \Lambda$ such that $f(x) \in V_{\alpha(x)}$. Since f is weakly $sg\alpha$-continuous, there exists $U(x) \in sg\alpha O(X, x)$ such that $f(U(x)) \subset sg\alpha$-Cl$(V_{\alpha(x)})$. The family $\{U(x) | x \in A\}$ is a cover of A by $sg\alpha$-open sets of X. Since A is $sg\alpha$-compact, there exists a finite number of points, say, x_1, x_2, ... x_n in A such that $A \subset \bigcup\{U(x_k) \mid x_k \in A, 1 \leq K \leq n\}$. Therefore, we obtain $f(A) \subset \bigcup \{f(U(x_k)) \mid x_k \in A, 1 \leq K \leq n\} \subset \bigcup \{sg\alpha\text{-Cl}(V_{\alpha(x_k)}) \mid x_k \in A, 1 \leq K \leq n\}$. This shows that $f(A)$ is $sg\alpha$-closed relative to (Y, σ).

Corollary 4.10 *If $f : (X, \tau) \to (Y, \sigma)$ is a weakly $sg\alpha$-continuous surjective function and the space (X, τ) is $sg\alpha$-compact, then (Y, σ) is a $sg\alpha$-closed space.*

Definition 4.11 *Let A be a subset of a topological space (X, τ). Then the $sg\alpha$-frontier [5] of A, denoted by $sg\alpha$-$Fr(A)$ is defined as $sg\alpha$-$Fr(A) = sg\alpha$-Cl(A) $\cap$ $sg\alpha$-Cl$(X \backslash A)$.*

Theorem 4.12 *The set of all points $x \in X$ at which a function $f : (X, \tau) \to (Y, \sigma)$ is not weakly $sg\alpha$-continuous if and only if the union of $sg\alpha$-frontier of the inverse images of the closure of open sets containing $f(x)$.*

Proof: **Necessity.** Suppose that f is not weakly $sg\alpha$-continuous at $x \in X$. Then there exists an open set V of Y containing $f(x)$ such that $f(U) \nsubseteq sg\alpha$-Cl(V) for every $U \in sg\alpha O(X, x)$. Then $U \cap (X \backslash f^{-1}(sg\alpha\text{-Cl}(V))) \neq \varnothing$ for every $U \in sg\alpha O(X, x)$ and hence by Lemma 2.4 $x \in sg\alpha$-Cl$(X \backslash f^{-1}(sg\alpha\text{-Cl}(V)))$. On the other hand, we have $x \in f^{-1}(V) \subset sg\alpha$-Cl$(f^{-1}(sg\alpha\text{-Cl}(V)))$ and hence $x \in sg\alpha$-$Fr(f^{-1}(sg\alpha\text{-Cl}(V)))$.
Sufficiency. Suppose that f is weakly $sg\alpha$-continuous at $x \in X$ and let V be any open set of Y containing $f(x)$. Then by Theorem 3.4, we have $x \in f^{-1}(V) \subset sg\alpha$-Int$(f^{-1}(sg\alpha\text{-Cl}(V)))$. Therefore, $x \in sg\alpha$-$Fr(f^{-1}(\text{Cl}(V)))$ for each open set V of Y containing $f(x)$.

References

[1] S.G. Crossley and S.K. Hildebrand, Semi-closure, *Texas J. Sci.*, 22(1971), 99-112.

[2] N. Levine, Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly*, 70(1963), 36-41.

[3] O. Nijastad, On some classes nearly open sets, *Pacific J. Math.*, 15(1965), 961-970.

[4] N. Rajesh and B. Krsteska, Semigeneralized α-closed sets, *Antarctica J. Math.*, 6(1) (2009), 1-12.

[5] N. Rajesh, B. Krsteska and G. Shanmugam, Semigeneralized α-continuous functions (submitted).

[6] N. Rajesh, B. Krsteska and G. Shanmugam, On new types of generalized homeomorphisms (submitted).

[7] N. Rajesh and G. Shanmugam, On $sg\alpha$-regular and $sg\alpha$-normal spaces (submitted).

[8] N. Rajesh and G. Shanmugam, Characterizations of $sg\alpha$-T_0, $sg\alpha$-T_1 and $sg\alpha$-T_2 topological spaces (submitted).