Gen. Math. Notes, Vol. 6, No. 1, September 2011, pp.25-39

ISSN 2219-7184;

www.i-csrs.org

Available free online at http://www.geman.in

Combined Effect of MHD and Radiation on Unsteady Transient Free Convection Flow between Two Long Vertical Parallel Plates with Constant Temperature and Mass Diffusion

U.S. Rajput[1] and P.K. Sahu[2]

[1]Department of Mathematics and Astronomy, University of Lucknow, Lucknow
U.P. -226007, India
E-mail: usrajput@sify.com
[2]Department of Mathematics and Astronomy, University of Lucknow, Lucknow
U.P. -226007, India
E-mail: rajputpradeeplko@gmail.com

(Received: 19-4-11/Accepted:30-6-11)

Abstract

The paper studies combined effects of MHD and radiation on unsteady transient free convection flow of a viscous, incompressible, electrically conducting and radiating fluid between two long vertical parallel plates with constant temperature and mass diffusion, under the assumption that the induced magnetic field is negligible. The Laplace transform method has been used to find the solutions for the velocity, temperature and concentration profiles. The velocity, temperature, concentration and skin-friction are studied for different parameters like Prandtl number, Schmidt number, magnetic parameter, buoyancy ratio parameter and time.

Keywords: *Heat and Mass Transfer, MHD, Mass Diffusion, Thermal Radiation and Transient free convection.*

1 Introduction

MHD is related to engineering problems such as plasma confinement, liquid-metal cooling of nuclear reactors, magnetic control of molten iron flow in steel industry and electromagnetic casting (among others). The fluid core of the Earth and other planets is theorized to be a huge MHD dynamo that generates the Earth's magnetic field due to the motion of liquid iron. On the other hand, radiation in free convection has been studied by many authors because of its applications in many engineering and industrial processes such as nuclear power plant, solar power technology, steel industry, fossil fuel combustion, etc. Ostrach[24] has studied laminar free convection flow of a viscous incompressible fluid between two vertical walls with constant wall temperature. Ostrach[25] and Sparrow et al.[11] have studied the combined effect of a steady free and forced convection laminar flow and heat transfer between two vertical parallel walls. Bodoia and Osterle[14], Aung[26] and Aung et al.[27], Miyatake and Fujii[20-22], Miyatake et al.[23], Lee and Yan[16], Higuera and Ryazantsev[12], Campo et al.[3], Pantokratoras[4] have presented their results for a steady free convection flow between vertical parallel plates by considering different conditions on the wall tenperature. Kettleborought[15] has described numerically the transient laminar two-dimensional motion of a viscous incompressible fluid between two heated vertical plates in which the motion is generated by a temperature gradient perpendicular to the direction of the body force.

Nelson and Wood[8-10] have presented numerical analysis of developing laminar flow between vertical parallel plates for combined heat and mass transfer natural convection with uniform wall temperature/concentration and uniform heat/mass flux boundary conditions. They also have presented an analytical solution for the fully developed combined heat and mass transfer natural convection between vertical parallel plates with asymmetric boundary conditions. Unsteady free convection couette flow between two vertical parallel plates has been studied by Singh[1]. Singh et al.[2] have studied the transient free convection flow of a viscous incompressible fluid between two vertical parallel plates when the walls are heated asymmetrically. Lee[17] has studied a combined numerical and theoretical investigation of laminar natural convection heat and mass transfer in open vertical parallel plates with unheated entry and unheated exit for various thermal and concentration boundary conditions. Unsteady MHD free convection couette flow between two vertical parallel plates has been studied by Jha.[6].

Desrayaud and Lauriat[13] have studied the heat and mass transfer analogy for condensation of humid air in a vertical parallel plate channel. Narahari et al.[18] have studied the transient free convection flow between two vertical parallel plates with constant heat flux at one boundary and the other maintained at constant temperature. Jha et al.[7] have presented the transient free convection flow in a vertical channel as a result of symmetric heating of the channel walls. Sing and Paul[5] have presented the transient free convection flow of a viscous and incompressible fluid between two vertical parallel walls as a result of asymmetric heating or cooling of the walls. Narahari[19] has studied the transient free convection flow of a viscous incompressible fluid between two infinite vertical parallel plates in the presence of constant temperature and mass diffusion.

The object of the present work is to study the combined effects of MHD and radiation on unsteady transient free convection flow between two long vertical parallel plates with constant temperature and mass diffusion, when the fluid is viscous, incompressible and electrically conducting.

2 Mathematical Analysis

We consider an unsteady transient free convection flow of a viscous, incompressible, electrically conducting and radiating fluid between two long vertical parallel plates with constant temperature and mass diffusion in the presence of transverse magnetic field. In the present problem, we assume that the magnetic Reynolds number is so small that the induced magnetic field can be neglected in comparison to the applied one. A magnetic field (fixed relative to the plates) of uniform strength B_0 is assumed to be applied transversely to the plates. The $x^{'}$- axis is considered along one of the vertical plates and the $y^{'}$- axis is taken normal to the plates. Initially, the temperature of the fluid and the plates are same as $T_d^{'}$ and the concentration of the fluid is $C_d^{'}$. At time $t^{'} > 0$, the temperature of the plate and concentration of the fluid at $y^{'} = 0$ are raised to $T_w^{'}$ and $C_w^{'}$ respectively, causing the flow of free convection currents. The governing equations under the usual Boussinesq's approximation are as follows:

$$\frac{\partial u^{'}}{\partial t^{'}} = g\beta \left(T^{'} - T_d^{'}\right) + g\beta^* \left(C^{'} - C_d^{'}\right) + \nu \frac{\partial^2 u^{'}}{\partial y^{'2}} - \frac{\sigma\beta_0^2 u^{'}}{\rho}, \tag{1}$$

$$\rho C_p \frac{\partial T^{'}}{\partial t^{'}} = k \frac{\partial^2 T^{'}}{\partial y^{'2}} - \frac{\partial q_r}{\partial y^{'}}, \tag{2}$$

$$\frac{\partial C^{'}}{\partial t^{'}} = D \frac{\partial^2 C^{'}}{\partial y^{'2}}. \tag{3}$$

The initial and boundary condition are as follows:

$$\left.\begin{array}{c} t^{'} \leq 0 : u^{'} = 0, T^{'} = T_d^{'}, C^{'} = C_d^{'} \text{ for } 0 \leq y^{'} \leq d, \\ t^{'} > 0 : u^{'} = 0, T^{'} = T_w^{'}, C^{'} = C_w^{'} \text{ at } y^{'} = 0, \\ u^{'} = 0, T^{'} = T_d^{'}, C^{'} = C_d^{'} \text{ at } y^{'} = d. \end{array}\right\} \tag{4}$$

Where $u^{'}$ is the velocity of the fluid, g-the acceleration due to gravity, β-volumetric coefficient of thermal expansion, $t^{'}$-time, d-the distance between two vertical plates, $T^{'}$-the temperature of the fluid, $T^{'}_{d}$-the temperature of the plate at $y^{'} = d$, β^{*}-volumetric coefficient of concentration expansion, $C^{'}$-species concentration in the fluid, $C^{'}_{d}$-species concentration at the plate $y^{'} = d$, ν-the kinematic viscosity, $y^{'}$-the coordinate axis normal to the plates, ρ-the density, C_p- the specific heat at constant pressure, k-the thermal conductivity of the fluid, D- the mass diffusion coefficient, $T^{'}_{w}$- temperature of the plate at $y^{'} = 0$, $C^{'}_{w}$-species concentration at the plate $y^{'} = 0$, B_0-the uniform magnetic field, σ-electrical conductivity and q_r-radiative heat flux.

The local radiant in case of an optically thin gray fluid is expressed by

$$\frac{\partial q_r}{\partial y^{'}} = -4a^{*}\sigma(T^{'4}_{d} - T^{'4}) \tag{5}$$

It is assumed that the temperature difference within the flow are sufficiently small, so that $T^{'4}$ may be expressed as a linear function of the temperature. Thus expanding $T^{'4}$ in a Taylar's series about $T^{'}_{d}$ and neglecting higher order terms, we obtain

$$T^{'4} = 4T^{'3}_{d}T^{'} - 3T^{'4}_{d} \tag{6}$$

Using the equations (5) and (6), equation (2) becomes

$$\rho C_p \frac{\partial T^{'}}{\partial t^{'}} = k\frac{\partial^2 T^{'}}{\partial y^{'2}} - 16a^{*}\sigma T^{'3}_{d}(T^{'} - T^{'}_{d}) \tag{7}$$

Introducing the following non-dimensional quantities:

$$\left.\begin{aligned} &y = \tfrac{y^{'}}{d}, t = \tfrac{t^{'}\nu}{d^2}, u = \tfrac{u^{'}\nu}{d^2 g\beta(T^{'}_{w}-T^{'}_{d})} = \tfrac{u^{'}d}{\nu Gr}, Gr = \tfrac{g\beta(T^{'}_{w}-T^{'}_{d})d^3}{\nu^2},\\ &\theta = \tfrac{T^{'}-T^{'}_{d}}{T^{'}_{w}-T^{'}_{d}}, Pr = \tfrac{\mu C_p}{k}, C = \tfrac{C^{'}-C^{'}_{d}}{C^{'}_{w}-C^{'}_{d}}, Gm = \tfrac{g\beta^{*}(C^{'}_{w}-C^{'}_{d})d^3}{\nu^2},\\ &Sc = \tfrac{\nu}{D}, N = \tfrac{Gm}{Gr}, M = \tfrac{\sigma B_0^2 d^2}{\mu}, \mu = \rho\nu, F = \tfrac{16a^{*}\sigma d^2 T^{'3}_{d}}{k}. \end{aligned}\right\} \tag{8}$$

Where u is the dimensionless velocity, y-dimensionless coordinate axis normal to the plates, t-dimensionless time, θ-the dimensionless temperature, C-the dimensionless concentration, Gr-thermal Grashof number, Gm-mass Grashof number, μ-the coefficient of viscosity, Pr-the Prandtl number, Sc-the Schmidt number, N-the buoyancy ratio parameter, M-magnetic parameter, a^{*}-absorption coefficient and F-radiation parameter. Then the model is transformed in to the following non-dimensional form of equations:

$$\frac{\partial u}{\partial t} = \theta + NC + \frac{\partial^2 u}{\partial y^2} - Mu, \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} - \frac{F}{Pr}\theta, \tag{10}$$

$$Sc\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2}. \tag{11}$$

The initial and boundary conditions become:

$$\left.\begin{array}{l} t \le 0 : u = 0, \theta = 0, C = 0 \ for \ 0 \le y \le 1, \\ t > 0; u = 0, \theta = 1, C = 1 \ at \ y = 0, \\ \quad u = 0, \theta = 0, C = 0 \ at \ y = 1. \end{array}\right\} \tag{12}$$

The final solution of equations (9), (10) and (11) with boundary condition (12) is as under:

Case I : $Pr \neq 1, Sc \neq 1$

$$\begin{aligned} u(t,y) = \textstyle\sum_{n=0}^{\infty}[& \tfrac{e^{Rt}}{2L}\{F_1(a,1,c_1,t) - F_1(b,1,c_1,t)\} \\ & + \tfrac{Ne^{Qt}}{2M}\{F_1(a,1,c_2,t) - F_1(b,1,c_2,t)\} - \left(\tfrac{1}{2L} + \tfrac{N}{2M}\right)\{F_1(a,1,M,t) \\ & - F_1(b,1,M,t)\} - \tfrac{e^{Rt}}{2L}\{F_1(a,Pr,c_3,t) - F_1(b,Pr,c_3,t)\} \\ & + \tfrac{1}{2L}\{F_1(a,Pr,Z,t) - F_1(b,Pr,Z,t)\} - \tfrac{Ne^{Qt}}{2M}\{F_1(a,Sc,Q,t) \\ & - F_1(b,Sc,Q,t)\} + \tfrac{N}{2M}\{F_1(a,Sc,0,t) - F_1(b,Sc,0,t)\}], \end{aligned} \tag{13}$$

$$\theta(t,y) = \textstyle\sum_{n=0}^{\infty}[\tfrac{1}{2}\{F_1(a,Pr,Z,t) - F_1(b,Pr,Z,t)\}], \tag{14}$$

$$C(t,y) = \textstyle\sum_{n=0}^{\infty}[\tfrac{1}{2}\{F_1(a,Sc,0,t) - F_1(b,Sc,0,t)\}]. \tag{15}$$

Case II : $Pr \neq 1, Sc = 1$

$$\begin{aligned} u(t,y) = \textstyle\sum_{n=0}^{\infty}[& \tfrac{e^{Rt}}{2L}\{F_1(a,1,c_1,t) - F_1(b,1,c_1,t)\} \\ & - \left(\tfrac{1}{2L} + \tfrac{N}{2M}\right)\{F_1(a,1,M,t) - F_1(b,1,M,t)\} - \tfrac{e^{Rt}}{2L}\{F_1(a,Pr,c_3,t) \\ & - F_1(b,Pr,c_3,t)\} + \tfrac{1}{2L}\{F_1(a,Pr,Z,t) - F_1(b,Pr,Z,t)\} \\ & + \tfrac{N}{2M}\{F_1(a,1,0,t) - F_1(b,1,0,t)\}], \end{aligned} \tag{16}$$

$$\theta(t,y) = \textstyle\sum_{n=0}^{\infty}[\tfrac{1}{2}\{F_1(a,Pr,Z,t) - F_1(b,Pr,Z,t)\}], \tag{17}$$

$$C(t,y) = \textstyle\sum_{n=0}^{\infty}[\tfrac{1}{2}\{F_1(a,1,0,t) - F_1(b,1,0,t)\}]. \tag{18}$$

Case III : $Pr = 1, Sc \neq 1$

$$\begin{aligned} u(t,y) = \textstyle\sum_{n=0}^{\infty}[& \tfrac{Ne^{Qt}}{2M}\{F_1(a,1,c_2,t) - F_1(b,1,c_2,t)\} \\ & - \left(\tfrac{1}{2L} + \tfrac{N}{2M}\right)\{F_1(a,1,M,t) - F_1(b,1,M,t)\} + \tfrac{1}{2L}\{F_1(a,1,F,t) \\ & - F_1(b,1,F,t)\} - \tfrac{Ne^{Qt}}{2M}\{F_1(a,Sc,Q,t) - F_1(b,Sc,Q,t)\} \end{aligned}$$

$$+\frac{N}{2M}\left\{F_1(a,Sc,0,t)-F_1(b,Sc,0,t)\right\}], \tag{19}$$

$$\theta(t,y)=\sum_{n=0}^{\infty}[\tfrac{1}{2}\left\{F_1(a,1,F,t)-F_1(b,1,F,t)\right\}], \tag{20}$$

$$C(t,y)=\sum_{n=0}^{\infty}[\tfrac{1}{2}\left\{F_1(a,Sc,0,t)-F_1(b,Sc,0,t)\right\}]. \tag{21}$$

Case IV : $Pr=1, Sc=1$

$$u(t,y)=\sum_{n=0}^{\infty}[\left(\frac{1}{2L}+\frac{N}{2M}\right)\left\{F_1(b,1,M,t)-F_1(a,1,M,t)\right\} + \frac{1}{2L}\left\{F_1(a,1,F,t)-F_1(b,1,F,t)\right\}+\frac{N}{2M}\left\{F_1(a,1,0,t)-F_1(b,1,0,t)\right\}], \tag{22}$$

$$\theta(t,y)=\sum_{n=0}^{\infty}[\tfrac{1}{2}\left\{F_1(a,1,F,t)-F_1(b,1,F,t)\right\}], \tag{23}$$

$$C(t,y)=\sum_{n=0}^{\infty}[\tfrac{1}{2}\left\{F_1(a,1,0,t)-F_1(b,1,0,t)\right\}]. \tag{24}$$

3 Skin-Friction

The skin-friction has been studied for $Sc \neq 1$ and $Pr \neq 1$. Therefore using the expressions (13) the skin-friction τ_0 and τ_1 in non-dimensional form are given by:

$$\tau_0=\frac{\tau_0'\nu}{dg\beta(T_w'-T_d')}=\left(\frac{du}{dy}\right)_{y=0}$$
$$=\sum_{n=0}^{\infty}[-\frac{e^{Rt}}{2L}\left\{F_2(d_1,1,c_1,t)+F_2(d_2,1,c_1,t)\right\}$$
$$-\frac{Ne^{Qt}}{2M}\left\{F_2(d_1,1,c_2,t)+F_2(d_2,1,c_2,t)\right\}+\left(\frac{1}{2L}+\frac{N}{2M}\right)\left\{F_2(d_1,1,M,t)\right.$$
$$\left.+F_2(d_2,1,M,t)\right\}+\frac{e^{Rt}}{2L}\left\{F_2(d_1,Pr,c_3,t)+F_2(d_2,Pr,c_3,t)\right\}$$
$$-\frac{1}{2L}\left\{F_2(d_1,Pr,Z,t)+F_2(d_2,Pr,Z,t)\right\}+\frac{Ne^{Qt}}{2M}\left\{F_2(d_1,Sc,Q,t)\right.$$
$$\left.+F_2(d_2,Sc,Q,t)\right\}-\frac{N}{2M}\left\{F_2(d_1,Sc,0,t)+F_2(d_2,Sc,0,t)\right\}],$$

$$\tau_1=-\left(\frac{du}{dy}\right)_{y=1}$$
$$=\sum_{n=0}^{\infty}[\frac{e^{Rt}}{L}F_2(d_3,1,c_1,t)+\frac{Ne^{Qt}}{M}F_2(d_3,1,c_2,t)-\left(\frac{1}{L}+\frac{N}{M}\right)F_2(d_3,1,M,t)$$
$$-\frac{e^{Rt}}{L}F_2(d_3,Pr,c_3,t)+\frac{1}{L}F_2(d_3,Pr,Z,t)-\frac{Ne^{Qt}}{M}F_2(d_3,Sc,Q,t)+\frac{N}{M}F_2(d_3,Sc,0,t)].$$

Where $a=2n+y, b=2+2n-y, d_1=2n, d_2=2+2n, d_3=1+2n, L=M-F, R=\frac{L}{Pr-1}, Q=\frac{M}{Sc-1}, Z=\frac{F}{Pr}, c_1=M+R, c_2=M+Q$, and $c_3=Z+R$. Other symbols/expressions are defined in appendix.

4 Result and Discussions

The numerical values of the velocity, concentration, temperature and skin-friction are computed for different parameters like Prandtl number Pr, Schmidt number Sc, magnetic parameter M, buoyancy ratio parameter N, time t and Radiation parameter F. When $N = 0$, there is no mass transfer and the buoyancy force is due to the thermal diffusion only. $N > 0$ means that mass buoyancy force acts in the same direction of thermal buoyancy force, while $N < 0$ means that mass buoyancy force acts in the opposite direction. The values of the main parameters considered are: the magnetic parameter $M = 1.0, 2.0, 3.0$; time $t = 0.2, 0.4, 0.6$; buoyancy ratio parameter $N = 0.2, 0.4, -0.2, -0.4$; Prandtl number $Pr = 0.71$(for air), 7 (for water) and 3 (for the saturated liquid Freon at $273.3K$); Schmidt number $Sc = 0.22$ (for Hydrogen), 0.78 (for Ammonia) and 2.01 (for Ethyl Benzene) and radiation parameter $F = 2.0, 3.0, 4.0, 7.0, 10$. Graphs have been plotted for the velocity, concentration and temperature profiles to show the effects of different parameters.

Figure 1 and 2 show the effect of time t and Schmidt number Sc on the concentration of fluid respectively. It is observed that the concentration increases with decrease of Schmidt number, but it increases with increase of time t.

Figure 3, 4 and 5 show the effect of time t, Prandtl numberPr and Radiation parameter F on the temperature of fluid respectively. It is observed that the temperature increases when Prandtl number and Radiation Parameter are decreased, but it increases with increase of time t.

Figure 6, 7, 8, 9, 10 and 11 show the effect of Prandtl numberPr, buoyancy ratio parameter N, time t, Radiation Parameter F, magnetic parameter M and Schmidt number Sc on the velocity of fluid respectively. It is observed that velocity increases when Prandtl number, Radiation Parameter, magnetic parameter and Schmidt number are decreased, but it increases with increase of time t. It is also observed that velocity increases in case of aiding flows ($N > 0$) and decreases in case of opposing flows ($N < 0$).

The numerical values of skin-friction τ_0 and τ_1 are presented in Table -1. From Table, it is observed that the skin-friction on the both plates decrease with of increase of magnetic parameter M, Prandtl number Pr, Schmidt number Sc and Radiation Parameter F, but it increases with increase of time t. It is also observed that skin-friction increases in the presence of aiding flows ($N > 0$) and decreases in the presence of opposing flows ($N < 0$).

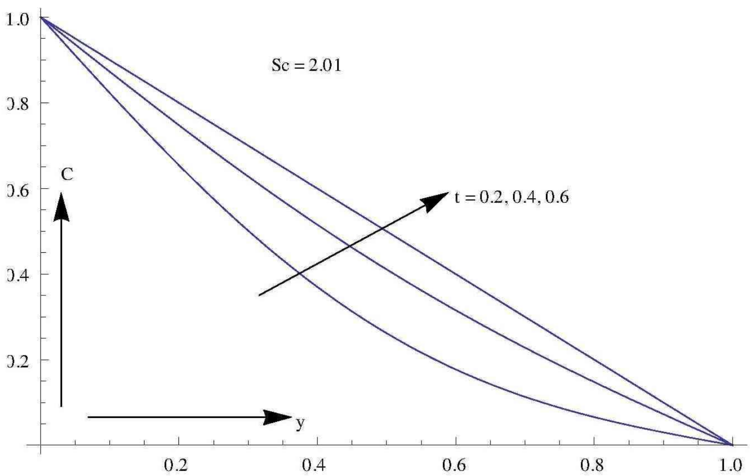

Figure 1: Concentration profiles

Table 1: Skin-friction for $Pr \neq 1$ and $Sc \neq 1$

M	N	Sc	Pr	F	t	τ_0	τ_1
1.0	0.2	0.22	7.0	2.0	0.2	0.191118	0.0335431
1.0	0.2	0.22	3.0	2.0	0.2	0.232295	0.0497192
1.0	0.2	0.22	0.71	2.0	0.2	0.296192	0.1047280
1.0	0.4	0.22	0.71	2.0	0.2	0.353273	0.1290260
1.0	−0.2	0.22	0.71	2.0	0.2	0.182029	0.0561312
1.0	−0.4	0.22	0.71	2.0	0.2	0.124948	0.0318331
1.0	0.2	0.22	7.0	2.0	0.4	0.243912	0.0626498
1.0	0.2	0.22	7.0	2.0	0.6	0.274312	0.0858964
1.0	0.2	0.22	0.71	10.0	0.2	0.247005	0.0722934
1.0	0.2	0.22	0.71	7.0	0.2	0.261847	0.0814779
1.0	0.2	0.22	0.71	4.0	0.2	0.280637	0.0938945
1.0	0.2	0.22	0.71	3.0	0.2	0.288058	0.0990061
3.0	0.2	0.22	7.0	4.0	0.2	0.180857	0.0288656
2.0	0.2	0.22	7.0	4.0	0.2	0.184936	0.0310091
1.0	0.2	0.22	7.0	4.0	0.2	0.189322	0.0333807
1.0	0.2	2.01	7.0	2.0	0.2	0.174576	0.0177436
1.0	0.2	0.78	7.0	2.0	0.2	0.184792	0.0272370

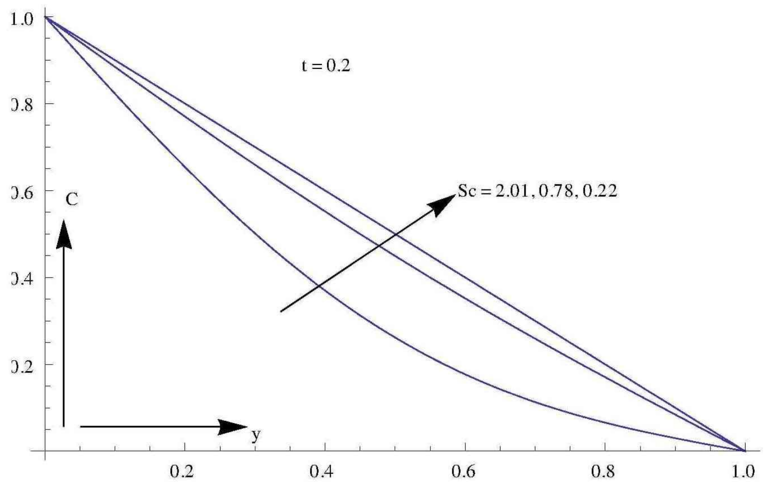

Figure 2: Concentration profiles

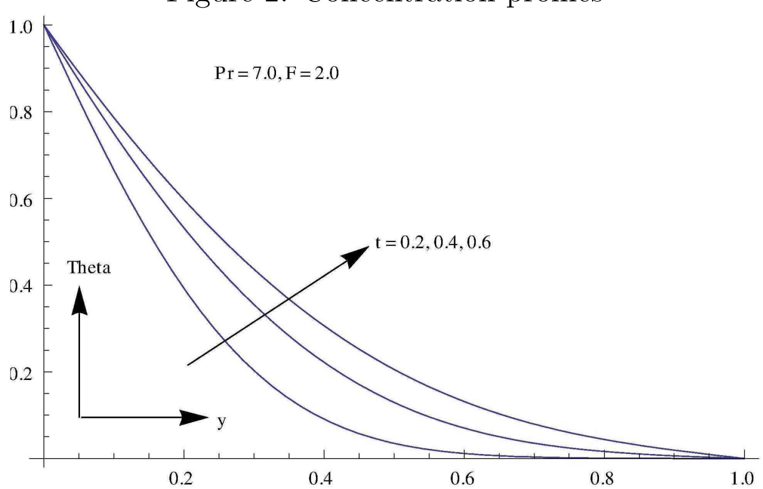

Figure 3: Temperature profiles

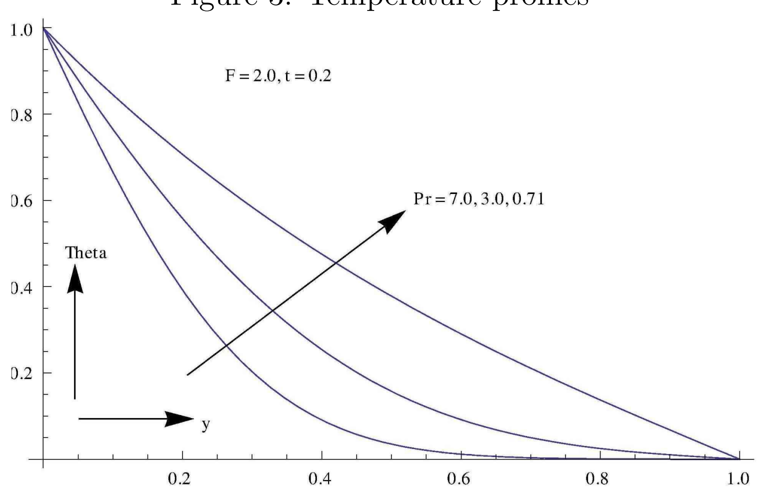

Figure 4: Temperature profiles

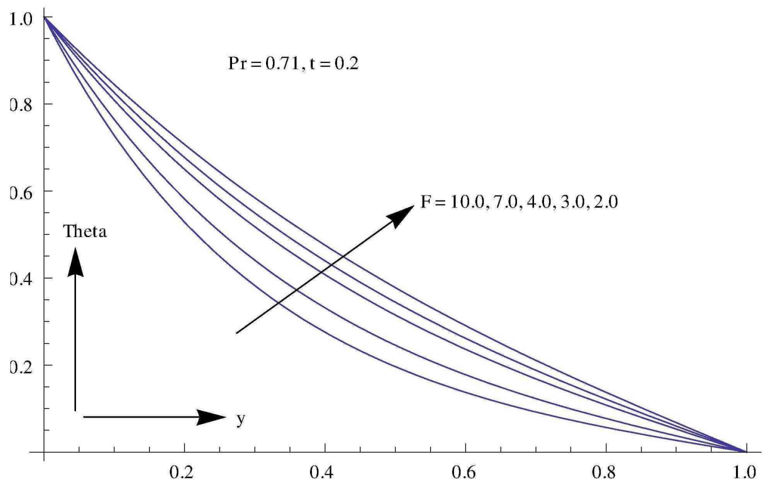

Figure 5: Temperature profiles

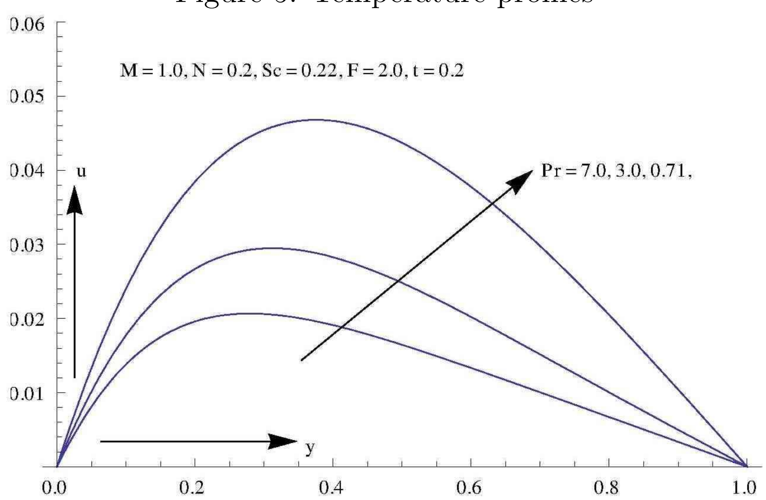

Figure 6: Velocity profiles

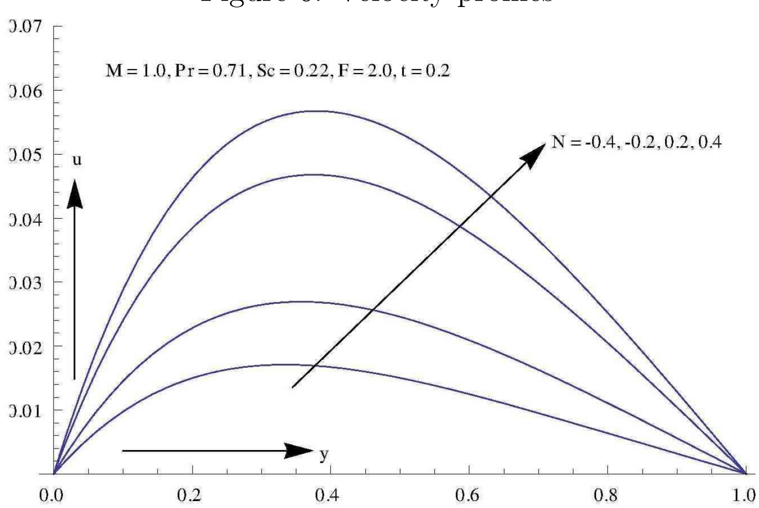

Figure 7: Velocity profiles

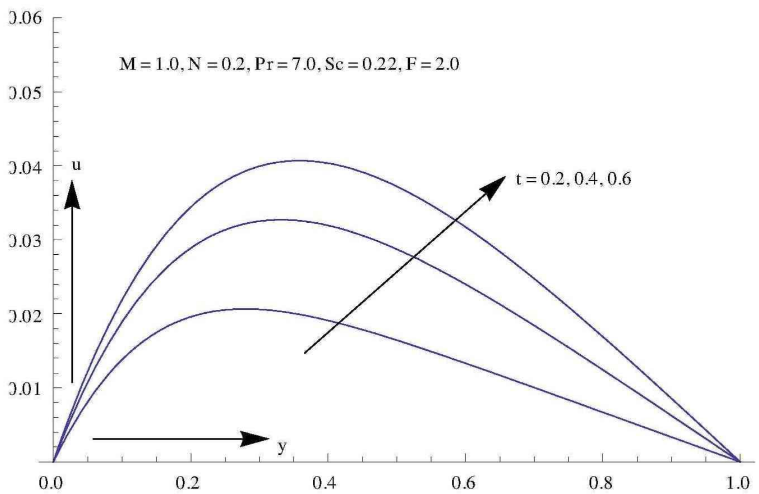

Figure 8: Velocity profiles

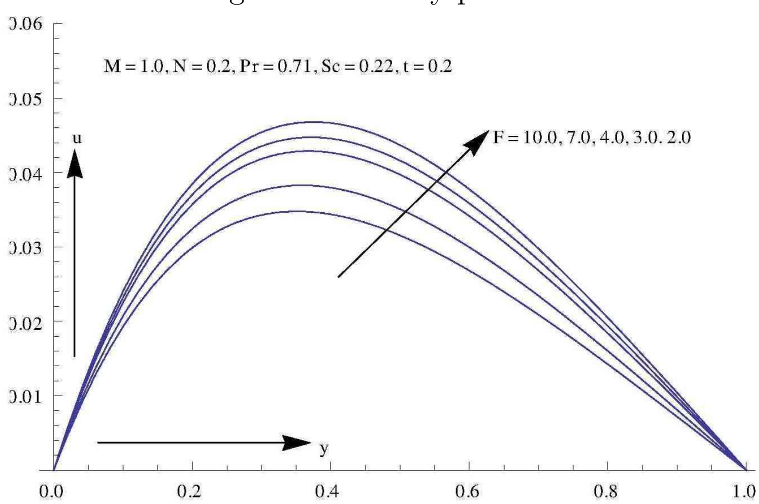

Figure 9: Velocity profiles

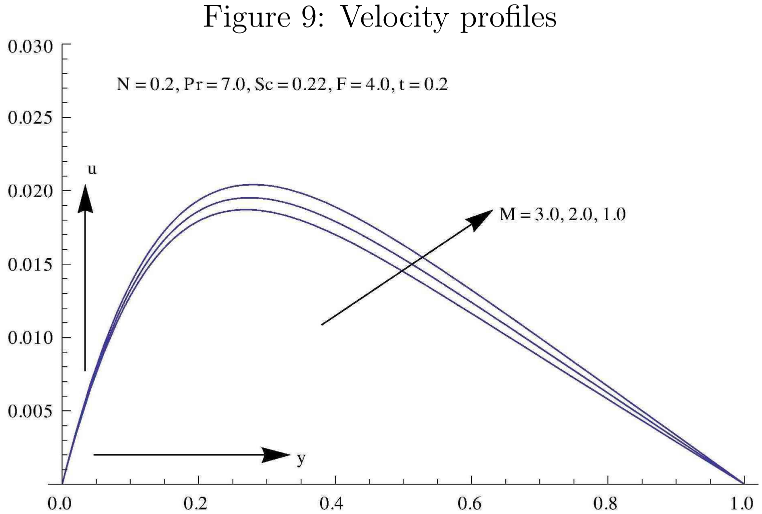

Figure 10: Velocity profiles

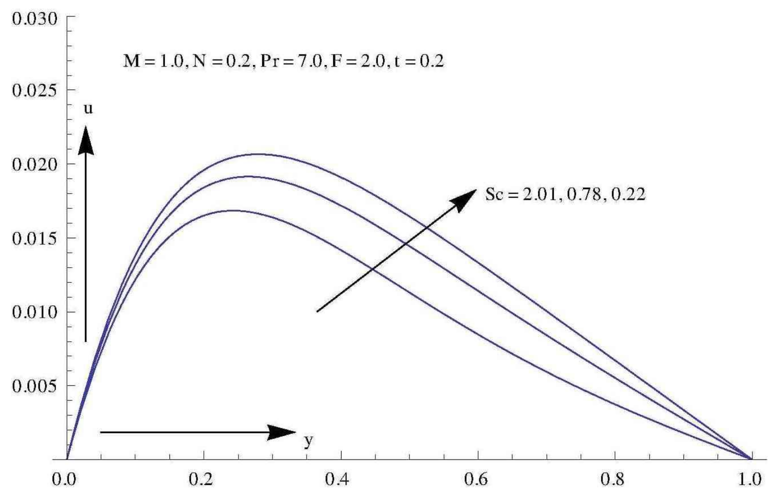

Figure 11: Velocity profiles

5 Conclusion

In the present paper, a theoretical analysis has been done to study the combined effects of MHD and radiation on unsteady transient free convection flow of a viscous, incompressible, electrically conducting fluid between two long vertical parallel plates with constant temperature and mass diffusion. The solutions for the model have been determined by using Laplace transform method. The conclusions of the study are as follows:

- The cocentration and Temperature of the fluid increase with increase of time t.
- The cocentration of the fluid increases with decrease of Schmidt number Sc.
- The Temperature of the fluid increases with decrease of Prandtl number Pr and radiation parameter F.
- The velocity and skin-friction of the fluid increase in case of aiding flows $(N > 0)$ and decrease with opposing flows $(N < 0)$.
- The velocity and skin-friction of the fluid increase with increasing the value of time t.
- The velocity and skin-friction of the fluid increase with decrease of Prandtl numberPr, Schmidt number Sc, magnetic parameter M and radiation parameter F.

Appendix:

$F_1(D_1, D_2, D_3, D_4) = e^{-a_3} erfc(a_1) + e^{a_3} erfc(a_2),$

$$F_2(D_1, D_2, D_3, D_4) = \frac{1}{\sqrt{\pi D_4}} e^{-a_3 - a_1^2} \sqrt{D_2} + \frac{1}{\sqrt{\pi D_4}} e^{a_3 - a_2^2} \sqrt{D_2} + e^{-a_3} \sqrt{D_2 D_3} erfc(a_1) - e^{a_3} \sqrt{D_2 D_3} erfc(a_2).$$

Here $a_1 = \left(\frac{D_1 \sqrt{D_2}}{2\sqrt{D_4}} - \sqrt{D_3 D_4} \right)$, $a_2 = \left(\frac{D_1 \sqrt{D_2}}{2\sqrt{D_4}} + \sqrt{D_3 D_4} \right)$ and $a_3 = D_1 \sqrt{D_2 D_3}$

References

[1] A.K. Singh, Natural convection in Unsteady Couette Motion, *Def Sci J*, (38)(1988), 35-41.

[2] A.K. Singh, H.R. Gholami and V.M. Soundalgekar, Transient free convection flow between two vertical parallel plates, Warme-und Stoffubertragung, *Heat and Mass Transfer*, (31)(1996), 329-331.

[3] A. Campo, O. Manka and B. Morron, Numerical inveatigation of the natural convection flows for low-Prandtl fluids in vertical parallel-plates channels, *ASME J. Heat Tranfer*, (73)(2006), 96-107.

[4] A. Pantokratoras, Fully developed laminar free covection with variable thermophysical properties between two open-ended vertical parallel plates heated asymmetrically with large temperature differences, *ASME J. Heat Tranfer*, (128)(2006), 405-408.

[5] A.K. Singh and T. Paul, Transient natural convection between tow vertical walls heated/cooled asymmetrically, *Int. J. Applied Mechanics and Engineering*, 11(1) (2006), 143-154.

[6] B.K. Jha, Natural convection in Unsteady MHD Couette flow, *Heat and Mass Transfer*, (37)(2001), 329-331.

[7] B.K. Jha, A.K. Singh and H.S. Takhar, Transient free convection flow in a vertical channel due to symmetric heating, *Int. J. Applied Mechanics and Engineering*, 8(3) (2003), 497-502.

[8] D.J. Nelson and B.D. Wood, Combined heat and mass transfer natural convection between vertical parallel plates with uniform heat flux boundary conditions, *Int. J. Heat and Mass Transfer*, (4)(1986), 1587-1592.

[9] D.J. Nelson and B.D. Wood, Combined heat and mass transfer natural convection between vertical parallel plates, *Int. J. Heat and Mass Transfer*, (32)(1989a), 1779-1787.

[10] D.J. Nelson and B.D. Wood, Fully developed Combined heat and mass transfer natural convection between vertical parallel plates with asymmetric boundary conditions, *Int. J. Heat and Mass Transfer*, (32)(1989b), 1789-1792.

[11] E.M. Sparrow, T. Eichhorn and J.L. Gregg, Combined forced and free convection in a boundary layer flow, *Physics of Fluids*, (2)(1959, 319-328.

[12] F.J. Higuera and Yu.S. Ryazantsev, Natural convection flow due to a heat source in a vertical channel, *Int. J. Heat and Mass Transfer*, (45)(2002), 2207-2212.

[13] G. Desrayaud and G. Lauriat, Heat and mass transfer analogy for condensation of humid air in a vertical channel, *Heat and Mass Transfer*, (37)(2001), 67-76.

[14] J.R. Bodoia and J.F. Osterle, The development of the free convection between heated vertical plates, *ASME J. Heat Tranfer*, (84)(1962), 40-44.

[15] Kettleborough, Transfer laminar free convection between heated vertical plates including entrance effects, *Int. J. Heat and Mass Transfer*, (15)(1972), 883-896.

[16] K.T. Lee and W.M. Yan, Laminar natural convection between patially heated vertical parallel plates, Warme-und Stoffubertragung, *Heat and Mass Tranfer*, (29)(1994), 145-151.

[17] K.T. Lee, Natural convection heat and mass transfer in partially heated vertical parallel plates, *Int. J. Heat and Mass Transfer*, (42)(1999), 4417-4425.

[18] M. Narahari, S. Sreenadh and V.M. Soundalgekar, Transient free convection flow between long vertical parallel plates with constant heat flux at one boundary, *Heat and Mass Transfer*, (31)(2002), 287-293.

[19] Narahari Marneni, Transient free convection flow between tow long vertical parallel plates with constant temperature and mass diffusion, *Proceedings of the World Congress on Engineering (WCE)*, London, U.K., (11)(2008).

[20] O. Miyatake and T. Fujii, Free convection heat transfer between vertical parallel plates-One plate isothermally heated and the other thermally insulated, *Heat Transfer-Jpn. Res.*, 1(1)(1972), 30-38.

[21] O. Miyatake and T. Fujii, Natural convection heat transfer between vertical parallel plates at unequal uniform temperatures, *Heat Transfer-Jpn. Res.*, 2(4)(1973), 79-88.

[22] O. Miyatake and T. Fujii, Natural convection heat transfer between vertical parallel plates with unequal unequal heat fluxes, *Heat Transfer-Jpn. Res.*, 3(3) (1974), 29-33.

[23] O. Miyatake, H. Tanaka, T. Fujii and M. Fujii, Natural convection heat transfer between vertical parallel plates-One plate with a uniform heat flux and the other thermally insulated, *Heat Transfer-Jpn. Res.*, 2(1) (1973), 25-33.

[24] S. Ostrach, Laminar natural-convection flow and heat transfer of fluids with and without heat sourses in channels with constant wall temperature, *NASA, Report NO. NACA-TN-2863*, (1952).

[25] S. Ostrach,Combined natural and forced convection laminal flow and heat transfer of fluids with and without heat sources in channels with linearly varying wall temperature, *NASA, Report No. NACA-TN-3141*, (1954).

[26] W. Aung, Fully developed laminar free convection between vertical plates heated asymmetrically, *Int. J. Heat Mass Tranfer*, (15)(1972), 1577-1580.

[27] W. Aung, L.S. Fletcher and V. Sernas, Developed laminar free convection between vertical flat plates asymmetric heating, *Int. J. Heat Mass Tranfer*, (15)(1972), 2293-2308.