Gen. Math. Notes, Vol. 8, No. 1, January 2012, pp. 29-34
ISSN 2219-7184;
www.i-csrs.org
Available free online at http://www.geman.in

Construction of a New Airport in a Developing Country, Using Entropy Optimization Method to the Model

Kakali Karmakar (Sur)[1] and Sanat Kumar Majumder[2]

[1]Department of Mathematics,
B.P.Poddar Institute of Management and Technology
137, VIP Road, Kolkata: 700052, India
E-mail: chumkiks@gmail.com
[2]Department of Mathematics,
Bengal Engineering and Science University
Shibpur, Howrah: 711103, West Bengal, India
E-mail: majumder sk@yahoo.co.in

(Received: 9-10-11/Accepted: 11-12-11)

Abstract

In recent years owing to the unpredictable growth in air travel, considerable attention has been developed to the planning and construction of airports. In this paper firstly we discuss about mathematical programming models regarding airport construction, and then optimize parameters using entropy optimization method.

Keywords: *Entropy, Percapita construction cost, Optimization.*

1 Introduction

In the formulation of certain location model (airport site selection) the problem of unavailability of road transports between the two cities has to be considered. There are various costs and benefits associated to the airport site selection problem. However, each nation generally has its own concept of the government, of what constitutes public benefits, and of the role of private sector agencies and evaluation procedures.De Neufville(1976) presented a country's concept of the public interest, who participates in the decisions, what kind of evaluation will occur and where power will lie in relation to airport site selection problem. This problem is all the more important in developing countries where resources are scarce, and data necessary to make rational decisions are either insufficient or inaccurate.[3] Air transportation system is represented by a network with airports and routes being the nodes and links, respectively. Planned airport constructions in appropriate cities or regions are treated as additions of nodes and links to the transportation network. Airport site selection problem is one of the macro-level planning problems of the transportation system and has to be treated within the entire system domain. However,because of the complexity of the transportation system and the lack of certain socio-economic indications and parameters. This particular problem is treated in isolation. In this paper, mathematical entropy model is developed for the airport site selection problem in developing countries. Then optimization has been made using entropy optimization method. To support our mathematical discussion numerical example has been cited.

2 Model

[1] Let $p_{1,}\ p_{2,}\ p_{3,}$…………,p_n be the proportions of population of n cities country under developing (the bus services between the n cities are not frequent) and let the corresponding generalized cost of traveling from i th city to j th city be $c_{1,}\ c_2$ c_3,……….c_n by air services.

If c is the expected travel budget by air service, then we have to maximize the entropy

$$-\sum_{i=1}^{n} p_i \ln p_i \qquad \text{……………[A]}$$

[i.e. to maximize the number of people traveling from i city to j city]

subject to $\sum_{i=1}^{n} p_i = 1; \sum_{i=1}^{n} p_i c_i = c$

In the above model, we have assumed that the budget of travel is given to us, but this is seldom the cost. However (people of a city) community tries to minimize its travel budget and at the same time it wants to be maximally unbiased about the

information not available to it. The community has two goals maximization of entropy $-\sum_{i=1}^{n} p_i \ln p_i$ and minimizing of cost $\sum_{i=1}^{n} p_i c_i$.
We combine it in a single goal of maximizing

$$-\sum_{i=1}^{n} p_i \ln p_i \; -\mu \sum_{i=1}^{n} p_i c_i \text{ …………..[B]}$$

μ depends on the attitude of the community to cost of travel. We can formulate the above model as [1] & [4]
Minimize

$$\mu \sum_{i=1}^{n} p_i c_i + \sum_{i=1}^{n} p_i \ln p_i \quad \text{…………………..[C]}$$

subject to

Minimization of the above model minimize the travel cost between the cities and maximize the number of people having the facility of traveling from i th city to j th city by air services. The above entropy model [c] minimizes the percapita construction cost of an airport at the i th city and [c] can be expressed as follows:

Minimize $\sum_{j=1}^{J} P_j Y_j$ …………………….. [D]

subject to the condition

$$\sum d_{ij} Y_j \geq 1 \qquad \text{for} \quad i=1, \quad 2, \quad 3,\text{……..}I \quad ;$$

Y_j =1 if an airport is constructed in j th city

Y_j = 0, otherwise

P_j is the per-capita construction cost of the airport in city j.
d_{ij} =1 if air transportation is feasible between cities i and j.
d_{ij} = 0, otherwise. [3]

The objective function gives the total per-capita airport construction cost in the cities of the region. The underlying assumption is that the airport construction cost per person can be used as a criterion to evaluate the alternatives. One way of estimating the cost parameters, P_j, of the objective function is to divide the airport construction costs by the total population to be served by an airport.
The constraint of the model generation alternative airport location patterns based on a specific criterion. This criterion could be distance, travel time or travel cost between cities, average bus fare between two cities. It was assumed that a passenger would prefer air travail over bus services beyond a certain distance and corresponding bus fare. So bus fare between two cities has been chosen as the criterion (or parameter) to identify the potential cities. We construct this

constrained entropy model to optimize (i.e. minimize) per capita construction cost, using entropy optimization method.

3 Mathematical Calculation

The Lagrangian of model has been made as follows:

$L_1 = \sum_{j=1}^{J} P_j Y_j - \lambda(\sum_j d_{ij} Y_j - 1)$ λ is lagrangian multiplier.

Hence $\frac{\partial L_1}{\partial Y_j} = 0$ gives

$$\sum_{j=1}^{J} p_i - \lambda \sum_j d_{ij} = 0$$

$$\lambda = \frac{1}{\sum_j d_{ij}} \sum_{j=1}^{J} p_i$$

4 Numerical Example

The model are tested for the eastern India where road transportation is difficult..Socio-economic interactions between the cities of the region were taken into consideration.
We consider 10 cities A,B,C,D,E,F,G,H,I,J of eastern India.[4] &[5]

Table-I: Population and Cost Estimated

Cities	Population in million	Airport construction cost[million US $]	Percapita construction cost[million US $] Pi
A	24.5	52.20	2.13
B	18.5	40.0	2.16
C	19.7	70.20	3.56
D	26.66	25.17	0.94
E	82.99	41.60	0.50
F	20.83	11.00	0.53
G	2.1	25.69	12.23
H	2.31	23.64	10.23
I	0.88	10.00	11.36

J	0.54	11.70	21.67

Table-II: Estimation of the Lagrangian multiplier

Cities	Percapita construction cost[million US $] Pi	$\sum_i d_{ij}$	λ
A	2.13	1	2.13
B	2.16	1	2.16
C	3.56	1	3.56
D	0.94	1	0.94
E	**0.50**	**1**	**0.50**
F	0.53	1	0.53
G	12.23	1	12.23
H	10.23	1	10.23
I	11.36	1	11.36
J	21.67	1	21.67

5 Conclusion

From the above tables it is clear that E city is the appropriate site for airport construction. In this paper, it is shown that airport site selection problem in developing countries can be approached through mathematical entropy model. In the above model, airport construction cost per person was used to determine the optimal airport sites. Using entropy optimization method we optimize the above entropy model and select the optimum site for airport construction.

Acknowledgement

The authors thankful to the Department of Mathematics,Bengal Engineering and Science University,Shibpur, Howrah-711103, West Bengal, India.

References

[1] J.N. Kapur, *Maximum Entropy Models in Science and Engineering*, Wiley Eastern Limited, (1989).

[2] J.N. Kapur and H.K. Kesavan, *Entropy Optimization Principles with Applications,* Academic Press, 23-54

[3] S. Ramanathan, *Airport Management*, (1982), 40-42.

[3] S. Ramanathan, *Airport Management*, (1982), 40-42.

[4] O. Sattcioglu, Mathematical programming models for airport site selection, *Transpn Res*-B, 16B(6) (1982), 435-447.

[5] http://www.censusindia.gov.in.