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Intuitionistic (T, S)-Fuzzy Filters on Residuated Lattices

Ya Qin[1] and Yi Liu[2]

[1, 2] College of Mathematics and Information Sciences
Neijiang Normal University, Neijiang
641000 Sichuan, P.R. China
[1]E-mail: qinyaqy@126.com.
[2]E-mail: liuyiyl@126.com

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Abstract

The aim of this paper is further to develop the filter theory on residuated lattices. The concept of interval valued intuitionistic (T, S)-fuzzy filters on residuated lattices is introduced by linking the intuitionistic fuzzy set, t-norm, s-norm and filter theory of residuated lattices; The properties and equivalent characterizations of Interval valued intuitionistic (T, S)-fuzzy filters are investigated.

Keywords: *Residuated lattices, Intuitionistic fuzzy set, t-norm, s-norm(t-conorm), Fuzzy filters*

1 Introduction

Intelligent information processing is one important research direction in artificial intelligence. Information processing dealing with certain information is based on the classical logic. However, non-classical logics including logics behind fuzzy reasoning handle information with various facets of uncertainty such as fuzziness, randomness, etc. Therefore, non-classical logic have become as a formal and useful tool for computer science to deal with uncertain information. Many-valued logic[1], a great extension and development of classical logic, has always been a crucial direction in non-classical logic. In the field of many-valued logic, lattice-valued logic plays an important role for the following two aspects: One is that it extends the chain-type truth-valued field of some well known present logic to some relatively general lattice. The other is that the incompletely comparable property of truth value characterized by general lattice can more efficiently reflect the uncertainty of human being's thinking, judging and decision. Hence, lattice-valued logic is becoming an active research field which strongly influences the development of algebraic logic, computer science and artificial intelligent technology. Various logical algebras have been proposed as the semantical systems of non-classical logic systems, such as residuated lattices[2], lattice implication algebras[3, 4, 5, 6], BL-algebras, MV-algebras, MTL-algebras, etc. Among these logical algebras, residuated lattices are very basic and important algebraic structure because the other logical algebras are all particular cases of residuated lattices.

The concept of fuzzy set was introduced by Zadeh (1965)[7]. Since then this idea has been applied to other algebraic structures such as groups, semigroups, rings, modules, vector spaces and topologies. With the development of fuzzy set, it is widely used in many fields. The concept of intuitionistic fuzzy sets was first introduced by Atanassov[19] in 1986 which is a generalization of the fuzzy sets. Many authors applied the concept of intuitionistic fuzzy sets to other algebraic structure such as groups, fuzzy ideals of BCK-algebras, filter theory of lattice implication and BL-algebras,etc[8, 9, 10, 11, 12, 14, 15, 16, 17, 18].

As for lattice implication algebras, BL-algebras, R_0-algebras, MTL-algebras, MV-algebras, etc, they all are particular types of residuated lattices. Therefore, it is meaningful to establish the fuzzy filter theory of general residuated lattice for studying the common properties of the above-mentioned logical algebras. This paper, as a continuation of above work, we will apply the interval-valued intuitionistic fuzzy subset and t-norm T, s-norm S on $D[0,1]$ to filter theory of residuated lattices, proposed the concept interval-valued intuitionistic (T,S)-fuzzy filters of residuated lattices and some equivalent results are obtained.

2 Preliminaries

Definition 2.1 *[2] A residuated lattice is an algebraic structure* $\mathcal{L}=(L,\vee,\wedge,\otimes,\rightarrow,0,1)$ *of type (2,2,2,2,0,0) satisfying the following axioms:*

(C1) $(L,\vee,\wedge,0,1)$ *is a bounded lattice.*

(C2) $(L,\otimes,1)$ *is a commutative semigroup (with the unit element 1).*

(C3) $(\otimes,\rightarrow)$ *is an adjoint pair, i.e., for any* $x,y,z,w\in L$,

(R1) *if* $x\le y$ *and* $z\le w$, *then* $x\otimes z\le y\otimes w$.

(R2) *if* $x\le y$, *then* $y\rightarrow z\le x\rightarrow z$ *and* $z\rightarrow x\le z\rightarrow y$.

(R3) *(adjointness condition)* $x\otimes y\le z$ *if and only if* $x\le y\rightarrow z$.

In what follows, let $\mathcal{L}$ denote a residuated lattice unless otherwise specified.

Theorem 2.2 *[2, 20] In each residuated lattice* $\mathcal{L}$*, the following properties hold for all* $x, y, z \in L$:

(P1) $(x \otimes y) \to z = x \to (y \to z)$. (P2) $z \leq x \to y \Leftrightarrow z \otimes x \leq y$.
(P3) $x \leq y \Leftrightarrow z \otimes x \leq z \otimes y$. (P4) $x \to (y \to z) = y \to (x \to z)$.
(P5) $x \leq y \Rightarrow z \to x \leq z \to y$. (P6) $x \leq y \Rightarrow y \to z \leq x \to z, y' \leq x'$.
(P7) $y \to z \leq (x \to y) \to (x \to z)$. (P8) $y \to x \leq (x \to z) \to (y \to z)$.
(P9) $1 \to x = x, x \to x = 1$. (P10) $x^m \leq x^n$, $m, n \in N$, $m \geq n$.
(P11) $x \leq y \Leftrightarrow x \to y = 1$. (P12) $0' = 1, 1' = 0, x' = x''', x \leq x''$.
(P13) $x \vee y \to z = (x \to z) \wedge (y \to z)$. (P14) $x \otimes x' = 0$.
(P15) $x \to (y \wedge z) = (x \to y) \wedge (x \to z)$.

Definition 2.3 *[13] A non-empty subset* F *of a residuated lattice* $\mathcal{L}$ *is called a* **filter** *of* $\mathcal{L}$ *if it satisfies*

(F1) $x, y \in F \Rightarrow x \otimes y \in F$.
(F2) $x \in F, x \leq y \Rightarrow y \in F$.

Theorem 2.4 *[13] A non-empty subset* F *of a residuated lattice* $\mathcal{L}$ *is called a* **filter** *of* $\mathcal{L}$ *if it satisfies, for any* $x, y \in L$,

(F3) $1 \in F$;
(F4) $x \in F, x \to y \in F \Rightarrow y \in F$.

A fuzzy set A on a residuated lattice $\mathcal{L}$ is a mapping from $\mathcal{L}$ to $[0, 1]$.[11]

Definition 2.5 *[13]* (1) *A fuzzy set* A *of a residuated lattice* $\mathcal{L}$ *is called a* **fuzzy filter***, if it satisfies, for any* $x, y \in L$

(FF1) $A(1) \geq A(x)$;
(FF2) $A(x \otimes y) \geq min\{A(x), A(y)\}$.

Theorem 2.6 *[13] A fuzzy set* A *of a residuated lattice* $\mathcal{L}$ *is a* **fuzzy filter***, if and only if it satisfies, for any* $x, y \in L$,

(FF3) $A(1) \geq A(x)$.
(FF4) $A(y) \geq min\{A(x \to y), A(x)\}$.

Definition 2.7 *[18] Let* δ *be a mapping from* $[0, 1] \times [0, 1]$ *to* $[0, 1]$. δ *is called a t-norm (resp. s-norm) on* $[0, 1]$*, if it satisfies the following conditions: for any* $x, y, z \in [0, 1]$,

(1) $\delta(x, 1) = x$ *(resp.* $\delta(x, 0) = x$*),*
(2) $\delta(x, y) = \delta(y, x)$,
(3) $\delta(\delta(x, y), z) = \delta(x, \delta(y, z))$,
(4) if $x \leq y$*, then* $\delta(x, z) \leq \delta(y, z)$.

The set of all δ-idempotent elements $D_\delta = \{x \in [0,1] | \delta(x,x) = x\}$.

An intuitionistic fuzzy set on X is defined as an object of the form $A = \{(x, M_A(x), x, N_A(x)) | x \in X\}$, where M_A, N_A are fuzzy sets on X such that $[0,0] \leq M_A(x) + N_A(x) \leq [1,1]$. For the sake of simplicity, in the following, such intuitionistic fuzzy sets will be denoted by $A = (M_A, N_A)$.

3 Intuitionistic (T,S)-Fuzzy Filters

In this section, all theorems are discussed under the condition that t-norm, s-norm are all nilpotent.

Definition 3.1 *An intuitionistic fuzzy set A of $\mathcal{L}$ is called an intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$, if for any $x, y, z \in L$:*

(V1) $M_A(I) \geq M_A(x)$ and $N_A(I) \leq N_A(x)$;

(V2) $M_A(y) \geq T(M_A(x \to y), M_A(x))$ and $N_A(y) \leq S(N_A(x \to y), N_A(x))$.

Remark 3.2 *In Definition 3.1, taking $T = min, S = max$, then intuitionistic (T,S)-fuzzy filter is intuitionistic fuzzy filter. So intuitionistic (T,S)-fuzzy filter is a generalization of intuitionistic fuzzy filter.*

Theorem 3.3 *Let A be an intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$. Then, for any $x, y \in L$:*

(V3) if $x \leq y$, then $M_A(x) \leq M_A(y)$ and $N_A(y) \leq N_A(x)$.

Proof. Since $x \leq y$, it follows that $x \to y = I$. By A is an intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$, we have $M_A(y) \geq T(M_A(x \to y), M_A(x))$ and $N_A(y) \leq S(N_A(x \to y), N_A(x))$. By(V1), $M_A(I) \geq M_A(x), N_A(I) \leq N_A(x)$ for any $x \in L$, therefore, $M(y) \geq T(M_A(x \to y), M_A(x)) = T(M_A(I), M_A(x)) \geq T(M_A(x), M_A(x)) = M_A(x)$, $N_A(y) \leq S(N_A(x \to y), N_A(x)) \leq S(N_A(I), N_A(x)) \leq S(N_A(x), N_A(x)) = N_A(x)$ as T, S are idempotent interval t-norm and s-norm. And so V(3) is valid.

Theorem 3.4 *Let A be an intuitionistic set on $\mathcal{L}$. Then A be an intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$, if and only if, for any $x, y, z \in L$, (V1) holds and*

(V4) $M_A(x \to z) \geq T(M_A(y \to (x \to z)), M_A(y))$ and $N_A(x \to z) \leq S(N_A(y \to (x \to z)), N_A(y))$.

Proof. Let A be an intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$, obviously, (V1)and (V4) hold. Conversely, assume that (V4) hold, taking $x = I$ in (V4), we have $M_A(z) = M_A(I \to z) \geq T(M_A(y \to (I \to z)), M_A(y)) = T(M_A(y \to z), M_A(y))$ and $N_A(z) = N_A(I \to z) \leq S(N_A(y \to (I \to z)), N_A(y)) = S(N_A(y \to z), N_A(y))$. Since (V1)hold, and so A is an intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$.

Theorem 3.5 *Let A be an intuitionistic set on $\mathcal{L}$. Then A be an intuitionistic (T, S)-fuzzy filter of $\mathcal{L}$, if and only if, for any $x, y, z \in L$, A satisfies (V3) and*

(V5) $M_A(x \otimes y) \geq T(M_A(x), M_A(y))$ and $N_A(x \otimes y) \leq S(N_A(x), N_A(y))$.

Proof. Assume that A is an intuitionistic (T, S)-fuzzy filter of $\mathcal{L}$, obviously (V3)holds. Since $x \leq y \to (x \otimes y)$, we have $M_A(y \to (x \otimes y)) \geq M_A(x)$ and $N_A(y \to (x \otimes y)) \leq N_A(x)$. By (V2), it follows that $M_A(x \otimes y) \geq T(M_A(y), M_A(y \to (x \otimes y))) \geq T(M_A(y), M_A(x))$ and $N_A(x \otimes y) \leq S(N_A(y), N_A(y \to (x \otimes y))) \leq S(N_A(y), N_A(x))$.

Conversely, assume (V3) and (V5)holds. Taking $y = I$ in (V3), then (V1)holds. As $x \otimes (x \to y) \leq y$, thus $M_A(y) \geq M_A(x \otimes (x \to y))$ and $N_A(y) \leq N_A(x \otimes (x \to y))$. By (V5), we have $M_A(y) \geq T(M_A(x), M_A(x \to y))$ and $N_A(y) \leq S(N_A(x), N_A(x \to y))$. Therefore (V2) is valid, so A is an intuitionistic (T, S)-fuzzy filter of $\mathcal{L}$.

Corollary 3.6 *An intuitionistic set on $\mathcal{L}$ is an intuitionistic (T, S)-fuzzy filter of $\mathcal{L}$, if and only if, for any $x, y, z \in L$:*

(V6) if $x \to (y \to z) = I$, then $M_A(z) \geq T(M_A(x), M_A(y))$ and $N_A(z) \leq S(N_A(x), N_A(y))$.

Corollary 3.7 *An intuitionistic fuzzy set on $\mathcal{L}$ is an intuitionistic (T, S)-fuzzy filter of $\mathcal{L}$, if and only if, for any $x, y, z \in L$:*

(V7) If $a_n \to (a_{n-1} \to \cdots \to (a_1 \to x) \cdots) = I$, then $M_A(x) \geq T(M_A(a_n), \cdots, M_A(a_1))$ and $N_A(x) \leq S(N_A(a_n), \cdots, N_A(a_1))$.

Theorem 3.8 *An intuitionistic fuzzy set on $\mathcal{L}$ is an intuitionistic (T, S)-fuzzy filter of $\mathcal{L}$, if and only if, for any $x, y, z \in L$, A satisfies (V1) and*

(V8) $M_A((x \to (y \to z)) \to z) \geq T(M_A(x), M_A(y))$ and $N_A((x \to (y \to z)) \to z) \leq S(N_A(x), N_A(y))$.

Proof. If A is an intuitionistic (T, S)-fuzzy filter of $\mathcal{L}$, (V1) is obvious. Since $M_A((x \to (y \to z)) \to z) \geq T(M_A((x \to (y \to z)) \to (y \to z)), M_A(y))$ and $N_A((x \to (y \to z)) \to z) \leq S(N_A((x \to (y \to z)) \to (y \to z)), N_A(y))$. As $(x \to (y \to z)) \to (y \to z) = x \vee (y \to z) \geq x$, by(V3), we have $M_A((x \to (y \to z)) \to (y \to z)) \geq M_A(x)$ and $N_A((x \to (y \to z)) \to (y \to z)) \leq N_A(x)$. Therefore, $M_A((x \to (y \to z)) \to z) \geq T(M_A(x), M_A(y))$ and $N_A((x \to (y \to z)) \to z) \leq S(N_A(x), N_A(y))$.

Conversely, suppose (V8) is valid. Since $M_A(y) = M_A(I \to y) = M_A(((x \to y) \to (x \to y)) \to y) \geq T(M_A(x \to y), M_A(x))$ and $N_A(y) = N_A(I \to y) = N_A(((x \to y) \to (x \to y)) \to y) \leq S(N_A(x \to y), N_A(x))$. we have (V2). By(V1), A is an intuitionistic (T, S)-fuzzy filter of $\mathcal{L}$.

Theorem 3.9 *Let A be an intuitionistic fuzzy set on $\mathcal{L}$. Then A is an intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$, for any $x,y,z \in L$, A satisfies (V1) and*
(V9) $M_A(x \to z) \geq T(M_A(x \to y), M_A(y \to z))$ and $N_A(x \to z) \leq S(N_A(x \to y), N_A(y \to z))$.

Proof. Assume that A is an intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$. Since $(x \to y) \leq (y \to z) \to (x \to z)$, it follows from Theorem 3.3 that $M_A((y \to z) \to (x \to z)) \geq M_A(x \to y)$ and $N_A((y \to z) \to (x \to z)) \leq N_A(x \to y)$. As A is an intuitionistic (T,S)-fuzzy filter, so $M_A(x \to z) \geq T(M_A(y \to z), M_A((y \to z) \to (x \to z)))$ and $N_A(x \to z) \leq S(N_A(y \to z), N_A((y \to z) \to (x \to z)))$. We have $M_A(x \to z) \geq T(M_A(y \to z), M_A(x \to z))$ and $N_A(x \to z) \leq S(N_A(y \to z), N_A(x \to z))$.

Conversely, if $M_A(x \to z) \geq T(M_A(x \to y), M_A(y \to z))$ and $N_A(x \to z) \leq S(N_A(x \to y), N_A(y \to z))$ for any $x,y,z \in L$, then $M_A(I \to z) \geq T(M_A(I \to y), M_A(y \to z))$ and $N_A(I \to z) \leq S(N_A(I \to y), N_A(y \to z))$, that is $M_A(z) \geq T(M_A(y), M_A(y \to z))$ and $N_A(z) \leq S(N_A(y), N_A(y \to z))$. By (V1), we have A is an intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$.

Theorem 3.10 *Let A be an intuitionistic fuzzy set on $\mathcal{L}$. Then A is an intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$, if and only if, for any $\alpha, \beta \in [0,1]$ and $\alpha + \beta \leq 1$, the sets $U(M_A;\alpha)(\neq \emptyset)$ and $L(N_A;\beta)(\neq \emptyset)$ are filters of $\mathcal{L}$, where $U(M_A;\alpha) = \{x \in L | M_A(x) \geq \alpha\}$, $L(N_A;\beta) = \{x \in L | N_A(x) \leq \beta\}$.*

Proof. Assume A is an intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$, then $M_A(I) \geq M_A(x)$. By the condition $U(M_A,\alpha) \neq \emptyset$, it follows that there exists $a \in L$ such that $M_A(a) \geq \alpha$, and so $M_A(I) \geq \alpha$, hence $I \in U(M_A;\alpha)$.

Let $x, x \to y \in U(M_A;\alpha)$, then $M_A(x) \geq \alpha, M_A(x \to y) \geq \alpha$. Since A is a v-filter of $\mathcal{L}$, then $M_A(y) \geq T(M_A(x), M_A(x \to y)) \geq T(\alpha,\alpha) = \alpha$. Hence $y \in U(M_A;\alpha)$.Therefore $U(M_A;\alpha)$ is a filter of $\mathcal{L}$.

We will show that $L(N_A;\beta)$ is a filter of $\mathcal{L}$.

Since A is an intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$, then $N_A(I) \leq N_A(x)$. By the condition $L(N_A,\beta) \neq \emptyset$, it follows that there exists $a \in L$ such that $N_A(a) \leq \beta$, and so $N_A(a) \leq \beta$, we have $N_A(I) \leq N_A(a) \leq \beta$, hence $I \in L(N_A;\beta)$.

Let $x, x \to y \in L(N_A;\beta)$, then $N_A(x) \leq \beta; N_A(x \to y) \leq \beta$. Since A is an intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$, then $N_A(y) \leq S(N_A(x), N_A(x \to y)\} \leq S(\beta,\beta) = \beta$. It follows that $N_A(y) \leq \beta$, hence $y \in L(N_A;\beta)$. Therefore $L(N_A;\beta)$ is a filter of $\mathcal{L}$.

Conversely, suppose that $U(M_A;\alpha)(\neq \emptyset)$ and $L(N_A;\beta)(\neq \emptyset)$ are filters of $\mathcal{L}$, then, for any $x \in L$, $x \in U(M_A; M_A(x))$ and $x \in L(N_A; N_A(x))$. By $U(M_A, M_A(x))(\neq \emptyset)$ and $L(N_A, N_A(x))(\neq \emptyset)$ are filters of $\mathcal{L}$, it follows that $I \in U(M_A, M_A(x))$ and $I \in L(N_A, \tilde{M}N_A(x))$, and so $M_A(I) \geq M_A(x)$ and $N_A(I) \leq N_A(x)$.

For any $x, y \in L$, let $\alpha = T(M_A(x), M_A(x \to y))$ and $\beta = S(N_A(x), N_A(x \to y)\}$, then $x, x \to y \in U(M_A; \alpha)$ and $x, x \to y \in L(N_A; \beta)$. And so $y \in U(M_A; \alpha)$ and $y \in L(N_A; \beta)$. Therefore $M_A(y) \geq \alpha = T(M_A(x), M_A(x \to y))$ and $N_A(y) \leq \beta = S(N_A(x), N_A(x \to y)\}$. From Theorem 3.2, we have A is an intuitionistic (T, S)-fuzzy filter of $\mathcal{L}$.

Let A, B be two intuitionistic fuzzy sets on $\mathcal{L}$, denote C by the intersection of A and B, i.e. $C = A \cap B$, where

$$\begin{aligned} M_C(x) &= T(M_A(x), M_B(x)), \\ N_C(x) &= S(N_A(x), N_B(x)) \end{aligned}$$

for any $x \in L$.

Theorem 3.11 *Let A, B be two intuitionistic (T, S)-fuzzy filters of $\mathcal{L}$, then $A \cap B$ is also an intuitionistic (T, S)-fuzzy filter of $\mathcal{L}$.*

Proof. Let $x, y, z \in L$ such that $z \leq x \to y$, then $z \to (x \to y) = I$. Since A, B be two intuitionistic (T, S)-fuzzy filters of $\mathcal{L}$, we have $M_A(y) \geq T(M_A(z), M_A(x))$, $N_A(y) \leq S(N_A(z), N_A(x))$ and $M_B(y) \geq T(M_B(z), M_B(x)\}$, $N_B(y) \leq S(N_B(z), N_B(x)\}$. Since

$$\begin{aligned} M_{A \cap B}(y) &= T(M_A(y), M_B(y)\} \geq T(T(M_A(z), M_A(x)), T(M_B(z), M_B(x))) \\ &= T(T(M_A(z), M_B(z)), T(M_A(x), M_B(x))) \\ &= T(M_{A \cap B}(z), M_{A \cap B}(x)) \end{aligned}$$

and

$$\begin{aligned} N_{A \cap B}(y) &= S(N_A(y), N_B(y)\} \leq S(S(N_A(z), N_A(x)), S(N_B(z), N_B(x))) \\ &= S(S(N_A(z), N_B(z)), S(N_A(x), N_B(x))) \\ &= S(N_{A \cap B}(z), N_{A \cap B}(x)) \end{aligned}$$

Since A, B be two intuitionistic (T, S)-fuzzy filters of $\mathcal{L}$, we have $M_A(I) \geq M_A(x)), N_A(I) \leq N_A(x))$ and $M_B(I) \geq M_B(x)$, $N_B(I) \leq N_B(x)$. Hence $M_{A \cap B}(I) = T(M_A(I), M_B(I)) \geq T(M_A(x), M_B(x)) = M_{A \cap B}(x)$. Similarly, we have $N_{A \cap B}(I) = S(N_A(I), N_B(I)) \leq S(N_A(x), N_B(x)) = N_{A \cap B}(x)$. Then $A \cap B$ is an intuitionistic (T, S)-fuzzy filters of $\mathcal{L}$.

Let A_i be a family intuitionistic fuzzy sets on $\mathcal{L}$, where i is an index set. Denoting C by the intersection of A_i, i.e. $\cap_{i \in I} A_i$, where

$$\begin{aligned} M_C(x) &= T(M_{A_1}(x), M_{A_2}(x), \cdots), \\ N_C(x) &= S(N_{A_1}(x), N_{A_2}(x), \cdots) \end{aligned}$$

for any $x \in L$.

Corollary 3.12 *Let A_i be a family intuitionistic (T,S)-fuzzy filters of $\mathcal{L}$, where $i \in I$, I is an index set. then $\cap_{i\in I} A_i$ is also an intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$.*

Suppose A is an intuitionistic fuzzy set on $\mathcal{L}$ and $\alpha, \beta \in [0,1]$. Denoting $A_{(\alpha,\beta)}$ by the set $\{x \in L | M_A(x) \geq \alpha, N_A(x) \leq \beta\}$.

Theorem 3.13 *Let A be an intuitionistic fuzzy set on $\mathcal{L}$. Then*
(1) for any $\alpha, \beta \in [0,1]$, if $A_{(\alpha,\beta)}$ is a filter of $\mathcal{L}$. Then, for any $x, y, z \in L$,
(V10) $M_A(z) \leq T(M_A(x \to y), M_A(x))$ and $N_A(z) \geq S(N_A(x \to y), N_A(x))$ imply $M_A(z) \leq M_A(y)$ and$N_A(z) \geq N_A(y)$.
(2) If A satisfy (V1)and (V10), then, for any $\alpha, \beta \in [0,1]$, $A_{(\alpha,\beta)}$ is a filter of $\mathcal{L}$.

Proof. (1) Assume that $A_{(\alpha,\beta)}$ is a filter of $\mathcal{L}$ for any $\alpha, \beta \in [0,1]$. Since $M_A(z) \leq T(M_A(x \to y), M_A(x))$ and $N_A(z) \geq S(N_A(x \to y), N_A(x))$, it follows that $M_A(z) \leq M_A(x \to y), M_A(z) \leq M_A(x$ and $N_A(z) \geq N_A(x \to y), N_A(z) \geq N_A(x)$. Therefore, $x \to y \in A_{(M_A(z),N_A(z))}, x \in A_{(M_A(z),N_A(z))}$. As $M_A(z), N_A(z) \in [0,1]$, and $A_{(M_A(z),N_A(z))}$ is a filter of $\mathcal{L}$, so $y \in A_{(M_A(z),N_A(z))}$. Thus $M_A(z) \leq M_A(y)$ and$N_A(z) \geq N_A(y)$.

(2) Assume A satisfy (V1) and (V10). For any $x, y \in L$, $\alpha, \beta \in [0,1]$, we have $x \to y \in A_{(\alpha,\beta)}, x \in A_{(\alpha,\beta)}$, therefore $M_A(x \to y) \geq \alpha$, $N_A(x \to y) \leq \beta$ and $M_A(x) \geq \alpha$, $N_A(x) \leq \beta$, and so $T(M_A(x \to y), M_A(x)) \geq T(\alpha,\alpha) = \alpha$, $S(N_A(x \to y), N_A(x)) \leq S(\beta,\beta) = \beta$. By (V10), we have $M_A(y) \geq \alpha$ and $N_A(y) \leq \beta$, that is, $y \in A_{(\alpha,\beta)}$.

Since $M_A(I) \geq M_A(x)$ and $N_A(I) \leq N_A(x)$ for any $x \in L$, it follows that $M_A(I) \geq \alpha$ and $N_A(I) \leq \beta$, that is, $I \in A_{(\alpha,\beta)}$. Then, for any $\alpha, \beta \in [0,1]$, $A_{(\alpha,\beta)}$ is a filter of $\mathcal{L}$.

Theorem 3.14 *Let A be an intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$, then, for any $\alpha, \beta \in [0,1]$, $A_{(\alpha,\beta)}(\neq \phi)$ is a filter of $\mathcal{L}$.*

Proof. Since $A_{(\alpha,\beta)} \neq \phi$, there exist $\alpha, \beta \in [0,1]$ such that $M_A(x) \geq \alpha, N_A(x) \leq \beta$. And A is an intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$, we have $M_A(I) \geq M_A(x) \geq \alpha, N_A(I) \leq N_A(x) \leq \beta$, therefore $I \in A_{(\alpha,\beta)}$.

Let $x, y \in L$ and $x \in A_{(\alpha,\beta)}, x \to y \in A_{(\alpha,\beta)}$, therefore $M_A(x) \geq \alpha, N_A(x) \leq \beta, M_A(x \to y) \geq \alpha, M_A(x \to y) \leq \beta$. Since A is an intuitionistic (T,S)-fuzzy filter $\mathcal{L}$, thus $M_A(y) \geq T(M_A(x \to y), M_A(x)) \geq \alpha$ and $N_A(y) \leq S(N_A(x \to y), N_A(x)) \leq \beta$, it follows that $y \in A_{(\alpha,\beta)}$. Therefore, $A_{(\alpha,\beta)}$ is a filter of $\mathcal{L}$.

In the Theorem 3.14, the filter $A_{(\alpha,\beta)}$ is also called **intuitionistic-cut** filter of $\mathcal{L}$.

Theorem 3.15 *Any filter F of $\mathcal{L}$ is a intuitionistic-cut filter of some intuitionistic (T,S)-fuzzy filter of $\mathcal{L}$.*

Proof. Consider the intuitionistic fuzzy set A of $\mathcal{L}$: $A = \{(x, M_A(x), x, N_A(x))|x \in L\}$, where

If $x \in F$,

$$M_A(x) = \alpha, N_A(x) = 1 - \alpha. \tag{1}$$

If $x \notin F$,

$$M_A(x) = 0, N_A(x) = 1. \tag{2}$$

where $\alpha \in [0, 1]$. Since F is a filter of $\mathcal{L}$, we have $I \in F$. Therefore $M_A(I) = \alpha \geq M_A(x)$ and $N_A(I) = 1 - \alpha \leq N_A(x)$.

For any $x, y \in L$, if $y \in F$, then $M_A(y) = \alpha = T(\alpha, \alpha) \geq T(M_A(x \to y), M_A(x))$ and $N_A(y) = 1 - \alpha = S(1 - \alpha, 1 - \alpha) \leq S(N_A(x \to y), N_A(x))$.

If $y \notin F$, then $x \notin F$ or $x \to y \notin F$. And so $M_A(y) = 0 = T(0, 0) = T(M_A(x \to y), M_A(x))$ and $N_A(y) = 1 = S(1, 1) = S(N_A(x \to y), N_A(x))$. Therefore A is an intuitionistic (T, S)-fuzzy filter of $\mathcal{L}$.

Theorem 3.16 *Let A be an intuitionistic (T, S)-fuzzy filter of $\mathcal{L}$. Then $F = \{x \in L|M_A(x) = M_A(I), N_A(x) = N_A(I)\}$ is a filter of $\mathcal{L}$.*

Proof. Since $F = \{x \in L|M_A(x) = M_A(I), N_A(x) = N_A(I)\}$, obviously $I \in F$. Let $x \to y \in F, x \in F$, so $M_A(x \to y) = M_A(x) = M_A(I)$ and $N_A(x \to y) = N_A(x) = N_A(I)$, Therefore $M_A(y) \geq T(M_A(x \to y), M_A(x)\} = M_A(I)$. And $M_A(I) \geq M_A(y)$, then $M_A(y) = M_A(I)$. Similarly, we have $N_A(y) = N_A(I)$. Thus $y \in F$. It follows that F is a filter of $\mathcal{L}$.

4 Conclusions

Filter theory plays an very important role in studying logical systems and the related algebraic structures. In this paper, we develop the intuitionistic (T, S)-fuzzy filter theory of residuated lattices. Mainly, we give some new characterizations of intuitionistic (T, S)-fuzzy filters in residuated lattices. The theory can be used in implicative filters, Boolean filters, positive implicative filters, MV filters, regular filters of residuated lattices. We desperately hope that our work would serve as a foundation for enriching corresponding many-valued logical system.

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