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On Super Edge Magic and Bimagic Labeling for Duplication Graphs

A. Amara Jothi[1], N.G. David[2] and J. Baskar Babujee[3]

[1,2]Department of Mathematics, Madras Christian College
Chennai – 600 059, India
[1]E-mail: amarajothia@gmail.com
[2]E-mail: ngdmcc@gmail.com
[3]Department of Mathematics, Anna University
Chennai – 600 025, India
E-mail: baskarbabujee@yahoo.com

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Abstract

An edge magic total labeling of a graph G(V, E) with p vertices and q edges is a bijection $f : V(G) \cup E(G) \to \{1, 2, ..., p+q\}$ such that for every edge uv in E, f(u) + f(uv) + f(v) is a constant k. If there exist two constants k_1 and k_2 such that the above sum is either k_1 or k_2, it is said to be an edge bimagic total labeling. In this paper we study and investigate super edge magic and bimagic labeling for duplication graphs of cycles and paths.

Keywords: *Graph, labeling, magic labeling, bimagic labeling, bijective function.*

1 Introduction:

A labeling of a graph G is an assignment f of labels to either the vertices or the edges or both subject to certain conditions. Labeled graphs are becoming an increasingly useful family of Mathematical Models from a broad range of applications. Graph labeling was first introduced in the late 1960's. A useful survey on graph labeling by J.A. Gallian (2012) can be found in [3]. All graphs considered here are finite, simple and undirected.

A (p, q)-graph G = (V, E) with p vertices and q edges is called total edge magic if there is a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, .., p+q\}$ such that there exists a constant k for any edge uv in E, f(u) + f(uv) + f(v) = k. The original concept of total edge-magic graph is due to Kotzig and Rosa [4]. They called it magic graph. A total edge-magic graph is called a super edge-magic if f(V(G)) = {1, 2, ..., p}. Wallis [5] called super edge-magic as strongly edge-magic.

It becomes interesting when we arrive with magic type labeling summing to exactly two distinct constants say k_1 or k_2. Edge bimagic total labeling was introduced by J. Baskar Babujee [1] and studied in [2] as (1, 1) edge bimagic labeling.

Definition 1.1: *A graph G(V, E) with p vertices and q edges is called total edge magic if there is a bijection $f: V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ such that there exists a constant k for any edge uv in E, f(u) + f(v) + f(uv) = k. A total edge magic graph is called super edge magic if f(V(G)) = {1, 2, …, p}.*

Definition 1.2: *A graph G(V, E) with p vertices and q edges is called total edge bimagic if there is a bijection $f: V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ such that for any edge uv in E, we have two constants k_1 and k_2 with f(u) + f(v) + f(uv) = k_1 or k_2. A total edge bimagic graph is called super edge bimagic if f(V(G)) = {1, 2, …, p}.*

Definition 1.3[6]: *Duplication of an edge $e = v_iv_{i+1}$ by a new vertex v^1 in a graph G produces a new graph G^1 such that the neighborhood of v^1 that is $N(v^1) = \{v_i, v_{i+1}\}$.*

Definition 1.4[6]: *Duplication of a vertex v_k by a new edge $e = v^1 v^{11}$ in a graph G produces a new graph G^1 such that the neighborhood of v^1 and v^{11} are respectively $N(v^1) = \{v_k, v^{11}\}$ and $N(v^{11}) = \{v_k, v^1\}$.*

In this paper we prove that super edge magic and bimagic labeling for some cycles and paths related duplication graphs.

2 Main Result

Theorem 2.1: *The graph G obtained by duplication of a vertex by an edge in C_n: $n \equiv 1 (mod 2)$ has super edge bimagic total labeling.*

Proof: Let $u_1, u_2, \ldots, u_n$ be vertices and $e_1, e_2, \ldots, e_n$ be edges of cycle C_n. Without loss of generality we duplicate the vertex u_{n-1} by an edge e_{n+1} with end vertices as

v_1 and v_2. Let the graph so obtained by (V, E). Then vertex set V = {v_1, v_2, u_i; $1 \le i \le n$} and edge set

$E = E_1 \cup E_2$ where $E_1 = \{u_1u_n;\ u_iu_{i+1};\ 1 \le i \le n-1\}$, $E_2 = \{u_{n-1}v_1;\ u_{n-1}v_2;\ v_1v_2\}$.

We define a bijective function f: $V(G) \cup E(G) \to \{1, 2, ..., 2n+5\}$ as follows.

For i = 1 to n; $i \equiv 1 (\text{mod}2)$, $f(u_i) = \frac{i+1}{2}$. For i = 2 to n-1; $i \equiv 0$ (mod2), $f(u_i) = \frac{n+1}{2} + \frac{i}{2}$.

For i = 1 to n-1; $f(u_iu_{i+1}) = 2n+5-i$. $f(u_1) = 1$, $f(u_n) = \frac{n+1}{2}$, $f(v_1) = n+1$, $f(v_2) = n+2$, $f(u_{n-1}) = n$, $f(u_1u_n) = 2n+5$, $f(v_1v_2) = n+3$, $f(v_1u_{n-1}) = n+5$, $f(v_2u_{n-1}) = n+4$.

Case (i): For every edge $u_iu_{i+1} \in E_1$

Subcase (i): $i \equiv 1$ (mod2)

$$f(u_i) + f(u_{i+1}) + f(u_iu_{i+1}) = \frac{i+1}{2} + \frac{n+1}{2} + \frac{i+1}{2} + 2n+5-i = \frac{5n+13}{2} = k_1$$

Subcase (ii): $i \equiv 0$ (mod2)

$$f(u_i) + f(u_{i+1}) + f(u_iu_{i+1}) = \frac{n+1}{2} + \frac{i}{2} + \frac{i+2}{2} + 2n+5-i = \frac{5n+13}{2} = k_1$$

Subcase (iii): For an edge $u_1u_n \in E_1$

$$f(u_1) + f(u_n) + f(u_1u_n) = 1 + \frac{n+1}{2} + 2n+5 = \frac{5n+13}{2} = k_1$$

Case (ii): For an edge $u_{n-1}v_1 \in E_2$

$$f(v_1) + f(u_{n-1}) + f(v_1u_{n-1}) = n+1+n+n+5 = 3n+6 = k_2$$

For an edge $u_{n-1}v_2 \in E_2$

$$f(v_2) + f(u_{n-1}) + f(v_2u_{n-1}) = n+2+n+n+4 = 3n+6 = k_2$$

For an edge $v_1v_2 \in E_2$

$f(v_1) + f(v_2) + f(v_1v_2) = n+1+n+2+n+3 = 3n+6 = k_2$

For all the above two cases the edge counts are $k_1 = \frac{5n+13}{2}$ and $k_2 = 3n+6$. Hence, the graph obtained by duplication of an arbitrary vertex by a new edge in cycle C_n: $n \equiv 1(mod2)$ is super edge bimagic total labeling.

Theorem 2.2: *The graph G obtained by duplication of an edge by a vertex in C_n: $n \equiv 1(mod2)$ admits super edge bimagic total labeling.*

Proof: Let $u_1, u_2, \ldots, u_n$ be vertices and $e_1, e_2, \ldots, e_n$ be edges of cycle C_n: $n \equiv 1(mod2)$. Without loss of generality we duplicate the edge u_nu_{n-1} by a vertex v_1. Let the graph so obtained by (V, E). Then the vertex set $V = \{v_1, u_i; 1 \le i \le n\}$ and edge set $E = E_1 \cup E_2$ where $E_1 = \{u_iu_{i+1}; 1 \le i \le n-1\}$, $E_2 = \{ u_1u_n ; v_1u_n ; u_{n-1}v_1\}$.

We define a bijective function $f: V(G) \cup E(G) \to \{1, 2, ..., 2n+3\}$ as follows.

For $i = 1$ to n; $i \equiv 1(mod2)$, $f(u_i) = \frac{i+1}{2}$.

For $i = 2$ to $n-1$; $i \equiv 0(mod2)$, $f(u_i) = \frac{n+1}{2}+\frac{i}{2}$,

For $i = 1$ to $n-1$; $f(u_iu_{i+1}) = 2n+3-i$. $f(u_1) = 1$, $f(u_n) = \frac{n+1}{2}$, $f(v_1) = n+1$,

$f(u_{n-1}) = n$, $f(v_1u_{n-1}) = n+2$, $f(v_1u_n) = n+3$, $f(u_1u_n) = 2n+3$.

Case (i): For any edge $u_iu_{i+1} \in E_1$

Subcase (i): $i \equiv 1$ (mod2)

$$f(u_i) + f(u_{i+1}) + f(u_iu_{i+1}) = \frac{i+1}{2}+\frac{n+1}{2}+\frac{i+1}{2}+2n+3-i = \frac{5n+9}{2} = k_1$$

Subcase (ii): $i \equiv 0$ (mod2)

$$f(u_i) + f(u_{i+1}) + f(u_iu_{i+1}) = \frac{n+1}{2}+\frac{i}{2}+\frac{i+2}{2}+2n+3-i = \frac{5n+9}{2} = k_1$$

Case (ii): For an edge $u_1u_n \in E_2$

$$f(u_1) + f(u_n) + f(u_1u_n) = 1+\frac{n+1}{2}+2n+3 = \frac{5n+9}{2} = k_1$$

For an edge $v_1u_n \in E_2$

$$f(v_1) + f(u_n) + f(v_1u_n) = n+1+\frac{n+1}{2}+n+3 = \frac{5n+9}{2} = k_1$$

For an edge $u_{n-1}v_1 \in E_2$

$$f(v_1) + f(u_{n-1}) + f(v_1u_{n-1}) = n+1+n+n+2 = 3n+3 = k_2$$

For all the above two cases the edge counts are $k_1 = \frac{5n+9}{2}$ and $k_2 = 3n+3$.

Hence the graph obtained by duplication of an arbitrary edge by a new vertex in cycle C_n: $n \equiv 1(mod2)$ is super edge bimagic total labeling.

Theorem 2.3: *The graph G obtained by duplicating all the vertices by edges in C_n: $n \equiv 1(mod2)$ admits super edge magic total labeling.*

Proof: Let $u_1, u_2, \ldots, u_n$ be vertices and $e_1, e_2, \ldots, e_n$ be edges of cycle C_n. Let the graph obtained by duplicating all the vertices by edges in cycle C_n. Then vertex set $V = \{v_i, u_i, w_i;\ 1 \le i \le n\}$ and edge set $E = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5$ where $E_1 = \{u_iu_{i+1};\ 1 \le i \le n-1\}$, $E_2 = \{v_iw_i;\ 1 \le i \le n\}$, $E_3 = \{u_iw_i;\ 1 \le i \le n-1\}$, $E_4 = \{v_iu_i;\ 1 \le i \le n-1\}$ and $E_5 = \{u_1u_n;\ w_nu_n;\ v_nw_n;\ u_nv_n\}$. We define a bijective function $f: V(G) \cup E(G) \to \{1, 2, ..., 7n\}$ as follows.

For $i = 1$ to n; $i \equiv 1(mod2)$, $f(u_i) = \frac{i+1}{2}$. For $i = 2$ to $n-1$; $i \equiv 0\ (mod2)$, $f(u_i) = \frac{n+1}{2}+\frac{i}{2}$,

For $i = 1$ to $n-1$; $f(u_iu_{i+1}) = 7n-i$ .For $i = 1$ to n; $f(v_i) = 2n-i$

For $i = 1$ to n; $i \equiv 1(mod2)$, $f(w_i) = 2n+\frac{n+1}{2}+\frac{i+1}{2}$.

For $i = 2$ to $n-1$; $i \equiv 0(mod2)$, $f(w_i) = 2n+1+\frac{i}{2}$

For $i = 1$ to n; $i \equiv 1(mod2)$, $f(v_iw_i) = 3n+\frac{i+1}{2}$.

For $i = 2$ to $n-1$; $i \equiv 0(mod2)$, $f(v_iw_i) = 3n+\frac{n+1}{2}+\frac{i}{2}$

For i = 1 to n-2; i ≡ 1(mod2), $f(u_i v_i) = 5n + \frac{i+1}{2} + \frac{n+1}{2}$.

For i = 2 to n-1; i ≡ 0(mod2), $f(u_i v_i) = 5n + 1 + \frac{i}{2}$

$f(u_n) = \frac{n+1}{2}$, $f(u_1 u_n) = 7n$, $f(w_n) = 2n+1$, $f(u_n w_n) = 5n$, $f(v_n w_n) = 3n + \frac{n+1}{2}$,

$f(u_n v_n) = 5n+1$, $f(v_n) = 2n$.

Case (i): For any edge $u_i u_{i+1} \in E_1$

Subcase (i): i ≡ 1 (mod2)

$$f(u_i) + f(u_{i+1}) + f(u_i u_{i+1}) = \frac{i+1}{2} + \frac{n+1}{2} + \frac{i+1}{2} + 7n - i = \frac{15n+3}{2} = k_1$$

Subcase (ii): i ≡ 0 (mod2)

$$f(u_i) + f(u_{i+1}) + f(u_i u_{i+1}) = \frac{n+1}{2} + \frac{i}{2} + \frac{i+2}{2} + 7n - i = \frac{15n+3}{2} = k_1$$

Case (ii): For any edge $v_i w_i \in E_2$

Subcase (i): i ≡ 1 (mod2)

$$f(v_i) + f(w_i) + f(v_i w_i) = 2n - i + 2n + \frac{n+1}{2} + \frac{i+1}{2} + 3n + \frac{i+1}{2} = \frac{15n+3}{2} = k_1$$

Subcase (ii): i ≡ 0 (mod2)

$$f(v_i) + f(w_i) + f(v_i w_i) = 2n - i + 2n + \frac{i}{2} + 1 + 3n + \frac{n+1}{2} + \frac{i}{2} = \frac{15n+3}{2} = k_1$$

Case (iii): For any edge $u_i w_i \in E_3$

Subcase (i): i ≡ 1 (mod2)

$$f(u_i) + f(w_i) + f(u_i w_i) = \frac{i+1}{2} + 2n + \frac{n+1}{2} + \frac{i+1}{2} + 5n - i = \frac{15n+3}{2} = k_1$$

Subcase (ii): $i \equiv 0$ (mod2)

$$f(u_i) + f(w_i) + f(u_iw_i) = \frac{i}{2} + \frac{n+1}{2} + 2n + \frac{i}{2} + 1 + 5n - i = \frac{15n+3}{2} = k_1$$

Case (iv): For any edge $u_iv_i \in E_4$

Subcase (i): $i \equiv 1$ (mod2)

$$f(u_i) + f(v_i) + f(u_iv_i) = \frac{i+1}{2} + 2n - i + 5n + \frac{n+1}{2} + \frac{i+1}{2} = \frac{15n+3}{2} = k_1$$

Subcase (ii): $i \equiv 0$ (mod2)

$$f(u_i) + f(v_i) + f(u_iv_i) = \frac{i}{2} + \frac{n+1}{2} + 2n - i + 5n + 1 + \frac{i}{2} = \frac{15n+3}{2} = k_1$$

Case (v): For an edge $u_1u_n \in E_5$

$$f(u_1) + f(u_n) + f(u_1u_n) = 1 + \frac{n+1}{2} + 7n = \frac{15n+3}{2} = k_1$$

For an edge $u_nw_n \in E_5$

$$f(u_n) + f(w_n) + f(u_nw_n) = \frac{n+1}{2} + 2n + 1 + 5n = \frac{15n+3}{2} = k_1$$

For an edge $v_nw_n \in E_5$

$$f(v_n) + f(w_n) + f(v_nw_n) = 2n + 2n + 1 + 3n + \frac{n+1}{2} = \frac{15n+3}{2} = k_1$$

For edge $u_nv_n \in E_5$

$$f(u_n) + f(v_n) + f(u_nv_n) = \frac{n+1}{2} + 2n + 5n + 1 = \frac{15n+3}{2} = k_1.$$

For all the above five cases the edge count is a constant $k_1 = \frac{15n+3}{2}$.

Hence the graph obtained by duplication of all the vertices by edges in cycle C_n: $n \equiv 1$(mod2) is super edge magic total labeling.

Illustration 2.4: Super edge magic labeling of a graph obtained by duplicating all the vertices by edges in C_5 is shown in figure 1.

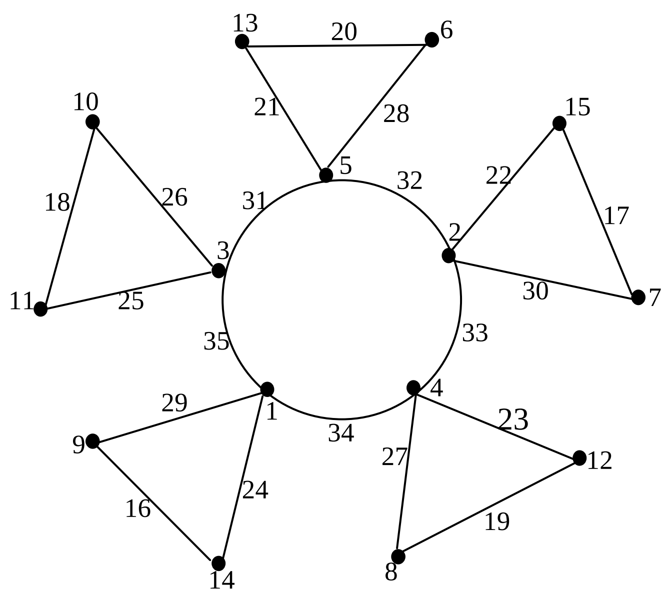

Figure 1: $k_1 = 39$

Theorem 2.5: *The graph G obtained by duplicating all the vertices by edges in path P_n admits super edge magic total labeling for $n \equiv 1 (mod2)$.*

Proof: Let $u_1, u_2, \ldots, u_n$ be vertices and $e_1, e_2, \ldots, e_{n-1}$ be edges of path P_n. Let the graph obtained by duplicating all the vertices by edges in path P_n in G. Then the vertex set $V = \{v_i, u_i, w_i;\ 1 \le i \le n\}$ and edge set $E = E_1 \cup E_2 \cup E_3 \cup E_4$ where

$E_1 = \{u_iu_{i+1};\ 1 \le i \le n\text{-}1\}$,

$E_2 = \{u_iv_i;\ 1 \le i \le n\}$, $E_3 = \{v_iw_i;\ 1 \le i \le n\}$, $E_4 = \{u_iw_i;\ 1 \le i \le n\}$.

We define a bijective function $f: V(G) \cup E(G) \to \{1, 2, \ldots, 7n\text{-}1\}$ as follows.

Case (i): For any edge $u_iu_{i+1} \in E_1$

Subcase (i): $i \equiv 1$ (mod2)

$$f(u_i) + f(u_{i+1}) + f(u_iu_{i+1}) = 3n+1-\frac{i+1}{2}+2n+\frac{n+1}{2}-\frac{i+1}{2}+3n+i = \frac{17n+1}{2} = k_1$$

Subcase (ii): $i \equiv 0$ (mod2)

$$f(u_i) + f(u_{i+1}) + f(u_iu_{i+1}) = 2n+\frac{n+1}{2}-\frac{i}{2}+3n-\frac{i+2}{2}+1+3n+i = \frac{17n+1}{2} = k_1$$

Case (ii): For any edge $u_i v_i \in E_2$

Subcase (i): i ≡ 1 (mod2)

$$f(u_i) + f(v_i) + f(u_i v_i) = 3n - \frac{i+1}{2} + 1 + i + 5n + \frac{n+1}{2} - \frac{i+1}{2} = \frac{17n+1}{2} = k_1$$

Subcase (ii): i ≡ 0 (mod2)

$$f(u_i) + f(v_i) + f(u_i v_i) = 2n + \frac{n+1}{2} - \frac{i}{2} + i + 6n - \frac{i}{2} = \frac{17n+1}{2} = k_1$$

Case (iii): For any edge $v_i w_i \in E_3$

Subcase (i): i ≡ 1 (mod2)

$$f(w_i) + f(v_i) + f(w_i v_i) = i + n + \frac{n+1}{2} + 1 - \frac{i+1}{2} + 7n - \frac{i+1}{2} = \frac{17n+1}{2} = k_1$$

Subcase (ii): i ≡ 0 (mod2)

$$f(w_i) + f(v_i) + f(w_i v_i) = i + 2n + 1 - \frac{i}{2} + 6n + \frac{n-1}{2} - \frac{i}{2} = \frac{17n+1}{2} = k_1$$

Case (iv): For every edge $u_i w_i \in E_4$

Subcase (i): i ≡ 1 (mod2)

$$f(w_i) + f(u_i) + f(w_i u_i) = 3n+1 - \frac{i+1}{2} + n + \frac{n+1}{2} + 1 - \frac{i+1}{2} + 4n - 1 + i = \frac{17n+1}{2} = k_1$$

Subcase (ii): i ≡ 0 (mod2)

$$f(w_i) + f(u_i) + f(w_i u_i) = 2n + \frac{n+1}{2} - \frac{i}{2} + 2n + 1 - \frac{i}{2} + 4n - 1 + i = \frac{17n+1}{2} = k_1$$

For all the above four cases the edge count is a constant $k_1 = \frac{17n+1}{2}$. Hence the graph obtained by duplication of all the vertices by edges in path P_n: n ≡ 1(mod2) is super edge magic total labeling.

Theorem 2.6: *The graph G obtained by duplicating all the vertices by edges in path P_n admits super edge bimagic total labeling for n ≡ 0 (mod2).*

Proof: Let $u_1, u_2, \ldots, u_n$ be vertices and $e_1, e_2, \ldots, e_{n-1}$ be edges of path P_n. Let the graph obtained by duplicating all the vertices by edges in path P_n in G. Then the vertex set $V = \{v_i, u_i, w_i; 1 \le i \le n-1\} \cup \{u^1, v^1, w^1\}$ and edge set $E = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5$ where $E_1 = \{u_iu_{i+1}; 1 \le i \le n-2\}$, $E_2 = \{u_iv_i; 1 \le i \le n-1\}$, $E_3 = \{v_iw_i; 1 \le i \le n-1\}$, $E_4 = \{u_iw_i; 1 \le i \le n-1\}$ and $E_5 = \{u^1u_1; u^1v^1; u^1w^1; v^1w^1\}$. We define a bijective function $f: V(G) \cup (E) \rightarrow \{1, 2, ..., 7n-1\}$ as follows.

For i = 1 to n-1; $i \equiv 1 \pmod 2$, $f(u_i) = 3n-2-\frac{i+1}{2}$.

For i = 2 to n-2; $i \equiv 0 \pmod 2$, $f(u_i) = 2n+\frac{n}{2}-2-\frac{i}{2}$.

For i = 1 to n-2; $f(u_iu_{i+1}) = 3n+4+i$. For i = 1 to n-1; $f(v_i) = i$.

For i = 1 to n-1; $i \equiv 1 \pmod 2$, $f(w_i) = n+\frac{n}{2}-\frac{i+1}{2}$.

For i = 2 to n-2; $i \equiv 0 \pmod 2$, $f(w_i) = 2n-1-\frac{i}{2}$.

For i = 1 to n-1; $i \equiv 1 \pmod 2$, $f(u_iv_i) = 5n+\frac{n}{2}+2-\frac{i+1}{2}$.

For i = 2 to n-2; $i \equiv 0 \pmod 2$, $f(u_iv_i) = 6n+1-\frac{i}{2}$.

For i = 1 to n-1; $i \equiv 1 \pmod 2$, $f(v_iw_i) = 7n-\frac{i+1}{2}$.

For i = 2 to n-2; $i \equiv 0 \pmod 2$, $f(v_iw_i) = 6n+\frac{n}{2}-\frac{i}{2}$.

For i = 1 to n-1; $f(u_iw_i) = 4n+2+i$.

$f(u^1) = 3n-1$, $f(v^1) = 3n$, $f(w^1) = 3n-2$, $f(u_1u^1) = 3n+4$, $f(u^1v^1) = 3n+1$, $f(v^1w^1) = 3n+2$, $f(u^1w^1) = 3n+3$.

Case (i): For any edge $u_iu_{i+1} \in E_1$

Subcase (i): $i \equiv 1 \pmod 2$

$$f(u_i) + f(u_{i+1}) + f(u_iu_{i+1}) = 3n-2-\frac{i+1}{2}+2n+\frac{n}{2}-2-\frac{i+1}{2}+3n+4+i = \frac{17n-2}{2} = k_1$$

Subcase (ii): i ≡ 0 (mod2)

$$f(u_i) + f(u_{i+1}) + f(u_iu_{i+1}) = 2n+\frac{n}{2}-2-\frac{i}{2}+3n-2-\frac{i+2}{2}+3n+4+i = \frac{17n-2}{2} = k_1$$

Case (ii): For any edge $u_iv_i \in E_2$

Subcase (i): i ≡ 1 (mod2)

$$f(u_i) + f(v_i) + f(u_iv_i) = 3n-2-\frac{i+1}{2}+i+5n+\frac{n}{2}+2-\frac{i+1}{2} = \frac{17n-2}{2} = k_1$$

Subcase (ii): i ≡ 0 (mod2)

$$f(u_i) + f(v_i) + f(u_iv_i) = 2n+\frac{n}{2}-2-\frac{i}{2}+i+6n+1-\frac{i}{2} = \frac{17n-2}{2} = k_1$$

Case (iii): For any edge $v_iw_i \in E_3$

Subcase (i): i ≡ 1 (mod2)

$$f(w_i) + f(v_i) + f(w_iv_i) = i+n+\frac{n}{2}-\frac{i+1}{2}+7n-\frac{i+1}{2} = \frac{17n-2}{2} = k_1$$

Subcase (ii): i ≡ 0 (mod2)

$$f(w_i) + f(v_i) + f(w_iv_i) = i+2n-1-\frac{i}{2}+6n+\frac{n}{2}-\frac{i}{2} = \frac{17n-2}{2} = k_1$$

Case (iv): For any edge $u_iw_i \in E_4$

Subcase (i): i ≡ 1 (mod2)

$$f(w_i) + f(u_i) + f(w_iu_i) = 3n-2-\frac{i+1}{2}+n+\frac{n}{2}-\frac{i+1}{2}+4n+2+i = \frac{17n-2}{2} = k_1$$

Subcase (ii): i ≡ 0 (mod2)

$$f(w_i) + f(u_i) + f(w_iu_i) = 2n+\frac{n}{2}-2-\frac{i}{2}+2n-1-\frac{i}{2}+4n+2+i = \frac{17n-2}{2} = k_1$$

Case (v): For an edge $u^1u_1 \in E_5$

$f(u^1) + f(u_1) + f(u^1u_1) = 3n - 1 + 3n - 3 + 3n + 4 = 9n = k_2$

For an edge $u^1v^1 \in E_5$

$f(u^1) + f(v^1) + f(u^1v^1) = 3n - 1 + 3n + 3n + 1 = 9n = k_2$

For an edge $u^1w^1 \in E_5$

$f(u^1) + f(w^1) + f(u^1w^1) = 3n - 1 + 3n - 2 + 3n + 3 = 9n = k_2$

For an edge $w^1v^1 \in E_5$

$f(w^1) + f(v^1) + f(w^1v^1) = 3n - 2 + 3n + 3n + 2 = 9n = k_2$.

For all the above five cases the edge counts are $k_1 = \frac{17n-2}{2}$ and $k_2 = 9n$. Hence the graph obtained by duplication of all the vertices by edges in path P_n: $n \equiv 0 \pmod 2$ is super edge bimagic total labeling.

Illustration 2.7: Super edge magic labeling of a graph obtained by duplicating all the vertices by edges in P_5 is shown in figure 2.

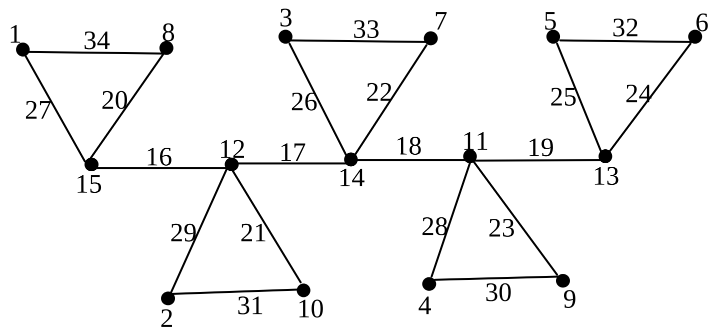

Figure 2: k = 43

Theorem 2.8: *The graph G obtained by duplicating all the edges by vertices in path P_n: $n \equiv 1 \pmod 2$ admits super edge bimagic total labeling.*

Proof: Let $u_1, u_2, \ldots, u_n$ be vertices and $e_1, e_2, \ldots, e_{n-1}$ be edges of path P_n. Let the graph obtained by duplicating all the edges by vertices in path P_n. Then the vertex set $V = \{u_i; 1 \le i \le n+1\} \cup \{v_i; 1 \le i \le n\}$ and edge set $E = E_1 \cup E_2 \cup E_3$

where $E_1 = \{u_iu_{i+1}; 1 \le i \le n\}$,

$E_2 = \{u_iv_i;\ 1 \le i \le n\}$, $E_3 = \{v_iu_{i+1};\ 1 \le i \le n\}$.

We define a bijective function f: $V(G) \cup E(G) \rightarrow \{1, 2, ...,5n+1\}$ as follows.

For i = 1 to n; $i \equiv 1(\text{mod}2)$, $f(u_i) = \frac{i+1}{2}$.

For i = 2 to n+1; $i \equiv 0(\text{mod}2)$, $f(u_i) = \frac{n+1}{2} + \frac{i}{2}$.

For i = 1 to n; $f(u_iu_{i+1}) = 4n+3-i$. For i = 1 to n; $f(v_i) = 2n+2-i$.

For i = 1 to n; $i \equiv 1(\text{mod}2)$, $f(u_iv_i) = 2n+\frac{n+1}{2}+1+\frac{i+1}{2}$.

For i = 2 to n-1; $i \equiv 0(\text{mod}2)$, $f(u_iv_i) = 4n+2+\frac{i}{2}$.

For i = 1 to n; $i \equiv 1(\text{mod}2)$, $f(u_{i+1}v_i) = 2n+1+\frac{n+1}{2}$.

For i = 2 to n-1; $i \equiv 0(\text{mod}2)$, $f(u_{i+1}v_i) = 4n+\frac{n+1}{2}+1+\frac{i}{2}$.

Case (i): For any edge $u_iu_{i+1} \in E_1$

Subcase (i): $i \equiv 1$ (mod2)

$$f(u_i) + f(u_{i+1}) + f(u_iu_{i+1}) = \frac{i+1}{2}+\frac{n+1}{2}+\frac{i+1}{2}+4n+3-i = \frac{9(n+1)}{2} = k_1$$

Subcase (ii): $i \equiv 0$ (mod2)

$$f(u_i) + f(u_{i+1}) + f(u_iu_{i+1}) = \frac{n+1}{2}+\frac{i}{2}+\frac{i+2}{2}+4n+3-i = \frac{9(n+1)}{2} = k_1$$

Case (ii): For any edge $u_iv_i \in E_2$

Subcase (i): $i \equiv 1$ (mod2)

$$f(u_i) + f(v_i) + f(u_iv_i) = \frac{i+1}{2}+2n+2-i+2n+\frac{i+1}{2}+1+\frac{i+1}{2} = = \frac{9(n+1)}{2} = k_1$$

Subcase (ii): i ≡ 0 (mod2)

$$f(u_i) + f(v_i) + f(u_iv_i) = \frac{n+1}{2} + \frac{i}{2} + 2n+2-i+4n+2+\frac{i}{2} = \frac{13n+9}{2} = k_2$$

Case (iii): For any edge $u_{i+1}v_i \in E_3$

Subcase (i): i ≡ 1 (mod2)

$$f(u_{i+1}) + f(v_i) + f(u_{i+1}v_i) = 2n+2-i+\frac{n+1}{2}+\frac{i+1}{2}+2n+1+\frac{i+1}{2} = \frac{9(n+1)}{2} = k_1$$

Subcase (ii): i ≡ 0 (mod2)

$$f(u_{i+1}) + f(v_i) + f(u_{i+1}v_i) = 2n+2-i+\frac{i+2}{2}+4n+\frac{n+1}{2}+1+\frac{i}{2} = \frac{13n+9}{2} = k_2$$

For all the above three cases the edge counts are $k_1 = \frac{9(n+1)}{2}$ and $k_2 = \frac{13n+9}{2}$. Hence the graph obtained by duplication of all the edges by vertices in path P_n: n ≡ 1(mod2) is super edge bimagic total labeling.

3 Conclusion:

In our present study, we investigated super edge magic and bimagic labeling for duplication graphs of cycles and paths. In this direction, we are interested in establishing the following results. (i) The graph G obtained by duplicating all the edges by vertices in path P_n: n ≡ 0 (mod2) has super edge bimagic labeling. (ii) The graph G obtained by duplicating all the vertices by edges in cycle C_n: n ≡ 0 (mod2) has super edge magic labeling.

References

[1] J.B. Babujee, Bimagic labeling in path graphs, *The Mathematics Education*, 38(1) (2004), 12-16.

[2] J.B. Babujee, On edge bimagic labeling, *Journal of Combinatorics Information and System Sciences*, 28(1-4) (2004), 239-244.

[3] J.A. Gallian, A dynamic survey of graph labeling, *Electronic Journal of Combinatorics*, 17(1) (2012), 1-246.

[4] A. Kotzig and A. Rosa, Magic valuations of finite graphs, *Canadian Mathematical Bulletin*, 13(1970), 451-461.

[5] W.D. Wallis, *Magic Graphs*, Birkhauser-Basel, (2001).

[6] S.K. Vaidya and C.M. Barasara, Product cordial graphs in the context of some graph operations, *International Journal of Mathematics and Scientific Computing*, 1(2) (2011), 1-6.