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Homomorphism of Multi L-Fuzzy Subgroup

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Abstract

In this paper we introduce the notion of homomorphism and anti homomorphism of a multi L-fuzzy subgroup and investigate some of its properties.

Keywords: *Fuzzy set, multi-L-fuzzy subgroup, homomorphism of multi L-fuzzy group, anti homomorphism of multi L-fuzzy group.*

1 Introduction

L. A. Zadeh introduced the notion of a fuzzy subset A of a set X as a function from X into I = [0, 1]. Rosenfeld [21] and Kuroki [14] applied this concept in group theory and semi group theory, and developed the theory of fuzzy subgroups and fuzzy sub semi groupoids respectively. J.A. Goguen [8] replaced the valuations set [0, 1], by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. In fact it seems in order to obtain a complete analogy of crisp mathematics in terms of fuzzy mathematics, it is necessary to replace the valuation set by a system having more rich algebraic structure. The concept of anti – fuzzy subgroup was introduced by

Biswas [3]. The concept of multi fuzzy subgroups was introduced by Souriar Sebastian and S. Babu Sundar [13]. In all these studies, the closed unit interval [0, 1] is taken as the Membership lattice.

We introduce the notion of a multi L-fuzzy sub group G and discussed some of its properties. The characterizations of a Multi L-fuzzy subgroup under homomorphism and anti homomorphism are discussed.

2 Preliminaries

In this section, we review some definitions and some results of Multi L-fuzzy subgroups which will be used in the later sections. Throughout this section we mean that $(G,*)$ is a group, e is the identity of G and xy as $x * y$.

Definition 2.1: *A L-fuzzy subset λ of X is a mapping from X into L, where L is a complete lattice satisfying the infinite meet distributive law. If L is the unit interval [0,1] of real numbers, there are the usual fuzzy subset of X.*

A L-fuzzy subset $\lambda:X\rightarrow L$ is said to be a nonempty, if it is not the constant map which assumes the values 0 of L.

Definition 2.2: *Let X be a non – empty set. A Multi L – fuzzy set λ in X is defined as a set of ordered sequences $\lambda = \{ (x, \mu_1(x), \mu_2(x), ..., \mu_i(x), ...) : x \in X\}$, where $\mu_i : X \rightarrow L$ for all i.*

Definition 2.3: *A L-fuzzy subset λ of G is said to be a L-fuzzy subgroup of G, if for all x, y $\in$ G,*

i. $\lambda(xy) \geq \lambda(x)\ \lambda(y)$
ii. $\lambda(x^{-1}) = \lambda(x)$.

Definition 2.4: *A Multi L – Fuzzy subset λ of G is called an Multi L – Fuzzy subgroup (MLFS) of G if for every x, y $\in$ G,*

i. $\lambda(xy) \geq \lambda(x) \wedge \lambda(y)$
ii. ii. $\lambda(x^{-1}) = \lambda(x)$.

Definition 2.5: *A multi L-fuzzy subset λ of G is said to be an anti multi L-fuzzy subgroup of G, if , $\forall$ x, y $\in$ G*

i. $\lambda(xy) \leq \lambda(x) \vee \lambda(y)$
ii. $\lambda(x^{-1}) = \lambda(x)$

Definition 2.6: *The function f: $G \to G'$ is said to be a homomorphism if f(xy)= f(x)f(y) $\forall$ x, y$\in$ G .*

Definition 2.7: *The function f: $G \to G'$ (G and G' are not necessarily commutative) is said to be an anti homomorphism if f(xy)= f(y)f(x) $\forall$ x, y $\in$ G.*

Definition 2.8: *Let f be any function from a set X to a set Y, and let λ be any L-fuzzy subset of X. Then λ is called f-invariant if f(x) = f(y) implies $\lambda(x) = \lambda(y)$, where x, y $\in$ X.*

3 Properties of Multi L-Fuzzy Subgroup under Homomorphism

In this section we study about properties of multi L-fuzzy subgroup under homomorphism.

Theorem 3.1: *Let G and G' be any two groups. Let f: $G \to G'$ be a homomorphism and onto. Let $\lambda : G \to L$ be a multi L-fuzzy subgroup of G. Then $f(\lambda)$ is a multi L-fuzzy subgroup of G', if λ has sup property and λ is f- invariant.*

Proof: Let λ be a multi L-fuzzy subgroup of G.

i. $f(\lambda)(xy) = \vee\{\lambda(xy)/xy \in G, f(xy) = x_0y_0\}$

$$= \lambda(x_0y_0)$$

$$\geq \lambda(x_0) \wedge \lambda(y_0)$$

$$\geq (\vee\{\lambda(x)/x \in G, f(x) = x_0\}) \wedge (\vee\{\lambda(y)/y \in G, f(y) = y_0\}$$

$$\geq (f(\lambda)(x)) \wedge (f(\lambda)(y))$$

$$f(\lambda)(xy) \geq (f(\lambda)(x)) \wedge (f(\lambda)(y)).$$

ii. $f(\lambda)(x^{-1}) = \vee\{\lambda(x^{-1})/x^{-1} \in G, f(x^{-1}) = x_0\}$

$$= \vee\{\lambda(x)/x \in G, f(x) = x_0\}$$

$$= \lambda(x_0)$$

$$= \vee\{\lambda(x)/x \in G, f(x_0) = x_0\}$$

$$= f(\lambda)(x)$$

$f(\lambda)(x^{-1}) = f(\lambda)(x)$.

Hence $f(\lambda)$ is a multi L- fuzzy subgroup of G^1.

Theorem 3.2: *Let G and G^1 be any two groups. Let f: $G \rightarrow G'$ be a homomorphism and onto. Let $\mu : G' \rightarrow L$ be a multi L-fuzzy subgroup of G^1. Then $f^{-1}(\mu)$ is a multi L-fuzzy subgroup of G.*

Proof: Let μ be a multi L-fuzzy subgroup of G'

i. $f^{-1}(\mu)(xy) = \mu(f(xy))$

$= \mu(f(x)f(y))$

$\geq \mu(f(x) \wedge \mu(f(y)$

$\geq f^{-1}(\mu)(x) \wedge f^{-1}(\mu)(y)$

$f^{-1}(\mu)(xy) \geq f^{-1}(\mu)(x) \wedge f^{-1}(\mu)(y)$

ii. $f^{-1}(\mu)(x^{-1}) = \mu(f(x^{-1}))$

$= \mu(f(x^{-1}))$

$= \mu(f(x))$

$= f^{-1}(\mu)(x)$

$f^{-1}(\mu)(x^{-1}) = f^{-1}(\mu)(x)$.

Hence $f^{-1}(\mu)$ is a multi L-fuzzy subgroup of G.

Theorem 3.3: *Let G and G^1 be any two groups. Let f: $G \rightarrow G'$ be an anti homomorphism and onto. Let λ: G→L be a multi L-fuzzy subgroup of G. Then f(λ) is a multi L-fuzzy subgroup of G^1, if λ has sup property and λ is f- invariant.*

Proof: Let λ be a multi L-fuzzy subgroup of G.

i. $f(\lambda)(xy) = \wedge\{\lambda(x_0y_0)/ x_0y_0 \in G, f(x_0y_0) = xy\} = \lambda(x_0y_0)$

$\leq \lambda(x_0) \vee (\lambda(y_0)$

$\leq (\wedge\{\lambda(x_0)/ x_0 \in G, f(x_0) = x\}) \vee (\wedge\{\lambda(y_0)/ y_0 \in G, f(y_0) = y\})$

$\leq (f(\lambda)(x)) \vee (f(\lambda)(y))$

$f(\lambda)(xy) \leq (f(\lambda)(x)) \vee (f(\lambda)(y))$.

ii. $f(\lambda)(x^{-1}) = \wedge\{\lambda(x_0^{-1}) / x_0^{-1} \in G, f(x_0^{-1}) = x^{-1}\}$

$= \lambda(x_0^{-1})$

$= \lambda(x_0)$

$= \wedge\{\lambda(x_0) / x_0 \in G, f(x_0) = x\}$

$= f(\lambda)(x)$

$f(\lambda)(x^{-1}) = f(\lambda)(x)$

Hence $f(\lambda)$ is a multi L- fuzzy subgroup of G'.

Theorem 3.4: *Let G and G^1 be any two groups. Let f: $G \rightarrow G'$ be an anti homomorphism and onto. Let $\mu : G' \rightarrow L$ be an multi L-fuzzy subgroup of G'. Then $f^{-1}(\mu)$ is a multi L-fuzzy subgroup of G.*

Proof: Let μ be a multi L-fuzzy subgroup of G'.

i. $f^{-1}(\mu)(xy) = \mu(f(xy))$

$= \mu(f(y)f(x))$

$\geq \mu(f(y) \wedge \mu(f(x)$

$\geq f^{-1}(\mu)(y) \wedge f^{-1}(\mu)(x)$

$f^{-1}(\mu)(xy) \geq f^{-1}(\mu)(y) \wedge f^{-1}(\mu)(x)$

ii. $f^{-1}(\mu)(x^{-1}) = \mu(f(x^{-1}))$

$= \mu(f(x^{-1}))$

$= \mu(f(x))$

$= f^{-1}(\mu)(x)$

$f^{-1}(\mu)(x^{-1}) = f^{-1}(\mu)(x)$.

Hence $f^{-1}(\mu)$ is a multi L-fuzzy subgroup of G.

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