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Properties of L Fuzzy Normal Sub λ- Groups

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Abstract

This paper contains some definitions and results of L fuzzy normal sub λ-group of λ- groups and generalized characteristics of L fuzzy normal sub λ-group of a λ- group.

Keywords: *Fuzzy set, L-fuzzy set, L-fuzzy sub λ- group, L-fuzzy normal sub λ-group, Homomorphism of L-fuzzy normal sub λ-group.*

1 Introduction

L.A. Zadeh [10] introduced the notion of a fuzzy subset μ of a set S as a function from X into I = [0, 1]. Rosenfeld [7] applied this concept in group theory and semi group theory, and developed the theory of fuzzy subgroups and fuzzy subsemigroupoids respectively J.A. Goguen [2], replaced the valuations set [0, 1], by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. In fact it seems in order to obtain a complete analogy of crisp mathematics in terms of fuzzy mathematics, it is necessary to replace the valuation set by a system having more rich algebraic structure. These concepts λ-groups play a major role in mathematics and fuzzy mathematics. G.S.V Satya Saibaba [9] introduced the concept of L- fuzzy λ-group and L-fuzzy λ-ideal of λ-group. In this paper, we initiate the study of L-fuzzy normal sub λ-groups. In this paper we study the properties of an L-fuzzy normal sub λ- groups under homomorphism are discussed.

2 Preliminaries

This section contains some definitions and results to be used in the sequel.

2.1 Definition [1, 2, 6] *A lattice ordered group is a system* $G = (G, *, \leq)$ *where*

(i) $(G, *)$ *is a group,*

(ii) $(G, \leq)$ *is a lattice and*

(iii) *the inclusion is invariant under all translations* $x \; \alpha \; a * x * b$.

i.e, $x \leq y \Rightarrow a * x * b \leq a * y * b$ *for all* $a, b \in G$.

2.2 Definition [10]: *Let S be any non-empty set. A fuzzy subset* μ *of S is a function* μ*: S* $\rightarrow$ *[0, 1].*

2.3 Definition [7]: *A L-fuzzy subset* μ *of G is said to be a L-fuzzy subgroup of G if for any x , y* $\in$ *G,*

i. $\mu(xy) \geq \mu(x) \wedge \mu(y)$,

ii $\mu(x^{-1}) = \mu(x)$.

2.4 Definition [9]: *A L-fuzzy subset* μ *of G is said to be a L-fuzzy sub* λ*-group of G if for any x, y* $\in$ *G,*

i. $\mu(xy) \geq \mu(x) \wedge \mu(y)$,

ii. $\mu(x^{-1}) = \mu(x)$,

iii. $\mu(x \vee y) \geq \mu(x) \wedge \mu(y)$,

iv. $\mu(x \wedge y) \geq \mu(x) \wedge \mu(y)$.

2.5 Definition [3, 4]: *If* a *is an element of* λ *group* G , *then* $a \vee (-a)$ *is called the absolute value of* a *and is denoted by* $|a|$. *Any element* a *of an* λ *-group* G *can be written as* $a = (a \vee 0) * (a \wedge 0)$ *i.e,* $a = a^{+} * a^{-}$, *where* a^{+} *is called positive part of* a *and* a^{-} *is called negative part of* a .

2.6 Definition [8]: *Let G and G^1 be any two λ- groups. Then the function* $f : G \to G^1$ *is said to be a homomorphism if* $f(xy) = f(x)f(y)$ *for all x, y∈ G.*

2.7 Definition [8]: *Let G and G^1 be any two λ- groups. Then the function* $f : G \to G^1$ *is said to be anti homomorphism if* $f(xy) = f(y)f(x)$ *for all* $x, y \in G$.

2.8 Definition [9]: *Let G be a λ- group. A L fuzzy sub λ- group A of G is said to be L fuzzy normal sub λ group (LFNSλ-G) of G if $\mu_A(xy) = \mu_A(yx)$, for all x and y ∈ G.*

2.9 Definition [5, 9]: *Let G be a λ- group. A L fuzzy sub λ-group μ of G is said to be a L fuzzy characteristic sub λ- group (LFCS λ- G) of G if $\mu_A(x) = \mu_A(f(x))$, for all x∈G and f∈Aut G.*

2.10 Definition [9]: *A fuzzy subset μ of a set X is said to be normalized if there exist x∈X such that $\mu_A(x) = 1$.*

3 Properties of L Fuzzy Normal Sub λ- Groups

3.1 Theorem: *Let G be an λ-group. If A and B are two L fuzzy normal sub λ groups of G, then their intersection A∩B is a L fuzzy normal sub λ- group of G.*

Proof: Let x and y∈ G.

Let A ={ ⟨ x, $\mu_A(x)$ ⟩ / x∈ G }and B = { ⟨ x, $\mu_B(x)$ ⟩ / x∈ G } be a L fuzzy normal sub λ-groups of a λ-group G.

Let C = A ∩B and C = { ⟨ x, $\mu_C(x)$ ⟩ / x∈ G}.

Then,

Clearly C is a L fuzzy sub λ-group of a λ-group G,

Since A and B are two L fuzzy sub λ-groups of a λ-group G.
And,

(i) $\mu_C(xy) = \mu_A(xy) \wedge \mu_B(xy)$
as A and B are LFNS λ Gs of a λ group G.
$= \mu_A(yx) \wedge \mu_B(yx)$
$= \mu_C(yx)$.

Therefore, $\mu_C(xy) = \mu_C(yx)$.

Hence $A \cap B$ is an L fuzzy normal sub λ-group of a λ-group G.

3.2 Theorem: *Let G be a M-group. The intersection of a family of L fuzzy normal sub λ-group of G is an L fuzzy normal sub λ-group of G.*

Proof: Let $\{A_i\}_{i \in I}$ be a family of L fuzzy normal sub λ groups of a λ group G and let $A = \cap A_i$

Then for x and $y \in G$.

Clearly the intersection of a family of L fuzzy sub λ- groups of a λ- group G is a L fuzzy sub λ-group of a λ- group G.

(i) $\mu_A(xy) = \wedge \mu_{A_i}(xy)$
$= \wedge \mu_{A_i}(yx)$, as $\{A_i\}_{i \in I}$ are LFNS λ- Gs of a λ- group G
$= \mu_A(yx)$.

Therefore, $\mu_A(xy) = \mu_A(yx)$.

Hence the intersection of a family of L fuzzy normal sub λ-groups of a λ- group G is a L fuzzy normal sub λ-group of a λ-group G.

3.3 Theorem: *If A is a fuzzy characteristic of L fuzzy sub λ-group of a λ- group G, then A is a L fuzzy normal sub λ- group of a λ-group G.*

Proof: Let A be a L fuzzy characteristic sub λ- group of a λ-group G and let $x, y \in G$.

Consider the map $f : G \to G$ defined by $f(x) = yxy^{-1}$.

Clearly, $f \in AutG$.

Now, $\mu_A(xy) = \mu_A(f(xy))$, as A is a LFCS λ G of a λ group G
$= \mu_A(y(xy)y^{-1})$
$= \mu_A(yx)$.

Therefore, $\mu_A(xy) = \mu_A(yx)$.

Hence A is a L fuzzy normal sub λ group of a λ group G.

3.4 Theorem: *A L fuzzy sub λ-group A of a λ-group G is a L fuzzy normal sub λ- group of G if and only if A is constant on the conjugate classes of G.*

Proof: Suppose that A is a L fuzzy normal sub λ-group of a λ-group G and let x and $y \in G$.

Now, $\mu_A(y^{-1}xy) = \mu_A(xyy^{-1})$, since A is a LFNS λ G of G
$= \mu_A(x)$.

Therefore, $\mu_A(y^{-1}xy) = \mu_A(x)$.

Hence (x) = { $y^{-1}xy$ / $y \in G$ }.

Hence A is constant on the conjugate classes of G.

Conversely, suppose that A is constant on the conjugate classes of G.

Then, $\mu_A(xy) = \mu_A(xyxx^{-1})$
$= \mu_A(x(yx)x^{-1})$, as A is constant on the conjugate classes of G
$= \mu_A(yx)$.

Therefore, $\mu_A(xy) = \mu_A(yx)$.

Hence A is a L fuzzy normal sub λ- group of a group G.

3.5 Theorem: *Let A be a L fuzzy normal sub λ group of a λ- group G. Then for any $y \in G$ we have $\mu_A(yxy^{-1}) = \mu_A(y^{-1}xy)$, for every $x \in G$.*

Proof: Let A be a L fuzzy normal sub λ-group of a λ-group G.

For any $y \in G$, we have,

$\mu_A(yxy^{-1}) = \mu_A(x)$, since A is a FNS λ-G of G
$= \mu_A(xyy^{-1})$, since A is a FNS λ-G of G
$= \mu_A(y^{-1}xy)$.

Therefore, $\mu_A(yxy^{-1}) = \mu_A(y^{-1}xy)$.

3.6 Theorem: *A L fuzzy sub λ-group A of a λ-group G is normalized if and only if $\mu_A(e) = 1$, where e is the identity element of the λ group G.*

Proof: If A is normalized, then there exists $x \in G$ such that $\mu_A(x) = 1$, but by properties of L fuzzy sub λ-group A of the λ-group G, $\mu_A(x) \leq \mu_A(e)$, for every $x \in G$.

Since $\mu_A(x) = 1$ and $\mu_A(x) \leq \mu_A(e)$, $1 \leq \mu_A(e)$.

But $1 \geq \mu_A(e)$.

Hence $\mu_A(e) = 1$.

Conversely, if $\mu_A(e) = 1$, then by the definition of normalized fuzzy subset, A is normalized.

3.7 Theorem: *Let A and B be L fuzzy sub λ- groups of λ-groups G and H, respectively. If A and B are L fuzzy normal sub λ- groups, then AxB is a L fuzzy normal sub λ-group of GxH.*

Proof: Let A and B be L fuzzy normal sub λ groups of the λ groups G and H respectively.

Clearly AxB is a L fuzzy sub λ-group of GxH.

Let x_1 and x_2 be in G, y_1 and y_2 be in H.

Then (x_1, y_1) and (x_2, y_2) are in GxH.

Now,

$$\begin{aligned} \mu_{AxB} [(x_1, y_1)(x_2, y_2)] &= \mu_{AxB} (x_1x_2, y_1y_2) \\ &= \mu_A(x_1x_2) \wedge \mu_B(y_1y_2) \\ &= \mu_A(x_2x_1) \wedge \mu_B(y_2y_1) \end{aligned}$$

Since A and B are LFNS λ Gs of the groups G and H

$$\begin{aligned} &= \mu_{AxB} (x_2x_1, y_2y_1) \\ &= \mu_{AxB} [(x_2, y_2)(x_1, y_1)]. \end{aligned}$$

Therefore, $\mu_{AxB} [(x_1, y_1)(x_2, y_2)] = \mu_{AxB} [(x_2, y_2)(x_1, y_1)]$.

Hence AxB is a L fuzzy normal sub λ group of GxH.

3.8 Theorem: *Let the L fuzzy normal sub λ- group A of a λ-group G be conjugate to a L fuzzy normal sub λ-group of G and a L fuzzy normal sub λ-group B of a λ-group H be conjugate to a L fuzzy normal sub λ- group N of H. Then a L fuzzy normal sub λ-group AxB of a λ group GxH is conjugate to a L fuzzy normal sub λ- group MxN of GxH.*

Proof: It is trivial.

3.9 Theorem: *Let A and B be L fuzzy subsets of the λ- groups G and H, respectively. Suppose that e and* e^{l} *are the identity element of G and H, respectively. If AxB is a L fuzzy normal sub λ- group of GxH, then at least one of the following two statements must hold.*

(i) $\mu_B(e^{l}) \geq \mu_A(x)$, *for all x in G,*
(ii) $\mu_A(e) \geq \mu_B(y)$, *for all y in H.*

Proof: It is trivial.

3.10 Theorem: *Let A and B be L fuzzy subsets of the λ- groups G and H, respectively and AxB is a L fuzzy normal sub λ- group of GxH. Then the following are true:*

(i) *if $\mu_A(x) \leq \mu_B(e^{\prime})$, then A is a L fuzzy normal sub λ- group of G.*
(ii) *if $\mu_B(x) \leq \mu_A(e)$, then B is a L fuzzy normal sub λ-group of H.*
(iii) *either A is a L fuzzy normal sub λ group of G or B is a L fuzzy normal sub λ- group of H.*

Proof: It is trivial.

References

[1] S.K. Bhakat and P.S. Das, Fuzzy sub algebras of a universal algebra, *Bull. Cal. Math. Soc*, 85(1993), 79-92.
[2] G. Birkhof, *Lattice Theory (Volume XXV)*, American Mathematical Society Colloquium Publications, (1940).
[3] J.A. Goguen, L-fuzzy sets, *J. Math. Anal. Appl.*, 18(1967), 145-174.
[4] L. Filep, Study of fuzzy algebras and relations from a general view point, *Acto Mathematica Academiae Paedagogocae Nyiregyhaziensis Tamus*, 14(1998), 49-55.
[5] L. Xiaoping, The intuitionistic fuzzy group and its homomorphic image, *Fuzzy Systems and Math.*, 14(1) (2000), 45-50.
[6] V. Murali, Lattice of fuzzy algebras and closure systems in IX, *Fuzzy Sets and Systems*, 41(1991), 101-111.
[7] A. Rosenfeld, Fuzzy groups, *J. Math. Anal. Appl.*, 35(1971), 512-517.
[8] U.M. Swamy and D.V. Raju, Algebraic fuzzy systems, *Fuzzy Sets and Systems*, 41(1991), 187-194.
[9] L.A. Zadeh, Fuzzy sets, *Inform and Control*, 8(1965), 338-353.
[10] G.S.V.S. Saibaba, Fuzzy lattice ordered groups, *Southeast Asian Bulletin of Mathematics*, 32(2008), 749-766.