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Research Article

Weighted Composition Operators on Some Weighted Spaces in the Unit Ball

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Let $\mathbb{B}_n$ be the unit ball of $\mathbb{C}^n$, $H(\mathbb{B}_n)$ the space of all holomorphic functions in $\mathbb{B}_n$. Let $u \in H(\mathbb{B}_n)$ and α be a holomorphic self-map of $\mathbb{B}_n$. For $f \in H(\mathbb{B}_n)$, the weigthed composition operator uC_α is defined by $(uC_\alpha f)(z) = u(z)f(\alpha(z)), z \in \mathbb{B}_n$. The boundedness and compactness of the weighted composition operator on some weighted spaces on the unit ball are studied in this paper.

1. Introduction

Let $\mathbb{B}_n$ be the unit ball of $\mathbb{C}^n$, $H(\mathbb{B}_n)$ the space of all holomorphic functions in $\mathbb{B}_n$, and $H^\infty = H^\infty(\mathbb{B}_n)$ the space of all bounded holomorphic functions in the unit ball. For $f \in H(\mathbb{B}_n)$, let

$$\Re f(z) = \sum_{j=1}^{n} z_j \frac{\partial f}{\partial z_j}(z) \tag{1.1}$$

be the radial derivative of f.

A positive continuous function μ on $[0,1)$ is called normal if there exist positive numbers α and $\beta, 0 < \alpha < \beta$, and $\delta \in [0,1)$ such that (see, e.g., [1, 2])

$$\begin{aligned} &\frac{\mu(r)}{(1-r)^\alpha} \text{ is decreasing on } [\delta,1), \qquad \lim_{r\to 1}\frac{\mu(r)}{(1-r)^\alpha} = 0, \\ &\frac{\mu(r)}{(1-r)^\beta} \text{ is increasing on } [\delta,1), \qquad \lim_{r\to 1}\frac{\mu(r)}{(1-r)^\beta} = \infty. \end{aligned} \tag{1.2}$$

An $f \in H(\mathbb{B}_n)$ is said to belong to the weighted-type space, denoted by $H_\mu^\infty = H_\mu^\infty(\mathbb{B}_n)$, if

$$\|f\|_{H_\mu^\infty} = \sup_{z\in\mathbb{B}_n} \mu(|z|)\left|f(z)\right| < \infty, \tag{1.3}$$

where μ is normal on $[0,1)$ (see [3]). H_μ^∞ is a Banach space with the norm $\|\cdot\|_{H_\mu^\infty}$.

The little weighted-type space, denoted by $H^\infty_{\mu,0}$, is the subspace of H^∞_μ consisting of those $f \in H^\infty_\mu$ such that

$$\lim_{|z|\to 1}\mu(|z|)\left|f(z)\right| = 0. \tag{1.4}$$

When $\mu(r) = (1-r^2)^\alpha$, the induced spaces H^∞_μ and $H^\infty_{\mu,0}$ become the (classical) weighted spaces H^∞_α and $H^\infty_{\alpha,0}$ respectively.

An $f \in H(\mathbb{B}_n)$ is said to belong to the logarithmic-type space $H^\infty_{\log}$ if

$$\|f\|_{H^\infty_{\log}} = \sup_{z\in\mathbb{B}_n}\frac{\left|f(z)\right|}{\log\left(e/(1-|z|^2)\right)} < \infty. \tag{1.5}$$

It is easy to see that $H^\infty_{\log}$ becomes a Banach space under the norm $\|\cdot\|_{H^\infty_{\log}}$, and that the following inclusions hold:

$$H^\infty \subset \mathcal{B} \subset H^\infty_{\log} \subset H^\infty_\alpha, \quad \alpha > 0, \tag{1.6}$$

where $\mathcal{B}$ is the Bloch space defined by

$$\mathcal{B} = \left\{ f \in H(\mathbb{B}_n) : \sup_{z\in\mathbb{B}_n}\left(1-|z|^2\right)\left|\Re f(z)\right| < \infty \right\}. \tag{1.7}$$

For some information on the Bloch and related spaces see, for example, [4–13] and the references therein. For some information on the space $H^\infty_{\log}$ in the unit disk see [14].

Let $u \in H(\mathbb{B}_n)$, and let φ be a holomorphic self-map of $\mathbb{B}_n$. For $f \in H(\mathbb{B}_n)$, the weighted composition operator uC_φ is defined by

$$\left(uC_\varphi f\right)(z) = u(z)f\left(\varphi(z)\right), \quad z \in \mathbb{B}_n. \tag{1.8}$$

The weighted composition operator can be regarded as a generalization of a multiplication operator and a composition operator, which is defined by $(C_\varphi f)(z) = f(\varphi(z))$. The work in [15] contains much information on this topic.

In the setting of the unit ball, Zhu studied the boundedness and compactness of the weighted composition operator between Bergman-type spaces and H^∞ in [16]. More general results can be found in [17, 18]. Some necessary and sufficient conditions for the weighted composition operator to be bounded and compact between the Bloch space and H^∞ are given in [19]. In the setting of the unit polydisk, some necessary and sufficient conditions for a weighted composition operator to be bounded and compact between the Bloch space and $H^\infty(\mathbb{B}_1^n)$ are given in [20, 21] (see also [22] for the case of composition operators). Other related results can be found, for example, in [3, 23–32].

In this paper, we study the weighted composition operator from $H^\infty_{\log}$ to the spaces H^∞_μ and $H^\infty_{\log}$. Some necessary and sufficient conditions for the weighted composition operator uC_φ to be bounded and compact are given.

Throughout the paper, constants are denoted by C; they are positive and may not be the same in every occurrence.

2. Main results and proofs

In this section, we give our main results and their proofs. Before stating these results, we need some auxiliary results, which are incorporated in the lemmas which follow.

Lemma 2.1. *Assume that* $u \in H(\mathbb{B}_n)$, φ *is a holomorphic self-map of* $\mathbb{B}_n$, *and* μ *is a normal function on* $[0,1)$. *Then,* $uC_\varphi : H^\infty_{\log} \to H^\infty_\mu$ *is compact if and only if* $uC_\varphi : H^\infty_{\log} \to H^\infty_\mu$ *is bounded, and for any bounded sequence* $(f_k)_{k\in\mathbb{N}}$ *in* $H^\infty_{\log}$ *which converges to zero uniformly on compact subsets of* $\mathbb{B}_n$ *as* $k \to \infty$, *one has* $\|uC_\varphi f_k\|_{H^\infty_\mu} \to 0$ *as* $k \to \infty$.

The proof of Lemma 2.1 follows by standard arguments (see, e.g., [15, Proposition 3.11] as well as the proofs of the corresponding results in [7, 22, 33, 34]). Hence, we omit the details.

Lemma 2.2. *Assume that* μ *is normal. A closed set* K *in* $H^\infty_{\mu,0}$ *is compact if and only if it is bounded and satisfies*

$$\lim_{|z|\to 1} \sup_{f\in K} \mu(|z|)\left|f(z)\right| = 0. \tag{2.1}$$

The proof of Lemma 2.2 is similar to the proof of Lemma 1 in [35]. We omit the details.

Now, we are in a position to state and prove our main results.

Theorem 2.3. *Assume that* $u \in H(\mathbb{B}_n)$, φ *is a holomorphic self-map of* $\mathbb{B}_n$, *and* μ *is normal on* $[0,1)$. *Then,* $uC_\varphi : H^\infty_{\log} \to H^\infty_\mu$ *is bounded if and only if*

$$M = \sup_{z\in\mathbb{B}_n} \mu(|z|)\left|u(z)\right| \log \frac{e}{1-\left|\varphi(z)\right|^2} < \infty. \tag{2.2}$$

Proof. Assume that $uC_\varphi : H^\infty_{\log} \to H^\infty_\mu$ is bounded. For $a \in \mathbb{B}_n$, set

$$f_a(z) = \log \frac{e}{1-\langle z, a\rangle}. \tag{2.3}$$

It is easy to see that $f_a \in H^\infty_{\log}$ and $\sup_{a\in\mathbb{B}_n} \|f_a\|_{H^\infty_{\log}} < \infty$.

For any $b \in \mathbb{B}_n$, we have

$$\begin{aligned} \infty &> \left\|uC_\varphi f_{\varphi(b)}\right\|_{H^\infty_\mu} \\ &= \sup_{z\in\mathbb{B}_n} \mu(|z|)\left|\left(uC_\varphi f_{\varphi(b)}\right)(z)\right| \\ &= \sup_{z\in\mathbb{B}_n} \mu(|z|)|u(z)||f_{\varphi(b)}(\varphi(z))| \\ &\geq \mu(|b|)|u(b)| \log \frac{e}{1-|\varphi(b)|^2}, \end{aligned} \tag{2.4}$$

which implies (2.2).

Conversely, assume that (2.2) holds. Then, for any $f \in H^\infty_{\log}$, we have

$$\mu(|z|)\left|(uC_\varphi f)(z)\right| = \mu(|z|)\left|u(z)\right|\left|f(\varphi(z))\right| \leq \mu(|z|)\left|u(z)\right| \log \frac{e}{1-\left|\varphi(z)\right|^2} \|f\|_{H^\infty_{\log}}. \tag{2.5}$$

Taking the supremum in (2.5) over $\mathbb{B}_n$ and using condition (2.2), the boundedness of the operator $uC_\varphi : H^\infty_{\log} \to H^\infty_\mu$ follows, as desired. □

Theorem 2.4. *Assume that $u \in H(\mathbb{B}_n)$, φ is a holomorphic self-map of $\mathbb{B}_n$, and μ is a normal function on $[0,1)$. Then, $uC_\varphi : H^\infty_{\log} \to H^\infty_\mu$ is compact if and only if $u \in H^\infty_\mu$ and*

$$\lim_{|\varphi(z)|\to 1} \mu(|z|)\left|u(z)\right| \log \frac{e}{1-\left|\varphi(z)\right|^2} = 0. \tag{2.6}$$

Proof. Assume that $uC_\varphi : H^\infty_{\log} \to H^\infty_\mu$ is compact. Then, it is obvious that $uC_\varphi : H^\infty_{\log} \to H^\infty_\mu$ is bounded. Taking the function $f(z) = 1 \in H^\infty_{\log}$, we see that $u \in H^\infty_\mu$. Let $(\varphi(z_k))_{k\in\mathbb{N}}$ be a sequence in $\mathbb{B}_n$ such that $\lim_{k\to\infty}|\varphi(z_k)| = 1$. Set

$$f_k(z) = \left(\log \frac{e}{1-\langle z, \varphi(z_k)\rangle}\right)^2 \left(\log \frac{e}{1-\left|\varphi(z_k)\right|^2}\right)^{-1}, \quad k \in \mathbb{N}. \tag{2.7}$$

It is easy to see that $\sup_{k\in\mathbb{N}}\|f_k\|_{H^\infty_{\log}} < \infty$. Moreover, $f_k \to 0$ uniformly on compact subsets of $\mathbb{B}_n$ as $k \to \infty$. By Lemma 2.1,

$$\lim_{k\to\infty} \left\|uC_\varphi f_k\right\|_{H^\infty_\mu} = 0. \tag{2.8}$$

We have

$$\left\|uC_\varphi f_k\right\|_{H^\infty_\mu} = \sup_{z\in\mathbb{B}_n} \mu(|z|)\left|u(z) f_k(\varphi(z))\right| \ge \mu(\left|z_k\right|)\left|u(z_k)\right| \log \frac{e}{1-|\varphi(z_k)|^2}, \tag{2.9}$$

which together with (2.8) implies that

$$\lim_{k\to\infty} \mu(\left|z_k\right|)\left|u(z_k)\right| \log \frac{e}{1-\left|\varphi(z_k)\right|^2} = 0. \tag{2.10}$$

This proves that (2.6) holds.

Conversely, assume that $u \in H^\infty_\mu$ and (2.6) holds. From this, it follows that (2.2) holds; hence $uC_\varphi : H^\infty_{\log} \to H^\infty_\mu$ is bounded. In order to prove that $uC_\varphi : H^\infty_{\log} \to H^\infty_\mu$ is compact, according to Lemma 2.1, it suffices to show that if $(f_k)_{k\in\mathbb{N}}$ is a bounded sequence in $H^\infty_{\log}$ converging to 0 uniformly on compact subsets of $\mathbb{B}_n$, then

$$\lim_{k\to\infty} \left\|uC_\varphi f_k\right\|_{H^\infty_\mu} = 0. \tag{2.11}$$

Let $(f_k)_{k\in\mathbb{N}}$ be a bounded sequence in $H^\infty_{\log}$ such that $f_k \to 0$ uniformly on compact subsets of $\mathbb{B}_n$ as $k \to \infty$. By (2.6), we have that for any $\varepsilon > 0$, there is a constant $\delta \in (0,1)$ such that

$$\mu(|z|)\left|u(z)\right| \log \frac{e}{1-\left|\varphi(z)\right|^2} < \varepsilon \tag{2.12}$$

whenever $\delta < |\varphi(z)| < 1$. Let $K = \{w \in \mathbb{B}_n : |w| \le \delta\}$. Equation (2.12) along with the fact that $u \in H_\mu^\infty$ implies

$$\begin{aligned}
\|uC_\varphi f_k\|_{H_\mu^\infty} &= \sup_{z\in\mathbb{B}_n} \mu(|z|)\left|(uC_\varphi f_k)(z)\right| \\
&= \sup_{z\in\mathbb{B}_n} \mu(|z|)\left|u(z) f_k(\varphi(z))\right| \\
&\le \left(\sup_{\{z\in\mathbb{B}_n : |\varphi(z)|\le\delta\}} + \sup_{\{z\in\mathbb{B}_n : \delta\le|\varphi(z)|<1\}}\right)\mu(|z|)\left|u(z)\right|\left|f_k(\varphi(z))\right| \\
&\le \|u\|_{H_\mu^\infty}\sup_{w\in K}\left|f_k(w)\right| + \sup_{\{z\in\mathbb{B}_n : \delta\le|\varphi(z)|<1\}} \mu(|z|)\left|u(z)\right| \log\frac{e}{1-\left|\varphi(z)\right|^2}\|f_k\|_{H_{\log}^\infty} \\
&\le \|u\|_{H_\mu^\infty}\sup_{w\in K}\left|f_k(w)\right| + C\varepsilon.
\end{aligned} \tag{2.13}$$

Observe that K is a compact subset of $\mathbb{B}_n$ so that

$$\lim_{k\to\infty}\sup_{w\in K}\left|f_k(w)\right| = 0. \tag{2.14}$$

With the aid of the above inequality, we can deduce that

$$\limsup_{k\to\infty}\|uC_\varphi f_k\|_{H_\mu^\infty} \le C\varepsilon \tag{2.15}$$

by letting $k \to \infty$. Since ε is an arbitrary positive number, it follows that the last limit is equal to zero. Therefore, $uC_\varphi : H_{\log}^\infty \to H_\mu^\infty$ is compact. The proof is complete. □

Theorem 2.5. *Assume that* $u \in H(\mathbb{B}_n)$, φ *is a holomorphic self-map of* $\mathbb{B}_n$, *and* μ *is a normal function on* $[0,1)$. *Then,* $uC_\varphi : H_{\log}^\infty \to H_{\mu,0}^\infty$ *is compact if and only if* $u \in H_{\mu,0}^\infty$ *and*

$$\lim_{|\varphi(z)|\to 1}\mu(|z|)\left|u(z)\right|\log\frac{e}{1-\left|\varphi(z)\right|^2} = 0. \tag{2.16}$$

Proof. Assume that $uC_\varphi : H_{\log}^\infty \to H_{\mu,0}^\infty$ is compact. Then, it is clear that $uC_\varphi : H_{\log}^\infty \to H_\mu^\infty$ is compact, and hence (2.16) holds. In addition, taking the function given by $f(z) = 1$, we get $u \in H_{\mu,0}^\infty$.

Conversely, suppose that $u \in H_{\mu,0}^\infty$ and (2.16) holds. In the proof of the implication we follow the lines, for example, of the proof of Lemma 4.2 in [24]. From (2.16), it follows that for every $\varepsilon > 0$, there exists a $\delta \in (0,1)$ such that

$$\mu(|z|)\left|u(z)\right|\log\frac{e}{1-\left|\varphi(z)\right|^2} < \varepsilon \tag{2.17}$$

when $\delta < |\varphi(z)| < 1$. From the assumption $u \in H_{\mu,0}^\infty$, we have that for the above ε, there exists an $r \in (0,1)$ such that when $r < |z| < 1$,

$$\mu(|z|)\left|u(z)\right| < \frac{\varepsilon}{\log\left(e/(1-\delta^2)\right)}. \tag{2.18}$$

Therefore, if $r < |z| < 1$ and $\delta < |\varphi(z)| < 1$, we obtain

$$\mu(|z|)\left|u(z)\right| \log \frac{e}{1-\left|\varphi(z)\right|^2} < \varepsilon. \tag{2.19}$$

If $|\varphi(z)| \le \delta$ and $r < |z| < 1$, we have that

$$\mu(|z|)\left|u(z)\right| \log \frac{e}{1-\left|\varphi(z)\right|^2} \le \mu(|z|)\left|u(z)\right| \log \frac{e}{1-\delta^2} < \varepsilon. \tag{2.20}$$

Combining (2.19) with (2.20), we get

$$\lim_{|z|\to 1} \mu(|z|)\left|u(z)\right| \log \frac{e}{1-\left|\varphi(z)\right|^2} = 0. \tag{2.21}$$

On the other hand, from (1.5) we have that

$$\mu(|z|)\left|(uC_\varphi f)(z)\right| \le \mu(|z|)\left|u(z)\right| \log \frac{e}{1-\left|\varphi(z)\right|^2} \|f\|_{H^\infty_{\log}}.$$

Taking the supremum in the above inequality over all $f \in H^\infty_{\log}$ such that $\|f\|_{H^\infty_{\log}} \le 1$, then letting $|z| \to 1$, by (2.21) it follows that

$$\lim_{|z|\to 1} \sup_{\|f\|_{H^\infty_{\log}} \le 1} \mu(|z|)\left|(uC_\varphi(f))(z)\right| = 0. \tag{2.22}$$

From this and by employing Lemma 2.2, we see that $uC_\varphi : H^\infty_{\log} \to H^\infty_{\mu,0}$ is compact. The proof is complete. □

Similar to the proofs of Theorems 2.3 and 2.4, we easily get the following two results. We also omit their proofs.

Theorem 2.6. *Assume that $u \in H(\mathbb{B}_n)$ and φ is a holomorphic self-map of $\mathbb{B}_n$. Then, the following statements hold.*

(a) $uC_\varphi : H^\infty_{\log} \to H^\infty_{\log}$ *is bounded if and only if*

$$\sup_{z\in\mathbb{B}_n} \frac{\left|u(z)\right| \log\left(e/\left(1-\left|\varphi(z)\right|^2\right)\right)}{\log\left(e/\left(1-|z|^2\right)\right)} < \infty. \tag{2.23}$$

(b) $uC_\varphi : H^\infty_{\log} \to H^\infty_{\log}$ *is compact if and only if $u \in H^\infty_{\log}$ and*

$$\lim_{|\varphi(z)|\to 1} \frac{\left|u(z)\right| \log\left(e/\left(1-\left|\varphi(z)\right|^2\right)\right)}{\log\left(e/\left(1-|z|^2\right)\right)} = 0. \tag{2.24}$$

Theorem 2.7. *Assume that $u \in H(\mathbb{B}_n)$ and φ is a holomorphic self-map of $\mathbb{B}_n$. Then, the following statements hold.*

(a) $uC_\varphi : H^\infty_{\log} \to H^\infty$ *is bounded if and only if*

$$\sup_{z\in\mathbb{B}_n} \left|u(z)\right| \log \frac{e}{1-\left|\varphi(z)\right|^2} < \infty. \tag{2.25}$$

(b) $uC_\varphi : H^\infty_{\log} \to H^\infty$ *is compact if and only if $u \in H^\infty$ and*

$$\lim_{|\varphi(z)|\to 1} \left|u(z)\right| \log \frac{e}{1-\left|\varphi(z)\right|^2} = 0. \tag{2.26}$$

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