Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2009, Article ID 725846, 16 pages
doi:10.1155/2009/725846

Research Article

Stochastic Passivity of Uncertain Neural Networks with Time-Varying Delays

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Received 22 July 2009; Accepted 18 October 2009

Recommended by Elena Litsyn

The passivity problem is investigated for a class of stochastic uncertain neural networks with time-varying delay as well as generalized activation functions. By constructing appropriate Lyapunov-Krasovskii functionals, and employing Newton-Leibniz formulation, the free-weighting matrix method, and stochastic analysis technique, a delay-dependent criterion for checking the passivity of the addressed neural networks is established in terms of linear matrix inequalities (LMIs), which can be checked numerically using the effective LMI toolbox in MATLAB. An example with simulation is given to show the effectiveness and less conservatism of the proposed criterion. It is noteworthy that the traditional assumptions on the differentiability of the time-varying delays and the boundedness of its derivative are removed.

1. Introduction

During the last two decades, many artificial neural networks have been extensively investigated and successfully applied to various areas such as signal processing, pattern recognition, associative memory, and optimization problems [1]. In such applications, it is of prime importance to ensure that the designed neural networks are stable [2].

In hardware implementation, time delays are likely to be present due to the finite switching speed of amplifiers and communication time. It has also been shown that the processing of moving images requires the introduction of delay in the signal transmitted through the networks [3]. The time delays are usually variable with time, which will affect the stability of designed neural networks and may lead to some complex dynamic behavior such as oscillation, bifurcation, or chaos [4]. Therefore, the study of stability with consideration of time delays becomes extremely important to manufacture high quality neural networks [5]. Many important results on stability of delayed neural networks have been reported, see [1–10] and the references therein for some recent publications.

It is also well known that parameter uncertainties, which are inherent features of many physical systems, are great sources of instability and poor performance [11]. These uncertainties may arise due to the variations in system parameters, modelling errors, or some ignored factors [12]. It is not possible to perfectly characterize the evolution of an uncertain dynamical system as a deterministic set of state equations [13]. Recently, the problem on robust stability analysis of uncertain neural networks with delays has been extensively investigated, see [11–14] and the references therein for some recent publications.

Just as pointed out in [15], in real nervous systems, synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. In the implementation of artificial neural networks, noise is unavoidable and should be taken into consideration in modelling. Therefore, it is of significant importance to consider stochastic effects to the dynamical behavior of neural networks [16]. Some recent interest results on stability of stochastic neural networks can be found, see [15–26] and the references therein for some recent publications.

On the other hand, the passivity theory is another effective tool to the stability analysis of nonlinear system [27]. The main idea of passivity theory is that the passive properties of system can keep the system internal stability [27]. Thus, the passivity theory has received a lot of attention from the control community since 1970s [28–31]. Recently, the passivity theory for delayed neural networks was investigated, some criteria checking the passivity were provided for certain or uncertain neural networks, see [32–38] and references therein. In [32], the passivity-based approach is used to derive stability conditions for dynamic neural networks with different time scales. In [33–36], authors investigated the passivity of neural networks with time-varying delay. In [37, 38], stochastic neural networks with time-varying delays were considered, several sufficient conditions checking the passivity were obtained. It is worth pointing out that, the given criteria in [33–37] have been based on the following assumptions: (1) the time-varying delays are continuously differentiable; (2) the derivative of time-varying delay is bounded and is smaller than one; (3) the activation functions are bounded and monotonically nondecreasing. However, time delays can occur in an irregular fashion, and sometimes the time-varying delays are not differentiable. In such a case, the methods developed in [33–38] may be difficult to be applied, and it is therefore necessary to further investigate the passivity problem of neural networks with time-varying delays under *milder* assumptions. To the best of our knowledge, few authors have considered the passivity problem for stochastic uncertain neural networks with time-varying delays as well as generalized activation functions.

Motivated by the above discussions, the objective of this paper is to study the passivity of stochastic uncertain neural networks with time-varying delays as well as generalized activation functions by employing a combination of Lyapunov functional, the free-weighting matrix method and stochastic analysis technique. The obtained sufficient conditions require *neither* the differentiability of time-varying delays *nor* the monotony of the activation functions, and are expressed in terms of linear matrix inequalities (LMIs), which can be checked numerically using the effective LMI toolbox in MATLAB. An example is given to show the effectiveness and less conservatism of the proposed criterion.

2. Problem Formulation and Preliminaries

In this paper, we consider the following stochastic uncertain neural networks with time-varying delay:

$$dx(t) = \left[-(C + \Delta C(t))x(t) + (A + \Delta A(t))f(x(t)) + (B + \Delta B(t))f(x(t - \tau(t))) + u(t)\right]dt \\ + \sigma(t, x(t), x(t - \tau(t)))d\omega(t) \tag{2.1}$$

for $t \geq 0$, where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in R^n$ is the state vector of the network at time t, n corresponds to the number of neurons; $C = \operatorname{diag}(c_1, c_2, \ldots, c_n)$ is a positive diagonal matrix, $A = (a_{ij})_{n\times n}$, and $B = (b_{ij})_{n\times n}$ are known constant matrices; $\Delta C(t)$, $\Delta A(t)$ and $\Delta B(t)$ are time-varying parametric uncertainties; $\sigma(t, x(t), x(t - \tau(t))) \in R^{n\times n}$ is the diffusion coefficient matrix and $\omega(t) = (\omega_1(t), \omega_2(t), \ldots, \omega_n(t))^T$ is an n-dimensional Brownian motion defined on a complete probability space $(\Omega, F, \{F_t\}_{t\geq 0}, P)$ with a filtration $\{F_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and F_0 contains all P-null sets); $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^T$ denotes the neuron activation at time t; $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T \in R^n$ is a varying external input vector; $\tau(t) > 0$ is the time-varying delay, and is assumed to satisfy $0 \leq \tau(t) \leq \tau$, where τ is constant.

The initial condition associated with model (2.1) is given by

$$x(s) = \phi(s), \quad s \in [-\tau, 0]. \tag{2.2}$$

Let $x(t, \phi)$ denote the state trajectory of model (2.1) from the above initial condition and $x(t, 0)$ the corresponding trajectory with zero initial condition.

Throughout this paper, we make the following assumptions.

(H1) [33] The time-varying uncertainties $\Delta C(t)$, $\Delta A(t)$ and $\Delta B(t)$ are of the form

$$\Delta C(t) = H_1 G_1(t) E_1, \qquad \Delta A(t) = H_2 G_2(t) E_2, \qquad \Delta B(t) = H_3 G_3(t) E_3, \tag{2.3}$$

where H_1, H_2, H_3, E_1, E_2, and E_3 are known constant matrices of appropriate dimensions, $G_1(t)$, $G_2(t)$, and $G_3(t)$ are known time-varying matrices with Lebesgue measurable elements bounded by

$$G_1^T(t) G_1(t) \leq I, \qquad G_2^T(t) G_2(t) \leq I, \qquad G_3^T(t) G_3(t) \leq I. \tag{2.4}$$

(H2) [10] For any $j \in \{1, 2, \ldots, n\}$, $f_j(0) = 0$ and there exist constants F_j^- and F_j^+ such that

$$F_j^- \leq \frac{f_j(\alpha_1) - f_j(\alpha_2)}{\alpha_1 - \alpha_2} \leq F_j^+ \tag{2.5}$$

for all $\alpha_1 \neq \alpha_2$.

(H3) [15] There exist two scalars $\rho_1 > 0$, $\rho_2 > 0$ such that the following inequality:

$$\operatorname{trace}\left[\sigma^T(t, u, v)\sigma(t, u, v)\right] \leq \rho_1 u^T u + \rho_2 v^T v \tag{2.6}$$

holds for all $(t, u, v) \in R \times R^n \times R^n$.

Definition 2.1 (see [33]). System (2.1) is called globally passive in the sense of expectation if there exists a scalar $\gamma > 0$ such that

$$2E\left\{\int_0^{t_p} f^T(x(s))u(s)ds\right\} \geq -E\left\{\gamma\int_0^{t_p} u^T(s)u(s)ds\right\} \tag{2.7}$$

for all $t_p \geq 0$ and for all $x(t,0)$, where $E\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$.

To prove our results, the following lemmas that can be found in [39] are necessary.

Lemma 2.2 (see [39]). *For given matrices H, E, and F with $F^TF \leq I$ and a scalar $\varepsilon > 0$, the following holds:*

$$HFE + (HFE)^T \leq \varepsilon HH^T + \varepsilon^{-1}E^TE. \tag{2.8}$$

Lemma 2.3 (see [39]). *For any constant matrix $W \in R^{m\times m}$, $W > 0$, scalar $0 < h(t) < h$, vector function $\omega : [0,h] \rightarrow R^m$ such that the integrations concerned are well defined, then*

$$\left(\int_0^{h(t)} \omega(s)ds\right)^T W\left(\int_0^{h(t)} \omega(s)ds\right) \leq h(t)\int_0^{h(t)} \omega^T(s)W\omega(s)ds. \tag{2.9}$$

Lemma 2.4 (see [39]). *Given constant matrices P, Q, and R, where $P^T = P$, $Q^T = Q$, then*

$$\begin{bmatrix} P & R \\ R^T & -Q \end{bmatrix} < 0 \tag{2.10}$$

is equivalent to the following conditions:

$$Q > 0, \quad P + RQ^{-1}R^T < 0. \tag{2.11}$$

3. Main Results

For presentation convenience, in the following, we denote

$$F_1 = \mathrm{diag}\left(F_1^-F_1^+, F_2^-F_2^+, \dots, F_n^-F_n^+\right), \qquad F_2 = \mathrm{diag}\left(\frac{F_1^- + F_1^+}{2}, \frac{F_2^- + F_2^+}{2}, \dots, \frac{F_n^- + F_n^+}{2}\right). \tag{3.1}$$

Theorem 3.1. *Under assumptions* (H1)–(H3), *model* (2.1) *is passive in the sense of expectation if there exist two scalars $\gamma > 0$, $\lambda > 0$, three symmetric positive definite matrices P_i ($i = 1,2,3$), two* *positive diagonal matrices L and S, and matrices Q_i ($i = 1,2,3,4$) such that the following two LMIs hold:*

$$P_1 < \lambda I, \tag{3.2}$$

$$\Omega = \begin{bmatrix} \Omega_1 & \Omega_2 \\ * & \Omega_3 \end{bmatrix} < 0, \tag{3.3}$$

where

$$\Omega_1 = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & Q_2B & Q_3 & 0 & Q_2 \\ * & \Omega_{22} & Q_1A & Q_1B & 0 & 0 & Q_1 \\ * & * & \Omega_{33} & 0 & 0 & 0 & -I \\ * & * & * & \Omega_{44} & F_2S & 0 & 0 \\ * & * & * & * & \Omega_{55} & Q_4 & 0 \\ * & * & * & * & * & -P_2 & 0 \\ * & * & * & * & * & * & -\gamma I \end{bmatrix},$$

$$\Omega_2 = \begin{bmatrix} Q_2H_1 & Q_2H_2 & Q_2H_3 & Q_3 & Q_3 & Q_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Q_1H_1 & Q_1H_2 & Q_1H_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{3.4}$$

$$\Omega_3 = \operatorname{diag}\{-\varepsilon_4 I, -\varepsilon_5 I, -\varepsilon_6 I, -\tau P_3, -P_1, -P_1, -\varepsilon_1 I, -\varepsilon_2 I, -\varepsilon_3 I, -\tau P_3\},$$

in which $\Omega_{11} = P_2 - Q_2C - CQ_2^T + (\varepsilon_1+\varepsilon_4)E_1^TE_1 - Q_3 - Q_3^T - F_1L + (1+\tau)\lambda\rho_1 I$, $\Omega_{12} = P_1 - CQ_1^T - Q_2$, $\Omega_{13} = Q_2A + F_2L$, $\Omega_{22} = -Q_1 - Q_1^T + \tau P_3$, $\Omega_{33} = (\varepsilon_2+\varepsilon_5)E_2^TE_2 - L$, $\Omega_{44} = (\varepsilon_3+\varepsilon_6)E_3^TE_3 - S$, $\Omega_{55} = -Q_4 - Q_4^T - F_1S + (1+\tau)\lambda\rho_2 I$.

Proof. Let $y(t) = -(C+\Delta C(t))x(t) + (A+\Delta A(t))f(x(t)) + (B+\Delta B(t))f(x(t-\tau(t))) + u(t)$, $\alpha(t) = \sigma(t, x(t), x(t-\tau(t)))$, then model (2.1) is rewritten as

$$dx(t) = y(t)dt + \alpha(t)d\omega(t). \tag{3.5}$$

Consider the following Lyapunov-Krasovskii functional as

$$V(t,x(t)) = x^T(t)P_1x(t) + \int_{t-\tau}^{t} x^T(s)P_2x(s)ds + \int_{-\tau}^{0}\int_{t+\theta}^{t} y^T(s)P_3y(s)ds + \int_{-\tau}^{0}\int_{t+\theta}^{t} \operatorname{trace}\left[\alpha^T(s)P_1\alpha(s)\right]ds\,d\theta. \tag{3.6}$$

By Itô differential rule, the stochastic derivative of $V(t)$ along the trajectory of model (3.5) can be obtained as

$$\begin{aligned} dV(t,x(t)) = \Big\{ & 2x^T(t)P_1y(t) + \operatorname{trace}\left[\alpha^T(t)P_1\alpha(t)\right] + x^T(t)P_2x(t) - x^T(t-\tau)P_2x(t-\tau) \\ & + \tau y^T(t)P_3y(t) - \int_{t-\tau}^{t} y^T(s)P_3y(s)ds \\ & + \tau \operatorname{trace}\left[\alpha^T(t)P_1\alpha(t)\right] - \int_{t-\tau}^{t} \operatorname{trace}\left[\alpha^T(s)P_1\alpha(s)\right]ds \Big\} dt \\ & + \left[x^T(t)P_1\alpha(t) + \alpha^T(t)P_1x(t)\right]d\omega(t). \end{aligned} \tag{3.7}$$

From the definition of $y(t)$, we have

$$\begin{aligned} 0 = 2\left(y^T(t)Q_1 + x^T(t)Q_2\right)\big[& -y(t) - (C + \Delta C(t))x(t) + (A + \Delta A(t))f(x(t)) \\ & + (B + \Delta B(t))f(x(t-\tau(t))) + u(t)\big]. \end{aligned} \tag{3.8}$$

By assumption (H1) and Lemma 2.2, we get

$$\begin{aligned} -2y^T(t)Q_1\Delta C(t)x(t) &\le \varepsilon_1^{-1}y^T(t)Q_1H_1H_1^TQ_1^Ty(t) + \varepsilon_1x^T(t)E_1^TE_1x(t), \\ 2y^T(t)Q_1\Delta A(t)f(x(t)) &\le \varepsilon_2^{-1}y^T(t)Q_1H_2H_2^TQ_1^Ty(t) + \varepsilon_2f^T(x(t))E_2^TE_2f(x(t)), \\ 2y^T(t)Q_1\Delta B(t)f(x(t-\tau(t))) &\le \varepsilon_3^{-1}y^T(t)Q_1H_3H_3^TQ_1^Ty(t) + \varepsilon_3f^T(x(t-\tau(t)))E_3^TE_3f(x(t-\tau(t))), \\ -2x^T(t)Q_2\Delta C(t)x(t) &\le \varepsilon_4^{-1}x^T(t)Q_2H_1H_1^TQ_2^Tx(t) + \varepsilon_4x^T(t)E_1^TE_1x(t), \\ 2x^T(t)Q_2\Delta A(t)f(x(t)) &\le \varepsilon_5^{-1}x^T(t)Q_2H_2H_2^TQ_2^Tx(t) + \varepsilon_5f^T(x(t))E_2^TE_2f(x(t)), \\ 2x^T(t)Q_2\Delta B(t)f(x(t-\tau(t))) &\le \varepsilon_6^{-1}x^T(t)Q_2H_3H_3^TQ_2^Tx(t) + \varepsilon_6f^T(x(t-\tau(t)))E_3^TE_3f(x(t-\tau(t))). \end{aligned} \tag{3.9}$$

It follows from (3.8) and (3.9) that

$$\begin{aligned}
0 \leq\ & x^T(t)\Big[-2Q_2C + (\varepsilon_1+\varepsilon_4)E_1^TE_1 + \varepsilon_4^{-1}Q_2H_1H_1^TQ_2^T + \varepsilon_5^{-1}Q_2H_2H_2^TQ_2^T + \varepsilon_6^{-1}Q_2H_3H_3^TQ_2^T\Big]x(t) \\
& + 2x^T(t)\Big[-CQ_1^T - Q_2\Big]y(t) + 2x^T(t)Q_2Af(x(t)) + 2x^T(t)Q_2Bf(x(t-\tau(t))) + 2x^T(t)Q_2u(t) \\
& + y^T(t)\Big[-2Q_1 + \varepsilon_1^{-1}Q_1H_1H_1^TQ_1^T + \varepsilon_2^{-1}Q_1H_2H_2^TQ_1^T + \varepsilon_3^{-1}Q_1H_3H_3^TQ_1^T\Big]y(t) \\
& + 2y^T(t)Q_1Af(x(t)) + 2y^T(t)Q_1Bf(x(t-\tau(t))) + 2y^T(t)Q_1u(t) \\
& + (\varepsilon_2+\varepsilon_5)f^T(x(t))E_2^TE_2f(x(t)) + (\varepsilon_3+\varepsilon_6)f^T(x(t-\tau(t)))E_3^TE_3f(x(t-\tau(t))).
\end{aligned} \tag{3.10}$$

Integrating both sides of (3.5) from $t-\tau(t)$ to t, we have

$$x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} y(s)ds - \int_{t-\tau(t)}^{t} \alpha(s)d\omega(s) = 0. \tag{3.11}$$

Hence,

$$-2x^T(t)Q_3\left[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} y(s)ds - \int_{t-\tau(t)}^{t} \alpha(s)d\omega(s)\right] = 0. \tag{3.12}$$

By Lemmas 2.2 and 2.3, and noting $\tau(t) \leq \tau$, we get

$$\begin{aligned}
0 &= -2x^T(t)Q_3x(t) + 2x^T(t)Q_3x(t-\tau(t)) + 2x^T(t)Q_3\int_{t-\tau(t)}^{t} y(s)ds + 2x^T(t)Q_3\int_{t-\tau(t)}^{t} \alpha(s)d\omega(s) \\
&\leq -2x^T(t)Q_3x(t) + 2x^T(t)Q_3x(t-\tau(t)) + \tau x^T(t)Q_3P_3^{-1}Q_3^Tx(t) + \int_{t-\tau(t)}^{t} y^T(s)P_3y(s)ds \\
&\quad + x^T(t)Q_3P_1^{-1}Q_3^Tx(t) + \left(\int_{t-\tau(t)}^{t} \alpha(s)d\omega(s)\right)^T P_1\left(\int_{t-\tau(t)}^{t} \alpha(s)d\omega(s)\right).
\end{aligned} \tag{3.13}$$

Integrating both sides of (3.5) from $t-\tau$ to $t-\tau(t)$, we have

$$x(t-\tau(t)) - x(t-\tau) - \int_{t-\tau}^{t-\tau(t)} y(s)ds - \int_{t-\tau}^{t-\tau(t)} \alpha(s)d\omega(s) = 0. \tag{3.14}$$

Similarly, by using of the same way, and noting $\tau - \tau(t) \le \tau$, we get

$$\begin{aligned}
0 &= -2x^T(t-\tau(t))Q_4\left(x(t-\tau(t)) - x(t-\tau) - \int_{t-\tau}^{t-\tau(t)} y(s)ds - \int_{t-\tau}^{t-\tau(t)} \alpha(s)d\omega(s)\right) \\
&\le -2x^T(t-\tau(t))Q_4x(t-\tau(t)) + 2x^T(t-\tau(t))Q_4x(t-\tau) \\
&\quad + \tau x^T(t-\tau(t))Q_4P_3^{-1}Q_4^Tx(t-\tau(t)) + \int_{t-\tau}^{t-\tau(t)} y^T(s)P_3y(s)ds \\
&\quad + x^T(t)Q_4P_1^{-1}Q_4^Tx(t) + \left(\int_{t-\tau}^{t-\tau(t)} \alpha(s)d\omega(s)\right)^T P_1\left(\int_{t-\tau}^{t-\tau(t)} \alpha(s)d\omega(s)\right).
\end{aligned} \tag{3.15}$$

From assumption (H2), we have

$$\left(f_i(x_i(t)) - F_i^- x_i(t)\right)\left(f_i(x_i(t)) - F_i^+ x_i(t)\right) \le 0, \quad i = 1, 2, \ldots, n, \tag{3.16}$$

which are equivalent to

$$\begin{bmatrix} x_i(t) \\ f_i(x_i(t)) \end{bmatrix}^T \begin{bmatrix} F_i^- F_i^+ e_i e_i^T & -\dfrac{F_i^- + F_i^+}{2} e_i e_i^T \\ -\dfrac{F_i^- + F_i^+}{2} e_i e_i^T & e_i e_i^T \end{bmatrix} \begin{bmatrix} x_i(t) \\ f_i(x_i(t)) \end{bmatrix} \le 0, \quad i = 1, 2, \ldots, n, \tag{3.17}$$

where e_r denotes the unit column vector having 1 element on its rth row and zeros elsewhere. Let

$$L = \operatorname{diag}\{l_1, l_2, \ldots, l_n\}, \qquad S = \operatorname{diag}\{s_1, s_2, \ldots, s_n\}, \tag{3.18}$$

then

$$\sum_{i=1}^{n} l_i \begin{bmatrix} x_i(t) \\ f_i(x_i(t)) \end{bmatrix}^T \begin{bmatrix} F_i^- F_i^+ e_i e_i^T & -\dfrac{F_i^- + F_i^+}{2} e_i e_i^T \\ -\dfrac{F_i^- + F_i^+}{2} e_i e_i^T & e_i e_i^T \end{bmatrix} \begin{bmatrix} x_i(t) \\ f_i(x_i(t)) \end{bmatrix} \le 0, \tag{3.19}$$

that is

$$\begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} F_1L & -F_2L \\ -F_2L & L \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \le 0. \tag{3.20}$$

Similarly, one has

$$\begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix}^T \begin{bmatrix} F_1S & -F_2S \\ -F_2S & S \end{bmatrix} \begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix} \le 0. \tag{3.21}$$

It follows from (3.7), (3.10), (3.13), (3.15), (3.20) and (3.21) that

$$
\begin{aligned}
&dV(t,x(t)) \\
&\quad\le \Big\{ x^T(t)\Big[P_2 - 2Q_2C + (\varepsilon_1+\varepsilon_4)E_1^TE_1 + \varepsilon_4^{-1}Q_2H_1H_1^TQ_2^T + \varepsilon_5^{-1}Q_2H_2H_2^TQ_2^T \\
&\qquad\qquad + \varepsilon_6^{-1}Q_2H_3H_3^TQ_2^T - 2Q_3 + \tau Q_3P_3^{-1}Q_3^T + Q_3P_1^{-1}Q_3^T + Q_4P_1^{-1}Q_4^T - F_1L\Big]x(t) \\
&\qquad + 2x^T(t)\Big[P_1 - CQ_1^T - Q_2\Big]y(t) + 2x^T(t)[Q_2A + F_2L]f(x(t)) \\
&\qquad + 2x^T(t)Q_2Bf(x(t-\tau(t))) + 2x^T(t)Q_2u(t) + 2x^T(t)Q_3x(t-\tau(t)) \\
&\qquad + y^T(t)\Big[-2Q_1 + \varepsilon_1^{-1}Q_1H_1H_1^TQ_1^T + \varepsilon_2^{-1}Q_1H_2H_2^TQ_1^T + \varepsilon_3^{-1}Q_1H_3H_3^TQ_1^T + \tau P_3\Big]y^T(t) \\
&\qquad + 2y^T(t)Q_1Af(x(t)) + 2y^T(t)Q_1Bf(x(t-\tau(t))) + 2y^T(t)Q_1u(t) \\
&\qquad + f^T(x(t))\Big[(\varepsilon_2+\varepsilon_5)E_2^TE_2 - L\Big]f(x(t)) \\
&\qquad + f^T(x(t-\tau(t)))\Big[(\varepsilon_3+\varepsilon_6)E_3^TE_3 - S\Big]f(x(t-\tau(t))) \\
&\qquad + 2f^T(x(t-\tau(t)))F_2Sx(t-\tau(t)) \\
&\qquad + x^T(t-\tau(t))\Big(-2Q_4 + \tau Q_4P_3^{-1}Q_4^T - F_1S\Big)x(t-\tau(t)) \\
&\qquad + 2x^T(t-\tau(t))Q_4x(t-\tau) - x^T(t-\tau)P_2x(t-\tau) + (1+\tau)\operatorname{trace}\Big[\alpha^T(t)P_1\alpha(t)\Big] \\
&\qquad - \int_{t-\tau}^{t}\operatorname{trace}\Big[\alpha^T(s)P_1\alpha(s)\Big]ds + \left(\int_{t-\tau}^{t-\tau(t)}\alpha(s)d\omega(s)\right)^T P_1\left(\int_{t-\tau}^{t-\tau(t)}\alpha(s)d\omega(s)\right) \\
&\qquad + \left(\int_{t-\tau(t)}^{t}\alpha(s)d\omega(s)\right)^T P_1\left(\int_{t-\tau(t)}^{t}\alpha(s)d\omega(s)\right)\Big\}dt \\
&\qquad + \Big[x^T(t)P_1\alpha(t) + \alpha^T(t)P_1x(t)\Big]d\omega(t).
\end{aligned}
\tag{3.22}
$$

By assumption (H3) and inequality (3.2), we get

$$
\operatorname{trace}\Big[\alpha^T(t)P_1\alpha(t)\Big] \le \lambda\Big[\rho_1x^T(t)x(t) + \rho_2x^T(t-\tau(t))x(t-\tau(t))\Big]. \tag{3.23}
$$

From the proof of [19], we have

$$E\left\{\left(\int_{t-\tau}^{t-\tau(t)}\alpha(s)d\omega(s)\right)^T P_1\left(\int_{t-\tau}^{t-\tau(t)}\alpha(s)d\omega(s)\right)\right\} = E\left\{\int_{t-\tau}^{t-\tau(t)}\operatorname{trace}\left[\alpha^T(s)P_1\alpha(s)\right]ds\right\},$$
$$E\left\{\left(\int_{t-\tau(t)}^{t}\alpha(s)d\omega(s)\right)^T P_1\left(\int_{t-\tau(t)}^{t}\alpha(s)d\omega(s)\right)\right\} = E\left\{\int_{t-\tau(t)}^{t}\operatorname{trace}\left[\alpha^T(s)P_1\alpha(s)\right]ds\right\}. \tag{3.24}$$

Taking the mathematical expectation on both sides of (3.22), and noting (3.24), we get

$$E\left\{dV(t,x(t)) - 2f^T(x(t))u(t)dt - \gamma u^T(t)u(t)dt\right\} \le E\left\{\xi^T(t)\Pi\xi(t)dt\right\}, \tag{3.25}$$

where $\xi(t) = (x^T(t), y^T(t), f^T(x(t)), f^T(x(t-\tau(t))), x^T(t-\tau(t)), x^T(t-\tau), u^T(t))^T$, and

$$\Pi = \begin{bmatrix} \Pi_1 & P_1 - CQ_1^T - Q_2 & Q_2A + F_2L & Q_2B & Q_3 & 0 & Q_2 \\ * & \Pi_2 & Q_1A & Q_1B & 0 & 0 & Q_1 \\ * & * & \Pi_3 & 0 & 0 & 0 & -I \\ * & * & * & \Pi_4 & F_2S & 0 & 0 \\ * & * & * & * & \Pi_5 & Q_4 & 0 \\ * & * & * & * & * & -P_2 & 0 \\ * & * & * & * & * & * & -\gamma I \end{bmatrix} \tag{3.26}$$

with $\Pi_1 = P_2 - Q_2C - CQ_2^T + (\varepsilon_1+\varepsilon_4)E_1^TE_1 + \varepsilon_4^{-1}Q_2H_1H_1^TQ_2^T + \varepsilon_5^{-1}Q_2H_2H_2^TQ_2^T + \varepsilon_6^{-1}Q_2H_3H_3^TQ_2^T - Q_3 - Q_3^T + \tau Q_3P_3^{-1}Q_3^T + Q_3P_1^{-1}Q_3^T + Q_4P_1^{-1}Q_4^T - F_1L + (1+\tau)\lambda\rho_1 I$, $\Pi_2 = -Q_1 - Q_1^T + \varepsilon_1^{-1}Q_1H_1H_1^TQ_1^T + \varepsilon_2^{-1}Q_1H_2H_2^TQ_1^T + \varepsilon_3^{-1}Q_1H_3H_3^TQ_1^T + \tau P_3$, $\Pi_3 = (\varepsilon_2+\varepsilon_5)E_2^TE_2 - L$, $\Pi_4 = (\varepsilon_3+\varepsilon_6)E_3^TE_3 - S$, $\Pi_5 = -Q_4 - Q_4^T + \tau Q_4P_3^{-1}Q_4^T - F_1S + (1+\tau)\lambda\rho_2 I$.

It is easy to verify the equivalence of $\Pi < 0$ and $\Omega < 0$ by using Lemma 2.4. Thus, one can derive from (3.3) and (3.25) that

$$\frac{E\{dV(t,x(t))\}}{dt} - E\left\{2f^T(x(t))u(t) + \gamma u^T(t)u(t)\right\} \le 0. \tag{3.27}$$

From (3.27) and the definition of $V(t,x(t))$, we can get

$$2E\left\{\int_0^{t_p} f^T(x(s))u(s)ds\right\} \ge -\gamma E\left\{\int_0^{t_p} u^T(s)u(s)ds\right\}. \tag{3.28}$$

From Definition 2.1, we know that the stochastic neural networks (2.1) are globally passive in the sense of expectation, and the proof of Theorem 3.1 is then completed. □

Remark 3.2. Assumption (H2) was first proposed in [10]. The constants F_j^- and F_j^+ ($i = 1, 2, \ldots, n$) in assumption (H2) are allowed to be positive, negative or zero. Hence, Assumption (H2) is weaker than the assumption in [27–37]. In addition, the conditions in [32–37] that the time-varying delay is differentiable and the derivative is bounded or smaller than one have been removed in this paper.

Remark 3.3. In [36], authors considered the passivity of uncertain neural networks with both discrete and distributed time-varying delays. In [37], authors considered the passivity for stochastic neural networks with time-varying delays and random abrupt changes. It is worth pointing out that, the method in this paper can also analyze the passivity for models in [36, 37].

Remark 3.4. It is known that the obtained criteria for checking passivity of neural networks depend on the constructed Lyapunov functionals or Lyapunov-Krasovskii functionals in varying degrees. Constructing proper Lyapunov functionals or Lyapunov-Krasovskii functionals can reduce conservatism. Recently, the delay fractioning approach has been used to investigate global synchronization of delayed complex networks with stochastic disturbances, which has shown the potential of reducing conservatism [22]. Using the delay fractioning approach, we can also investigate the passivity of delayed neural networks. The corresponding results will appear in the near future.

Remark 3.5. When we do not consider the stochastic effect, model (2.1) turns into the following model:

$$\frac{dx(t)}{dt} = -(C + \Delta C(t))x(t) + (A + \Delta A(t))f(x(t)) + (B + \Delta B(t))f(x(t - \tau(t))) + u(t). \tag{3.29}$$

Furthermore, model (3.29) also comprises the following neural network model with neither stochastic effect nor uncertainty

$$\frac{dx(t)}{dt} = -Cx(t) + Af(x(t)) + Bf(x(t - \tau(t))) + u(t) \tag{3.30}$$

which have been considered in [33, 34]. For models (3.29) and (3.30), one can get the following results.

Corollary 3.6. *Under assumptions* (H1)-(H2), *model* (3.29) *is passive if there exist a scalar* $\gamma > 0$, *three symmetric positive definite matrices* P_i ($i = 1, 2, 3$), *two positive diagonal matrices* L *and* S, *and matrices* Q_i ($i = 1, 2, 3, 4$) *such that the following LMI holds:*

$$\Omega = \begin{bmatrix} \Omega_1 & \Omega_2 \\ * & \Omega_3 \end{bmatrix} < 0, \tag{3.31}$$

where

$$\Omega_1 = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & Q_2B & Q_3 & 0 & Q_2 \\ * & \Omega_{22} & Q_1A & Q_1B & 0 & 0 & Q_1 \\ * & * & \Omega_{33} & 0 & 0 & 0 & -I \\ * & * & * & \Omega_{44} & F_2S & 0 & 0 \\ * & * & * & * & \Omega_{55} & Q_4 & 0 \\ * & * & * & * & * & -P_2 & 0 \\ * & * & * & * & * & * & -\gamma I \end{bmatrix},$$

$$\Omega_2 = \begin{bmatrix} Q_2H_1 & Q_2H_2 & Q_2H_3 & Q_3 & Q_3 & Q_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Q_1H_1 & Q_1H_2 & Q_1H_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{3.32}$$

$$\Omega_3 = \operatorname{diag}\{-\varepsilon_4 I, -\varepsilon_5 I, -\varepsilon_6 I, -\tau P_3, -P_1, -P_1, -\varepsilon_1 I, -\varepsilon_2 I, -\varepsilon_3 I, -\tau P_3\},$$

in which $\Omega_{11} = P_2 - Q_2C - CQ_2^T + (\varepsilon_1 + \varepsilon_4)E_1^TE_1 - Q_3 - Q_3^T - F_1L$, $\Omega_{12} = P_1 - CQ_1^T - Q_2$, $\Omega_{13} = Q_2A + F_2L$, $\Omega_{22} = -Q_1 - Q_1^T + \tau P_3$, $\Omega_{33} = (\varepsilon_2 + \varepsilon_5)E_2^TE_2 - L$, $\Omega_{44} = (\varepsilon_3 + \varepsilon_6)E_3^TE_3 - S$, $\Omega_{55} = -Q_4 - Q_4^T - F_1S$.

Corollary 3.7. *Under assumption* (H2)*, model* (3.30) *is passive if there exist a scalar* $\gamma > 0$*, three symmetric positive definite matrices* P_i *(*$i = 1,2,3$*), two positive diagonal matrices* L *and* S*, and matrices* Q_i *(*$i = 1,2,3,4$*) such that the following LMI holds:*

$$\Pi = \begin{bmatrix} \Pi_1 & \Pi_2 & \Pi_3 & Q_2B & Q_3 & 0 & Q_2 & Q_3 & Q_3 & Q_4 & 0 \\ * & \Pi_4 & Q_1A & Q_1B & 0 & 0 & Q_1 & 0 & 0 & 0 & 0 \\ * & * & -L & 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 \\ * & * & * & -S & F_2S & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_5 & Q_4 & 0 & 0 & 0 & 0 & Q_4 \\ * & * & * & * & * & -P_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\gamma I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\tau P_3 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -P_1 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -P_1 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\tau P_3 \end{bmatrix} < 0, \tag{3.33}$$

where $\Pi_1 = P_2 - Q_2C - CQ_2^T - Q_3 - Q_3^T - F_1L$, $\Pi_2 = P_1 - CQ_1^T - Q_2$, $\Pi_3 = Q_2A + F_2L$, $\Pi_4 = -Q_1 - Q_1^T + \tau P_3$, $\Pi_5 = -Q_4 - Q_4^T - F_1S$.

4. An Example

Consider a two-neuron neural network (3.30), where

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 0.9 \end{bmatrix}, \qquad A = \begin{bmatrix} 2.3 & -0.1 \\ -6 & 2.8 \end{bmatrix}, \qquad B = \begin{bmatrix} -1.9 & -0.1 \\ -0.3 & -9.7 \end{bmatrix},$$
$$f_1(z) = \tanh(z), \quad f_2(z) = \tanh(z), \quad \tau(t) = 2|\sin t|, \quad u_1(t) = -0.2t\cos t, \quad u_2(t) = -0.5\sin t. \tag{4.1}$$

Figure 1 depicts the states of the considered network with initial conditions $x_1(t) = 0.5$, $x_2(t) = 0.45$, $t \in [-2, 0]$.

It can be verified that assumption (H2) is satisfied, and $F_1 = 0$, $F_2 = \mathrm{diag}\{0.5, 0.5\}$, $\tau = 2$. By the Matlab LMI Control Toolbox, we find a solution to the LMI in (3.33) as follows:

$$\begin{gathered}
P_1 = 10^{-10}\begin{bmatrix} 0.6744 & 0.1008 \\ 0.1008 & 0.4522 \end{bmatrix}, \qquad P_2 = 10^{-10}\begin{bmatrix} 0.2490 & 0.0755 \\ 0.0755 & 0.1154 \end{bmatrix}, \\
P_3 = 10^{-10}\begin{bmatrix} 0.1735 & 0.0193 \\ 0.0193 & 0.1175 \end{bmatrix}, \\
Q_1 = 10^{-10}\begin{bmatrix} 0.1724 & 0.0342 \\ 0.0246 & 0.0149 \end{bmatrix}, \qquad Q_2 = 10^{-10}\begin{bmatrix} 0.2615 & 0.0622 \\ 0.1216 & 0.0513 \end{bmatrix}, \\
Q_3 = 10^{-10}\begin{bmatrix} 0.4765 & 0.1639 \\ 0.0668 & 0.4203 \end{bmatrix}, \qquad Q_4 = 10^{-10}\begin{bmatrix} 0.4126 & 0.0233 \\ 0.0216 & 0.3631 \end{bmatrix}, \\
L = 10^{-9}\begin{bmatrix} 0.1100 & 0 \\ 0 & 0.0811 \end{bmatrix}, \qquad S = 10^{-9}\begin{bmatrix} 0.2772 & 0 \\ 0 & 0.2750 \end{bmatrix}, \\
\gamma = 6.7369 \times 10^{9}.
\end{gathered} \tag{4.2}$$

Therefore, by Corollary 3.7, we know that model (3.30) is passive. It should be pointed out that the conditions in [33–36] cannot be applied to this example since that require the differentiability of the time-varying delay.

5. Conclusions

In this paper, the passivity has been investigated for a class of stochastic uncertain neural networks with time-varying delay as well as generalized activation functions. By employing a combination of Lyapunov-Krasovskii functionals, the free-weighting

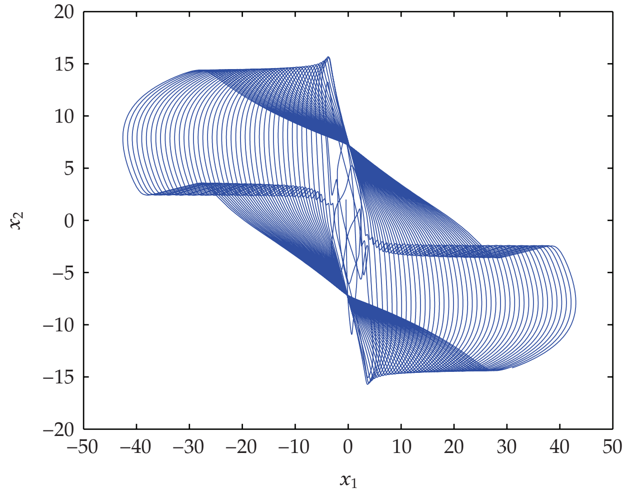

Figure 1: State responses of $x_1(t)$ and $x_2(t)$.

matrix method, Newton-Leibniz formulation, and stochastic analysis technique, a delay-independent criterion for checking the passivity of the addressed neural networks has been established in terms of linear matrix inequalities (LMIs), which can be checked numerically using the effective LMI toolbox in MATLAB. The obtained results generalize and improve the earlier publications and remove the traditional assumptions on the differentiability of the time-varying delay and the boundedness of its derivative. An example has been provided to demonstrate the effectiveness and less conservatism of the proposed criterion.

We would like to point out that it is possible to generalize our main results to more complex neural networks, such as neural networks with discrete and distributed delays [10, 26], and neural networks of neutral-type [7, 20], neural networks with Markovian jumping [24, 25]. The corresponding results will appear in the near future.

Acknowledgments

The authors would like to thank the reviewers and the editor for their valuable suggestions and comments which have led to a much improved paper. This work was supported by the National Natural Science Foundation of China under Grants 60974132 and 50608072, and in part by Natural Science Foundation Project of CQ CSTC2008BA6038 and 2008BB2351, and Scientific & Technological Research Projects of CQ KJ090406, and the Ministry of Education for New Century Excellent Talent Support Program.

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