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Research Article

Nearly Ternary Quadratic Higher Derivations on Non-Archimedean Ternary Banach Algebras: A Fixed Point Approach

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We investigate the stability and superstability of ternary quadratic higher derivations in non-Archimedean ternary algebras by using a version of fixed point theorem via quadratic functional equation.

1. Introduction

In 1897, Hensel [1] has introduced a normed space which does not have the Archimedean property. It turned out that non-Archimedean spaces have many nice applications [2, 3].

A non-Archimedean field is a field $\mathbb{K}$ equipped with a function (valuation) $|\cdot|$ from $\mathbb{K}$ into $[0,\infty)$ such that $|r| = 0$ if and only if $r = 0$, $|rs| = |r||s|$, and $|r+s| \le \max\{|r|, |s|\}$ for all $r, s \in \mathbb{K}$. Clearly, $|1| = |-1| = 1$ and $|n| \le 1$ for all $n \in \mathbb{N}$. An example of a non-Archimedean valuation is the mapping $|\cdot|$ taking everything but 0 into 1 and $|0| = 0$. This valuation is called trivial (see [4–12]).

Definition 1.1. Let X be a vector space over a scalar field $\mathbb{K}$ with a non-Archimedean nontrivial valuation $|\cdot|$. A function $\|\cdot\| : X \to \mathbb{R}$ is a non-Archimedean norm (valuation) if it satisfies the following conditions:

(NA_1) $\|x\| = 0$ if and only if $x = 0$;

(NA_2) $\|rx\| = |r|\|x\|$ for all $r \in \mathbb{K}$ and $x \in X$;

(NA_3) $\|x + y\| \leq \max\{\|x\|, \|y\|\}$ for all $x, y \in X$ (the strong triangle inequality).

Then, $(X, \|\cdot\|)$ is called a non-Archimedean normed space.

We note the inequality

$$\|x_m - x_l\| \leq \max\{\|x_{j+1} - x_j\| : l \leq j \leq m-1\} \quad (m > l), \tag{1.1}$$

where a sequence $\{x_m\}$ is Cauchy if and only if $\{x_{m+1} - x_m\}$ converges to zero in a non-Archimedean space. By a complete non-Archimedean space, we mean one in which every Cauchy sequence is convergent.

The most important examples of non-Archimedean spaces are p-adic numbers. A key property of p-adic numbers is that they do not satisfy the Archimedean axiom "for $x, y > 0$, there exists $n \in \mathbb{N}$ such that $x < ny$."

Let p be a prime number. For any nonzero rational number $x = (a/b)p^{n_x}$ such that a and b are integers not divisible by p, define the p-adic absolute value $|x|_p := p^{-n_x}$. Then $|\cdot|$ is a non-Archimedean norm on $\mathbb{Q}$. The completion of $\mathbb{Q}$ with respect to $|\cdot|$ is denoted by $\mathbb{Q}_p$ which is called the p-adic number field.

Note that if $p \geq 3$, then $|2^n| = 1$ in for each integer n.

Definition 1.2. Let X be a nonempty set and $d : X \times X \to [0, \infty]$ satisfy the following properties:

(D_1) $d(x, y) = 0$ if and only if $x = y$,

(D_2) $d(x, y) = d(y, x)$ (symmetry),

(D_3) $d(x, z) \leq \max\{d(x, y), d(y, z)\}$ (strong triangle inequality),

for all $x, y, z \in X$. Then (X, d) is called a non-Archimedean generalized metric space. (X, d) is called complete if every d-Cauchy sequence in X is d-convergent.

Example 1.3 (see [13]). For each nonempty set X, define

$$d(x, y) = \begin{cases} 0 & \text{if } x = y \\ \infty & \text{if } x \neq y. \end{cases} \tag{1.2}$$

Then d is a generalized non-Archimedean metric on X.

A non-Archimedean ternary algebra is a non-Archimedean vector space over a non-Archimedean field $\mathbb{K}$, endowed with a linear mapping, the so-called a ternary product, $(x, y, z) \to [xyz]$ of $A \times A \times A$ into A such that $[[xyz]tu] = [x[yzt]u] = [xy[ztu]]$ for all $x, y, z, t, u \in A$. If $(A, \cdot)$ is a usual binary non-Archimedean algebra, then an induced ternary multiplication can be defined by $[xyz] = (x \cdot y) \cdot z$. Hence, the non-Archimedean algebra is a natural generalization of the binary case. A normed non-Archimedean ternary algebra is a non-Archimedean ternary algebra with a norm $\|\cdot\|$ such that $\|[xyz]\| \leq \|x\|\|y\|\|z\|$

for $x, y, z \in A$. A Banach non-Archimedean ternary algebra is a normed non-Archimedean ternary algebra such that the normed non-Archimedean vector space with norm $\| \cdot \|$ is complete.

The ternary algebras have been studied in nineteenth century. Their structures appeared more or less naturally in various domains of mathematical physics and data processing. The discovery of the Nambu mechanics and the progress of quantum mechanics [14], as well as work of Okubo [15] on Yang-Baxter equation gave a significant development on ternary algebras (see also [16–20]).

We say that a functional equation (ξ) is stable if any function g satisfying the equation (ξ) *approximately* is near to true solution of (ξ). We say that a functional equation (ξ) is *superstable* if every approximately solution of (ξ) is an exact solution of it (see [21]).

The stability of functional equations was first introduced by Ulam [22] in 1940. In 1941, Hyers [23] gave a first affirmative answer to the question of Ulam for Banach spaces. In 1978, Rassias [24] generalized the theorem of Hyers for linear mappings by considering the stability problem with unbounded Cauchy differences $\|f(x+y) - f(x) - f(y)\| \le \epsilon(\|x\|^p + \|y\|^p)$, $(\epsilon > 0, p \in [0,1))$. In 1991, Gajda [25] answered the question for the case $p > 1$, which was raised by Rassias. This new concept is known as generalized Hyers-Ulam stability of functional equations (see [6–12, 17, 21, 25–58]). In 1982–1994, Rassias (see [59–66]) solved the Ulam problem for different mappings and for many Euler-Lagrange type quadratic mappings, by involving a product of different powers of norms. In addition, Rassias considered the mixed product sum of powers of norms control function [67].

In 1949, Bourgin [41] proved the following result, which is sometimes called the superstability of ring homomorphisms. Suppose that A and B are Banach algebras with unit. If $f : A \to B$ is a surjective mapping such that

$$\begin{aligned} \|f(x+y) - f(x) - f(y)\| &\le \epsilon, \\ \|f(xy) - f(x)f(y)\| &\le \delta \end{aligned} \tag{1.3}$$

for some $\epsilon \ge 0$, $\delta \ge 0$ and for all $x, y \in A$, then f is a ring homomorphism.

Badora [68] and Miura et al. [69] proved the Ulam-Hyers stability and the Isaac and Rassias type stability of derivations [30].

The functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y), \tag{1.4}$$

is related to a symmetric biadditive function [2, 27]. It is natural that this equation is called a *quadratic functional equation*. In particular, every solution of the quadratic equation (1.4) is said to be a *quadratic mapping*. It is well known that a mapping f between real vector spaces is quadratic if and only if there exists a unique symmetric bi-additive mapping B_1 such that $f(x) = B_1(x,x)$ for all x. The bi-additive mapping B_1 is given by $B_1(x,y) = (1/4)(f(x+y) - f(x-y))$.

The stability problem of quadratic functional equation (1.4) was proved by Skof [37] for functions $f : A \to B$, where A is normed space and B Banach space (see also [42, 43]).

Definition 1.4. A mapping $H : A \to B$ is called a ternary quadratic homomorphism between non-Archimedean ternary algebras A, B if

(1) H is a quadratic function,

(2) $H([xyz]) = [H(x)H(y)H(z)]$, for all $x, y, z \in A$.

For instance, let A be commutative ternary algebra, then the function $f : A \to A$ defined by $f(a) = a^2 (a \in A)$ is a quadratic homomorphism.

Definition 1.5. A mapping $D : A \to A$ is called a non-Archimedean ternary quadratic derivation on ternary non-Archimedean algebra A if

(1) D is a quadratic function,

(2) $D([xyz]) = [D(x)y^2z^2] + [x^2D(y)z^2] + [x^2y^2D(z)]$, for all $x, y, z \in A$.

For example, consider the algebra of 2×2 matrices

$$\mathcal{A} = \left\{ \begin{bmatrix} c_1 & c_2 \\ 0 & 0 \end{bmatrix} : c_1, c_2 \in \mathbb{C} \right\}, \tag{1.5}$$

then it is easy to see that $\mathcal{A}$ is a ternary algebra. Moreover, the function $f : \mathcal{A} \to \mathcal{A}$ defined by

$$f\left(\begin{bmatrix} c_1 & c_2 \\ 0 & 0 \end{bmatrix} \right) = \begin{bmatrix} 0 & c_2^2 \\ 0 & 0 \end{bmatrix}, \tag{1.6}$$

is a ternary quadratic derivation.

We note that ternary quadratic derivations and ternary ring derivations are different.

As another example, Let $\mathcal{A}$ be a Banach algebra. Then we take

$$\mathfrak{T} = \begin{bmatrix} 0 & \mathcal{A} & \mathcal{A} \\ 0 & 0 & \mathcal{A} \\ 0 & 0 & 0 \end{bmatrix}, \tag{1.7}$$

where $\mathfrak{T}$ is a ternary Banach algebra equipped with the usual matrix-like operations and the following norm:

$$\left\| \begin{bmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{bmatrix} \right\| = \|a\| + \|b\| + \|c\| \quad (a, b, c \in \mathcal{A}). \tag{1.8}$$

It is known that

$$\mathfrak{T}^* = \begin{bmatrix} o & \mathcal{A}^* & \mathcal{A}^* \\ 0 & 0 & \mathcal{A}^* \\ 0 & 0 & 0 \end{bmatrix} \tag{1.9}$$

is the dual of $\mathfrak{T}$ under the following norm:

$$\left\| \begin{bmatrix} 0 & f & g \\ 0 & 0 & h \\ 0 & 0 & 0 \end{bmatrix} \right\| = \max\{\|f\|, \|g\|, \|h\|\} \quad (f, g, h \in \mathcal{A}^*). \tag{1.10}$$

Let the left module action of $\mathfrak{T}$ on $\mathfrak{T}^*$ be trivial, and let the right module action of $\mathfrak{T}$ on $\mathfrak{T}^*$ is defined as follows:

$$\left\langle \begin{bmatrix} 0 & f & g \\ 0 & 0 & h \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{bmatrix} \right\rangle = g(acz), \tag{1.11}$$

for all $f, g, h \in \mathcal{A}^*$, $a, b, c, x, y, z \in \mathcal{A}$. Then $\mathfrak{T}^*$ is a Banach $\mathfrak{T}$-module. Let

$$\begin{bmatrix} 0 & k & g \\ 0 & 0 & h \\ 0 & 0 & 0 \end{bmatrix} \in \mathfrak{T}^*. \tag{1.12}$$

We define $D : \mathfrak{T} \to \mathfrak{T}^*$ by

$$D\left(\begin{bmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{bmatrix} \right) = \begin{bmatrix} 0 & k & g \\ 0 & 0 & h \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & ac \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (a, b, c \in \mathcal{A}). \tag{1.13}$$

Then it is easy to show that D is a ternary quadratic derivation from $\mathfrak{T}$ into $\mathfrak{T}^*$.

Definition 1.6. Let A, B be two non-Archimedean ternary normed algebras over a non-Archimedean field $\mathbb{K}$. For $m \in \mathbb{N} \cup \{0\} = \mathbb{N}_0$, a sequence $H = \{h_0, h_1, \ldots, h_m\}$ (resp., $H = \{h_0, h_1, \ldots, h_n, \ldots\}$) of quadratic mappings from A into B is called a ternary quadratic higher derivation of rank m (resp. infinite rank) from A into B if

$$h_n[xyz] = \sum_{i+j+k=n} [h_i(x)h_j(y)h_k(z)], \tag{1.14}$$

holds for each $n \in \{0, 1, \ldots, m\}$ (resp., $n \in \mathbb{N}_0$) and all $x, y, z \in A$. The ternary quadratic higher derivation H on A is said to be strong if $h_0(x) = x^2$ for all $x \in A$. Of course, a ternary quadratic higher derivation of rank 0 from A into B (resp., a strong ternary quadratic higher derivation of rank 1 on A) is a ternary quadratic homomorphism (resp., a ternary quadratic derivation). So a ternary quadratic higher derivation is a generalization of both a a ternary quadratic homomorphism and a ternary quadratic derivation.

Recently, Jung and Chang [70] have investigated the stability and superstability of higher derivations on rings. More recently, the first author of the present paper [46] has investigated the stability of homomorphisms and derivations on non-Archimedean Banach algebras, also Eshaghi Gordji and Alizadeh [47] by using fixed point methods, established the stability and superstability of derivations on non-Archimedean Banach algebras. In this paper, by using fixed point methods, we establish the stability and superstability of quadratic ternary higher derivations on non-Archimedean Banach ternary algebras.

We need the following fixed point theorem (see [13, 71]).

Theorem 1.7 (non-Archimedean alternative contraction principle). *Suppose that (X,d) is a non-Archimedean generalized complete metric space and $\Lambda : X \to X$ is a strictly contractive mapping, that is,*

$$d(\Lambda x, \Lambda y) \leq L d(x,y) \quad (x, y \in X), \tag{1.15}$$

for some $L < 1$. If there exists a nonnegative integer k such that $d(\Lambda^{k+1}x, \Lambda^k x) < \infty$ for some $x \in X$, then the followings are true:

(a) *the sequence $\{\Lambda^n x\}$ converges to a fixed point x^* of Λ;*

(b) *x^* is a unique fixed point of Λ in*

$$X^* = \left\{ y \in X \mid d\left(\Lambda^k x, y\right) < \infty \right\}, \tag{1.16}$$

(c) *If $y \in X^*$, then*

$$d(y, x^*) \leq d(\Lambda y, y). \tag{1.17}$$

2. Main Results

In this section, A denotes a non-Archimedean ternary normed algebra over a non-Archimedean field $\mathbb{K}$, and B is a non-Archimedean Banach ternary algebra over $\mathbb{K}$.

Theorem 2.1. *Let $\varphi : A \times A \times A \to [0,\infty)$ be a function. Suppose that $F = \{f_0, f_1, \ldots, f_n, \ldots\}$ be a sequence of mappings from A into B such that for each $n \in \mathbb{N}_0$, $f_n(0) = 0$,*

$$\| f_n(x+y) + f_n(x-y) - 2f_n(x) - 2f_n(y) \| \leq \varphi(x, y, o), \tag{2.1}$$

$$\left\| f_n[xyz] - \sum_{i+j+t=n} [f_i(x) f_j(y) f_t(z)] \right\| \leq \varphi(x, y, z), \tag{2.2}$$

for all $x, y, z \in A$. Suppose that there exist a natural number $k \in \mathbb{K}$ and $L \in (0,1)$, such that

$$|k|^2\varphi\left(k^{-1}x, k^{-1}y, k^{-1}z\right) \leq L\varphi(x,y,z), \tag{2.3}$$

for all $x, y, z \in A$. Then there exists a unique ternary quadratic higher derivation $H = \{h_0, h_1, \ldots, h_n, \ldots\}$ of any rank from A into B such that for each $n \in \mathbb{N}_0$,

$$\|f_n(x) - h_n(x)\| \leq \frac{L\psi(x)}{|k|^2}, \tag{2.4}$$

for all $x \in A$, where

$$\psi(x) = \max\{\varphi(0,0,0), \varphi(x,x,0), \varphi(2x,x,0), \ldots, \varphi((k-1)x,x,0)\} \quad (x \in A). \tag{2.5}$$

Proof. By induction on i, one can show that for each $n \in \mathbb{N}_0$, for all $x \in A$ and $i \geq 2$,

$$\left\|f_n(ix) - i^2 f_n(x)\right\| \leq \max\{\varphi(0,0,0), \varphi(x,x,0), \varphi(2x,x,0), \ldots, \varphi((i-1)x,x,0)\}. \tag{2.6}$$

Let $x = y$ in (2.1), then

$$\|f_n(2x) - 2f_n(x)\| \leq \max\{\varphi(0,0,0), \varphi(x,x,0)\}, \quad n \in \mathbb{N}_0, \ x \in A. \tag{2.7}$$

This proves (2.6) for $i = 2$. Let (2.6) holds for $i = 1, 2, \ldots J$. Replacing x with jx and y with x in (2.1) for each $n \in \mathbb{N}_0$, and for all $x \in A$, we get

$$\|f_n((j+1)x) + f_n((j-1)x) - 2f_n(jx) - 2f_n(x)\| \leq \max\{\varphi(0,0,0), \varphi(jx,x,0)\}. \tag{2.8}$$

Since

$$\begin{aligned} &f_n((j+1)x) + f_n((j-1)x) - 2f_n(jx) - 2f_n(x) \\ &\quad = f_n((j+1)x) - (j+1)^2 f_n(x) + f_n((j-1)x) - (j-1)^2 f_n(jx) - 2\left[f_n(jx) - j^2 f_n(x)\right], \end{aligned} \tag{2.9}$$

for all $x \in A$, it follows from induction hypothesis and (2.8) that for all $x \in A$,

$$\begin{aligned} &\left\|f_n((j+1)x) - (j+1)^2 f_n(x)\right\| \\ &\quad \leq \max\Big\{\|f_n((j+1)x) + f_n((j-1)x) - 2f_n(jx) - 2f_n(x)\|, \\ &\qquad\qquad \left\|f_n((j-1)x) - (j-1)^2 f_n(x)\right\|, |2|\left\|j^2 f_n(x) - f_n(jx)\right\|\Big\} \\ &\quad \leq \max\{\varphi(0,0,0), \varphi(x,x,0), \varphi(2x,x,0), \ldots, \varphi((j)x,x,0)\}. \end{aligned} \tag{2.10}$$

This proves (2.6) for all $i \geq 2$. In particular,

$$\left\| f_n(kx) - k^2 f_n(x) \right\| \leq \varphi(x) \quad (x \in A). \tag{2.11}$$

Replacing x with $k^{-1}x$ in (2.11), it follows that for each $x \in A$,

$$\left\| f_n(x) - k^2 f_n\left(k^{-1}x\right) \right\| \leq \varphi\left(k^{-1}x\right) \quad (x \in A). \tag{2.12}$$

Let

$$\begin{gathered} \mathrm{X} = \{h : A \longrightarrow B\}, \\ d(g,h) = \inf\{\alpha > 0 : \|g(x) - h(x)\| \leq \alpha\varphi(x) \ \forall x \in A\}, \qquad f, g \in \mathrm{X}. \end{gathered} \tag{2.13}$$

It is easy to see that d defines a generalized non-Archimedean complete metric on X. Define $J : \mathrm{X} \to \mathrm{X}$ by $J(h)(x) = k^2 h(k^{-1}x)$. Then J is strictly contractive on X, in fact if

$$\|g(x) - h(x)\| \leq \alpha\varphi(x), \quad (x \in A), \tag{2.14}$$

then by (2.3),

$$\|J(g)(x) - J(h)(x)\| = |k|^2 \left\| g\left(k^{-1}x\right) - h\left(k^{-1}x\right) \right\| \leq \alpha |k|^2 \varphi\left(k^{-1}x\right) \leq L\alpha\varphi(x), \quad (x \in A). \tag{2.15}$$

It follows that

$$d(J(g), J(h)) \leq L d(g,h) \quad (g, h \in \mathrm{X}). \tag{2.16}$$

Hence, J is strictly contractive mapping with Lipschitz constant L. By (2.12),

$$\|(J f_n)(x) - f_n(x)\| = \left\| k^2 f_n\left(k^{-1}x\right) - f_n(x) \right\| \leq \varphi\left(k^{-1}x\right) \leq |k|^{-2} L\varphi(x) \quad (x \in A). \tag{2.17}$$

This means that $d(J(f_n), f_n) \leq (L/|k|^2)$. By Theorem 1.7, J has a unique fixed point $h_n : A \to B$ in the set

$$U_n = \{g_n \in \mathrm{X} : d(g_n, J(f_n)) < \infty\}, \tag{2.18}$$

and for each $x \in A$,

$$h_n(x) = \lim_{m \to \infty} J^m(f_n(x)) = \lim k^{2m} f_n\left(k^{-m}x\right). \tag{2.19}$$

Therefore,

$$\begin{aligned}
&\left\|h_n(x+y)+h_n(x-y)-2h_n(x)-2h_n(y)\right\| \\
&\quad=\lim_{m\to\infty}|k|^{2m}\left\|f_n\left(k^{-m}(x+y)\right)+f_n\left(k^{-m}(x-y)\right)-2f_n\left(k^{-m}x\right)-2f_n\left(k^{-m}y\right)\right\| \\
&\quad\le\lim_{m\to\infty}|k|^{2m}\max\left\{\varphi(0,0,0),\varphi\left(k^{-m}x,k^{-m}y,0\right)\right\} \\
&\quad\le\lim_{m\to\infty}L^m\varphi(x,y,0)=0,
\end{aligned}\tag{2.20}$$

for all $x, y \in A$. This shows that h_n is quadratic. It follows from Theorem 1.7 that

$$d(f_n,h_n)\le d(J(f_n),f_n),\tag{2.21}$$

that is,

$$\left\|f_n(x)-h_n(x)\right\|\le\frac{L\varphi(x)}{|k|^2}\quad(x\in A,\ n\in\mathbb{N}_0).\tag{2.22}$$

The inequality (2.2) implies that the function $D_n : A \times A \times A \to B$, defined by

$$D_n(x,y,z)=f_n[xyz]-\sum_{i+j+t=n}\left[f_i(x)f_j(y)f_t(z)\right]\tag{2.23}$$

for each $n \in \mathbb{N}_0$ and for all $x, y, z \in A$, is bounded.

By (2.3), we see that

$$\begin{aligned}
\left\|k^{2m}D_n\left(k^{-m}x,k^{-m}y,k^{-m},k^{-m}z\right)\right\|&=|k|^{2m}\left\|D_n\left(k^{-m}x,k^{-m}y,k^{-m}z\right)\right\| \\
&\le|k|^{2m}\varphi\left(k^{-m}x,k^{-m}y,k^{-m}z\right) \\
&\le L^m\varphi(x,y,z),\ \ x,y\in A.
\end{aligned}\tag{2.24}$$

Taking the limit as $m \to \infty$, we obtain

$$\lim_{m\to\infty}k^{2m}D_n\left(k^{-m}x,k^{-m}y,k^{-m}z\right)=0,\tag{2.25}$$

for each $n \in \mathbb{N}_0$ and for all $x, y, z \in A$. Now, using (2.19), (2.23), and (2.25), we have

$$
\begin{aligned}
h_n[xyz] &= \lim_{m\to\infty} k^{2m} f_n(k^{-m}[xyz]) \\
&= \lim_{r\to\infty} k^{6r} f_n\left[\left(k^{-3r}\right)xyz\right] \\
&= \lim_{r\to\infty} k^{6r} f_n[(k^{-r}x)(k^{-r}y)(k^{-r}z)] \\
&= \lim_{r\to\infty}\left(\sum_{i+j+t=n}\left[k^{2r} f_i(k^{-r}x)k^{2r} f_j(k^{-r}y)k^{2r} f_t(k^{-r}z)\right] + k^{6r} D_n(k^{-r}x, k^{-r}y, k^{-r}z)\right) \\
&= \sum_{i+j+t=n}\left[\lim_{r\to\infty} k^{2r} f_i(k^{-r}x) \lim_{r\to\infty} k^{2r} f_j(k^{-r}y) \lim_{r\to\infty} k^{2r} f_t(k^{-r}z)\right] \\
&\quad + \lim_{r\to\infty} k^{6r} D_n(k^{-r}x, k^{-r}y, k^{-r}z) \\
&= \sum_{i+j+t=n}[h_i(x)h_j(y)h_t(z)],
\end{aligned}
\tag{2.26}
$$

for all $n \in \mathbb{N}_0$ and all $x, y, z \in A$. It follows that $H = \{h_0, h_1, \ldots, h_n, \ldots\}$ satisfies

$$
h_n[xyz] = \sum_{i+j+t=n}[h_i(x)h_j(y)h_t z], \tag{2.27}
$$

for all $n \in \mathbb{N}_0$ and all $x, y, z \in A$. This completes the proof of the theorem. □

By a same method as above, we have the following theorem.

Theorem 2.2. *Let $\varphi : A \times A \times A \to [0,\infty)$ be a function. Suppose that $F = \{f_0, f_1, \ldots, f_n, \ldots\}$ is a sequence of mappings from A into B such that for each $n \in \mathbb{N}_0$, $f_n(0) = 0$,*

$$
\begin{gathered}
\|f(x+y) + f(x-y) - 2f(x) - 2f(x)\| \le \varphi(x, y, 0), \\
\left\| f_n([xyz]) - \sum_{i+j+t=n}[f_i(x)f_j(y)f_t(z)] \right\| \le \varphi(x, y, z),
\end{gathered}
\tag{2.28}
$$

for all $x, y, z \in A$. If there exist $k \in \mathbb{K}$ and $0 < L < 1$ such that

$$
|k|^{-2}\varphi(kx, ky, kz) \le L\varphi(x, y, z) \tag{2.29}
$$

for all $x, y, z \in A$, then there exists a unique ternary quadratic higher derivation $H = \{h_0, h_1, \ldots, h_n, \ldots\}$ of any rank from A into B such that for each $n \in \mathbb{N}_0$,

$$
\|f_n(x) - h_n(x)\| \le \frac{\varphi(x)}{|k|^2}, \tag{2.30}
$$

for all $x \in A$, where

$$\psi(x) = \max\{\varphi(0,0,0), \varphi(x,x,0), \varphi(2x,x,0), \dots, \varphi((k-1)x,x,0)\} \quad (x \in A). \tag{2.31}$$

Proof. By a same reasoning of Theorem 2.1, for all $n \in \mathbb{N}_0$, and all $x \in A$ and $i \geq 2$, we have

$$\left\| f_n(ix) - i^2 f_n(x) \right\| \leq \max\{\varphi(0,0,0), \varphi(x,x,0), \varphi(2x,x,0), \dots, \varphi((i-1)x,x,0)\}. \tag{2.32}$$

In particular,

$$\left\| f_n(kx) - k^2 f_n(x) \right\| \leq \psi(x) \quad (x \in A). \tag{2.33}$$

Let

$$\begin{gathered} X = \{h : A \longrightarrow B\} \\ d(g,h) = \inf\{\alpha > 0 : \|g(x) - h(x)\| \leq \alpha\psi(x)\ \forall x \in A\}. \end{gathered} \tag{2.34}$$

It is easy to see that d defines a generalized complete metric on X. Define $J : X \to X$ by $J(h)(x) = k^{-2}h(kx)$. Then J is strictly contractive on X, in fact if

$$\|g(x) - h(x)\| \leq \alpha\psi(x), \quad (x \in A), \tag{2.35}$$

then by (2.29),

$$\|J(g)(x) - J(h)(x)\| = |k|^{-2}\|g(kx) - h(kx)\| \leq \alpha|k|^{-2}\psi(kx) \leq L\alpha\psi(x),) \quad (x \in A). \tag{2.36}$$

It follows that

$$d(J(g), J(h)) \leq Ld(g,h) \quad (g,h \in X). \tag{2.37}$$

Hence, J is strictly contractive mapping with Lipschitz constant L. By (2.33),

$$\begin{gathered} \|(Jf_n)(x) - f_n(x)\| = \left\| k^{-2} f_n(kx) - f_n(x) \right\|, \\ |k|^{-2}\left\| f_n(kx) - k^2 f_n(x) \right\| \leq |k|^{-2}\psi(x) \quad (x \in A). \end{gathered} \tag{2.38}$$

This means that $d(J(f_n), f_n) \leq (1/|k|^2)$. By Theorem 1.7, J has a unique fixed point $h_n : A \to B$ in the set

$$U_n = \{g_n \in X : d(g_n, J(f_n)) < \infty\}, \tag{2.39}$$

and for each $x \in A$,

$$h_n(x) = \lim_{m\to\infty} J^m(f_n(x)) = \lim k^{-2m} f_n(k^m x). \tag{2.40}$$

Therefore,

$$\begin{aligned}
&\|h_n(x+y) + h_n(x-y) - 2h_n(x) - 2h_n(y)\|) \\
&= \lim_{m\to\infty} |k|^{-2m} \|f_n(k^m(x+y)) + f_n(x-y) - 2f_n(k^m x) - 2f_n(k^m y)\| \\
&\leq \lim_{m\to\infty} |k|^{-2m} \max\{\varphi(0,0,0), \varphi(kx,ky,0)\} \\
&\leq \lim_{m\to\infty} L^m \varphi(x,y,0) = 0,
\end{aligned} \tag{2.41}$$

for all $x, y \in A$. This shows that h_n is quadratic. Again by Theorem 1.7, we have

$$d(f_n, h_n) \leq d(J(f_n), f_n), \tag{2.42}$$

that is,

$$\|f_n(x) - h_n(x)\| \leq \frac{\varphi(x)}{|k|^2} \quad (x \in A,\ n \in \mathbb{N}_0). \tag{2.43}$$

The rest of proof is similar to the proof of Theorem 2.1. □

In the following corollaries, $\mathbb{Q}_p$ is the field of p-adic numbers, where $p > 2$ is a prime number.

As a consequence of Theorem 2.1, we show the Hyers-Ulam-Rassias stability of ternary quadratic higher derivations.

Corollary 2.3. *Let $(A, \|\cdot\|_A)$ be a non-Archimedean normed ternary algebra over $\mathbb{Q}_p$ and $(B, \|\cdot\|_B)$ be a non-Archimedean Banach ternary algebra over $\mathbb{Q}_p$. Assume that $F = \{f_0, f_1, \ldots, f_n, \ldots\}$ is a sequence of mappings from A into B such that for each $n \in \mathbb{N}_0$, and for all $x, y \in A$,*

$$\|f_n(x+y) + f_n(x-y) - 2f_n(x) - 2f_n(y)\|_B \leq \theta(\|x\|_A^r + \|y\|_A^r), \tag{2.44}$$

$$\left\| f_n[xyz] - \sum_{i+j+t=n} [f_i(x) f_j(y) f_t(z)] \right\|_B \leq \theta(\|x\|_A^r + \|y\|_A^r + \|z\|_A^r), \tag{2.45}$$

for some $\theta > 0$ and $r < 2$. Then there exists a unique ternary quadratic higher derivation $H = \{h_0, h_1, \ldots, h_n, \ldots\}$ of any rank from A into B such that

$$\|f_n(x) - h_n(x)\| \leq 2\theta p^r \|x\|_A^r \quad (x \in A), \tag{2.46}$$

for all $n \in \mathbb{N}_0$.

Proof. By (2.44), we have $f_n(0) = 0$ for all $n \in \mathbb{N}_0$. Let $\varphi(x,y,z) = \theta(\|x\|_A^r + \|y\|_A^r + \|z\|_A^r)$ for all $x, y, z \in A$, then

$$|p|^2\varphi\left(p^{-1}x, p^{-1}y, p^{-1}z\right) = \theta|p|^{2-r}\left(\|x\|_A^r + \|y\|_A^r + \|z\|_A^r\right) = p^{r-2}\varphi(x,y,z) \quad (x,y,z \in A). \tag{2.47}$$

Moreover,

$$\psi(x) = \max\{\varphi(0,0,0), \varphi(x,x,0), \varphi(2x,x,0), \ldots, \varphi((p-1)x,x,0)\} = 2\theta\|x\|_A^r \quad (x \in A). \tag{2.48}$$

Put $L = p^{r-2}$. By Theorem 2.1, there exists a sequence $H = \{h_0, h_1, \ldots, h_n, \ldots\}$ with the required properties. □

The following corollary is similar to Corollary 2.3 for the case where $r > 2$.

Corollary 2.4. *Let $(A, \|\cdot\|_A)$ be a be a non-Archimedean normed ternary algebra over $\mathbb{Q}_p$ and let $(B, \|\cdot\|_B)$ be a non-Archimedean Banach ternary algebra over $\mathbb{Q}_p$. Assume that $F = \{f_0, f_1, \ldots, f_n, \ldots\}$ is a sequence of mappings from A into B such that for each $n \in \mathbb{N}_0$, for all $x, y \in A$,*

$$\|f_n(x+y) + f_n(x-y) - 2f_n(x) - 2f_n(y)\|_B \le \theta\left(\|x\|_A^r + \|y\|_A^r\right), \tag{2.49}$$

$$\left\| f_n([xyz]) - \sum_{i+j+t=n} [f_i(x)f_j(y)f_t(z)] \right\|_B \le \theta\left(\|x\|_A^r + \|y\|_A^r + \|z\|_A^r\right), \tag{2.50}$$

for some $\theta > 0$ and $r > 2$. Then there exists a unique ternary quadratic higher derivation $H = \{h_0, h_1, \ldots, h_n, \ldots\}$ of any rank from A into B such that

$$\|f_n(x) - h_n(x)\| \le 2\theta|p|^{-2}\|x\|_A^r \quad (x \in A), \tag{2.51}$$

for all $n \in \mathbb{N}_0$.

Proof. By (2.49), $f_n(0) = 0$ for all $n \in \mathbb{N}_0$. Let $\varphi(x,y) = \theta(\|x\|_A^r + \|y\|_A^r + \|z\|_A^r)$ for all $x, y, z \in A$, then

$$|p|^{-2}\varphi(px, py, pz) = \theta|p|^{r-2}\left(\|x\|_A^r + \|y\|_A^r + \|z\|_A^r\right) = p^{2-r}\varphi(x,y,z) \quad (x,y,z \in A). \tag{2.52}$$

Moreover,

$$\psi(x) = \max\{\varphi(0,0,0), \varphi(x,x,0), \varphi(2x,x,0), \ldots, \varphi((p-1)x,x,0)\} = 2\theta\|x\|_A^r \quad (x \in A). \tag{2.53}$$

Put $L = p^{2-r}$. By Theorem 2.2, there exists a sequence $H = \{h_0, h_1, \ldots, h_n, \ldots\}$ with the required properties. □

Corollary 2.5. *Let $(A, \|\cdot\|_A)$ be a non-Archimedean normed ternary algebra over $\mathbb{Q}_p$, and let $(B, \|\cdot\|_B)$ be a non-Archimedean Banach ternary algebra over $\mathbb{Q}_p$. Assume that $F = \{f_0, f_1, \dots, f_n, \dots\}$ is a sequence of mappings from A into B such that for each $n \in \mathbb{N}_0$, and for all $x, y \in A$,*

$$\begin{gathered} \left\| f_n(x+y) + f_n(x-y) - 2f_n(x) - 2f_n(y) \right\|_B \le \max\left\{ \|x\|_A^r, \|y\|_A^r \right\}, \\ \left\| f_n[xyz] - \sum_{i+j+t=n} \left[f_i(x) f_j(y) f_t(z) \right] \right\|_B \le \max\left\{ \|x\|_A^r, \|y\|_A^r, \|z\|_A^r \right\}, \end{gathered} \tag{2.54}$$

for some $r > 2$. Then there exists a unique ternary quadratic higher derivation $H = \{h_0, h_1, \dots, h_n, \dots\}$ of any rank from A into B such that

$$\left\| f_n(x) - h_n(x) \right\| \le p^r \|x\|_A^r \quad (x \in A), \tag{2.55}$$

for all $n \in \mathbb{N}_0$.

Corollary 2.6. *Let $(A, \|\cdot\|_A)$ be a non-Archimedean normed ternary algebra over $\mathbb{Q}_p$ and let $(B, \|\cdot\|_B)$ be a non-Archimedean Banach ternary algebra over $\mathbb{Q}_p$. Assume that $F = \{f_0, f_1, \dots, f_n, \dots\}$ is a sequence of mappings from A into B such that for each $n \in \mathbb{N}_0$, and for all $x, y \in A$,*

$$\begin{gathered} \left\| f_n(x+y) + f_n(x-y) - 2f_n(x) - 2f_n(y) \right\|_B \le \max\left\{ \|x\|_A^r, \|y\|_A^r \right\}, \\ \left\| f_n[xyz] - \sum_{i+j+t=n} \left[f_i(x) f_j(y) f_t(z) \right] \right\|_B \le \max\left\{ \|x\|_A^r, \|y\|_A^r, \|z\|_A^r \right\}, \end{gathered} \tag{2.56}$$

for some $r < 2$. Then there exists a unique ternary quadratic higher derivation $H = \{h_0, h_1, \dots, h_n, \dots\}$ of any rank from A into B such that

$$\left\| f_n(x) - h_n(x) \right\| \le |p|^{-2} \|x\|_A^r \quad (x \in A), \tag{2.57}$$

for all $n \in \mathbb{N}_0$.

As a consequence of Theorem 2.1, we have the following superstability results for ternary quadratic higher derivations.

Corollary 2.7. *Let r, s be two real numbers such that $r + s < -2$. Let $(A, \|\cdot\|_A)$ be a non-Archimedean normed ternary algebra over $\mathbb{Q}_p$ and $(B, \|\cdot\|_B)$ be a non-Archimedean Banach ternary algebra over $\mathbb{Q}_p$. Assume that $F = \{f_0, f_1, \dots, f_n, \dots\}$ is a sequence of mappings from A into B such that for each $n \in \mathbb{N}_0$, and for all $x, y \in A$,*

$$\begin{gathered} \left\| f_n(x+y) + f_n(x-y) - 2f_n(x) - 2f_n(y) \right\|_B \le \theta\left(\|x\|_A^r + \|y\|_A^r \right), \\ \left\| f_n[xyz] - \sum_{i+j+t=n} \left[f_i(x) f_j(y) f_t(z) \right] \right\|_B \le \theta\left(\|x\|_A^r + \|y\|_A^r \right) \|z\|_A^s, \end{gathered} \tag{2.58}$$

for some $\theta > 0$. Then F is a ternary quadratic higher derivation.

Proof. It follows from Theorem 2.1 by putting $\varphi(x,y,z) = \theta(\|x\|_A^r + \|y\|_A^r)\|z\|_A^s$ for all $x, y, z \in A$. □

We can prove a same result with condition $r + s > -2$ by using of Theorem 2.2.

Corollary 2.8. *Let r, s, t be real numbers such that $r+s+t < -2$. Let $(A, \|\cdot\|_A)$ be a non-Archimedean normed ternary algebra over $\mathbb{Q}_p$ and $(B, \|\cdot\|_B)$ be a non-Archimedean Banach ternary algebra over $\mathbb{Q}_p$. Assume that $F = \{f_0, f_1, \ldots, f_n, \ldots\}$ is a sequence of mappings from A into B such that for each $n \in \mathbb{N}_0$, and for all $x, y \in A$,*

$$\begin{aligned}
&\left\| f_n(x+y) + f_n(x-y) - 2f_n(x) - 2f_n(y) \right\|_B \le \theta\left(\|x\|_A^r + \|y\|_A^r\right), \\
&\left\| f_n[xyz] - \sum_{i+j+t=n} [f_i(x) f_j(y) f_t(z)] \right\|_B \le \theta\left(\|x\|_A^r \|y\|_A^s \|z\|_A^s\right),
\end{aligned} \tag{2.59}$$

for some $\theta > 0$. Then F is a ternary quadratic higher derivation.

Proof. It follows from Theorem 2.1 by putting $\varphi(x,y,z) = \theta(\|x\|_A^r \|y\|_A^r \|z\|_A^s)$ for all $x, y, z \in A$. □

Moreover, we can prove a same result with condition $r + s + t > -2$, by applying Theorem 2.2.

References

[1] K. Hensel, "Uber eine neue Begrundung der Theorie der algebraischen Zahlen," *Jahresbericht der Deutschen Mathematiker-Vereinigung*, vol. 6, pp. 83–88, 1897.

[2] A. Khrennikov, *Non-Archimedean Analysis: Quantum Paradoxes, Dynamical Systems and Biological Models*, Kluwer Academic, Dordrecht, 1997.

[3] A. C. M. van Rooij, *Non-Archimedean Functional Analysis*, vol. 51, Marcel Dekker, New York, NY, USA, 1978.

[4] J. A. Alvarez, "C*-algebras of operators in non-Archimedean Hilbert spaces," *Commentationes Mathematicae Universitatis Carolinae*, vol. 33, no. 4, pp. 573–580, 1992.

[5] L. M. Arriola and W. A. Beyer, "Stability of the Cauchy functional equation over p-adic fields," *Real Analysis Exchange*, vol. 31, no. 1, pp. 125–132, 2005-2006.

[6] M. Eshaghi Gordji and M. B. Savadkouhi, "Stability of cubic and quartic functional equations in non-Archimedean spaces," *Acta Applicandae Mathematicae*, vol. 110, no. 3, pp. 1321–1329, 2010.

[7] M. Eshaghi Gordji and M. B. Savadkouhi, "Stability of a mixed type cubic-quartic functional equation in non-Archimedean spaces," *Applied Mathematics Letters*, vol. 23, no. 10, pp. 1198–1202, 2010.

[8] M. Eshaghi Gordji, M. B. Savadkouhi, and M. Bidkham, "Stability of a mixed type additive and quadratic functional equation in non-Archimedean spaces," *Journal of Computational Analysis and Applications*, vol. 12, no. 2, pp. 454–462, 2010.

[9] M. Eshaghi Gordji and M. B. Savadkouhi, "Approximation of generalized homomorphisms in quasi-Banach algebras," *Mathematical Journal of the Ovidius University of Constantza*, vol. 17, no. 2, pp. 203–214, 2009.

[10] M. Eshaghi Gordji and M. Bavand Savadkouhi, "On approximate cubic homomorphisms," *Advances in Difference Equations*, Article ID 618463, 11 pages, 2009.

[11] M. Eshaghi Gordji, T. Karimi, and S. Kaboli Gharetapeh, "Approximately *n*-Jordan homomorphisms on Banach algebras," *Journal of Inequalities and Applications*, Article ID 870843, 8 pages, 2009.

[12] M. Eshaghi Gordji, R. Khodabakhsh, H. Khodaei, and S.-M. Jung, "AQCQ-functional equation in non-Archimedean normed spaces," *Abstract and Applied Analysis*, Article ID 741942, 22 pages, 2010.

[13] A. K. Mirmostafaee, "Non-Archimedean stability of quadratic equations," *Fixed Point Theory*, vol. 11, no. 1, pp. 67–75, 2010.

[14] Y. Nambu, "Generalized Hamiltonian dynamics," *Physical Review*, vol. 7, pp. 2405–2412, 1973.

[15] S. Okubo, "Triple products and Yang-Baxter equation. I. Octonionic and quaternionic triple systems," *Journal of Mathematical Physics*, vol. 34, no. 7, pp. 3273–3291, 1993.

[16] V. Abramov, R. Kerner, and B. Le Roy, "Hypersymmetry: a Z_3-graded generalization of supersymmetry," *Journal of Mathematical Physics*, vol. 38, no. 3, pp. 1650–1669, 1997.

[17] A. Cayley, "On the 34 concomitants of the ternary cubic," *American Journal of Mathematics*, vol. 4, no. 1–4, pp. 1–15, 1881.

[18] R. Kerner, "The cubic chessboard," *Classical and Quantum Gravity*, vol. 14, no. 1, pp. A203–A225, 1997, Geometry and physic.

[19] G. L. Sewell, *Quantum Mechanics and Its Emergent Macrophysics*, Princeton University, Princeton, NJ, USA, 2002.

[20] L. Takhtajan, "On foundation of the generalized Nambu mechanics," *Communications in Mathematical Physics*, vol. 160, no. 2, pp. 295–315, 1994.

[21] Th. M. Rassias, "On the stability of functional equations and a problem of Ulam," *Acta Applicandae Mathematicae*, vol. 62, no. 1, pp. 23–130, 2000.

[22] S. M. Ulam, *Problems in Modern Mathematics*, chapter 5, Science Edition, Wiley, New York, NY, USA, 1940.

[23] D. H. Hyers, "On the stability of the linear functional equation," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 27, pp. 222–224, 1941.

[24] Th. M. Rassias, "On the stability of the linear mapping in Banach spaces," *Proceedings of the American Mathematical Society*, vol. 72, no. 2, pp. 297–300, 1978.

[25] Z. Gajda, "On stability of additive mappings," *International Journal of Mathematics and Mathematical Sciences*, vol. 14, no. 3, pp. 431–434, 1991.

[26] S. Abbaszadeh, "Intuitionistic fuzzy stability of a quadratic and quartic functional equation," *International Journal of Nonlinear Analysis and Applications*, vol. 1, no. 2, pp. 100–124, 2010.

[27] J. Aczél and J. Dhombres, *Functional Equations in Several Variables*, Cambridge University Press, Cambridge, UK, 1989.

[28] T. Aoki, "On the stability of the linear transformation in Banach spaces," *Journal of the Mathematical Society of Japan*, vol. 2, pp. 64–66, 1950.

[29] D. H. Hyers, G. Isac, and Th. M. Rassias, *Stability of Functional Equations in Several Variables*, Birkhäuser, Basel, Switzerland, 1998.

[30] G. Isac and Th. M. Rassias, "On the Hyers-Ulam stability of ψ-additive mappings," *Journal of Approximation Theory*, vol. 72, no. 2, pp. 131–137, 1993.

[31] Pl. Kannappan, "Quadratic functional equation and inner product spaces," *Results in Mathematics*, vol. 27, no. 3-4, pp. 368–372, 1995.

[32] C. Park, "Generalized quadratic mappings in several variables," *Nonlinear Analysis*, vol. 57, no. 5-6, pp. 713–722, 2004.

[33] C. Park, "On the stability of the quadratic mapping in Banach modules," *Journal of Mathematical Analysis and Applications*, vol. 276, no. 1, pp. 135–144, 2002.

[34] C. Park and M. E. Gordji, "Comment on "Approximate ternary Jordan derivations on Banach ternary algebras" [Bavand Savadkouhi J. Math. Phys. 50, 042303 (2009)]," *Journal of Mathematical Physics*, vol. 51, no. 4, Article ID 046002JMP, pp. 1–7, 2010.

[35] C. Park and A. Najati, "Generalized additive functional inequalities in Banach algebras," *International Journal of Nonlinear Analysis and Applications*, vol. 1, no. 2, pp. 54–62, 2010.

[36] C. Park and Th. M. Rassias, "Isomorphisms in unital C^*-algebras," *International Journal of Nonlinear Analysis and Applications*, vol. 1, no. 2, pp. 1–10, 2010.

[37] F. Skof, "Local properties and approximation of operators," *Rendiconti del Seminario Matematico e Fisico di Milano*, vol. 53, pp. 113–129, 1983.

[38] M. B. Savadkouhi, M. E. Gordji, J. M. Rassias, and N. Ghobadipour, "Approximate ternary Jordan derivations on Banach ternary algebras," *Journal of Mathematical Physics*, vol. 50, no. 4, p. 042303, 9, 2009.

[39] C. Borelli and G. L. Forti, "On a general Hyers-Ulam stability result," *International Journal of Mathematics and Mathematical Sciences*, vol. 18, no. 2, pp. 229–236, 1995.

[40] D. G. Bourgin, "Classes of transformations and bordering transformations," *Bulletin of the American Mathematical Society*, vol. 57, pp. 223–237, 1951.

[41] D. G. Bourgin, “Approximately isometric and multiplicative transformations on continuous function rings,” *Duke Mathematical Journal*, vol. 16, pp. 385–397, 1949.

[42] P. W. Cholewa, “Remarks on the stability of functional equations,” *Aequationes Mathematicae*, vol. 27, no. 1-2, pp. 76–86, 1984.

[43] S. Czerwik, “On the stability of the quadratic mapping in normed spaces,” *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg*, vol. 62, pp. 59–64, 1992.

[44] A. Ebadian, N. Ghobadipour, and M. Eshaghi Gordji, “A fixed point method for perturbation of bimultipliers and Jordan bimultipliers in C^*-ternary algebras,” *Journal of Mathematical Physics*, vol. 51, 10 pages, 2010.

[45] A. Ebadian, A. Najati, and M. Eshaghi Gordji, “On approximate additive-quartic and quadratic-cubic functional equations in two variables on abelian groups,” *Results in Mathematics*, vol. 58, no. 1-2, pp. 39–53, 2010.

[46] M. Eshaghi Gordji, “Nearly ring homomorphisms and nearly ring derivations on non-Archimedean Banach algebras,” *Abstract and Applied Analysis*, Article ID 393247, 12 pages, 2010.

[47] M. Eshaghi Gordji and Z. Alizadeh, “Stability and superstability of ring homomorphisms on non-Archimedean Banach algebras,” *Abstract and Applied Analysis*, Article ID 123656, 10 pages, 2011.

[48] M. Eshaghi Gordji, H. Khodaei, and R. Khodabakhsh, “General quartic-cubic-quadratic functional equation in non-Archimedean normed spaces,” *University "Politehnica" of Bucharest, Scientific Bulletin Series A*, vol. 72, no. 3, pp. 69–84, 2010.

[49] M. Eshaghi Gordji and H. Khodaei, “Solution and stability of generalized mixed type cubic, quadratic and additive functional equation in quasi-Banach spaces,” *Nonlinear Analysis*, vol. 71, no. 11, pp. 5629–5643, 2009.

[50] M. Eshaghi Gordji, H. Khodaei, and R. Khodabakhsh, “General quartic-cubic-quadratic functional equation in non-Archimedean normed spaces,” *University "Politehnica" of Bucharest, Scientific Bulletin Series A*, vol. 72, no. 3, pp. 69–84, 2010.

[51] M. Eshaghi Gordji and A. Najati, “Approximately J^*-homomorphisms: a fixed point approach,” *Journal of Geometry and Physics*, vol. 60, no. 5, pp. 809–814, 2010.

[52] R. Farokhzad and S. A. R. Hosseinioun, “Perturbations of Jordan higher derivations in Banach ternary algebras: an alternative fixed point approach,” *International Journal of Nonlinear Analysis and Applications*, vol. 1, no. 1, pp. 42–53, 2010.

[53] P. Găvruţa, “A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings,” *Journal of Mathematical Analysis and Applications*, vol. 184, no. 3, pp. 431–436, 1994.

[54] P. Găvruta and L. Găvruta, “A new method for the generalized Hyers—Ulam—Rassias stability,” *International Journal of Nonlinear Analysis and Applications*, vol. 1, no. 2, pp. 11–18, 2010.

[55] M. Eshaghi Gordji and H. Khodaei, “On the generalized Hyers-Ulam-Rassias stability of quadratic functional equations,” *Abstract and Applied Analysis*, Article ID 923476, 11 pages, 2009.

[56] M. Eshaghi Gordji, S. Kaboli Gharetapeh, T. Karimi, E. Rashidi, and M. Aghaei, “Ternary Jordan derivations on C^*-ternary algebras,” *Journal of Computational Analysis and Applications*, vol. 12, no. 2, pp. 463–470, 2010.

[57] M. Eshaghi Gordji, J. M. Rassias, and N. Ghobadipour, “Generalized Hyers-Ulam stability of generalized (n, k)-derivations,” *Abstract and Applied Analysis*, Article ID 437931, 8 pages, 2009.

[58] A. Grabiec, “The generalized Hyers-Ulam stability of a class of functional equations,” *Publicationes Mathematicae Debrecen*, vol. 48, no. 3-4, pp. 217–235, 1996.

[59] J. M. Rassias, “On approximation of approximately linear mappings by linear mappings,” *Journal of Functional Analysis*, vol. 46, no. 1, pp. 126–130, 1982.

[60] J. M. Rassias, “On approximation of approximately linear mappings by linear mappings,” *Bulletin des Sciences Mathématiques*, vol. 108, no. 4, pp. 445–446, 1984.

[61] J. M. Rassias, “On a new approximation of approximately linear mappings by linear mappings,” *Discussiones Mathematicae*, vol. 7, pp. 193–196, 1985.

[62] J. M. Rassias, “Solution of a problem of Ulam,” *Journal of Approximation Theory*, vol. 57, no. 3, pp. 268–273, 1989.

[63] J. M. Rassias, “On the stability of the Euler-Lagrange functional equation,” *Chinese Journal of Mathematics*, vol. 20, no. 2, pp. 185–190, 1992.

[64] J. M. Rassias, “Solution of a stability problem of Ulam,” *Discussiones Mathematicae*, vol. 12, pp. 95–103, 1992.

[65] J. M. Rassias, “On the stability of the non-linear Euler-Lagrange functional equation in real normed linear spaces,” *Journal of Mathematical and Physical Sciences*, vol. 28, no. 5, pp. 231–235, 1994.

[66] J. M. Rassias, “On the stability of a multi-dimensional Cauchy type functional equation,” in *Geometry, Analysis and Mechanics*, pp. 365–376, World Scientific, 1994.

[67] K. Ravi, M. Arunkumar, and J. M. Rassias, “Ulam stability for the orthogonally general Euler-Lagrange type functional equation,” *International Journal of Mathematics and Statistics*, vol. 3, pp. 36–46, 2008.

[68] R. Badora, “On approximate derivations,” *Mathematical Inequalities & Applications*, vol. 9, no. 1, pp. 167–173, 2006.

[69] T. Miura, G. Hirasawa, and S.-E. Takahasi, “A perturbation of ring derivations on Banach algebras,” *Journal of Mathematical Analysis and Applications*, vol. 319, no. 2, pp. 522–530, 2006.

[70] Y.-S. Jung and I.-S. Chang, “On approximately higher ring derivations,” *Journal of Mathematical Analysis and Applications*, vol. 343, no. 2, pp. 636–643, 2008.

[71] A. K. Mirmostafaee, “Non-Archimedean stability of the monomial functional equations,” *Tamsui Oxford Journal of Mathematical Sciences*, vol. 26, no. 2, pp. 221–235, 2010.