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Research Article

Differential Subordinations for Certain Meromorphically Multivalent Functions Defined by Dziok-Srivastava Operator

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By making use of the Dziok-Srivastava operator, we introduce a new class of meromorphically multivalent functions. Some inclusion properties of functions belonging to this class are derived.

1. Introduction

Let $\Sigma(p)$ denote the class of functions of the form

$$f(z) = z^{-p} + \sum_{n=1}^{\infty} a_n z^{n-p} \quad \left(p \in \mathbb{N} := \{1, 2, 3, \dots\}\right), \tag{1.1}$$

which are analytic in the punctured open unit disk $\mathbb{U}_0 = \{z : 0 < |z| < 1\}$ with a pole at $z = 0$. Also let the Hadamard product (or convolution) of the following functions:

$$f_j(z) = z^{-p} + \sum_{n=1}^{\infty} a_{n,j} z^{n-p} \quad \left(j = 1, 2\right) \tag{1.2}$$

be given by

$$\left(f_1 * f_2\right)(z) := z^{-p} + \sum_{n=1}^{\infty} a_{n,1} a_{n,2} z^{n-p} = \left(f_2 * f_1\right)(z). \tag{1.3}$$

Given two functions $f(z)$ and $g(z)$, which are analytic in $\mathbb{U} = \mathbb{U}_0 \cup \{0\}$, we say that the function $g(z)$ is subordinate to $f(z)$ and write $g \prec f$ or (more precisely) $g(z) \prec f(z)$ $(z \in \mathbb{U})$, if there exists a Schwarz function $w(z)$, analytic in $\mathbb{U}$ with $w(0) = 0$ and $|w(z)| < 1$ $(z \in \mathbb{U})$ such that $g(z) = f(w(z))$ $(z \in \mathbb{U})$. In particular, if $f(z)$ is univalent in $\mathbb{U}$, we have the following equivalence:

$$g(z) \prec f(z) \quad (z \in \mathbb{U}) \Longleftrightarrow g(0) = f(0), \quad g(\mathbb{U}) \subset f(\mathbb{U}). \tag{1.4}$$

Let A be the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} b_n z^n, \tag{1.5}$$

which are analytic in $\mathbb{U}$. A function $f(z) \in A$ is said to be in the class $S^*(\alpha)$ if

$$\operatorname{Re}\left\{ \frac{zf'(z)}{f(z)} \right\} > \alpha \quad (z \in \mathbb{U}) \tag{1.6}$$

for some α $(\alpha < 1)$. When $0 \le \alpha < 1$, $S^*(\alpha)$ is the class of starlike functions of order α in $\mathbb{U}$. A function $f(z) \in A$ is said to be prestarlike of order α in $\mathbb{U}$ if

$$\frac{z}{(1-z)^{2(1-\alpha)}} * f(z) \in S^*(\alpha) \quad (\alpha < 1), \tag{1.7}$$

where the symbol $*$ means the familiar Hadamard product (or convolution) of two analytic functions in $\mathbb{U}$. We denote this class by $R(\alpha)$ (see [1]). Clearly a function $f(z) \in A$ is in the class $R(0)$ if and only if $f(z)$ is convex univalent in $\mathbb{U}$ and $R(1/2) = S^*(1/2)$.

For complex parameters

$$\alpha_1, \ldots, \alpha_q \text{ and } \beta_1, \ldots, \beta_s \quad \left(\beta_j \notin \mathbb{Z}_0^- := \{0, -1, -2, \ldots\};\ j = 1, 2, \ldots, s\right), \tag{1.8}$$

we define the generalized hypergeometric function ${}_qF_s(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z)$ by

$$\begin{gathered} {}_qF_s\left(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z\right) := \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \cdots (\alpha_q)_n}{(\beta_1)_n \cdots (\beta_s)_n} \cdot \frac{z^n}{n!} \\ \left(q \le s+1; q, s \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}; z \in \mathbb{U}\right), \end{gathered} \tag{1.9}$$

where $(x)_n$ is the Pochhammer symbol defined, in terms of the Gamma function $\Gamma(z)$,by

$$(x)_n := \frac{\Gamma(x+n)}{\Gamma(x)} = \begin{cases} 1 & (n = 0; x \in \mathbb{C} \setminus \{0\}) \\ x(x+1)\cdots(x+n-1) & (n \in \mathbb{N}; x \in \mathbb{C}). \end{cases} \tag{1.10}$$

Corresponding to a function $h_p(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z)$ defined by

$$h_p(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) = z^{-p} {}_qF_s(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z), \tag{1.11}$$

we now consider a linear operator

$$H_p(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s) : \Sigma(p) \longrightarrow \Sigma(p), \tag{1.12}$$

defined by means of the Hadamard product (or convolution) as follows:

$$H_p(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s) f(z) := h_p(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) * f(z). \tag{1.13}$$

For convenience, we write

$$H_{p,q,s}(\alpha_1) := H_p(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s). \tag{1.14}$$

Thus, after some calculations, we have

$$z\left(H_{p,q,s}(\alpha_1) f(z)\right)' = \alpha_1 H_{p,q,s}(\alpha_1 + 1) f(z) - (\alpha_1 + p) H_{p,q,s}(\alpha_1) f(z). \tag{1.15}$$

The operator $H_{p,q,s}(\alpha_1)$ is popularly known as the generalized Dziok-Srivastava operator. Many interesting subclasses of multivalent functions, associated with the operator $H_{p,q,s}(\alpha_1)$ and its various special cases, were investigated recently by (e.g.) Dziok and Srivastava [2–4], Liu [5], Liu and Srivastava [6, 7], Patel et al. [8], Wang et al. [9], and others.

Let P be the class of functions $h(z)$ with $h(0) = 1$, which are analytic and convex univalent in $\mathbb{U}$.

Definition 1.1. A function $f(z) \in \Sigma(p)$ is said to be in the class $T_{p,q,s}(\alpha_1, \lambda; h)$ if it satisfies the subordination condition

$$\frac{(\lambda - 1)}{p} z^{p+1} \left(H_{p,q,s}(\alpha_1) f(z)\right)' + \frac{\lambda}{p(p+1)} z^{p+2} \left(H_{p,q,s}(\alpha_1) f(z)\right)'' \prec h(z), \tag{1.16}$$

where λ is a complex number and $h(z) \in P$.

The main object of this paper is to present a systematic investigation of the class $T_{p,q,s}(\alpha_1, \lambda; h)$ defined above by means of the generalized Dziok-Srivastava operator $H_{p,q,s}(\alpha_1)$.

For our purpose, we shall need the following lemmas to derive our main results for the class $T_{p,q,s}(\alpha_1, \lambda; h)$.

Lemma 1.2 (see [10]). *Let $g(z)$ be analytic in $\mathbb{U}$ and $h(z)$ be analytic and convex univalent in $\mathbb{U}$ with $h(0) = g(0)$. If*

$$g(z) + \frac{1}{\mu} z g'(z) \prec h(z), \tag{1.17}$$

where Re $\mu > 0$, *then*

$$g(z) \prec \widetilde{h}(z) = \mu z^{-\mu} \int_{v}^{z} t^{\mu-1} h(t) dt \prec h(z) \tag{1.18}$$

and $\widetilde{h}(z)$ *is the best dominant of* (1.17).

Lemma 1.3 (see [1]). *Let* $\alpha < 1$, $f(z) \in S^{*}(\alpha)$ *and* $g(z) \in R(\alpha)$. *Then, for any analytic function* $F(z)$ *in* $\mathbb{U}$,

$$\frac{g * (fF)}{g * f}(\mathbb{U}) \subset \overline{\text{co}}(F(\mathbb{U})), \tag{1.19}$$

where $\overline{\text{co}}(F(\mathbb{U}))$ *denotes the closed convex hull of* $F(\mathbb{U})$.

2. Properties of the Class $T_{p,q,s}(\alpha_1, \lambda; h)$

Theorem 2.1. *Let* $\lambda_1 < \lambda_2 \leq 0$. *Then* $T_{p,q,s}(\alpha_1, \lambda_1; h) \subset T_{p,q,s}(\alpha_1, \lambda_2; h)$.

Proof. Let $\lambda_1 < \lambda_2 \leq 0$ and suppose that

$$g(z) = -\frac{z^{p+1}(H_{p,q,s}(\alpha_1) f(z))'}{p} \tag{2.1}$$

for $f(z) \in T_{p,q,s}(\alpha_1, \lambda_1; h)$. Then the function $g(z)$ is analytic in $\mathbb{U}$ with $g(0) = 1$. Differentiating both sides of (2.1) with respect to z and using (1.16), we have

$$\frac{(\lambda_1 - 1)}{p} z^{p+1}(H_{p,q,s}(\alpha_1) f(z))' + \frac{\lambda_1}{p(p+1)} z^{p+2}(H_{p,q,s}(\alpha_1) f(z))'' = g(z) - \frac{\lambda_1}{p+1} z g'(z) \prec h(z). \tag{2.2}$$

Hence an application of Lemma 1.2 yields

$$g(z) \prec h(z). \tag{2.3}$$

Noting that $0 < \lambda_2/\lambda_1 < 1$ and that $h(z)$ is convex univalent in $\mathbb{U}$, it follows from (2.1) to (2.3) that

$$\begin{aligned} &\frac{(\lambda_2 - 1)}{p} z^{p+1}(H_{p,q,s}(\alpha_1) f(z))' + \frac{\lambda_2}{p(p+1)} z^{p+2}(H_{p,q,s}(\alpha_1) f(z))'' \\ &\quad = \frac{\lambda_2}{\lambda_1}\left(\frac{(\lambda_1 - 1)}{p} z^{p+1}(H_{p,q,s}(\alpha_1) f(z))' + \frac{\lambda_1}{p(p+1)} z^{p+2}(H_{p,q,s}(\alpha_1) f(z))''\right) \\ &\qquad + \left(1 - \frac{\lambda_2}{\lambda_1}\right) g(z) \prec h(z). \end{aligned} \tag{2.4}$$

Thus $f(z) \in Tp, q, s(\alpha_1, \lambda_2; h)$ and the proof of Theorem 2.1 is completed. □

Theorem 2.2. *Let* $0 < b_1 < b_2$. *Then* $T_{p,q,s}(b_2, \lambda; h) \subset T_{p,q,s}(b_1, \lambda; h)$.

Proof. Define a function $g(z)$ by

$$g(z) = z + \sum_{n=1}^{\infty} \frac{(b_1)_n}{(b_2)_n} z^{n+1} \quad (z \in \mathbb{U}; 0 < b_1 < b_2). \tag{2.5}$$

Then

$$z^{p+1} h_p(b_1, \alpha_2, \ldots, \alpha_s, 1; b_2, \alpha_2, \ldots, \alpha_s; z) = g(z) \in A, \tag{2.6}$$

where

$$h_p(b_1, \alpha_2, \ldots, \alpha_s, 1; b_2, \alpha_2, \ldots, \alpha_s; z) \tag{2.7}$$

is defined as in (1.11), and

$$\frac{z}{(1-z)^{b_2}} * g(z) = \frac{z}{(1-z)^{b_1}}. \tag{2.8}$$

By (2.8), we see that

$$\frac{z}{(1-z)^{b_2}} * g(z) \in S^*\left(1 - \frac{b_1}{2}\right) \subset S^*\left(1 - \frac{b_2}{2}\right) \quad (0 < b_1 < b_2), \tag{2.9}$$

which implies that

$$g(z) \in R\left(1 - \frac{b_2}{2}\right). \tag{2.10}$$

Let $f(z) \in T_{p,q,s}(b_2, \lambda; h)$. It is easy to verify that

$$z^{p+1} \left(H_{p,q,s}(b_1) f(z)\right)' = \left(z^p h_p(b_1, \alpha_2, \ldots, \alpha_s, 1; b_2, \alpha_2, \ldots, \alpha_s; z)\right) * \left(z^{p+1} \left(H_{p,q,s}(b_2) f(z)\right)'\right) \tag{2.11}$$

$$z^{p+2} \left(H_{p,q,s}(b_1) f(z)\right)'' = \left(z^p h_p(b_1, \alpha_2, \ldots, \alpha_s, 1; b_2, \alpha_2, \ldots, \alpha_s; z)\right) * \left(z^{p+2} \left(H_{p,q,s}(b_2) f(z)\right)''\right). \tag{2.12}$$

From (2.11), (2.12), and (2.6), we deduce that

$$\begin{aligned} &\frac{(\lambda - 1)}{p} z^{p+1} \left(H_{p,q,s}(b_1) f(z)\right)' + \frac{\lambda}{p(p+1)} z^{p+2} \left(H_{p,q,s}(b_1) f(z)\right)'' \\ &\quad = \frac{g(z)}{z} * w(z) = \frac{g(z) * (z w(z))}{g(z) * z}, \end{aligned} \tag{2.13}$$

where

$$w(z) := \frac{(\lambda-1)}{p} z^{p+1}\left(H_{p,q,s}(b_2)f(z)\right)' + \frac{\lambda}{p(p+1)} z^{p+2}\left(H_{p,q,s}(b_2)f(z)\right)'' \prec h(z). \tag{2.14}$$

Since the function z belongs to the function class $S^*(1-b_2/2)$ and $h(z)$ is convex univalent in $\mathbb{U}$, it follows from (2.12), (2.13), (2.14), and Lemma 1.3 that

$$\frac{(\lambda-1)}{p} z^{p+1}\left(H_{p,q,s}(b_1)f(z)\right)' + \frac{\lambda}{p(p+1)} z^{p+2}\left(H_{p,q,s}(b_1)f(z)\right)'' \prec h(z). \tag{2.15}$$

Thus $f(z) \in T_{p,q,s}(b_1,\lambda;h)$ and the proof of Theorem 2.2 is completed. □

Theorem 2.3. *Let* $f(z) \in T_{p,q,s}(\alpha_1,\lambda;h)$, $g(z) \in \Sigma(p)$ *and*

$$\operatorname{Re}\left\{z^p g(z)\right\} > \frac{1}{2} \quad (z \in \mathbb{U}). \tag{2.16}$$

Then

$$(f * g)(z) \in T_{p,q,s}(\alpha_1,\lambda;h). \tag{2.17}$$

Proof. For $f(z) \in T_{p,q,s}(\alpha_1,\lambda;h)$ and $g(z) \in \Sigma(p)$, we have

$$\begin{aligned}
&\frac{(\lambda-1)}{p} z^{p+1}\left(H_{p,q,s}(\alpha_1)(f*g)(z)\right)' + \frac{\lambda}{p(p+1)} z^{p+2}\left(H_{p,q,s}(\alpha_1)(f*g)(z)\right)'' \\
&\quad = \frac{(\lambda-1)}{p}\left(z^p g(z)\right) * \left(z^{p+1}\left(H_{p,q,s}(\alpha_1)f(z)\right)'\right) + \frac{\lambda}{p(p+1)}\left(z^p g(z)\right) * \left(z^{p+2}\left(H_{p,q,s}(\alpha_1)f(z)\right)''\right) \\
&\quad = \left(z^p g(z)\right) * \psi(z),
\end{aligned} \tag{2.18}$$

where

$$\psi(z) = \frac{(\lambda-1)}{p} z^{p+1}\left(H_{p,q,s}(\alpha_1)f(z)\right)' + \frac{\lambda}{p(p+1)} z^{p+2}\left(H_{p,q,s}(\alpha_1)f(z)\right)''. \tag{2.19}$$

In view of (2.16), the function $z^p g(z)$ has the Herglotz representation

$$z^p g(z) = \int_{|x|=1} \frac{d\mu(x)}{1-xz} \quad (z \in \mathbb{U}), \tag{2.20}$$

where $\mu(x)$ is a probability measure defined on the unit circle $|x| = 1$ and

$$\int_{|x|=1} d\mu(x) = 1. \tag{2.21}$$

Since $h(z)$ is convex univalent in $\mathbb{U}$, it follows from (2.18) to (2.20) that

$$\begin{aligned}&\frac{(\lambda-1)}{p}z^{p+1}\left(H_{p,q,s}(\alpha_1)(f*g)(z)\right)'+\frac{\lambda}{p(p+1)}z^{p+2}\left(H_{p,q,s}(\alpha_1)(f*g)(z)\right)''\\&\quad=\int_{|x|=1}\varphi(xz)d\mu(x)\prec h(z).\end{aligned}\tag{2.22}$$

This shows that $(f*g)(z)\in T_{p,q,s}(\alpha_1,\lambda;h)$ and the theorem is proved. □

Theorem 2.4. *Let $f(z)\in T_{p,q,s}(\alpha_1,\lambda;h)$, $g(z)\in\Sigma(p)$ and*

$$z^{p+1}g(z)\in R(\alpha)\quad(\alpha<1).\tag{2.23}$$

Then

$$(f*g)(z)\in T_{p,q,s}(\alpha_1,\lambda;h).\tag{2.24}$$

Proof. For $f(z)\in T_{p,q,s}(\alpha_1,\lambda;h)$ and $g(z)\in\Sigma(p)$, from (2.18) we have

$$\begin{aligned}&\frac{(\lambda-1)}{p}z^{p+1}\left(H_{p,q,s}(f*g)(z)\right)'+\frac{\lambda}{p(p+1)}z^{p+2}\left(H_{p,q,s}(f*g)(z)\right)''\\&\quad=\frac{\left(z^{p+1}g(z)\right)*\left(z\varphi(z)\right)}{\left(z^{p+1}g(z)\right)*z}\quad(z\in\mathbb{U}),\end{aligned}\tag{2.25}$$

where $\varphi(z)$ is defined as in (2.19).

Since $h(z)$ is convex univalent in $\mathbb{U}$,

$$\varphi(z)\prec h(z),\quad z^{p+1}g(z)\in R(\alpha),\quad z\in S^*(\alpha)\ (\alpha<1),\tag{2.26}$$

it follows from (2.25) and Lemma 1.3 the desired result. □

Theorem 2.5. *Let $\lambda<0$, $\beta>0$ and $f(z)\in T_{p,q,s}(\lambda;\beta h+1-\beta)$. If $\beta\le\beta_0$, where*

$$\beta_0=\frac{1}{2}\left(1+\frac{p+1}{\lambda}\int_0^1\frac{u^{-((p+1)/\lambda)-1}}{1+u}du\right)^{-1},\tag{2.27}$$

then $f(z)\in T_{p,q,s}(0;h)$. The bound β_0 is sharp when $h(z)=1/(1-z)$.

Proof. Let us define

$$g(z)=-\frac{z^{p+1}\left(H_{p,q,s}(\alpha_1)f(z)\right)'}{p}\tag{2.28}$$

for $f(z) \in T_{p,q,s}(\lambda; \beta h + 1 - \beta)$ with $\lambda < 0$ and $\beta > 0$. Then we have

$$\begin{aligned} g(z) - \frac{\lambda}{p+1} z g'(z) &= \frac{(\lambda - 1)}{p} z^{p+1} \left(H_{p,q,s}(\alpha_1) f(z)\right)' + \frac{\lambda}{p(p+1)} z^{p+2} \left(H_{p,q,s}(\alpha_1) f(z)\right)'' \\ &\prec \beta h(z) + 1 - \beta. \end{aligned} \tag{2.29}$$

Hence an application of Lemma 1.2 yields

$$\begin{aligned} g(z) &\prec -\frac{\beta(p+1)}{\lambda} z^{(p+1)/\lambda} \int_0^z t^{-((p+1)/\lambda)-1} h(t) dt + 1 - \beta \\ &= (h * \varphi)(z), \end{aligned} \tag{2.30}$$

where

$$\varphi(z) = -\frac{\beta(p+1)}{\lambda} z^{(p+1)/\lambda} \int_0^z \frac{t^{-((p+1)/\lambda)-1}}{1-t} dt + 1 - \beta. \tag{2.31}$$

If $0 < \beta \leq \beta_0$, where β_0 (> 1) is given by (2.27), then it follows from (2.31) that

$$\begin{aligned} \operatorname{Re} \varphi(z) &= -\frac{\beta(p+1)}{\lambda} \int_0^1 u^{-((p+1)/\lambda)-1} \operatorname{Re}\left(\frac{1}{1-uz}\right) du + 1 - \beta \\ &> -\frac{\beta(p+1)}{\lambda} \int_0^1 \frac{u^{-((p+1)/\lambda)-1}}{1+u} du + 1 - \beta \\ &\geq \frac{1}{2} \quad (z \in \mathbb{U}; \lambda < 0). \end{aligned} \tag{2.32}$$

Now, by using the Herglotz representation for $\varphi(z)$, from (2.28) and (2.30), we arrive at

$$-\frac{z^{p+1}\left(H_{p,q,s}(\alpha_1) f(z)\right)'}{p} \prec (h * \varphi)(z) \prec h(z) \tag{2.33}$$

because $h(z)$ is convex univalent in $\mathbb{U}$. This shows that $f(z) \in T_{p,q,s}(0; h)$.

For $h(z) = 1/(1-z)$ and $f(z) \in \Sigma(p)$ defined by

$$-\frac{z^{p+1}\left(H_{p,q,s}(\alpha_1) f(z)\right)'}{p} = -\frac{\beta(p+1)}{\lambda} z^{(p+1)/\lambda} \int_0^z \frac{t^{-((p+1)/\lambda)-1}}{1-t} dt + 1 - \beta, \tag{2.34}$$

it is easy to verify that

$$\frac{(\lambda-1)}{p} z^{p+1} \left(H_{p,q,s}(\alpha_1) f(z)\right)' + \frac{\lambda}{p(p+1)} z^{p+2} \left(H_{p,q,s}(\alpha_1) f(z)\right)'' = \beta h(z) + 1 - \beta. \tag{2.35}$$

Thus $f(z) \in T_{p,q,s}(\lambda; \beta h + 1 - \beta)$. Also, for $\beta > \beta_0$, we have

$$\operatorname{Re}\left\{-\frac{z^{p+1}\left(H_{p,q,s}(\alpha_1) f(z)\right)'}{p}\right\} \longrightarrow -\frac{\beta(p+1)}{\lambda}\int_0^1 \frac{u^{-((p+1)/\lambda)-1}}{1+u}du + 1 - \beta < \frac{1}{2} \quad (z \longrightarrow -1), \tag{2.36}$$

which implies that $f(z) \notin T_{p,q,s}(0; h)$. Hence the bound β_0 cannot be increased when $h(z) = 1/(1-z)$. □

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