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Research Article

A Generalized Meir-Keeler-Type Contraction on Partial Metric Spaces

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We introduce a generalization of the Meir-Keeler-type contractions, referred to as generalized Meir-Keeler-type contractions, over partial metric spaces. Moreover, we show that every orbitally continuous generalized Meir-Keeler-type contraction has a fixed point on a 0-complete partial metric space.

1. Introduction

In 1992, Matthews introduced the notion of a partial metric space which is a generalization of usual metric space [1]. The main motivation behind the idea of a partial metric space is to transfer mathematical techniques into computer science. This is mostly apparent in the research areas of computer domains and semantics, which have many applications (see, e.g., [2–10]). Following this initial work, Matthews generalized the Banach contraction principle in the context of complete partial metric spaces. He proved that a self-mapping T on a complete partial metric space (X, p) has a unique fixed point if there exists $0 \le k < 1$ such that $p(Tx, Ty) \le kp(x, y)$ for all $x, y \in X$. After Matthews' innovative approach, many authors conducted further studies on partial metric spaces and their topological properties (see, e.g., [2–4, 6, 11–41]).

A partial metric is a function $p : X \times X \to [0, \infty)$ satisfying the following conditions:

(P1) $p(x, y) = p(y, x)$,

(P2) if $p(x, x) = p(x, y) = p(y, y)$, then $x = y$,

(P3) $p(x, x) \le p(x, y)$,

(P4) $p(x, z) + p(y, y) \le p(x, y) + p(y, z)$,

for all $x, y, z \in X$. Then (X, p) is called a partial metric space.

Example 1.1 (see [42]). Let (X,d) and (X,p) be a metric space and partial metric space, respectively. Mappings $\rho_i : X \times X \to \mathbb{R}^+$ $(i \in \{1,2,3\})$ defined by

$$\begin{gathered}\rho_1(x,y) = d(x,y) + p(x,y), \\ \rho_2(x,y) = d(x,y) + \max\{\omega(x),\omega(y)\}, \\ \rho_3(x,y) = d(x,y) + a\end{gathered} \tag{1.1}$$

induce partial metrics on X, where $\omega : X \to \mathbb{R}^+$ is an arbitrary function and $a \geq 0$.

Each partial metric p on X generates a T_0 topology τ_p on X with the family of open p-balls $\{B_p(x,\varepsilon) : x \in X, \varepsilon > 0\}$ as a base, where $B_p(x,\varepsilon) = \{y \in X : p(x,y) < p(x,x) + \varepsilon\}$ for all $x \in X$. Similarly, a closed p-ball is defined as $B_p[x,\varepsilon] = \{y \in X : p(x,y) \leq p(x,x) + \varepsilon\}$.

In [1, page 187], Matthews gave the characterization of convergence in partial metric space as follows: a sequence $\{x_n\}$ in a partial metric space (X,p) converges to $x \in X$ with respect to τ_p if and only if $\lim_{n\to\infty} p(x,x_n) = p(x,x)$.

Now we recall some basic concepts and useful facts on completeness of partial metric spaces. A sequence $\{x_n\}$ in a partial metric space (X,p) is called Cauchy whenever $\lim_{n,m\to\infty} p(x_n,x_m)$ exists (and is finite) [1, Definition 5.2].

A partial metric space (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x \in X$ such that $\lim_{n,m\to\infty} p(x_n,x_m) = p(x,x)$ [1, Definition 5.3].

In [35], Romaguera introduced the concepts 0-Cauchy sequence in a partial metric space and 0-complete partial metric space as follows.

Definition 1.2. A sequence $\{x_n\}$ in a partial metric space (X,p) is called 0-Cauchy if $\lim_{n,m\to\infty} p(x_n,x_m) = 0$. A partial metric space (X,p) is said to be 0-complete if every 0-Cauchy sequence in X converges, with respect to τ_p, to a point $x \in X$ such that $p(x,x) = 0$. In this case, p is said to be a 0-complete partial metric on X.

Notice that each 0-Cauchy sequence is also a Cauchy sequence in a partial metric space. In particular, each complete partial metric is a 0-complete partial metric on X. But the converse is not true. The following example shows that there exists a 0-complete partial metric which is not complete.

Example 1.3 (see [35, 39]). Let $(\mathbb{Q} \cap [0,\infty), p)$ be the partial metric space, where $\mathbb{Q}$ and $p(x,y)$ represent the set of rational numbers and the partial metric $\max\{x,y\}$, respectively.

A self-mapping F on a partial metric space (X,p) is continuous at $x \in X$ if and only if for every $\varepsilon > 0$, there exists $\delta > 0$ such that $F(B_p(x,\delta)) \subseteq B_p(Fx,\varepsilon)$ (see, e.g., [15]).

It is quite natural to consider characterizations of continuity of mappings in partial metric spaces. For example, Samet et al. [43] proved the following.

Lemma 1.4. *Let (X,p) be a partial metric space. $F : X \to X$ is continuous if given a sequence $\{x_n\} \in \mathbb{N}$ and $x \in X$ such that $p(x,x) = \lim_{n\to+\infty} p(x,x_n)$; then, $p(Fx,Fx) = \lim_{n\to+\infty} p(Fx,Fx_n)$.*

Very recently, Samet et al. [43] also observed the relationship between the continuity of a mapping in a partial metric space and in a metric space.

Lemma 1.5. *Consider $X = [0,\infty)$ endowed with the partial metric $p : X \times X \to [0,\infty)$ defined by $p(x,y) = \max\{x,y\}$ for all $x,y \geq 0$. Let $F : X \to X$ be a nondecreasing function. If F is continuous with respect to the standard metric $d(x,y) = |x-y|$ for all $x,y \geq 0$, then F is continuous with respect to the partial metric p.*

In 1971, Ćirić [44] introduced orbitally continuous maps on metric spaces as follows.

Definition 1.6. Let (X,d) be a metric space. A mapping T on X is orbitally continuous if $\lim_{i\to\infty} T^{n_i}x = u$ implies $\lim_{i\to\infty} TT^{n_i}x = Tu$ for each $x \in X$.

Recently, Karapınar and Erhan [28] renovated the definition above in the context of partial metric spaces in the following way.

Definition 1.7. Let (X,p) be a partial metric space, and let $T : X \to X$ be a self-map. One says that T is orbitally continuous whenever $\lim_{i\to\infty} p(T^{n_i}x, z) = p(z,z)$ implies that $\lim_{i\to\infty} p(TT^{n_i}x, Tz) = p(Tz,Tz)$ for each $x \in X$.

It is clear that continuous mappings are orbitally continuous.

We would like to point out the close relationship between metrics and partial metrics. In fact, if p is a partial metric on X, then the function $d_p : X \times X \to [0,\infty)$ given by

$$d_p(x,y) = 2p(x,y) - p(x,x) - p(y,y) \tag{1.2}$$

is a metric on X. Moreover,

$$\lim_{n\to\infty} d_p(x,x_n) = 0 \Longleftrightarrow \lim_{n\to\infty} p(x,x_n) = \lim_{n,m\to\infty} p(x_n,x_m) = p(x,x). \tag{1.3}$$

Lemma 1.8 (see, e.g., [1, 15]). *Let (X,p) be a partial metric space.*

(a) *A sequence $\{x_n\}$ is Cauchy if and only if $\{x_n\}$ is a Cauchy sequence in the metric space (X,d_p);*

(b) *(X,p) is complete if and only if the metric space (X,d_p) is complete.*

In 1969, Meir and Keeler [45] published their celebrated paper in which an interesting and general contraction condition for self-maps in metric spaces was considered.

Definition 1.9. Let (X,d) be a metric space, and let T be a self-map on X. Then T is called a Meir-Keeler-type contraction whenever for each $\varepsilon > 0$ there exists $\delta > 0$ such that

$$d(x,y) < \varepsilon + \delta \Longrightarrow d(Tx,Ty) < \varepsilon. \tag{1.4}$$

Many authors have discussed several variations, generalizations, and modifications of that condition both in metric spaces and other related structures (see, e.g., [46–49]). Following this trend, we introduce a generalized Meir-Keeler-type contraction on partial metric spaces. In this paper, we show an orbitally continuous self-mapping T on a 0-complete partial metric spaces satisfying that generalized Meir-Keeler-type contraction has a unique fixed point.

2. Main Results

We start this section by recalling the following two lemmas ([13]), which will be frequently used in the proofs of the main results.

Lemma 2.1. *Let (X, p) be a partial metric space. Then*

(a) *if $p(x, y) = 0$, then $x = y$,*

(b) *if $x \neq y$, then $p(x, y) > 0$,*

(c) *if $x_n \to z$ with $p(z, z) = 0$, then $\lim_{n\to\infty} p(x_n, y) = p(z, y)$ for all $y \in X$.*

We introduce the definition of a generalized Meir-Keeler-type contraction.

Definition 2.2. Let (X, p) be a partial metric space and T a self-map on X. Then T is called a generalized Meir-Keeler-type contraction whenever for each $\varepsilon > 0$ there exists $\delta > 0$ such that

$$\varepsilon \leq M(x, y) < \varepsilon + \delta \Longrightarrow p(Tx, Ty) < \varepsilon, \tag{2.1}$$

where $M(x, y) = \max\{p(x, y), p(Tx, x), p(Ty, y), (1/2)[p(Tx, y) + p(x, Ty)]\}$.

Remark 2.3. Note that if T is a generalized Meir-Keeler-type contraction, we have

$$p(Tx, Ty) \leq M(x, y) \quad \forall x, y \in X. \tag{2.2}$$

If $M(x, y) = 0$, it follows from (2.2) that $p(Tx, Ty) = 0$. On the other hand, if $M(x, y) > 0$, we get the strict inequality $p(Tx, Ty) < M(x, y)$ by (2.1).

Now, we are ready to state and prove our main results.

Proposition 2.4. *Let (X, p) be a partial metric space and $T : X \to X$ a generalized Meir-Keeler-type contraction. Then, $\lim_{n\to\infty} p(T^{n+1}x, T^n x) = 0$ for all $x \in X$.*

Proof. Take $x \in X$, and set $x_0 = x$. Define $x_{n+1} = Tx_n = T^{n+1}x_0$ for all $n \geq 0$. If $p(x_{n_0+1}, x_{n_0}) = 0$ for some $n_0 \geq 0$, then $Tx_{n_0} = x_{n_0+1} = x_{n_0}$ by Lemma 2.1. Then, $p(x_{k+1}, x_k) = 0$ for all $k \geq n_0$. In this case, the proposition follows. In the rest of the proof, we assume that $p(x_{n+1}, x_n) \neq 0$ for every $n \geq 0$. As a consequence, we have $M(x_{n+1}, x_n) > 0$ for every $n \geq 0$. By Remark 2.3,

$$\begin{aligned} p(x_{n+2}, x_{n+1}) = p(Tx_{n+1}, Tx_n) &\leq M(x_{n+1}, x_n) \\ &= \max\left\{p(x_{n+1}, x_n), p(Tx_{n+1}, x_{n+1}), p(Tx_n, x_n), \frac{1}{2}[p(Tx_{n+1}, x_n) + p(x_{n+1}, Tx_n)]\right\} \\ &\leq \max\{p(x_{n+1}, x_n), p(x_{n+2}, x_{n+1})\}. \end{aligned} \tag{2.3}$$

Since $M(x_{n+1}, x_n)$ is strictly positive for each n, we find that

$$p(x_{n+2}, x_{n+1}) < M(x_{n+1}, x_n) \leq \max\{p(x_{n+1}, x_n), p(x_{n+2}, x_{n+1})\} \tag{2.4}$$

by the use of Remark 2.3 again. Notice that the case where

$$\max\{p(x_{n+1},x_n),\ p(x_{n+2},x_{n+1})\} = p(x_{n+2},x_{n+1}) \tag{2.5}$$

is not possible. Hence we derive that

$$p(x_{n+2},x_{n+1}) < M(x_{n+1},x_n) \le p(x_{n+1},x_n) \tag{2.6}$$

for every n. Thus, $\{p(x_{n+1},x_n)\}_{n=0}^{\infty}$ is a decreasing sequence which is bounded below by 0. Hence, it converges to some $\varepsilon \in [0,\infty)$, that is,

$$\lim_{n\to\infty} p(x_{n+1},x_n) = \varepsilon. \tag{2.7}$$

In particular, we have

$$\lim_{n\to\infty} M(x_{n+1},x_n) = \varepsilon. \tag{2.8}$$

Notice that $\varepsilon = \inf\{p(x_n,x_{n+1}) : n \in \mathbb{N}\}$.

We claim that $\varepsilon = 0$. Suppose, to the contrary, that $\varepsilon > 0$. Regarding (2.8) together with the assumption that T is generalized Meir-Keeler-type contraction, for this ε, there exists $\delta > 0$ and a natural number m such that

$$\varepsilon \le M(x_{m+1},x_m) < \varepsilon+\delta \quad \text{implies that } p(Tx_{m+1},Tx_m) = p(x_{m+2},x_{m+1}) < \varepsilon. \tag{2.9}$$

This is a contradiction since $\varepsilon = \inf\{p(x_n,x_{n+1}) : n \in \mathbb{N}\}$. □

Theorem 2.5. *Let (X,p) be a 0-complete partial metric space, and let $T : X \to X$ be an orbitally continuous generalized Meir-Keeler-type contraction. Then, T has a unique fixed point, say $z \in X$. Moreover, $\lim_{n\to\infty} p(T^n x, z) = p(z,z)$ for all $x \in X$ and $p(z,z) = 0$.*

Proof. Take $x \in X$, and set $x_0 = x$. Define $x_{n+1} = Tx_n = T^{n+1}x_0$ for all $n \ge 0$. We claim that $\lim_{m,n\to\infty} p(x_n,x_m) = 0$. If this is not the case, then there exist a $\varepsilon > 0$ and a subsequence $\{x_{n(i)}\}$ of $\{x_n\}$ such that

$$p\left(x_{n(i)},x_{n(i+1)}\right) > 2\varepsilon. \tag{2.10}$$

For the same $\varepsilon > 0$ above, there exists $\delta > 0$ such that $\varepsilon \le M(x,y) < \varepsilon+\delta$ which implies that $p(Tx,Ty) < \varepsilon$. Set $r = \min\{\varepsilon,\delta\}$ and $d_n = p(x_n,x_{n+1})$ for all $n \ge 1$. By Proposition 2.4, one can choose a natural number n_0 such that

$$d_n = p(x_n,x_{n+1}) < \frac{r}{4} \tag{2.11}$$

for all $n \geq n_0$. Let $n(i) > n_0$. We have $n(i) \leq n(i+1) - 1$. If $p(x_{n(i)}, x_{n(i+1)-1}) \leq \varepsilon + (r/2)$, then by using (P4) we derive

$$\begin{aligned} p(x_{n(i)}, x_{n(i+1)}) &\leq p(x_{n(i)}, x_{n(i+1)-1}) + p(x_{n(i+1)-1}, x_{n(i+1)}) - p(x_{n(i+1)-1}, x_{n(i+1)-1}) \\ &\leq p(x_{n(i)}, x_{n(i+1)-1}) + p(x_{n(i+1)-1}, x_{n(i+1)}) \\ &< \varepsilon + \frac{r}{2} + d_{n(i+1)-1} < \varepsilon + \frac{3r}{4} < 2\varepsilon, \end{aligned} \tag{2.12}$$

which contradicts with assumption (2.10). Therefore, there are values of k such that $n(i) \leq k \leq n(i+1)$ and $p(x_{n(i)}, x_k) > \varepsilon + (r/2)$. Now if $p(x_{n(i)}, x_{n(i)+1}) \geq \varepsilon + (r/2)$, then

$$d_{n(i)} = p(x_{n(i)}, x_{n(i)+1}) \geq \varepsilon + \frac{r}{2} > r + \frac{r}{2} > \frac{r}{4}. \tag{2.13}$$

This is a contradiction because of (2.11). Hence, there are values of k with $n(i) \leq k \leq n(i+1)$ such that $p(x_{n(i)}, x_k) < \varepsilon + (r/2)$. Choose the smallest integer k with $k \geq n(i)$ such that $p(x_{n(i)}, x_k) \geq \varepsilon + (r/2)$. Thus, we find $p(x_{n(i)}, x_{k-1}) < \varepsilon + (r/2)$. So we see that

$$\begin{aligned} p(x_{n(i)}, x_k) &\leq p(x_{n(i)}, x_{k-1}) + p(x_{k-1}, x_k) - p(x_{k-1}, x_{k-1}) \\ &\leq p(x_{n(i)}, x_{k-1}) + p(x_{k-1}, x_k) < \varepsilon + \frac{r}{2} + \frac{r}{4} = \varepsilon + \frac{3r}{4}. \end{aligned} \tag{2.14}$$

Now, we can choose a natural number k satisfying $n(i) \leq k \leq n(i+1)$ such that

$$\varepsilon + \frac{r}{2} \leq p(x_{n(i)}, x_k) < \varepsilon + \frac{3r}{4}. \tag{2.15}$$

Therefore, we obtain the inequalities

$$p(x_{n(i)}, x_k) < \varepsilon + \frac{3r}{4} < \varepsilon + r, \tag{2.16}$$

$$\begin{aligned} p(x_{n(i)}, x_{n(i)+1}) &= d_{n(i)} < \frac{r}{4} < \varepsilon + r, \\ p(x_k, x_{k+1}) &= d_k < \frac{r}{4} < \varepsilon + r. \end{aligned} \tag{2.17}$$

Thus, we have

$$\begin{aligned} &\frac{1}{2}\left[p(x_{n(i)}, x_{k+1}) + p(x_{n(i)+1}, x_k)\right] \\ &\quad\leq \frac{1}{2}\left[p(x_{n(i)}, x_k) + p(x_k, x_{k+1}) - p(x_k, x_k) + p(x_{n(i)+1}, x_{n(i)}) + p(x_{n(i)}, x_k) - p(x_{n(i)}, x_{n(i)})\right] \\ &\quad\leq \frac{1}{2}\left[p(x_{n(i)}, x_k) + p(x_k, x_{k+1}) + p(x_{n(i)+1}, x_{n(i)}) + p(x_{n(i)}, x_k)\right] \\ &\quad= p(x_{n(i)}, x_k) + \frac{1}{2}\left[d_k + d_{n(i)}\right] < \varepsilon + \frac{3r}{4} + \frac{1}{2}\left[\frac{r}{4} + \frac{r}{4}\right] = \varepsilon + r. \end{aligned} \tag{2.18}$$

Now, inequalities (2.16)–(2.18) imply that $M(x_{n(i)}, x_k) < \varepsilon + r \le \varepsilon + \delta$. Hence, the fact that T is a generalized Meir-Keeler-type contraction yields $p(x_{n(i)+1}, x_{k+1}) < \varepsilon$. By using (P4), we obtain

$$\begin{aligned} p\left(T^{n(i)}x_0, T^k x_0\right) &\le p\left(T^{n(i)}x_0, T^{n(i)+1}x_0\right) + p\left(T^{n(i)+1}x_0, T^k x_0\right) \\ &\quad - p\left(T^{n(i)+1}x_0, T^{n(i)+1}x_0\right) \\ &\le p\left(T^{n(i)}x_0, T^{n(i)+1}x_0\right) + p\left(T^{n(i)+1}x_0, T^k x_0\right) \\ &\le p\left(T^{n(i)}x_0, T^{n(i)+1}x_0\right) + p\left(T^{n(i)+1}x_0, T^{k+1}x_0\right) \\ &\quad + p\left(T^{k+1}x_0, T^k x_0\right). \end{aligned} \tag{2.19}$$

We combine the inequality above with (2.15) and (2.17) to conclude

$$\begin{aligned} p(x_{n(i)+1}, x_{k+1}) &\ge p(x_{n(i)}, x_k) - p(x_{n(i)}, x_{n(i)+1}) - p(x_k, x_{k+1}) \\ &> \varepsilon + \frac{r}{2} - \frac{r}{4} - \frac{r}{4} = \varepsilon, \end{aligned} \tag{2.20}$$

which is a contradiction. Therefore, our claim is proved. So $\{x_n\} = \{T^n x_0\}$ is a 0-Cauchy sequence. Since (X, p) is 0-complete, then by Definition 1.2, the sequence $\{x_n\}$ converges with respect to τ_p to some $z \in X$ such that $p(z, z) = 0$. Thus

$$\lim_{n\to\infty} p(T^n x_0, z) = p(z, z) = 0. \tag{2.21}$$

Now, we will show that z is a fixed point of T.

Since T is orbitally continuous and $\lim_{n\to\infty} p(T^n x_0, z) = p(z, z)$, we get that

$$\lim_{n\to\infty} p(TT^n x_0, Tz) = p(Tz, Tz). \tag{2.22}$$

On the other hand, from Lemma 2.1, we have

$$\lim_{n\to\infty} p(TT^n x_0, Tz) = \lim_{n\to\infty} p(x_{n+1}, Tz) = p(z, Tz) \tag{2.23}$$

which follows from the fact that $\{x_{n+1}\}$ converges to z in (X, p) with $p(z, z) = 0$, where $x_{n+1} = TT^n x_0 = T^{n+1}x_0$. Combining this with (2.22), we get that $p(z, Tz) = p(Tz, Tz)$.

We aim to show that $p(z, Tz) = 0$. Assume that $p(z, Tz) > 0$. Then, we obtain $M(z, z) \ge p(z, Tz) > 0$. By (2.2), we have

$$p(Tz, Tz) < M(z, z) = \max\left\{p(z, z) = 0, p(z, Tz)\right\} = p(z, Tz) = p(Tz, Tz), \tag{2.24}$$

a contradiction. This implies $Tz = z$ by Lemma 2.1.

Finally, we show that T has a unique fixed point. If there exists $w \in X$ such that $Tw = w$ and $p(z,w) \neq 0$, then we get $M(z,w) \geq p(z,w) > 0$. Since T is a generalized Meir-Keeler-type contraction, we derive

$$\begin{aligned} 0 < p(z,w) = p(Tz,Tw) &< M(z,w) \\ &= \max\left\{p(z,w), p(Tz,z), p(Tw,w), \ \frac{1}{2}\left[p(Tz,w) + p(z,Tw)\right]\right\} \\ &= \max\{p(z,w), p(w,w)\} = p(w,z), \end{aligned} \tag{2.25}$$

which is a contradiction. Thus, we find that $p(z,w) = 0$. So by Lemma 2.1 we conclude that $z = w$. In particular, T has a unique fixed point. □

We state two examples to illustrate our results.

Example 2.6. Let (X,p) be the set $[0,\infty)$ equipped with the partial metric $p(x,y) = \max\{x,y\}$. Clearly, (X,p) is a 0-complete partial metric space. Consider $T : X \to X$ defined by $Tx = x/3(1+x)$. Given $\varepsilon > 0$, we will show that there exists $\delta = \delta(\varepsilon) \geq 0$ such that (2.1) holds for all $x, y \in X$. Without loss of generality, take $x \leq y$. Then, it is easy to show that

$$\begin{aligned} p(Tx,Ty) &= \frac{y}{3(1+y)} \\ M(x,y) &= \max\left\{p(x,y), p(Tx,x), p(Ty,y), \frac{1}{2}\left[p(Tx,y) + p(x,Ty)\right]\right\} = y. \end{aligned} \tag{2.26}$$

Thus, taking $\delta(\varepsilon) = 2\varepsilon$, we get that (2.1) holds. Also, by Lemma 1.5, the mapping T is continuous, and hence it is orbitally continuous. All hypotheses of Theorem 2.5 are satisfied and $z = 0$ is the unique fixed point of T.

Example 2.7. Let (X,p) be the interval $[0,2]$ equipped with the partial metric $p(x,y) = \max\{x,y\}$. Consider $T : X \to X$ defined by

$$Tx = \begin{cases} \dfrac{x}{2} & \text{if } 0 \leq x < 1, \\ \dfrac{1}{2} & \text{if } 1 \leq x \leq 2. \end{cases} \tag{2.27}$$

Take $x \leq y$. Given $\varepsilon > 0$, we have the two following cases.

Case 1 $(0 \leq x \leq y < 1)$. We have

$$p(Tx,Ty) = \frac{y}{2}, \qquad M(x,y) = y. \tag{2.28}$$

Case 2 $((0 \leq x < 1 \text{ and } 1 \leq y < 2) \text{ or } (1 \leq x \leq y \leq 2))$. We have

$$p(Tx,Ty) = \frac{1}{2}, \qquad M(x,y) = y. \tag{2.29}$$

In each case, it suffices to take $\delta = \varepsilon$ in order that (2.1) holds. Again, by Lemma 1.5, the mapping T is continuous, and hence it is orbitally continuous. All hypotheses of Theorem 2.5 are satisfied and $z = 0$ is the unique fixed point of T.

References

[1] S. G. Matthews, "Partial metric topology," *Annals of the New York Academy of Sciences*, vol. 728, pp. 183–197, 1994.

[2] R. D. Kopperman, S. G. Matthews, and H. Pajoohesh, "What do partial metrics represent?" in *Proceedings of the 19th Summer Conference on Topology and Its Applications*, University of CapeTown, 2004.

[3] H.-P. A. Künzi, H. Pajoohesh, and M. P. Schellekens, "Partial quasi-metrics," *Theoretical Computer Science*, vol. 365, no. 3, pp. 237–246, 2006.

[4] S. J. O'Neill, "Two topologies are better than one," Tech. Rep., University of Warwick, 1995.

[5] R. Heckmann, "Approximation of metric spaces by partial metric spaces," *Applied Categorical Structures*, vol. 7, no. 1-2, pp. 71–83, 1999.

[6] S. Romaguera and M. Schellekens, "Duality and quasi-normability for complexity spaces," *Applied General Topology*, vol. 3, no. 1, pp. 91–112, 2002.

[7] M. P. Schellekens, "A characterization of partial metrizability: domains are quantifiable," *Theoretical Computer Science*, vol. 305, no. 1–3, pp. 409–432, 2003.

[8] M. P. Schellekens, "The correspondence between partial metrics and semivaluations," *Theoretical Computer Science*, vol. 315, no. 1, pp. 135–149, 2004.

[9] P. Waszkiewicz, "Partial metrisability of continuous posets," *Mathematical Structures in Computer Science*, vol. 16, no. 2, pp. 359–372, 2006.

[10] P. Waszkiewicz, "Quantitative continuous domains," *Applied Categorical Structures*, vol. 11, no. 1, pp. 41–67, 2003.

[11] M. Abbas, T. Nazir, and S. Romaguera, "Fixed point results for generalized cyclic contraction mappings in partial metric spaces," *Revista de la Real Academia de Ciencias Exactas*. In press.

[12] T. Abdeljawad, "Fixed points for generalized weakly contractive mappings in partial metric spaces," *Mathematical and Computer Modelling*, vol. 54, no. 11-12, pp. 2923–2927, 2011.

[13] T. Abdeljawad, E. Karapınar, and K. Taş, "Existence and uniqueness of a common fixed point on partial metric spaces," *Applied Mathematics Letters*, vol. 24, no. 11, pp. 1900–1904, 2011.

[14] T. Abdeljawad, E. Karapinar, and K. Taş, "A generalized contraction principle with control functions on partial metric spaces," *Computers and Mathematics with Applications*, vol. 63, no. 3, pp. 716–719, 2012.

[15] I. Altun and A. Erduran, "Fixed point theorems for monotone mappings on partial metric spaces," *Fixed Point Theory and Applications*, vol. 2011, Article ID 508730, 10 pages, 2011.

[16] I. Altun, F. Sola, and H. Simsek, "Generalized contractions on partial metric spaces," *Topology and its Applications*, vol. 157, no. 18, pp. 2778–2785, 2010.

[17] H. Aydi, "Some coupled fixed point results on partial metric spaces," *International Journal of Mathematics and Mathematical Sciences*, vol. 2011, Article ID 647091, 11 pages, 2011.

[18] H. Aydi, "Fixed point results for weakly contractive mappings in ordered partial metric spaces," *Journal of Advanced Mathematical Studies*, vol. 4, no. 2, pp. 1–12, 2011.

[19] H. Aydi, "Fixed point theorems for generalized weakly contractive condition in ordered partial metric spaces," *Journal of Nonlinear Analysis and Optimization: Theory and Applications*, vol. 2, no. 2, pp. 33–48, 2011.

[20] H. Aydi, "Common fixed point results for mappings satisfying (ψ, ϕ)-weak contractions in ordered partial metric spaces," *International Journal of Applied Mathematics and Statistics*, vol. 12, no. 2, pp. 53–64, 2012.

[21] H. Aydi, E. Karapınar, and W. Shatanawi, "Coupled fixed point results for (ψ, φ)-weakly contractive condition in ordered partial metric spaces," *Computers & Mathematics with Applications*, vol. 62, no. 12, pp. 4449–4460, 2011.

[22] H. Aydi, "A common fixed point result by altering distances involving a contractive condition of integral type in partial metric spaces," *Demonstratio Mathematica*. In press.

[23] K. P. Chi, E. Karapinar, and T. D. Thanh, "A generalized contraction principle in partial metric spaces," *Mathematical and Computer Modelling*, vol. 55, no. 5-6, pp. 1673–1681, 2012.

[24] L. Ćirić, B. Samet, H. Aydi, and C. Vetro, "Common fixed points of generalized contractions on partial metric spaces and an application," *Applied Mathematics and Computation*, vol. 218, no. 6, pp. 2398–2406, 2011.

[25] D. Ilić, V. Pavlović, and V. Rakočević, "Some new extensions of Banach's contraction principle to partial metric space," *Applied Mathematics Letters*, vol. 24, no. 8, pp. 1326–1330, 2011.

[26] D. Ilić, V. Pavlović, and V. Rakoçević, "Extensions of the Zamfirescu theorem to partial metric spaces," *Mathematical and Computer Modelling*, vol. 55, no. 34, pp. 801–809, 2012.

[27] E. Karapinar, "Weak ϕ-contraction on partial metric spaces," *Journal of Computational Analysis and Applications*, vol. 14, no. 2, pp. 206–210, 2012.

[28] E. Karapınar and I. M. Erhan, "Fixed point theorems for operators on partial metric spaces," *Applied Mathematics Letters*, vol. 24, pp. 1900–1904, 2011.

[29] E. Karapinar, "Generalizations of Caristi Kirk's theorem on partial metric spaces," *Fixed Point Theory and Applications*, vol. 2011, article 4, 2011.

[30] E. Karapnar and U. Yüksel, "Some common fixed point theorems in partial metric spaces," *Journal of Applied Mathematics*, vol. 2011, Article ID 263621, 17 pages, 2011.

[31] E. Karapnar and U. Yüksel, "Some common fixed point theorems in partial metric spaces," *Journal of Applied Mathematics*, vol. 12, no. 2, pp. 185–191, 2011.

[32] H. K. Nashine, Z. Kadelburg, and S. Radenović, "Common fixed point theorems for weakly isotone increasing mappings in ordered partial metric spaces," *Mathematical and Computer Modelling*. In press.

[33] S. Oltra and O. Valero, "Banach's fixed point theorem for partial metric spaces," *Rendiconti dell'Istituto di Matematica dell'Università di Trieste*, vol. 36, no. 1-2, pp. 17–26, 2004.

[34] D. Paesano and P. Vetro, "Suzuki's type characterizations of completeness for partial metric spaces and fixed points for partially ordered metric spaces," *Topology and its Applications*, vol. 159, no. 3, pp. 911–920, 2012.

[35] S. Romaguera, "A Kirk type characterization of completeness for partial metric spaces," *Fixed Point Theory and Applications*, vol. 2010, Article ID 493298, 6 pages, 2010.

[36] S. Romaguera and M. Schellekens, "Partial metric monoids and semivaluation spaces," *Topology and its Applications*, vol. 153, no. 5-6, pp. 948–962, 2005.

[37] S. Romaguera and O. Valero, "A quantitative computational model for complete partial metric spaces via formal balls," *Mathematical Structures in Computer Science*, vol. 19, no. 3, pp. 541–563, 2009.

[38] S. Romaguera, "Fixed point theorems for generalized contractions on partial metric spaces," *Topology and its Applications*, vol. 159, no. 1, pp. 194–199, 2012.

[39] S. Romaguera, "Matkowskis type theorems for generalized contractions on (ordered) partial metric spaces," *Applications of General Topology*, vol. 12, no. 2, pp. 213–220, 2011.

[40] W. Shatanawi, B. Samet, and M. Abbas, "Coupled fixed point theorems for mixed monotone mappings in ordered partial metric spaces," *Mathematical and Computer Modelling*, vol. 55, no. 3-4, pp. 680–687, 2012.

[41] O. Valero, "On Banach fixed point theorems for partial metric spaces," *Applied General Topology*, vol. 6, no. 2, pp. 229–240, 2005.

[42] N. Shobkolaei, S.M. Vaezpour, and S. Sedghi, "A common fixed point theorem on ordered partial metric spaces," *Journal of Basic and Applied Sciences*, vol. 1, no. 12, pp. 3433–3439, 2011.

[43] B. Samet, M. Rajović, R. Lazović, and R. Stoiljković, "Common fixed point results for nonlinear contractions in ordered partial metric spaces," *Fixed Point Theory and Applications*, vol. 2011, article 71, 2011.

[44] L. Ćirić, "On contraction type mappings," *Mathematica Balkanica*, vol. 1, pp. 52–57, 1971.

[45] A. Meir and E. Keeler, "A theorem on contraction mappings," *Journal of Mathematical Analysis and Applications*, vol. 28, pp. 326–329, 1969.

[46] J. Jachymski, "Equivalent conditions and the Meir-Keeler type theorems," *Journal of Mathematical Analysis and Applications*, vol. 194, no. 1, pp. 293–303, 1995.

[47] Z. Kadelburg and S. Radenović, "Meir-Keeler-type conditions in abstract metric spaces," *Applied Mathematics Letters*, vol. 24, no. 8, pp. 1411–1414, 2011.

[48] B. Samet, "Coupled fixed point theorems for a generalized Meir-Keeler contraction in partially ordered metric spaces," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 72, no. 12, pp. 4508–4517, 2010.

[49] T. Suzuki, "Fixed-point theorem for asymptotic contractions of Meir-Keeler type in complete metric spaces," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 64, no. 5, pp. 971–978, 2006.