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*Research Article*

# Fractal Derivative Model for Air Permeability in Hierarchic Porous Media

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Air permeability in hierarchic porous media does not obey Fick's equation or its modification because fractal objects have well-defined geometric properties, which are discrete and discontinuous. We propose a theoretical model dealing with, for the first time, a seemingly complex air permeability process using fractal derivative method. The fractal derivative model has been successfully applied to explain the novel air permeability phenomenon of cocoon. The theoretical analysis was in agreement with experimental results.

## 1. Introduction

Air permeation and moisture vapor diffusion in textile fabric have a close relation to the comfort ability of clothing [1–3]. Since the skin is "breathing" all the time, water vapor is continuously released from the skin. If the moisture cannot be effectively delivered to the outside atmosphere, the body will feel sultry in summer and clammy in winter.

Clothing exhibits a multiscale inner structure constructed by textile fiber. The micropores among the thin fibers provide a winding passage for air and moisture vapor exchange between the microclimate (between the skin and fabric) and atmosphere. For certain textile apparel, the arrangement style of the fiber in the fabric may result in different air permeation efficiency. Recently, Blossman-Myer and Burggren investigated the water loss and oxygen diffusion through the silk cocoon. And the experimental data indicated that the cocoon does not have any impact on the oxygen and water vapor transportation [4]. This interesting finding suggests that cocoon has some special functional performance in gas permeation besides its basic purpose for protecting pupa against predation, biodegradation, and dehydration [5]. The fantastic air permeation performance of cocoon might be related to its configuration. Investigation on the mechanism of gas permeation in such a natural porous media composed by fiber is an inspiration in developing new functional clothing.

As a transport property, mechanism of gas permeation in different porous media has received continuous attention due to its significance in both science and engineering [6, 7], such as gas separation or gas purification [8, 9], chemical catalyst industry [10], proton exchange membrane fuel cell [11, 12], and textile material [1–3]. Since the microstructures of the hierarchic porous media are usually disordered and extremely complicated, it is very difficult to appropriately describe the random structures analytically.

In the past two decades, fractal theory has been introduced to solve problems of diffusion in media with hierarchic configuration since Mandelbrot's pioneering work [13]. The fractal dimensions have been very effective in characterization of objects in terms of their self-similar properties, which allows analysis of physical phenomena in disordered structures to become much easier.

A number of fractal geometry models have been developed by many researchers. However, most of these models include continuous parameters, which were against the basic property of fractal geometry that fractal objects are discontinuous. As alternative approaches, fractional and fractal derivatives have been found effective in modeling permeation problem in hierarchic porous media [14–16]. There are many definitions of fractional derivative, but most fractional derivatives are very complex for engineering applications [17]. In order to better model an engineering problem in a discontinuous media, a new derivative is much needed. Comparing it with fractional derivative, fractal derivative is a much easier and efficient new method in developing the fractal model to deal with these complex problems [18].

In this work, a fractal model for air permeation in hierarchic porous media was developed based on a new fractal derivative theory. And the excellent gas permeation of cocoon was investigated with assistant of the novel fractal derivative model.

## 2. Definition of the Fractal Derivative

Chen [19] firstly developed fractal derivative from fractal concept defined as

$$\frac{du(x)}{dx^D} = \lim_{s \to x} \frac{u(x) - u(s)}{x^D - s^D}. \tag{2.1}$$

As an alternative modeling formalism of fractional derivative, fractal derivative presents the fractal space-time transforms to display explicitly how the fractal metric space-time influences physical behaviors in statistical description of the anomalous diffusion in diverse engineering fields.

Recently, He [20] introduced a new fractal derivative defined as

$$\frac{Du(t)}{Dx^\alpha} = \lim_{\Delta x \to L_0} \frac{u(A) - u(B)}{\text{The distance between two points}} = \frac{du}{ds} = \lim_{\Delta x \to L_0} \frac{u(A) - u(B)}{kL_0^\alpha}, \tag{2.2}$$

where $k$ is a constant, $\alpha$ is the fractal dimension, $L_0$ is the smallest measure, and $u(t)$ is the distance between two points in the porous media. The distance between two points in a discontinuous space is

$$ds = kL_0^{\alpha}, \tag{2.3}$$

where $k$ is a constant.

The new fractal derivative can convert fractional differential equations to ordinary differential equations, so that nonmathematicians can easily deal with fractional calculus.

## 3. Fractal Model for Gas Permeation in Hierarchic Porous Media

In continuous media, gas permeation obeys Fick's equation:

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}\left(D\frac{\partial c}{\partial x}\right) = 0, \tag{3.1}$$

where $\partial c/\partial x$ is the concentration gradient of gas and $D$ is the diffusion coefficient of gas.

When the concentration of gas does not change with time, the gas diffusion can be regarded as in one-dimensional steady-state case. Equation (3.1) became

$$D\frac{dc}{dx} = q. \tag{3.2}$$

The solution of (3.2) is

$$c = c_0 + \frac{q}{D}x, \tag{3.3}$$

where $q$ is the gas flux and $c_0$ is the initial gas concentration.

Equation (3.2) depicts the gas permeation in a continuous diffusion path. However, gas diffusion path through the hierarchic porous media was rather different.

Gas permeation in the discontinuous hierarchic porous media can be expressed as

$$D\frac{D\overline{c}}{Dl^{\alpha}} = \overline{q}, \tag{3.4}$$

where $\overline{c}$ is the gas concentration in porous media and $\overline{q}$ is the gas flux in the porous media.

According to the definition of fractal derivative

$$\frac{D\overline{c}}{Dl^{\alpha}} = \lim_{\Delta l \to x}\frac{c_2 - c_1}{x_2 - x_1} = \lim_{\Delta l \to x}\frac{\Delta c}{\Delta s} = \lim_{\Delta l \to x}\frac{\Delta c}{kx^{\alpha}}, \tag{3.5}$$

where $\Delta l$ tends to zero, but to the smallest measure size $x$, $k$ is a constant.

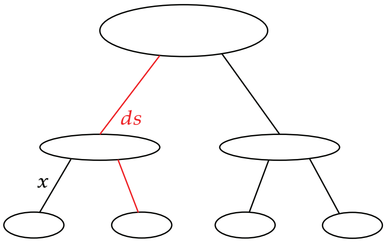

**Figure 1:** Schematic diagram of hierarchic gas diffusion path in cocoon wall.

Submit (3.5) into (3.4), we get the solution of (3.4),

$$\overline{c} = \overline{c}_0 + \frac{\overline{q}}{D} k x^{\alpha}, \tag{3.6}$$

where $D$ is the diffusion coefficient of gas, $\overline{c}_0$ is the initial gas concentration and $\alpha$ is the fractal dimension of the gas path.

Suppose the gas concentration of the inner side of a porous medium equals to that of the outer side of the porous medium, that is, $c = \overline{c}$, $c_0 = \overline{c}_0$. Dividing (3.6) by (3.3), we obtain

$$\frac{\overline{q}}{q} = \frac{x^{1-\alpha}}{k}. \tag{3.7}$$

Equation (3.7) indicated that the gas flux strongly depends on the configuration of the gas diffusion path.

## 4. Discussion

For (3.7), if the fractal dimension of the porous medium $\alpha = 1 - \ln k / \ln x$, then $\overline{q}/q = 1$. The porous medium has no effect on preventing the gas to permeate through the medium. Air can permeate through the medium freely as if the medium does not exist.

For the case the smallest measuring size $x$ is small, $x \ll e$, that is, $\ln x < 0$.

If $\alpha > 1 - \ln k / \ln x$, then $\overline{q}/q > 1$, which means that gas permeation became even faster with the existence of the porous medium.

If $\alpha < 1 - \ln k / \ln x$, then $\overline{q}/q < 1$, which means that the hierarchic porous medium performs a blocking effect on gas permeation.

To investigate the novel gas diffusion phenomenon of cocoon, we need first to analyze the structure of cocoon. SEM observation shows that the cocoon was composed of mainly three layers. Silk fiber in the out layer of cocoon wall was relatively thick with a mean diameter of 26 $\mu$m, while the silk fiber in the inner layer was relatively thin with a mean diameter of 16 $\mu$m. Moreover, the thin fiber in the inner layer of cocoon wall was densely packed, resulting in the tiny pores for gas and vapor diffusion [4]. The size of pores in different layers along the thickness direction of cocoon wall gradually changed, which endows the cocoon with a hierarchic gas path, shown in Figure 1.

In Figure 1, the rings stand for the pores at different fiber layers of cocoon. The bars between the rings stand for the gas path between different layers in the cocoon, and the length of the bar is equal to the fiber diameter. Suppose the included angle between two bars is $\pi/3$, then the fractal dimension of the gas diffusion path in cocoon can be calculated as

$$\alpha = \frac{\ln 3}{\ln\left(1 + \sqrt{3}/2\right)} = 1.76. \tag{4.1}$$

The constant $k$ can be expressed as

$$k = \frac{N^n}{l_n^{1-\alpha}} = \frac{N^n}{\left(l_0\varphi^n\right)^{1-\alpha}}, \tag{4.2}$$

where $N$ is the bifurcation number, $n$ is the number of hierarchic level, $l_0$ is the length of gas path in the primary level, and $\varphi$ is the length ratio of gas path between the daughter level and parent level.

Considering the construction of cocoon, the bifurcation number equals to 2. Since the thickness of the cocoon is about 500 $\mu$m and the average diameter of fiber is about 21 $\mu$m, the cocoon is composed of about 24 layers of fiber, which means that the number of hierarchic level $n = 24$. The length of gas path in the primary level is $l_0 = 26\,\mu$m. Suppose the diameter of silk fiber gradually decreases in a linear relationship from the outermost layer of fiber to the innermost layer of fiber, the length ratio of gas path is $\varphi = 0.9746$. In this case, the smallest measuring scale of the gas diffusion path is $x = 16\,\mu$m, which is equal to the smallest diameter of silk fiber. Submit the above value into the following equation, we obtain

$$1 - \frac{\ln k}{\ln x} = 1 - \frac{\ln\left(2^{24}/\left(26 \times 0.9746^{24} \times 10^{-6}\right)^{1-1.76}\right)}{\ln(16 \times 10^{-6})} = 1.746 \approx \alpha. \tag{4.3}$$

The value of (4.3) is in close proximity to the fractal dimension of the gas diffusion path in cocoon, indicating that the porous medium of cocoon has almost no effect on preventing air or moisture vapor from passing through the multilayer silk fiber mat. And the air flux ratio became

$$\frac{\overline{q}}{q} = \frac{x^{1-\alpha}}{k} = \frac{\left(16 \times 10^{-6}\right)^{1-1.76}}{2^{24}/\left(26 \times 0.9746^{24} \times 10^{-6}\right)^{1-1.76}} = 1.16. \tag{4.4}$$

The value of (4.4) is close to 1, but slightly larger than 1, which suggests that the air flux of cocoon is slightly larger than that of the continuous medium. The cocoon performs excellent gas permeability. In addition, the cocoon can somewhat promote exchange of gas between both sides of the cocoon wall.

The result obtained in this presentation can explain the experimental phenomenon observed by Blossman-Myer and Burggren. The gas permeation property of cocoon is attributed to the inner hierarchic porous configuration constructed by hierarchic assemble of silk fiber. The structure feature of cocoon provides us with an optimized template, which could be duplicated in biomimic fabric design to improve the heat-moisture comfort ability of apparel.

## 5. Conclusions

A fractal derivative model for gas permeation has been proposed for the first time in hierarchic space. This model is able to describe a complex dynamic process completely from the theory, which is of critical importance for biomimic design in any fields, such as industry and biomaterial, functional textiles. The validity of the model has been proved in explaining the fascinating gas diffusion phenomenon of cocoon discovered by scientific experiments.

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