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Research Article

Weak Sharp Minima in Set-Valued Optimization Problems

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We introduce the notion of a weak ψ-sharp minimizer for set-valued optimization problems. We present some sufficient and necessary conditions that a pair point is a weak ψ-sharp minimizer through the outer limit of set-valued map and develop the characterization of the weak ψ-sharp minimizer in terms of a generalized nonlinear scalarization function. These results extend the corresponding ones in Studniarski (2007).

1. Introduction

The notion of weak sharp minima in general mathematical program problems was first introduced by Ferris in [1]. It is a generalization of a sharp minimum in [2] to include the possibility of nonunique solution set. The study of weak sharp minima is motivated primarily by applications in convex and convex composite programming, where such minima commonly occur. Weak sharp minima plays an important role in the sensitivity analysis [3, 4] and convergence analysis of a wide range of optimization algorithms [5]. Recently, the study of weak sharp solution set covers real-valued optimization problems [5–8] and piecewise linear multiobjective optimization problems [9, 10].

In [11], Bednarczuk defined weak sharp Pareto minima of order m for vector-valued mappings and used weak sharp Pareto minima to prove upper Hölderness and Hölder calmness of the solution set-valued mappings for parametric vector optimization problems. In [12], Studniarski gave the definition of weak ψ-sharp local Pareto minima in multiobjective optimization problems and presented necessary and sufficient conditions. In [13], Xu and Li established a sufficient and necessary condition for weak ψ-sharp local

Pareto minima in vector optimization problems in infinite spaces, the approach is that they transformed weak ψ-sharp local Pareto minima of a vector-valued function to weak ψ-sharp local minima of a family of scalar functions. Most recently, Durea and Strugariu [14] introduced the definition of weak ψ-sharp local minima by an oriented distance function in set-valued optimization problems and established necessary optimality conditions in terms of Mordukhovich coderivative.

In the paper, motivated by the work in [15, 16], we also introduce the notion of weak ψ-sharp minima, which is different from one in [14], and establish some sufficient and necessary conditions through the outer limit of set-valued map. In particular, we develop the characterization of the weak ψ-sharp minimizer in terms of the generalized nonlinear scalarization function.

This paper is organized as follows. In Section 2, we recall some basic definitions and give the notion of the weak ψ-sharp local minimizer for set-valued optimization problems. In Section 3, we present some sufficient and necessary conditions through the outer limit of the set-valued map. In Section 4, we establish a characterization of weak ψ-sharp local minima in terms of the generalized nonlinear scalarization function.

2. Preliminary Results

Throughout this paper, let X, Y be real normed spaces. $B(x,\delta)$ denotes the open ball with center $x \in X$ and radius $\delta > 0$, $\mathcal{N}(x)$ is the family of all neighborhoods of x, and $\operatorname{dist}(x,W)$ is the distance from the point x to the set $W \subset X$. The symbols S^c, $\operatorname{cl} S$, and $\operatorname{int} S$ denote, respectively, the complement, closure, and interior of S. Let $D \subset Y$ be a convex cone (containing 0) with nonempty interior $\operatorname{int} D$ and let Y be partially ordered by D.

Let $F : X \to 2^Y$ be a set-valued map. We denote the graph and domain of F, respectively, by

$$\operatorname{Gr} F = \{(x,y) \in X \times Y : y \in F(x)\}, \qquad \operatorname{Dom} F = \{x \in X : F(x) \neq \emptyset\}. \tag{2.1}$$

If S is a subset of X, then $F(S) = \cup_{x\in S} F(x)$ and the inverse set-valued map of F is $F^{-1} : Y \to 2^X$ given by $(y,x) \in \operatorname{Gr} F^{-1}$ if and only if $(x,y) \in \operatorname{Gr} F$.

Definition 2.1. Suppose that D is a closed convex pointed $((-D) \cap D = \{0\})$ cone. A point $y_0 \in A \subset Y$ is called a strict efficient (resp., weak) point of A, denoted by $y_0 \in \operatorname{Str}_D A$ (resp., $y_0 \in W \operatorname{Min}_D A$) if

$$(A - y_0) \cap (-D \setminus \{0\}) = \emptyset \quad (\text{resp. } (A - y_0) \cap (-\operatorname{int} D) = \emptyset). \tag{2.2}$$

Given a set-valued map $F : X \to 2^Y$ and a subset S of X, the following abstract optimization is considered:

$$\min F(x), \quad \text{s.t. } x \in S. \tag{2.3}$$

Definition 2.2. Suppose that D is a closed convex pointed cone. A point $(x_0, y_0) \in \operatorname{Gr} F$, with $x_0 \in S$, is said to be a local strict (resp., weak) minimizer of F over S, written as

$(x_0, y_0) \in L\,\mathrm{Str}_D(F, S)$ (resp., $(x_0, y_0) \in LW\,\mathrm{Min}_D(F, S)$), if there exists a neighborhood U of x_0 in X such that

$$y_0 \in \mathrm{Str}_D(F(U \cap S)) \quad (\text{resp., } y_0 \in W\,\min_D(F(U \cap S))),$$
$$\text{that is}, \forall x \in S \cap U, (F(x) - y_0) \cap (-D \setminus \{0\}) = \emptyset \quad (\text{resp. } \forall x \in S \cap U, (F(x) - y_0) \cap (-\operatorname{int} D) = \emptyset). \tag{2.4}$$

We will say that (x_0, y_0) is a global strict (global weak) minimizers when $U = X$. The set of all global strict minimizers (resp., weak minimizers) is denoted by Str_D (F, S) (resp., $W\,\mathrm{Min}_D(F, S)$).

Definition 2.3. Let $\psi : [0, +\infty) \to [0, +\infty)$ be a nondecreasing function with the property $\psi(t) = 0 \Leftrightarrow t = 0$ (such a family of functions is denoted by $\boldsymbol{\Psi}$) and $x_0 \in S$. We say that a point pair $(x_0, y_0) \in \mathrm{Gr}\,F \cap (S \times Y)$ is a weak ψ-sharp local Pareto minimizer for (2.3), denoted by $(x_0, y_0) \in W\mathrm{SL}(\psi, F, S)$, if there exists a constant $\alpha > 0$ and $U \in \mathcal{N}(x_0)$ such that

$$(F(x) + D) \cap B(y_0, \alpha\psi(\mathrm{dist}(x, W))) = \emptyset, \quad \forall x \in S \cap U \setminus W, \tag{2.5}$$

where

$$W = \{x \in S : y_0 \in F(x)\} = S \cap F^{-1}(y_0). \tag{2.6}$$

If we choose $U = X$, we will say the point pair $(x_0, y_0) \in \mathrm{Gr}\,F \cap (S \times Y)$ is a weak ψ-sharp minimizer for (2.3), denoted by $(x_0, y_0) \in WS(\psi, F, S)$. In particular, let $\psi_m(t) = t^m$ for $m = 1, 2, \ldots$. Then, we say $(x_0, y_0) \in \mathrm{Gr}\,F \cap (S \times Y)$ is a weak ψ-sharp local minimizer of order m for (2.3) if $(x_0, y_0) \in W\mathrm{SL}(\psi_m, F, S)$.

Obviously, condition (2.5) can be expressed in the following equivalent form:

$$F(x) \subset (B(y_0, \alpha\psi(\mathrm{dist}(x, W))) - D)^c, \quad \forall x \in S \cap U \setminus W. \tag{2.7}$$

Remark 2.4. Clearly, if the map F is a vector-valued function, the notion is equivalent to Definition 8.2.3 with $\psi = \psi_m$ in [17] and the weak ψ-sharp local minimizer for vector optimizations in [12].

Remark 2.5. In [14], the definition of weak ψ-sharp local minimizer for set-valued optimization is given by the oriented distance function $\triangle$. However, we establish the definition by the map F. When the map F is the real-valued function and the cone $D = R_+$, our definition is equivalent to Definition 2.1 in [14].

3. Optimality Conditions for Weak ψ-Sharp Minimizer for Set-Valued Optimization

In this section, we present sufficient and necessary conditions that a point pair is a weak ψ-sharp local minimizer in set-valued optimization problems.

Theorem 3.1. *Let $F : X \to 2^Y$, $x_0 \in S$, and $\varphi \in \mathbf{\Psi}$. Assume that W defined in* (2.6) *is a closed set. Then, $(x_0, y_0) \in WSL(\varphi, F, S)$ if and only if*

$$0 \notin \limsup_{x \to x_0, x \in S} \frac{F(x) - y_0 + D}{\varphi(\mathrm{dist}(x, W))}. \tag{3.1}$$

Proof. Part "only if": suppose that (3.1) is false, then there exist sequences $x_k \in S \setminus W$, $y_k \in F(x_k)$, $d_k \in D$ such that $x_k \to x_0$ and

$$\lim_{k \to \infty} \frac{y_k + d_k - y_0}{\varphi(\mathrm{dist}(x_k, W))} = 0. \tag{3.2}$$

Hence, for any $\epsilon > 0$, there is $k_0 = k_0(\epsilon)$ such that

$$\left\| y_k - y_0 + d_k \right\| < \epsilon \varphi(\mathrm{dist}(x_k, W)), \quad \forall k \geq k_0. \tag{3.3}$$

Namely, $y_k + d_k \in B(y_0, \epsilon\varphi(\mathrm{dist}(x_k, W)))$.

By assumption, there exist $\alpha > 0$ and $U = B(x_0, \delta)$ such that (2.5) holds. In particular, for $\epsilon = \min\{\alpha, \delta\}$, there exists $k_0 = k_0(\epsilon)$ such that for each $k \geq k_0$, we have that $x_k \in S \cap B(x_0, \delta) \setminus W$

$$y_k + d_k \in B\big(y_0, \epsilon\varphi(\mathrm{dist}(x_k, W))\big) \subset B\big(y_0, \alpha\varphi(\mathrm{dist}(x_k, W))\big), \tag{3.4}$$

which is contradiction to (2.5).

Part "if": suppose that the relation (2.5) is false, then for any $\delta > 0$ and $\alpha > 0$, there exist $x \in S \cap B(x_0, \delta) \setminus W$ and $y \in F(x)$ such that

$$(y + D) \cap B\big(y_0, \alpha\varphi(\mathrm{dist}(x, W))\big) \neq \emptyset. \tag{3.5}$$

In particular, choosing $\alpha = \delta = 1/k$, there exist $x_k \in S \cap B(x_0, 1/k) \setminus W$ and $y_k \in F(x_k)$ and $d_k \in D$ such that

$$(y_k + d_k) \in B\left(y_0, \frac{1}{k}\varphi(\mathrm{dist}(x_k, W))\right), \tag{3.6}$$

that is,

$$\frac{\left\| y_k + d_k - y_0 \right\|}{\varphi(\mathrm{dist}(x_k, W))} < \frac{1}{k}. \tag{3.7}$$

Hence, for sufficiently large k, we have

$$\frac{\left\| y_k + d_k - y_0 \right\|}{\varphi(\mathrm{dist}(x_k, W))} \longrightarrow 0, \tag{3.8}$$

which contradicts (3.1). □

From Theorem 3.1, we easily obtain the following result.

Corollary 3.2. *Let $F : X \to 2^Y$, $x_0 \in S$, and $\varphi \in \boldsymbol{\Psi}$. Assume that W defined in* (2.6) *is a closed set. If $(x_0, y_0) \in WSL(\varphi, F, S)$, then,*

$$\left(\limsup_{x \to x_0, x \in S} \frac{F(x) - y_0}{\varphi(\operatorname{dist}(x, W))}\right) \cap (-D) = \emptyset. \tag{3.9}$$

Theorem 3.3. *Let $Y = R^p$, $D = R^p_+ = [0, +\infty)^p$, and $\overline{R}^p_+ = [0, +\infty]^p$. Let $F : X \to 2^Y$, $x_0 \in S$. Assume that W defined in* (2.6) *is a closed set and $\varphi \in \boldsymbol{\Psi}$. Then, the following statements are equivalent:*

(i) $(x_0, y_0) \notin WSL(\varphi, F, S)$,

(ii) $(\limsup_{x \to x_0, x \in S}((F(x) - y_0)/\varphi(\operatorname{dist}(x, W)))) \cap (-\overline{R}^p_+) \neq \emptyset$.

Proof. (i $\Rightarrow$ ii) By assumption and Theorem 3.1, there exist sequences $x_k \in S \setminus W$, $y_k \in F(x_k)$, $d_k \in R^p_+$ such that $x_k \to x_0$ and

$$\lim_{k \to \infty} \frac{y_k + d_k - y_0}{\varphi(\operatorname{dist}(x_k, W))} = 0. \tag{3.10}$$

Let $b_k = a_k + c_k$, where

$$a_k = \frac{y_k - y_0}{\varphi(\operatorname{dist}(x_k, W))}, \qquad c_k = \frac{d_k}{\varphi(\operatorname{dist}(x_k, W))} \in R^p_+. \tag{3.11}$$

Consider the first component of the vector $c_k = (c_k^1, c_k^2, \ldots, c_k^p)$. Let $c^1 = \limsup_{k \to \infty} c_k^1$. Then, there is an infinite set $K_1 \subset \mathcal{N}$ such that $\lim_{K_1 \ni k \to \infty} c_k^1 = c^1$. We have $c^1 \geq 0$ (it can be taken $+\infty$), since $c_k^1 \geq 0$. Now, let us consider the second component of sequence $(c_k)_{k \in K_1}$. Let $c^2 = \limsup_{K_1 \ni k \to \infty} c_k^2$. Hence, there exists an infinite set $K_2 \subset K_1$ such that $\lim_{K_2 \ni k \to \infty} c_k^2 = c^2$. We still have $c^2 \geq 0$ (it can be taken $+\infty$). So, we have $\lim_{K_2 \ni k \to \infty}(c_k^1, c_k^2) = (c^1, c^2)$. Continuing the process, we obtain a vector $c = (c^1, c^2, \ldots, c^p) \in \overline{R}^p_+$ and an infinite set $K_p \subset \mathcal{N}$ such that $c = \lim_{K_p \ni k \to \infty} c_k$.

Since $b_k = a_k + c_k$, taking the limit on both sides of the equation, we have

$$0 = \lim_{K_p \ni k \to \infty} b_k = \lim_{K_p \ni k \to \infty} a_k + \lim_{K_p \ni k \to \infty} c_k. \tag{3.12}$$

Therefore, $\overline{d} := \lim_{K_p \ni k \to \infty} a_k = -c \leq 0$. Namely,

$$\left(\limsup_{x \to x_0, x \in S} \frac{F(x) - y_0}{\varphi(\operatorname{dist}(x, W))}\right) \cap \left(-\overline{R}^p_+\right) \neq \emptyset. \tag{3.13}$$

(i $\Leftarrow$ ii) If $-\overline{d} \in R^p_+$, by Theorem 3.1, the result is true. So, we suppose that some components of $\overline{d}$ are $-\infty$. Reordering to $\overline{d}$, let $\overline{d} = (\overline{d}^1, \overline{d}^2, \ldots, \overline{d}^n, \overline{d}^{n+1}, \ldots, \overline{d}^p)$ with $\overline{d}^i = -\infty$ for

$i = 1, 2, \ldots, n$ and $\overline{d}^i \in (-\infty, 0]$ for $i > n$, with $n \geq 1$. Hence, from relation (3.13), we see that there exist $x_k \in S \setminus W, x_k \to x_0,\ y_k \in F(x_k)$ such that

$$\lim_{k \to \infty} \frac{y_k^i - y_0^i}{\psi(\operatorname{dist}(x_k, W))} = -\infty, \quad i = 1, 2, \ldots, n. \tag{3.14}$$

Since, for sufficiently large k and for $i = 1, 2, \ldots, n$, $d_k^i := -(y_k^i - y_0^i) > 0$. Let

$$d_k = \left(d_k^1, \ldots, d_k^n, -\psi(\operatorname{dist}(x_k, W))\overline{d}^{n+1}, \ldots, -\psi(\operatorname{dist}(x_k, W))\overline{d}^p\right) \in R_+^p. \tag{3.15}$$

Clearly, one has

$$\lim_{k \to \infty} \frac{y_k - y_0 + d_k}{\psi(\operatorname{dist}(x_k, W))} = 0. \tag{3.16}$$

Namely,

$$0 \in \limsup_{x \to x_0, x \in S} \frac{F(x) - y_0 + R_+^p}{\psi(\operatorname{dist}(x, W))}. \tag{3.17}$$

By Theorem 3.1, we derive the result. □

4. Scalarization

Scalarization is one of the most important procedures in vector optimization. In this section, we apply a generalized nonlinear scalarization function introduced by Hernández and Rodríguez-Marín in [18] to discuss the weak ψ-sharp minimizer in set-valued optimization problems.

Let D be a proper closed convex cone and $\operatorname{int} D \neq \emptyset$. Let $e \in \operatorname{int} D$ be a fixed point.

Definition 4.1 (see [18]). The generalized nonlinear scalarization function $G : 2^Y \to R \cup \{-\infty\}$ is defined by

$$G(A) = \inf\{t \in R : te \in A + D\}. \tag{4.1}$$

A nonempty set $A \subset Y$ is said to be D-proper if $A + D \neq Y$.

Next, we present several properties about the generalized nonlinear scalarization function G.

Lemma 4.2 (see [18]). *A is D-proper if and only if $G(A) > -\infty$.*

Lemma 4.3 (see [16]). *Let $r \in R$ and A be a nonempty subset of Y. Then,*

$$G(A) > r \Longleftrightarrow re \notin \operatorname{cl}(A + D). \tag{4.2}$$

Lemma 4.4 (see [18]). *Let $A \subset Y$ and $y_0 \in A$. If $y_0 \in Str_D\, A$, then $G(A - y_0) = 0$.*

Given a set-valued map $F : X \to 2^Y$ and $(x_0, y_0) \in \mathrm{Gr}\, F$. Define $\overline{F} : X \to 2^Y$ by

$$\overline{F}(x) = F(x) - y_0. \tag{4.3}$$

Now, we consider weak ψ-sharp local minimizer for a set-valued map F through the weak sharp local minimizer of the scalarization function $G \circ \overline{F} : X \to R \cup \{-\infty\}$.

Theorem 4.5. *Let $x_0 \in S$ and $(x_0, y_0) \in Gr\, F$. Suppose that W defined in (2.6) is a closed set and $y_0 \in Str_D\, F(x_0)$. Then,*

$$(x_0, y_0) \in WSL(\psi, F, S) \Longleftrightarrow x_0 \in WSL\left(\psi, G \circ \overline{F}, S\right). \tag{4.4}$$

Proof. Part "only if": assume that $(x_0, y_0) \in \mathrm{WSL}(\psi, F, S)$, there exist $\alpha > 0$ and $U \in \mathcal{N}(x_0)$ such that

$$\left(F(x) - y_0 + D\right) \cap B\left(0, \alpha\psi(\mathrm{dist}(x, W))\right) = \emptyset, \quad \forall x \in S \cap U \setminus W. \tag{4.5}$$

Since $B(0, \alpha\psi(\mathrm{dist}(x, W))$ is an open set,

$$B\left(0, \alpha\psi(\mathrm{dist}(x, W))\right) \subset \left(\mathrm{cl}\left(F(x) - y_0 + D\right)\right)^c. \tag{4.6}$$

Note that, when W is a closed set,

$$\frac{\alpha}{4\|e\|}\psi(\mathrm{dist}(x, W))e \in B\left(0, \alpha\psi(\mathrm{dist}(x, W))\right). \tag{4.7}$$

Hence,

$$\frac{\alpha}{4\|e\|}\psi(\mathrm{dist}(x, W))e \notin \mathrm{cl}\left(F(x) - y_0 + D\right). \tag{4.8}$$

By Lemma 4.3, we have

$$G\left(F(x) - y_0\right) > \frac{\alpha}{4\|e\|}\psi(\mathrm{dist}(x, W)). \tag{4.9}$$

On the other hand, since $y_0 \in \mathrm{Str} F(x_0)$, in terms of Lemma 4.4, we get

$$G\left(F(x_0) - y_0\right) = 0. \tag{4.10}$$

This relation, together with (4.9), yields

$$G\left(F(x) - y_0\right) > G\left(F(x_0) - y_0\right) + \frac{\alpha}{4\|e\|}\psi(\mathrm{dist}(x, W)), \quad \forall x \in S \cap U \setminus W. \tag{4.11}$$

Namely,

$$\left(G\circ\overline{F}\right)(x) > \left(G\circ\overline{F}\right)(x_0) + \frac{\alpha}{4\|e\|}\psi(\mathrm{dist}(x,W)),\quad \forall x\in S\cap U\setminus W, \tag{4.12}$$

that is, $x_0 \in WSL(\psi, G\circ\overline{F}, S)$.

Part "if": by assumption, there exist $\beta > 0$ and $U \in \mathcal{N}(x_0)$ such that

$$G\left(\overline{F}(x)\right) > G\left(\overline{F}(x_0)\right) + \beta\psi(\mathrm{dist}(x,W)),\quad \forall x\in S\cap U\setminus W. \tag{4.13}$$

Since $y_0 \in \mathrm{Str}_D F(x_0)$, by applying Lemma 4.4, we get $G(F(x_0) - y_0) = 0$. Thus, we have

$$G\left(F(x) - y_0\right) > \beta\psi(\mathrm{dist}(x,W)),\quad \forall x\in S\cap U\setminus W. \tag{4.14}$$

Once more using Lemma 4.3, one has

$$\beta\psi(\mathrm{dist}(x,W))e \notin \mathrm{cl}\left(F(x) - y_0 + D\right). \tag{4.15}$$

Furthermore,

$$\beta\psi(\mathrm{dist}(x,W))e \notin F(x) - y_0 + D, \tag{4.16}$$

which implies that

$$\left(\beta\psi(\mathrm{dist}(x,W))e - D\right)\cap\left(F(x) - y_0 + D\right) = \emptyset,\quad \forall x\in S\cap U\setminus W. \tag{4.17}$$

Since $e \in \mathrm{int}\, D$, there exists a number $\epsilon > 0$ such that $B(0,\epsilon) \subset e - D$. Moreover,

$$B(0,\lambda\epsilon) \subset \lambda e - D,\quad \forall\lambda > 0. \tag{4.18}$$

Hence, from (4.18), we obtain

$$B\left(0, \epsilon\beta\psi(\mathrm{dist}(x,W))\right) \subset \beta\psi(\mathrm{dist}(x,W))e - D. \tag{4.19}$$

Combining it with relation (4.17), we deduce that

$$B\left(0,\epsilon\beta\psi(\mathrm{dist}(x,W))\right)\cap\left(F(x) - y_0 + D\right) = \emptyset,\quad \forall x\in S\cap U\setminus W. \tag{4.20}$$

By the definition of weak ψ-sharp local minimum, we have $(x_0, y_0) \in WSL(\psi, F, S)$. □

In Theorem 4.5, if the map F is a vector-valued function and the function G becomes the nonlinear scalarization function g, we easily obtain the following result, which is Theorem 3.4 in [13].

Corollary 4.6. *Let $x_0 \in S$, $f : X \to Y$, and $\overline{f}(x) = f(x) - f(x_0)$. Then,*

$$x_0 \in WSL(\psi, f, S) \Longleftrightarrow x_0 \in WSL\left(\psi, g \circ \overline{f}, S\right). \tag{4.21}$$

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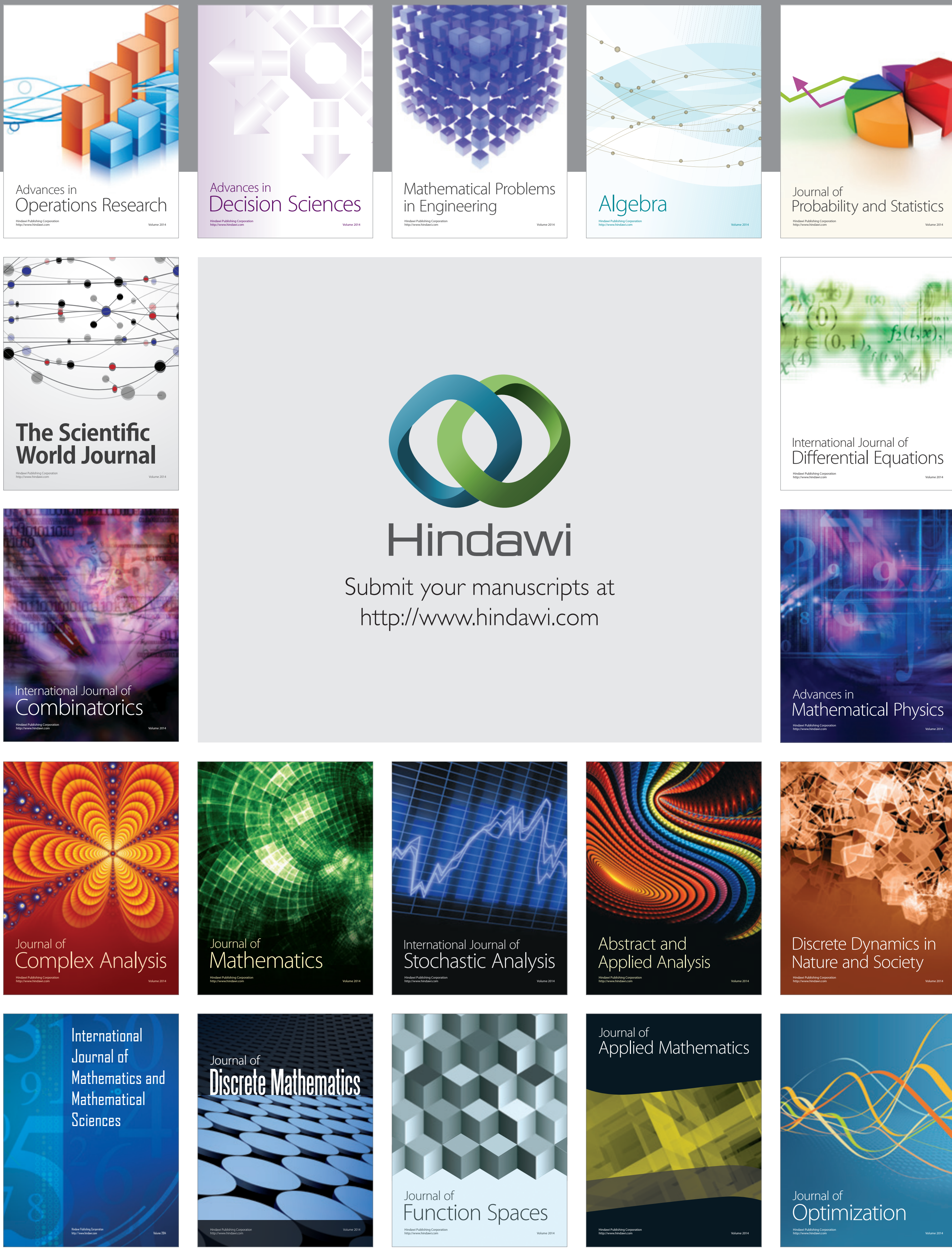
References

[1] M. C. Ferris, *Weak sharp minima and penalty functions in mathematical programming [Ph.D. thesis]*, Universiy of Cambridge, Cambridge, UK, 1988.
[2] B. T. Polyak, "Sharp minima, Institue of control sciences lecture notes, Moscow," in *Presented at the IIASA Workshop Om Generalized Lagrangians and Their Applications*, IIASA, Laxenburg, Austria, 1979.
[3] R. Henrion and J. Outrata, "A subdifferential condition for calmness of multifunctions," *Journal of Mathematical Analysis and Applications*, vol. 258, no. 1, pp. 110–130, 2001.
[4] A. S. Lewis and J. S. Pang, "Eror bounds for convex inequality systems," in *Proceedings of the 5th symposium on generalized convexity*, J. P. Crouzeix, Ed., Marseille, France, 1996.
[5] J. V. Burke and M. C. Ferris, "Weak sharp minima in mathematical programming," *SIAM Journal on Control and Optimization*, vol. 31, no. 5, pp. 1340–1359, 1993.
[6] J. V. Burke and S. Deng, "Weak sharp minima revisited. I. Basic theory," *Control and Cybernetics*, vol. 31, no. 3, pp. 439–469, 2002.
[7] J. V. Burke and S. Deng, "Weak sharp minima revisited. II. Application to linear regularity and error bounds," *Mathematical Programming B*, vol. 104, no. 2-3, pp. 235–261, 2005.
[8] J. V. Burke and S. Deng, "Weak sharp minima revisited. III. Error bounds for differentiable convex inclusions," *Mathematical Programming B*, vol. 116, no. 1-2, pp. 37–56, 2009.
[9] S. Deng and X. Q. Yang, "Weak sharp minima in multicriteria linear programming," *SIAM Journal on Optimization*, vol. 15, no. 2, pp. 456–460, 2004.
[10] X. Y. Zheng and X. Q. Yang, "Weak sharp minima for piecewise linear multiobjective optimization in normed spaces," *Nonlinear Analysis: Theory, Methods & Applications* , vol. 68, no. 12, pp. 3771–3779, 2008.
[11] E. M. Bednarczuk, "Weak sharp efficiency and growth condition for vector-valued functions with applications," *Optimization*, vol. 53, no. 5-6, pp. 455–474, 2004.
[12] M. Studniarski, "Weak sharp minima in multiobjective optimization," *Control and Cybernetics*, vol. 36, no. 4, pp. 925–937, 2007.
[13] S. Xu and S. J. Li, "Weak ψ-sharp minima in vector optimization problems," *Fixed Point Theory and Applications*, vol. 2010, Article ID 154598, 10 pages, 2010.
[14] M. Durea and R. Strugariu, "Necessary optimality conditions for weak sharp minima in set-valued optimization," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 73, no. 7, pp. 2148–2157, 2010.
[15] B. Jiménez, "Strict efficiency in vector optimization," *Journal of Mathematical Analysis and Applications*, vol. 265, no. 2, pp. 264–284, 2002.
[16] F. Flores-Bazán and B. Jiménez, "Strict efficiency in set-valued optimization," *SIAM Journal on Control and Optimization*, vol. 48, no. 2, pp. 881–908, 2009.
[17] E. Bednarczuk, "Stability analysis for parametric vector optimization problems," *Dissertationes Mathematicae*, vol. 442, pp. 1–126, 2006.
[18] E. Hernández and L. Rodríguez-Marín, "Nonconvex scalarization in set optimization with set-valued maps," *Journal of Mathematical Analysis and Applications*, vol. 325, pp. 1–18, 2007.



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