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Research Article

ψ-Exponential Stability of Nonlinear Impulsive Dynamic Equations on Time Scales

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The purpose of this paper is to present the sufficient ψ-exponential, uniform exponential, and global exponential stability conditions for nonlinear impulsive dynamic systems on time scales.

1. Introduction

In recent years, a significant progress has been made in the stability theory of impulsive systems [1, 2], and in [3] authors studied the ψ-exponential stability for nonlinear impulsive differential equations. There are various types of stability of dynamic systems on time scales such as asymptotic stability [4, 5], exponential and uniform exponential stability [6–8], and h-stability [9]. In the past decade, many authors studied impulsive dynamic systems on time scales [10–14]. There are some papers on the theory of the stability of impulsive dynamic systems on time scales. In [15], stability criteria for impulsive systems are given and in [16], authors studied ψ-uniform stability of linear impulsive dynamic systems.

In this paper, we consider the ψ-exponential stability of the zero solution of the first-order nonlinear impulsive dynamic system

$$\begin{gathered} x^{\Delta}(t) = f(t, x(t)), \quad t \in \mathbb{T}^{+}_{t_0},\ t \neq t_k, \\ x(t_k^{+}) - x(t_k^{-}) = I_k(x(t_k^{-})), \quad t = t_k,\ k = 1, 2, \ldots, n, \\ x(t_0^{+}) = x_0, \end{gathered} \tag{1}$$

where $\mathbb{T}$ is a time scale which has at least finitely many right-dense points of impulsive t_k, $f : [0, \infty) \times \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function and rd continuous in $(t_{k-1}, t_k] \times \mathbb{R}^n$, $I_k \in C_{\mathrm{rd}}[\mathbb{R}^n, \mathbb{R}^n]$, and $0 \le t_0 < t_1 < t_2 < \cdots < t_n < t$ are fixed moments of impulsive effect. Let $\psi_i : \mathbb{T} \to (0, \infty)$, $i = 1, 2, \ldots, n$, be rd continuous functions and let $\psi = \operatorname{diag}[\psi_1, \psi_2, \ldots, \psi_n]$. Throughout the paper, we assume that $f(t, 0) = 0$, for all t in the time scale interval $[0, \infty)$, and call the zero function the trivial solution of (1) and we consider $\mathbb{T}^{+}_{t_0} = \{t \in \mathbb{T} : t \ge t_0\}$. Existence and uniqueness of solutions of (1) have been studied in [10].

In the following part we present some basic concepts about time scale calculus and we refer the reader to resource [17] for more detailed information on dynamic equations on time scales.

2. Preliminaries

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$. For $t \in \mathbb{T}$ we define the *forward jump operator* $\sigma : \mathbb{T} \to \mathbb{T}$ by

$$\sigma(t) := \inf\{s \in \mathbb{T} : s > t\} \tag{2}$$

while the *backward jump operator* $\rho : \mathbb{T} \to \mathbb{T}$ is defined by

$$\rho(t) := \sup\{s \in \mathbb{T} : s < t\}. \tag{3}$$

If $\sigma(t) > t$, we say that t is *right scattered*, while if $\rho(t) < t$, we say that t is *left scattered*. Also, if $\sigma(t) = t$, then t is called *right dense*, and if $\rho(t) = t$, then t is called *left dense*. The *graininess function* $\mu : \mathbb{T} \to [0, \infty)$ is defined by

$$\mu(t) := \sigma(t) - t. \tag{4}$$

We introduce the set $\mathbb{T}^\kappa$ which is derived from the time scale $\mathbb{T}$ as follows. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^\kappa = \mathbb{T} - \{m\}$; otherwise $\mathbb{T}^\kappa = \mathbb{T}$.

A function f on $\mathbb{T}$ is said to be delta differentiable at some point $t \in \mathbb{T}$ if there is a number $f^\Delta(t)$ such that for every $\varepsilon > 0$ there is a neighborhood $U \subset \mathbb{T}$ of t such that

$$\left| f(\sigma(t)) - f(s) - f^\Delta(t)(\sigma(t) - s) \right| \le \varepsilon |\sigma(t) - s|, \quad s \in U. \tag{5}$$

The function $p : \mathbb{T} \to \mathbb{R}$ is said to be regressive provided $1 + \mu(t)p(t) \ne 0$ for all $t \in \mathbb{T}^\kappa$. The set of all regressive rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ is denoted by $\mathfrak{R}$.

Let $p \in \mathfrak{R}$ and $\mu(t) \ne 0$ for all $t \in \mathbb{T}$. The exponential function on $\mathbb{T}$, defined by

$$e_p(t, s) = \exp\left(\int_s^t \frac{1}{\mu(z)} \log(1 + \mu(z)p(z)) \Delta z \right), \tag{6}$$

is the solution to the initial value problem $y^\Delta = p(t)y$, $y(s) = 1$. Properties of the exponential function on $\mathbb{T}$ are given in [6].

In [6] authors defined the Lyapunov function on time scales, type I Lyapunov function V as,

$$V(x) = \sum_{i=1}^n V_i(x_i) = V_1(x_1) + \cdots + V_n(x_n), \tag{7}$$

and Δ derivative of type I Lyapunov function as follows:

$$[V(x(t))]^\Delta = \begin{cases} \displaystyle\sum_{i=1}^n \frac{[V_i(x_i + \mu(t) f_i(t, x)) - V_i(x_i)]}{\mu(t)} & \text{for } \mu(t) \ne 0, \\ \nabla V(x) \cdot f(t, x) & \text{for } \mu(t) = 0. \end{cases} \tag{8}$$

We start introducing notations that will be used in the following sections. In the Euclidean n-space, norm of a vector $x = \{x_1, x_2, \ldots, x_n\}^T$ is given by $\|x\| = \max\{|x_1|, |x_2|, \ldots, |x_n|\}$. The induced norm of an $n \times n$ matrix A is defined to be $\|A\| = \sup_{\|x\| \le 1} \|Ax\|$.

Now, we give definition of ψ-exponential, ψ-uniform exponential, ψ-global exponential stability, and stability conditions for the solution of nonlinear impulsive dynamic system (1).

3. ψ-Exponential Stability

Definition 1. The trivial solution to (1) is ψ exponentially stable on $[0, \infty)$ if any solution $x(t, t_0, x_0)$ of the system (1) satisfies for all $t \in [t_{k-1}, t_k)$, $k = 1, 2, \ldots, n$,

$$\|\psi(t) x(t, t_0, x_0)\| \le C(\|x_0\|, t_0)(e_{\ominus M}(t, t_0))^d, \tag{9}$$

where d is a positive constant and $C(h, t) \in \mathbb{R}^+ \times \mathbb{T}^+_{t_0} \to \mathbb{R}^+$ is a nonnegative increasing function, $M > 0$. If the function C is independent of t_0, then the trivial solution to system (1) is said to be ψ uniformly exponentially stable on $[0, \infty)$.

Definition 2. The trivial solution to (1) is ψ globally exponentially stable on $[0, \infty)$ if there exist some constants $\delta > 0$ and $M \ge 1$ such that any solution $x(t, t_0, x_0)$ of (1), for all $t \in [t_{k-1}, t_k)$, $k = 1, 2, \ldots, n$, we have

$$\|\psi(t) x(t, t_0, x_0)\| \le M e_{\ominus\delta}(t, t_0). \tag{10}$$

Now, we shall present sufficient conditions for the ψ-exponential stability, ψ uniformly exponential stability, and ψ globally exponentially stability of (1).

Theorem 3. *Assume that $D \subset \mathbb{R}^n$ contains the origin and there exists a type I Lyapunov function $V : \mathbb{T}^+_{t_0} \times D \to [0, \infty)$ such that, for all $(t, x) \in \mathbb{T}^+_{t_0} \times D$ and $t \in [t_{k-1}, t_k)$, $k = 1, 2, \ldots, n$,*

$$\lambda_1(t) \|\psi(t) x(t)\|^p \le V(t, x) \le \lambda_2(t) \|\psi(t) x(t)\|^q, \tag{11}$$

$$V^\Delta(t, x) \le \frac{-\lambda_3(t) \|\psi(t) x(t)\|^r - L(M \ominus \delta) e_{\ominus\delta}(t, t_0)}{1 + M\mu(t)}, \tag{12}$$

$$V(t, x) - V^{r/q}(t, x) \le \gamma e_{\ominus\delta}(t, t_0), \tag{13}$$

where $\lambda_1(t)$, $\lambda_2(t)$, and $\lambda_3(t)$ are positive functions, where $\lambda_1(t)$ is nondecreasing; p, q, r, and γ are positive constants; L is a nonnegative constant, and $\delta > M := \inf_{t \ge 0} \lambda_3(t)/[\lambda_2(t)]^{r/q} > 0$. Then the trivial solution to (1) is ψ exponentially stable on $[0, \infty)$.

Proof. Let x be a solution to (1) that stays in D for all $t \ge t_0$. As $M := \inf_{t \ge 0} \lambda_3(t)/[\lambda_2(t)]^{r/q} > 0$, $e_M(t, t_0)$ is well defined and positive. Thus $\lambda_3(t)/[\lambda_2(t)]^{r/q} \ge M$. Consider

$$\begin{aligned}
&[V(t, x(t)) e_M(t, t_0)]^\Delta \\
&\quad = V^\Delta(t, x(t)) e_M^\sigma(t, t_0) + V(t, x(t)) e_M^\Delta(t, t_0), \\
&\quad \le \left(-\lambda_3(t) \|\psi(t) x(t)\|^r - L(M \ominus \delta) e_{\ominus\delta}(t, t_0) \right) e_M(t, t_0) \\
&\qquad + MV(t, x(t)) e_M(t, t_0) \\
&\quad = \left(-\lambda_3(t) \|\psi(t) x(t)\|^r + MV(t, x(t)) - L(M \ominus \delta) e_{\ominus\delta}(t, t_0) \right) \\
&\qquad \times e_M(t, t_0) \\
&\quad \le \left(\frac{-\lambda_3(t)}{[\lambda_2(t)]^{r/q}} V^{r/q}(t, x(t)) + MV(t, x(t)) \right. \\
&\qquad \left. - L(M \ominus \delta) e_{\ominus\delta}(t, t_0) \right) e_M(t, t_0)
\end{aligned}$$

$$\leq \left(M\left(V\left(t,x\left(t\right)\right)-V^{r/q}\left(t,x\left(t\right)\right)\right)-L\left(M\ominus\delta\right)e_{\ominus\delta}\left(t,t_0\right)\right) \times e_M\left(t,t_0\right) \leq \left(M\gamma - L\left(M\ominus\delta\right)\right)e_{M\ominus\delta}\left(t,t_0\right). \tag{14}$$

Integrating both sides of *above inequality* from t_0 to t with $x_0 = x(t_0)$, we obtain, for $t \in [t_{k-1}, t_k)$,

$$\begin{aligned} V\left(t,x\right)e_M\left(t,t_0\right) &\leq V\left(t_0,x_0\right) + \int_{t_0}^{t}\left(M\gamma - L\left(M\ominus\delta\right)\right)e_{M\ominus\delta}\left(\tau,t_0\right)\Delta\tau \\ &= V\left(t_0,x_0\right) + \left(\frac{M\gamma}{M\ominus\delta} - L\right)e_{M\ominus\delta}\left(t,t_0\right) + \frac{M\gamma}{\delta\ominus M} + L \\ &\leq V\left(t_0,x_0\right) + \frac{M\gamma}{\delta\ominus M} + L. \end{aligned} \tag{15}$$

From condition $V(t_0,x_0) \leq \lambda_2(t_0)\|\psi(t_0)x_0\|^q$

$$V\left(t,x\right)e_M\left(t,t_0\right) \leq \lambda_2\left(t_0\right)\left\|\psi\left(t_0\right)x_0\right\|^q + \frac{M\gamma}{\delta\ominus M} + L. \tag{16}$$

Letting

$$\lambda_2\left(t_0\right)\left\|\psi\left(t_0\right)x_0\right\|^q + \frac{M\gamma}{\delta\ominus M} + L = C\left(\left\|x_0\right\|,t_0\right) > 0 \tag{17}$$

we get,

$$V\left(t,x\right)e_M\left(t,t_0\right) \leq C\left(\left\|x_0\right\|,t_0\right). \tag{18}$$

By condition (11), we have

$$\left\|\psi\left(t\right)x\left(t\right)\right\| \leq \lambda_1^{-1/p}\left(t\right)\left(V\left(t,x\right)\right)^{1/p} \tag{19}$$

And by the fact that $\lambda_1(t) \geq \lambda_1(t_0)$, we obtain

$$\left\|\psi\left(t\right)x\left(t\right)\right\| \leq \lambda_1^{-1/p}\left(t_0\right)\left(V\left(t,x\right)\right)^{1/p}. \tag{20}$$

From (18) and (20) we obtain the result for all, $t \in [t_{k-1}, t_k)$, $k = 1, 2, \ldots, n$,

$$\left\|\psi\left(t\right)x\left(t\right)\right\| \leq \lambda_1^{-1/p}\left(t_0\right)\left(C\left(\left\|x_0\right\|,t_0\right)\right)^{1/p}e_{\ominus M}(t,t_0)^{1/p}. \tag{21}$$

By Definition 1 system (1) is ψ exponentially stable. □

If we consider ψ as scaler function independent of t, then we get a sufficient condition for ψ uniformly exponential stability as stated below.

Theorem 4. *In Theorem 3 if ψ is a constant function independent of t and $\lambda_i(t) = \lambda_i$, $i = 1, 2, 3$, are positive constants, then the trivial solution to system* (1) *is ψ uniformly exponentially stable on* $[0, \infty)$.

Proof. The proof is similar to proof of Theorem 3 by taking $\delta > \lambda_3/[\lambda_2]^{r/q}$ and $M = \lambda_3/[\lambda_2]^{r/q}$, hence omitted. □

Theorem 5. *Assume that $D \subset \mathbb{R}^n$ contains the origin and there exists a type I Lyapunov function $V : \mathbb{T}_{t_0}^+ \times D \to [0, \infty)$ such that, for all $(t, x) \in \mathbb{T}_{t_0}^+ \times D$ and $t \in [t_{k-1}, t_k)$, $k = 1, 2, \ldots, n$,*

$$\lambda_1\left\|\psi x\left(t\right)\right\|^p \leq V\left(x\right), \tag{22}$$

$$V^{\Delta}\left(t,x\right) \leq \frac{-\lambda_2 V\left(x\right) - L\left(M\ominus\delta\right)e_{\ominus\delta}\left(t,0\right)}{1 + M\mu\left(t\right)}, \tag{23}$$

where ψ is a constant function independent of t. $\lambda_1, \lambda_2, p, \delta > 0$, $L \geq 0$ are constants and $0 < M < \min\{\lambda_2, \delta\}$. Then the trivial solution to (1) *is ψ uniformly exponentially stable on* $[0, \infty)$.

Proof. Let x be a solution to (1) that stays in D for all $t \geq t_0$. Since $M \in \mathfrak{R}^+$, $e_M(t, 0)$ is well defined and positive. Now consider

$$\begin{aligned} &\left[V\left(x\left(t\right)\right)e_M\left(t,0\right)\right]^{\Delta} \\ &= V^{\Delta}\left(t,x\left(t\right)\right)e_M^{\sigma}\left(t,0\right) + MV\left(x\left(t\right)\right)e_M\left(t,0\right), \\ &\leq \left(-\lambda_2 V\left(x\left(t\right)\right) - L\left(M\ominus\delta\right)e_{\ominus\delta}\left(t,0\right)\right)e_M\left(t,0\right) + MV\left(x\left(t\right)\right)e_M\left(t,0\right) \\ &= \left(-\lambda_2 V\left(x\left(t\right)\right) + MV(x(t)) - L\left(M\ominus\delta\right)e_{\ominus\delta}\left(t,0\right)\right)e_M\left(t,0\right) \\ &\leq \left(\left(M-\lambda_2\right)V\left(x\left(t\right)\right) - L\left(M\ominus\delta\right)e_{\ominus\delta}\left(t,0\right)\right)e_M\left(t,0\right) \\ &\leq -L\left(M\ominus\delta\right)e_{\ominus\delta}\left(t,0\right)e_M\left(t,0\right) \\ &= -L\left(M\ominus\delta\right)e_{M\ominus\delta}\left(t,0\right). \end{aligned} \tag{24}$$

Integrating both sides of the above inequality from t_0 to t, we obtain, for $t \in [t_{k-1}, t_k)$,

$$\begin{aligned} V\left(x\left(t\right)\right)e_M\left(t,0\right) &\leq V\left(x_0\right)e_M\left(t_0,0\right) - Le_{M\ominus\delta}\left(t,0\right) + Le_{M\ominus\delta}\left(t_0,0\right) \\ &\leq V\left(x_0\right)e_M\left(t_0,0\right) + Le_{M\ominus\delta}\left(t_0,0\right) \\ &\leq \left(V\left(x_0\right) + L\right)e_M\left(t_0,0\right). \end{aligned} \tag{25}$$

This implies that

$$\begin{aligned} V\left(x\left(t\right)\right) &\leq \left(\left(V\left(x_0\right) + L\right)e_M\left(t_0,0\right)\right)e_{\ominus M}\left(t,0\right) \\ &= \left(V\left(x_0\right) + L\right)e_{\ominus M}\left(t,t_0\right). \end{aligned} \tag{26}$$

From (26) and by invoking condition (22) we obtain, for all $t \in [t_{k-1}, t_k)$, $k = 1, 2, \ldots, n$,

$$\left\|\psi x\left(t\right)\right\| \leq \lambda_1^{-1/p}\left(\left(V\left(x_0\right) + L\right)e_{\ominus M}\left(t,t_0\right)\right)^{1/p}. \tag{27}$$

By Definition 1 system (1) is ψ uniformly exponentially stable. □

Theorem 6. *Assume that $D \subset \mathbb{R}^n$ contains the origin and there exists a type I Lyapunov function $V : \mathbb{T}^+_{t_0} \times D \to [0,\infty)$ such that, for all $(t,x) \in \mathbb{T}^+_{t_0} \times D$ and $t \in [t_{k-1}, t_k)$, $k = 1,2,\ldots,n$,*

$$\lambda_1 \|\psi(t)x(t)\|^p \le V(x) \le \lambda_2 \|\psi(t)x(t)\|^p, \tag{28}$$

$$V^\Delta(t,x) \le \frac{-\lambda_3 \|\psi(t)x(t)\|^p - L(K\ominus\delta)e_{\ominus\delta}(t,0)}{1+K\mu(t)}, \tag{29}$$

where $\lambda_1, \lambda_2, \lambda_3$, and p are positive constants, $K = \lambda_3/\lambda_2$, $L \ge \lambda_1$ is a nonnegative constant, and $\delta > \lambda_3/\lambda_2$. Then the trivial solution to (1) is ψ globally exponentially stable on $[0,\infty)$.

Proof. Let x be a solution to (1) that stays in D for all $t \ge t_0$. Since $K = \lambda_3/\lambda_2$, $e_K(t,0)$ is well defined and positive. For all $t \in [t_{k-1}, t_k)$, $k = 1,2,\ldots,n$, consider

$$\begin{aligned}
&[V(x(t))e_K(t,0)]^\Delta \\
&= V^\Delta(t,x(t))e_K^\sigma(t,0) + V(x(t))e_K^\Delta(t,0), \\
&\le \left(-\lambda_3\|\psi(t)x(t)\|^p - L(K\ominus\delta)e_{\ominus\delta}(t,0)\right)e_K(t,0) \\
&\quad + KV(x(t))e_K(t,0) \\
&= \left(-\lambda_3\|\psi(t)x(t)\|^p + KV(x(t)) - L(K\ominus\delta)e_{\ominus\delta}(t,0)\right) \\
&\quad \times e_K(t,0) \\
&\le \left(\frac{-\lambda_3}{\lambda_2}V(x(t)) + KV(x(t)) - L(K\ominus\delta)e_{\ominus\delta}(t,0)\right)e_K(t,0) \\
&= \left(-L(K\ominus\delta)e_{\ominus\delta}(t,0)\right)e_K(t,0) \\
&= -L(K\ominus\delta)e_{K\ominus\delta}(t,0).
\end{aligned} \tag{30}$$

Integrating both sides of the above inequality from t_0 to $t, t \ne t_k$, with $x_0 = x(t_0)$, we obtain,

$$\begin{aligned}
V(x(t))e_K(t,0) &\le V(x_0)e_K(t_0,0) \\
&\quad + L\left(e_{K\ominus\delta}(t_0,0) - e_{K\ominus\delta}(t,0)\right) \\
&\le V(x_0)e_K(t_0,0) + Le_{K\ominus\delta}(t_0,0) \\
&\le (V(x_0)+L)e_K(t_0,0).
\end{aligned} \tag{31}$$

This implies that

$$\begin{aligned}
V(x(t)) &\le \left((V(x_0)+L)e_K(t_0,0)\right)e_{\ominus K}(t,0) \\
&= (V(x_0)+L)e_{\ominus K}(t,t_0).
\end{aligned} \tag{32}$$

From (32), and by invoking condition (28), we obtain, for all $t \in [t_{k-1}, t_k)$, $k = 1,2,\ldots,n$,

$$\begin{aligned}
\|\psi(t)x(t)\| &\le \lambda_1^{-1/p}\left((V(x_0)+L)e_{\ominus K}(t,t_0)\right)^{1/p} \\
&\le \lambda_1^{-1/p}\left((V(x_0)+L)e_{\ominus K}(t,t_0)\right)^{1/p}.
\end{aligned} \tag{33}$$

If we set $M := ((V(x_0)+L)/\lambda_1)^{1/p}$, then (33) can be written as

$$\|\psi(t)x(t)\| \le M(e_{\ominus K}(t,t_0))^{1/p}. \tag{34}$$

Since $M \ge 1$, by Definition 2 system (1) is ψ globally exponentially stable. □

4. Examples

Example 7. We consider Example (35) in [7] and extend the example by using impulse condition,

$$x^\Delta = -x + \frac{1}{5}x^{1/3}e_{\ominus\delta}(t,0), \quad t \ne t_k, \ t \in \mathbb{T}, \tag{35}$$

$$x(t_k^+) = -\frac{1}{3}, \quad t = k, \ k = 1,2,\ldots,n, \tag{36}$$

where $\delta > 0$ is a constant $x_0 \in \mathbb{R}$. If there is a constant $0 < M < \delta$ such that

$$(\mu(t)-1)(1+M\mu(t)) \le -M, \tag{37}$$

$$\begin{aligned}
&\left(\frac{2}{3}\left(\frac{1}{25}\mu(t)\right)^{3/2} + \frac{|(2/5)-(2/5)\mu(t)|^3}{3}\right)(1+M\mu(t)) \\
&\quad \le -L(M\ominus\delta)(t),
\end{aligned} \tag{38}$$

for some constant $L \ge 0$ and all $t \ne k$, (35) is ψ uniformly exponentially stable.

Under above assumptions, we will show that the conditions of Theorem 4 are satisfied. Let $\psi(t) = 1/2$, choose $D = \mathbb{R}$ and $V(x) = x^2$, $t \ne k$, then (11) holds with $p = q = 2$, $\lambda_1 = \lambda_2 = 4$. If we calculate V^Δ, for all $t \ne k$,

$$\begin{aligned}
V^\Delta &= 2x\left(-x + \frac{1}{5}x^{1/3}e_{\ominus\delta}(t,0)\right) \\
&\quad + \mu(t)\left(-x + \frac{1}{5}x^{1/3}e_{\ominus\delta}(t,0)\right)^2,
\end{aligned} \tag{39}$$

we have the following comparison:

$$\begin{aligned}
V^\Delta &= 2x\left(-x + \frac{1}{5}x^{1/3}e_{\ominus\delta}(t,0)\right) \\
&\quad + \mu(t)\left(-x + \frac{1}{5}x^{1/3}e_{\ominus\delta}(t,0)\right)^2 \\
&\le (\mu(t)-1)x^2 \\
&\quad + \left[\frac{2}{3}\left(\frac{1}{25}\mu(t)\right)^{3/2} + \frac{|(2/5)-(2/5)\mu(t)|^3}{3}\right]e_{\ominus\delta}(t,0).
\end{aligned} \tag{40}$$

Dividing and multiplying the right-hand side by $(1+M\mu(t))$, we see that (12) holds under the above assumptions with $r = 2$ and $\lambda_3 = 4M$. Also, since $p = q = 2$, we have

$$V(x) - V^{r/q}(x) = x^2 - \left(x^2\right)^{2/2} = 0 \le \gamma e_{\ominus\delta}(t,t_0), \tag{41}$$

for all $t \neq k$. Therefore (13) is satisfied. Hence, all hypotheses of Theorem 4 are satisfied and we conclude that the trivial solution to (35) is ψ uniformly exponentially stable. We consider following two special cases of (35).

Case 1. If $\mathbb{T} = \mathbb{R}$, then $\mu(t) = 0$. It is easy to see that (37) holds for $M = 1$. Also for $L = 8/[375(\delta - M)]$, condition (38) is satisfied. Hence, we conclude that if $\delta > 1$, then the trivial solution to (35) is ψ uniformly exponentially stable.

Case 2. If $\mathbb{T} = (1/2)\mathbb{Z}$, then $\mu(t) = 1/2$. In this case rewriting (37) we have

$$\left(-\frac{1}{2}\right)\left(1 + \frac{M}{2}\right) \leq -M, \tag{42}$$

then (37) holds for $2/3 > M > 0$. Also for $L = ((6 + \sqrt{2})/2250(\delta - M))(1 - (M/2))(1 - (\delta/2))$, condition (38) is satisfied. Therefore for $\delta > 2/3$, then the trivial solution to (35) is ψ uniformly exponentially stable.

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