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Research Article

The Multisoliton Solutions for the (2 + 1)-Dimensional Sawada-Kotera Equation

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Applying bilinear form and extended three-wavetype of ansätz approach on the (2 + 1)-dimensional Sawada-Kotera equation, we obtain new multisoliton solutions, including the double periodic-type three-wave solutions, the breather two-soliton solutions, the double breather soliton solutions, and the three-solitary solutions. These results show that the high-dimensional nonlinear evolution equation has rich dynamical behavior.

1. Introduction

As is well known that the exact solutions of nonlinear evolution equations play an important role in nonlinear science field, especially in nonlinear physical science since they can provide much physical information and more insight into the physical aspects of the problem and thus lead to further applications. The search for exact solutions of nonlinear partial differential equations has long been an interesting and hot topic in nonlinear mathematical physics. Consequently, many methods are available to look for exact solutions of nonlinear evolution equations, such as the inverse scattering method, the Lie group method, the mapping method, Exp-function method, and ansätz technique [1–4]. Very recently, Wang et al. [5] proposed a new technique called extended three-wave approach to seek multiwave solutions for integrable equations, and this method has been used to investigate several equations [6, 7]. In this paper, we consider the following Sawada-Kotera equation:

$$u_t = \left(u_{xxxx} + 5uu_{xx} + \frac{5}{3}u^3 + 5u_{xy}\right)_x - 5\int\left(u_{yy}\right)dx + 5uu_y + 5u_x\int\left(u_y\right)dx. \quad (1)$$

Equation (1) was derived by B. G. Konopelchenko and V. G. Dubrovsky, and was called the Sawada-Kotera (SK) equation; for example, see [8]. By means of the two-soliton method, the exact periodic soliton solutions, N-soliton solutions, and traveling wave solutions of the SK equation were found [8–10].

In this paper, we discuss further the (2 + 1)-dimensional SK equation, by using bilinear form and extended three-wave type of ansätz approach, respectively [5, 11–15], and some new multisoliton solutions are obtained.

2. The Multisoliton Solutions

We assume

$$u = -2(\ln f)_{xx}, \quad (2)$$

where $f = f(x, y, t)$ is an unknown real function. Substituting (2) into (1), we can reduce (1) into the following equation [8]:

$$\left(D_x^6 + 5D_yD_x^3 - 5D_y^2 + D_xD_t\right) f \cdot f = 0, \quad (3)$$

where the Hirota bilinear operator D is defined by $(n, m \geq 0)$

$$D_x^m D_t^n f(x,t) \cdot g(x,t) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n \times \left[f(x,t)\, g(x',t')\right]\Big|_{x'=x,t'=t}. \quad (4)$$

Now we suppose the solution of (3) as

$$f = e^{-\xi} + \delta_1 \cos(\eta) + \delta_2 \cosh(\gamma) + \delta_3 e^{\xi}, \quad (5)$$

where $\xi = a_1 x + b_1 y + c_1 t$, $\eta = a_2 x + b_2 y + c_2 t$, $\gamma = a_3 x + b_3 y + c_3 t$, and a_i, b_i, and c_i $(i = 1, 2, 3)$ are some constants to be determined later. Substituting (5) into (3) and equating all the coefficients of different powers of e^{ξ}, $e^{-\xi}$, $\sin(\eta)$, $\cos(\eta)$, $\sinh(\gamma)$, $\cosh(\gamma)$, and the constant term to zero, we can obtain a set of algebraic equations for a_i, b_i, c_i, and δ_j $(i = 1, 2, 3;\ j = 1, 2, 3)$. Solving the system with the aid of Maple, we get the following results.

Case 1. If $a_2 = 0$, then

$$\begin{gathered} b_1 = -\frac{1}{4} a_1 \left(4a_1^2 + 3a_3^2\right), \qquad b_2 = \frac{3}{2}\, i a_1^2 a_3, \\ b_3 = -\frac{1}{4} a_3 \left(6a_1^2 + a_3^2\right), \qquad \delta_2 = -\frac{\delta_1 a_1^2}{a_1^2 - a_3^2}, \\ \delta_3 = \delta_3, \\ c_1 = \frac{9}{16} a_1 \left(5a_3^4 + 40a_1^2 a_3^2 + 16a_1^4\right), \\ c_2 = -\frac{45}{4}\, i a_1^2 a_3 \left(2a_1^2 + a_3^2\right), \\ c_3 = \frac{9}{16} a_3 \left(a_3^4 + 20a_1^2 a_3^2 + 40a_1^4\right), \end{gathered} \quad (6)$$

where a_1, a_3, δ_1, and δ_3 are free real constants. Substituting (6) into (5) and taking $\delta_3 > 0$, we have

$$\begin{aligned} f_1 &= 2\sqrt{\delta_3} \cosh\left(a_1 x + K_1 y + L_1 t + \frac{1}{2}\ln(\delta_3)\right) \\ &\quad - \delta_1 \cosh(M_1 y + N_1 t) - \frac{\delta_1 a_1^2}{a_1^2 - a_3^2} \\ &\quad \times \cosh(a_3 x - H_1 y + J_1 t), \end{aligned} \quad (7)$$

where $K_1 = (1/4)a_1(4a_1^2 + 3a_3^2)$, $L_1 = (9/16)a_1(5a_3^4 + 40a_1^2 a_3^2 + 16a_1^4)$, $M_1 = -(3/2)a_1^2 a_3$, $N_1 = (45/4)a_1^2 a_3(2a_1^2 + a_3^2)$, $H_1 = (1/4)a_3(6a_1^2 + a_3^2)$, and $J_1 = (9/16)a_3(a_3^4 + 20a_1^2 a_3^2 + 40a_1^4)$. Substituting (7) into (2) yields the three-soliton solution of SK equation as follows:

$$\begin{aligned} u_1 = &-\Bigg(2\left[2\sqrt{\delta_3} a_1^2 \cosh\left(\xi_1 + \frac{1}{2}\ln(\delta_3)\right) - \frac{\delta_1 a_1^2 a_3^2 \cosh(\eta_1)}{a_1^2 - a_3^2}\right]\Bigg) \\ &\times \Bigg(2\sqrt{\delta_3} \cosh\left(\xi_1 + \frac{1}{2}\ln(\delta_3)\right) - \frac{\delta_1 a_1^2 \cosh(\eta_1)}{a_1^2 - a_3^2} - \delta_1 \cosh(\gamma_1)\Bigg)^{-1} \\ &+ \Bigg[\Bigg(2\Bigg(2\sqrt{\delta_3} a_1 \sinh\left(\xi_1 + \frac{1}{2}\ln(\delta_3)\right) - \frac{\delta_1 a_1^2 a_3 \sinh(\eta_1)}{(a_1^2 - a_3^2)}\Bigg)\Bigg) \\ &\times \Bigg(2\sqrt{\delta_3} \cosh\left(\xi_1 + \frac{1}{2}\ln(\delta_3)\right) - \frac{\delta_1 a_1^2 \cosh(\eta_1)}{a_1^2 - a_3^2} - \delta_1 \cosh(\gamma_1)\Bigg)^{-1}\Bigg]^2, \end{aligned} \quad (8)$$

where $\xi_1 = a_1 x + K_1 y + L_1 t$, $\eta_1 = a_3 x - H_1 y + J_1 t$, and $\gamma_1 = M_1 y + N_1 t$.

If taking $a_3 = iA_3$ in (7), then we have

$$\begin{aligned} f_2 &= 2\sqrt{\delta_3} \cosh\left(a_1 x + K_2 y + L_2 t + \frac{1}{2}\ln(\delta_3)\right) \\ &\quad + \delta_1 \cos(M_2 y + N_2 t) \\ &\quad - \frac{\delta_1 a_1^2 \cos(A_3 x - H_2 y + J_2 t)}{a_1^2 + A_3^2}, \end{aligned} \quad (9)$$

where $\delta_3 > 0$, $K_2 = -(1/4)a_1(4a_1^2 - 3A_3^2)$, $L_2 = (9/16)a_1(5A_3^4 - 40a_1^2 A_3^2 + 16a_1^4)$, $M_2 = (3/2)a_1^2 A_3$, $N_2 = -(45/4)a_1^2 A_3(2a_1^2 - A_3^2)$, $H_2 = A_3 x - (1/4)A_3(6a_1^2 - A_3^2)$, and $J_2 = (9/16)A_3(A_3^4 - 20a_1^2 A_3^2 + 40a_1^4)$. Substituting (9) into (2) yields the double breather soliton solution of SK equation as follows:

$$\begin{aligned} u_2 = &-\Bigg(2\left[2a_1^2\sqrt{\delta_3} \cosh\left(\xi_2 + \frac{1}{2}\ln(\delta_3)\right) + \frac{\delta_1 a_1^2 A_3^2 \cos(\eta_2)}{a_1^2 + A_3^2}\right]\Bigg) \\ &\times \Bigg(2\sqrt{\delta_3} \cosh\left(\xi_2 + \frac{1}{2}\ln(\delta_3)\right) + \delta_1 \cos(\gamma_2) - \frac{\delta_1 a_1^2 \cos(\eta_2)}{a_1^2 + A_3^2}\Bigg)^{-1} \end{aligned}$$

$$+2\Bigg[\Bigg(\Big(2a_1\sqrt{\delta_3}\sinh\Big(\xi_2+\frac{1}{2}\ln(\delta_3)\Big)+\frac{\delta_1 a_1^2 A_3\sin(\eta_2)}{a_1^2+A_3^2}\Big)\times\Big(2\sqrt{\delta_3}\cosh\Big(\xi_2+\frac{1}{2}\ln(\delta_3)\Big)+\delta_1\cos(\gamma_2)-\frac{\delta_1 a_1^2\cos(\eta_2)}{a_1^2+A_3^2}\Big)^{-1}\Bigg]^2, \tag{10}$$

where $\xi_2 = a_1x + K_2y + L_2t$, $\eta_2 = A_3x - H_2y + J_2t$, and $\gamma_2 = M_2y + N_2t$.

Case 2. If $a_2 \neq 0$, then

$$\begin{gathered} b_1 = -a_1^3, \qquad b_2 = a_2^3, \qquad b_3 = -a_3^3,\\ \delta_1 = \delta_1, \qquad \delta_2 = \delta_2, \qquad \delta_3 = \delta_3,\\ c_1 = 9a_1^5, \qquad c_2 = 9a_2^5, \qquad c_3 = 9a_3^5, \end{gathered} \tag{11}$$

where a_1, a_2, a_3, δ_1, δ_2, and δ_3 are free real constants. Substituting (11) into (5) and taking $\delta_3 > 0$, we have

$$\begin{aligned} f_3 = {} & 2\sqrt{\delta_3}\cosh\Big(a_1x - a_1^3y + 9a_1^5t + \frac{1}{2}\ln(\delta_3)\Big)\\ & + \delta_1\cos\big(a_2x + a_2^3y + 9a_2^5t\big)\\ & + \delta_2\cosh\big(a_3x - a_3^3y + 9a_3^5t\big). \end{aligned} \tag{12}$$

Substituting (12) into (2) yields the breather two-soliton solution of SK equation as follows:

$$\begin{aligned} u_3 = {} & -\Big(2\Big[2\sqrt{\delta_3}a_1^2\cosh\Big(\xi_3+\frac{1}{2}\ln(\delta_3)\Big) - \delta_1 a_2^2\cos(\eta_3) + \delta_2 a_3^2\cosh(\gamma_3)\Big]\Big)\\ & \times\Big(2\sqrt{\delta_3}\cosh\Big(\xi_3+\frac{1}{2}\ln(\delta_3)\Big) + \delta_1\cos(\eta_3) + \delta_2\cosh(\gamma_3)\Big)^{-1}\\ & + \Bigg[\Big(2\Big(2\sqrt{\delta_3}a_1\sinh\Big(\xi_3+\frac{1}{2}\ln(\delta_3)\Big) - \delta_1 a_2\sin(\eta_3) + \delta_2 a_3\sinh(\gamma_3)\Big)\Big)\\ & \times\Big(2\sqrt{\delta_3}\cosh\Big(\xi_3+\frac{1}{2}\ln(\delta_3)\Big) + \delta_1\cos(\eta_3) + \delta_2\cosh(\gamma_3)\Big)^{-1}\Bigg]^2, \end{aligned} \tag{13}$$

where $\xi_3 = a_1x - a_1^3y + 9a_1^5t$, $\eta_3 = a_2x + a_2^3y + 9a_2^5t$, and $\gamma_3 = a_3x - a_3^3y + 9a_3^5t$.

The expression (u_3) is the breather two-soliton solution of SK equation which is a periodic wave in x, y and meanwhile is a two-soliton in x, y (refer to Figure 1(b)).

Case 3. If $a_2 = b_1 = 0$, then

$$\begin{gathered} a_1 = 2a_3, \qquad b_2 = \sqrt{21}a_3^3, \qquad b_3 = -\frac{3}{2}a_3^3,\\ c_1 = -\frac{169}{2}a_3^5,\\ c_2 = -20\sqrt{21}a_3^5, \qquad c_3 = -\frac{349}{4}a_3^5,\\ \delta_3 = \frac{5}{152}\delta_2^2 - \frac{7}{228}\delta_1^2, \end{gathered} \tag{14}$$

where a_3, δ_1, and δ_2 are free real constants. Substituting (14) into (5) and taking $\delta_3 > 0$, we have

$$\begin{aligned} f_4 = {} & 2\sqrt{\frac{5}{152}\delta_2^2 - \frac{7}{228}\delta_1^2}\\ & \times\cosh\Big(-2a_3x + \frac{169}{2}a_3^5t - \frac{1}{2}\ln\Big(\frac{5}{152}\delta_2^2 - \frac{7}{228}\delta_1^2\Big)\Big)\\ & + \delta_1\cos\big(-\sqrt{21}a_3^3y + 20\sqrt{21}a_3^5t\big)\\ & + \delta_2\cosh\Big(-a_3x + \frac{3}{2}{a_3}^3y + \frac{349}{4}a_3^5t\Big), \end{aligned} \tag{15}$$

where $(5/152)\delta_2^2 - (7/228)\delta_1^2 > 0$. Substituting (15) into (2) yields the breather two-soliton solution of SK equation as follows:

$$\begin{aligned} u_4 = {} & -\Big(2\Big[8\sqrt{K_4}a_3^2\cosh\Big(\xi_4 - \frac{1}{2}\ln(K_4)\Big) + \delta_2 a_3^2\cosh(\eta_4)\Big]\Big)\\ & \times\Big(2\sqrt{K_4}\cosh\Big(\xi_4 - \frac{1}{2}\ln(K_4)\Big) + \delta_1\cos(\gamma_4) + \delta_2\cosh(\eta_4)\Big)^{-1}\\ & + 2\Bigg[\Big(4\sqrt{K_4}a_3\sinh\Big(\xi_4 - \frac{1}{2}\ln(K_4)\Big) + \delta_2 a_3\sinh(\eta_4)\Big)\\ & \times\Big(2\sqrt{K_4}\cosh\Big(\xi_4 - \frac{1}{2}\ln(K_4)\Big) + \delta_1\cos(\gamma_4) + \delta_2\cosh(\eta_4)\Big)^{-1}\Bigg]^2, \end{aligned} \tag{16}$$

where $K_4 = (5/152)\delta_2^2 - (7/228)\delta_1^2$, $\xi_4 = -2a_3x + (169/2)a_3^5t$, $\eta_4 = -a_3x + (3/2)a_3^3y + (349/4)a_3^5t$, and $\gamma_4 = -\sqrt{21}a_3^3y + 20\sqrt{21}a_3^5t$.

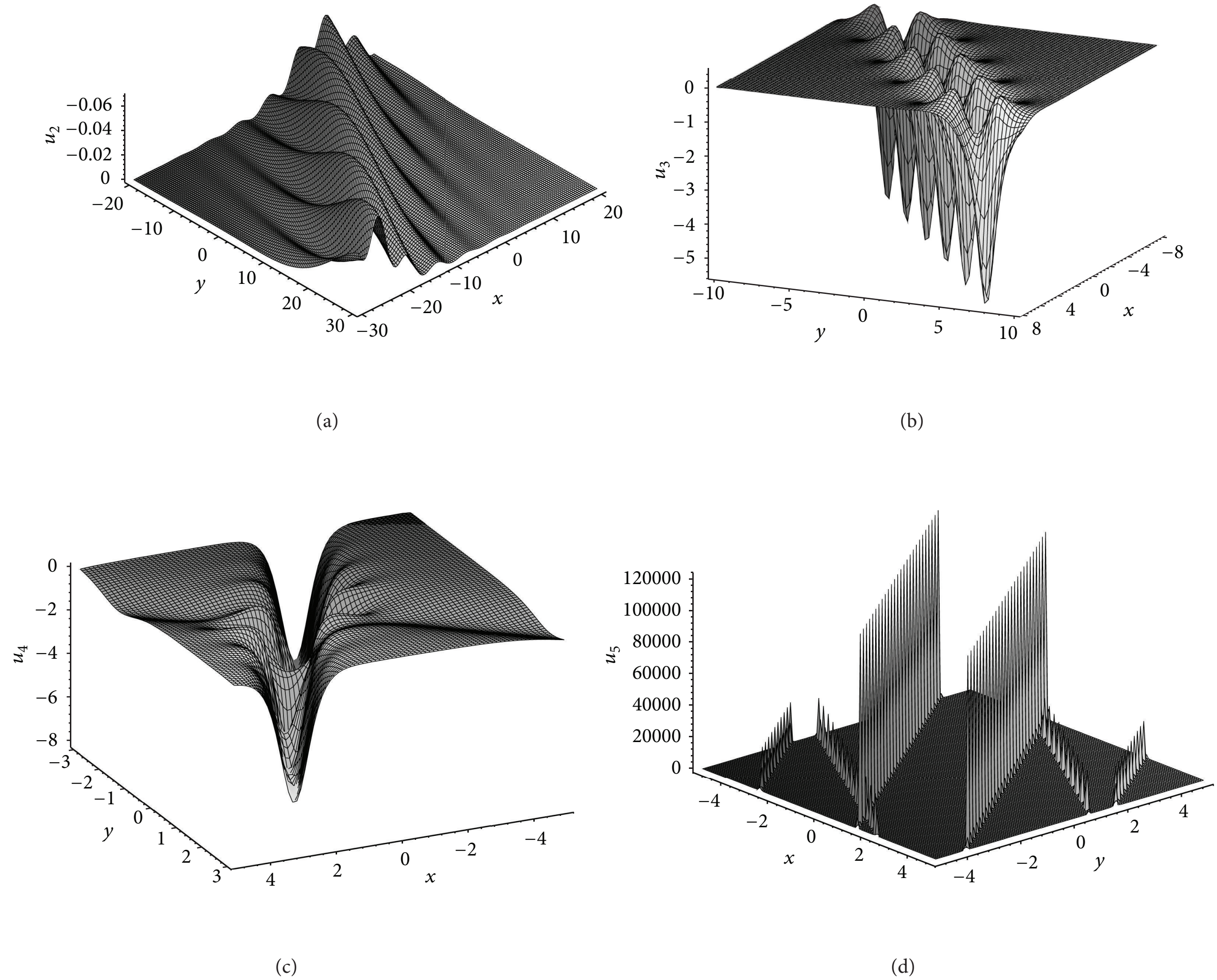

Figure 1: (a) The figure of u_2 as $\delta_1 = 1$, $\delta_3 = 1$, and $t = 1$. (b) The figure of u_3 as $\delta_1 = \sqrt{2}$, $\delta_2 = 1$, and $t = 0$. (c) The figure of u_4 as $\delta_1 = \sqrt{2}$, $\delta_2 = \sqrt{5}$, and $t = 0.005$. (d) The figure of u_5 as $\delta_1 = 1$, $\delta_2 = 1$, and $t = 0$.

The expression (u_4) is the breather two-soliton solution of SK equation which is a periodic wave in y-t and meanwhile is a two-soliton in x, y and in x-t, respectively (refer to Figure 1(c)).

Notice that u_3 and u_4 are also the breather two-soliton solutions, but their structure is different, because the two wave propagation directions are different in the u_3 and u_4, respectively (refer to Figures 1(b) and 1(c)).

If taking $a_1 = iA_1$, $a_3 = iA_3$ in (12), then we have

$$f_5 = 2\cos\left(A_1x + A_1^3y + 9A_1^5t\right) + \delta_1\cos\left(a_2x + a_2^3y + 9a_2^5t\right) + \delta_2\cos\left(A_3x + A_3^3y + 9A_3^5t\right), \tag{17}$$

when $\delta_3 = 1$. Substituting (17) into (2) gives the double-periodic three-wave solution of SK equation as follows:

$$u_5 = \frac{2\left[2A_1^2\cos(\xi_5) + \delta_1 a_2^2\cos(\eta_5) + \delta_2 A_3^2\cos(\gamma_5)\right]}{2\cos(\xi_5) + \delta_1\cos(\eta_5) + \delta_2\cos(\gamma_5)} + 2\left[\frac{2A_1\sin(\xi_5) + \delta_1 a_2\sin(\eta_5) + \delta_2 A_3\sin(\gamma_5)}{2\cos(\xi_5) + \delta_1\cos(\eta_5) + \delta_2\cos(\gamma_5)}\right]^2, \tag{18}$$

where $\xi_5 = A_1x + A_1^3y + 9A_1^5t$, $\eta_5 = a_2x + a_2^3y + 9a_2^5t$, and $\gamma_5 = A_3x + A_3^3y + 9A_3^5t$.

3. Conclusion

By using bilinear form and extended three-wave type of ansätz approach, we discuss further the (2 + 1)-dimensional

Sawada-Kotera equation and find some new multisoliton solutions. The result shows that the extended three-wave type of ansätz approach may provide us with a straightforward and effective mathematical tool for seeking multiwave solutions of high-dimensional nonlinear evolution equations.

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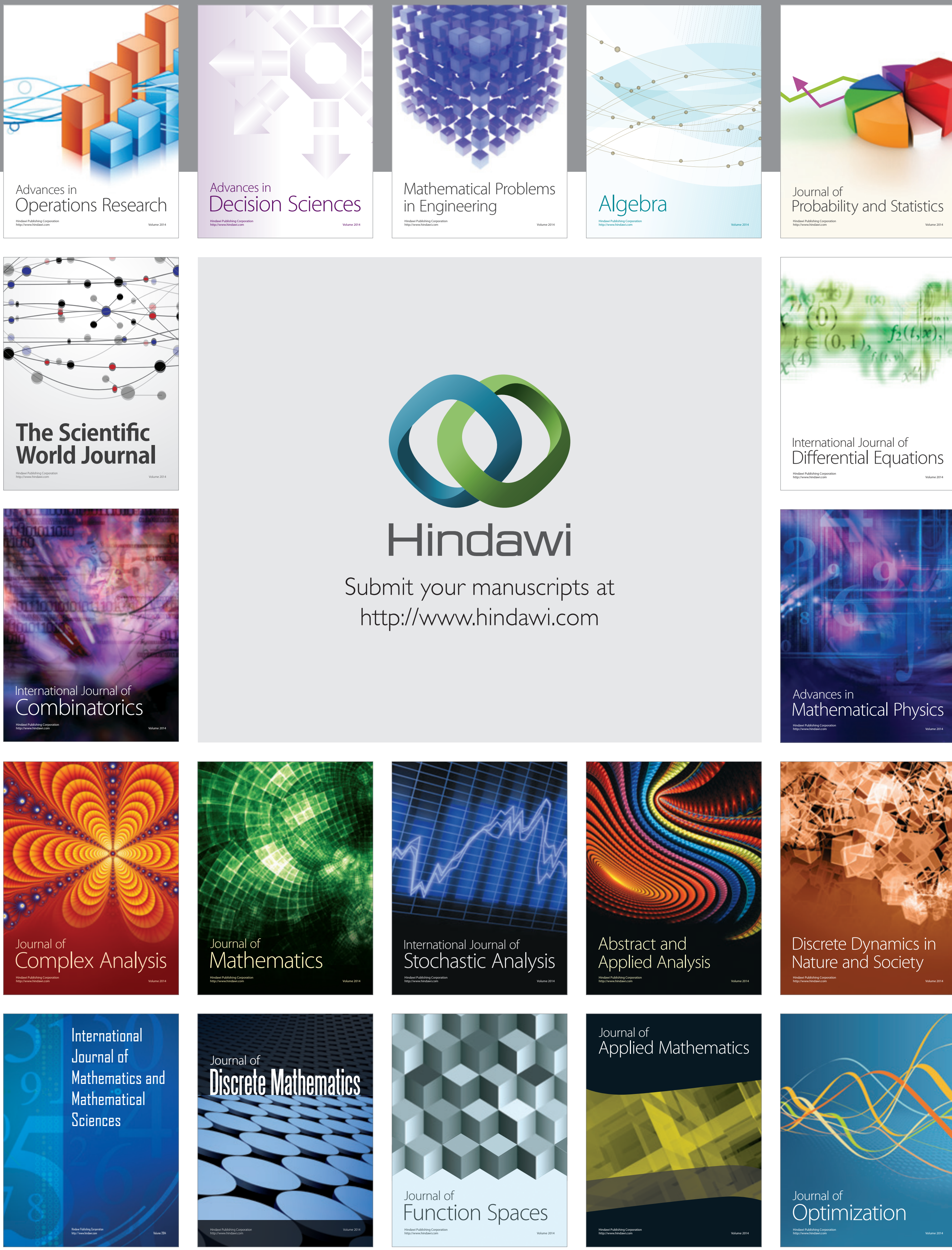
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