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Research Article

Resilient L_2-L_∞ Filtering of Uncertain Markovian Jumping Systems within the Finite-Time Interval

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This paper studies the resilient L_2-L_∞ filtering problem for a class of uncertain Markovian jumping systems within the finite-time interval. The objective is to design such a resilient filter that the finite-time L_2-L_∞ gain from the unknown input to an estimation error is minimized or guaranteed to be less than or equal to a prescribed value. Based on the selected Lyapunov-Krasovskii functional, sufficient conditions are obtained for the existence of the desired resilient L_2-L_∞ filter which also guarantees the stochastic finite-time boundedness of the filtering error dynamic systems. In terms of linear matrix inequalities (LMIs) techniques, the sufficient condition on the existence of finite-time resilient L_2-L_∞ filter is presented and proved. The filter matrices can be solved directly by using the existing LMIs optimization techniques. A numerical example is given at last to illustrate the effectiveness of the proposed approach.

1. Introduction

More recently, the finite-time stability and control problems have received great attention in the literature; see [1–6]. Compared with the Lyapunov stable dynamical systems, a finite-time stable dynamical system does not require the steady-state behavior of control dynamics over an infinite-time interval and the asymptotic pattern of system trajectories. The main attention may be related to the transient characteristics of the dynamical systems over a fixed finite-time interval, for instance, keeping the acceptable values in a prescribed bound in the presence of saturations. However, more details are related to the stability and control problems of various dynamic systems, and very few reports in the literature consider the filtering problems.

Since the Kalman filtering theory [7] has been introduced in the early 1960s, the filtering problem has been extensively investigated. In the filtering scheme, its objective is to estimate the unavailable state variables (or a linear combination of the states) of a given system. During the past decades, many filtering schemes have been developed, such as Kalman filtering [8], H_∞ filtering [9], reduced-order H_∞ filtering [10], and L_2-L_∞ filtering [11]. Then, extension of this effort to the problem of resilient Kalman filtering with respect to estimator gain perturbations was considered in [12]. And the resilient H_∞ filtering [13] was also raised. Among the filtering schemes, the resilient L_2-L_∞ filtering was not considered. In practical engineering applications, the peak values of filtering error should always be considered. Compared with the H_∞ filtering scheme, the external disturbances are both assumed to be energy bounded; but L_2-L_∞ filtering setting requires the mapping from the external disturbances to the filtering error is minimized or no larger than some prescribed level in terms of the L_2-L_∞ performance norm.

In this paper, we have studied the resilient finite-time L_2-L_∞ filtering problem for uncertain Markouvian Jumping Systems (MJSs). Firstly, the augmented filtering error dynamic system is constructed based on the state estimated filter with resilient filtering parameters. Secondly, a sufficient condition is established on the existence of the robust finite-time filter such that the filtering error dynamic MJSs are finite-time bounded and satisfy a prescribed level of L_2-L_∞ disturbance attenuation with the finite-time interval. And the design criterion is presented by means of LMIs techniques. Subsequently, the robust finite-time L_2-L_∞ filter matrices can be solved directly by using the existing LMIs optimization algorithms. In order to illustrate the proposed result, a numerical example is given at last.

Let us introduce some notations. The symbols $\mathfrak{R}^n$ and $\mathfrak{R}^{n\times m}$ stand for an n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively, A^T and A^{-1} denote the matrix transpose and matrix inverse, $\operatorname{diag}\{A \;\; B\}$ represents the block-diagonal matrix of A and B, $\sigma_{\max}(A)$ and $\sigma_{\min}(A)$, respectively, denote the maximal and minimal eigenvalues of a real matrix A, $\|*\|$ denotes the Euclidean norm of vectors, $\mathbf{E}\{*\}$ denotes the mathematics statistical expectation of the stochastic process or vector, $P > 0$ stands for a positive-definite matrix, I is the unit matrix with appropriate dimensions, 0 is the zero matrix with appropriate dimensions, and $*$ means the symmetric terms in a symmetric matrix.

2. Problem Formulation

Given a probability space (Ω, F, P_r) where Ω is the sample space, F is the algebra of events, and P_r is the probability measure defined on F. Let us consider a class of linear uncertain MJSs defined in the probability space (Ω, F, P_r) and described by the following differential equations:

$$\begin{gathered}\dot{x}(t)=\left[A(r_t)+\Delta A(r_t)\right]x(t)+B(r_t)w(t),\\ y(t)=\left[C(r_t)+\Delta C(r_t)\right]x(t)+D(r_t)w(t),\\ z(t)=E(r_t)x(t),\\ x(t)=x_0,\quad r_t=r_0,\quad t=0,\end{gathered}\tag{1}$$

where $x(t)\in\mathfrak{R}^n$ is the state, $y(t)\in\mathfrak{R}^l$ is the measured output, $w(t)\in L_2^m[0\;\; +\infty)$ is the unknown input, $z(t)\in\mathfrak{R}^q$ is the controlled output, and x_0 and r_0 are, respectively, the initial states and mode. $A(r_t)$, $B(r_t)$, $C(r_t)$, $D(r_t)$, and $E(r_t)$ are known mode-dependent constant matrices with appropriate dimensions. The jump parameter r_t represents a continuous-time discrete-state Markov stochastic process taking values on a finite set $\mathbf{M}=\{1,2,\ldots,N\}$ with transition rate matrix $\Pi=\{\pi_{ij}\}$, $i,j\in\mathbf{M}$, and has the following transition probability from mode i at time t to mode j at time $t+\Delta t$ as

$$P_{ij}=P_r\{r_{t+\Delta t}=j\mid r_t=i\}=\begin{cases}\pi_{ij}\Delta t+o(\Delta t), & i\neq j,\\ 1+\pi_{ii}\Delta t+o(\Delta t), & i=j,\end{cases}\tag{2}$$

where $\Delta t>0$ and $\lim_{\Delta t\to 0}(o(\Delta t)/\Delta t)\to 0$.

In this relation, $\pi_{ij}\geq 0$ is the transition probability rates from mode i at time t to mode j $(i\neq j)$ at time $t+\Delta t$, and

$$\sum_{j=1,j\neq i}^{N}\pi_{ij}=-\pi_{ii}\quad\text{for } i,j\in\mathbf{M},\; i\neq j.\tag{3}$$

For presentation convenience, we denote $A(r_t)$, $B(r_t)$, $C(r_t)$, $D(r_t)$, $E(r_t)$, $\Delta A(r_t)$, and $\Delta C(r_t)$ as A_i, B_i, C_i, D_i, E_i, ΔA_i, and ΔC_i, respectively.

And the matrices with the symbol $\Delta(*)$ are considered as the uncertain matrices satisfying the following conditions:

$$\begin{bmatrix}\Delta A_i\\ \Delta C_i\end{bmatrix}=\begin{bmatrix}M_{1i}\\ M_{2i}\end{bmatrix}\Gamma_i(t)N_{1i},\tag{4}$$

where M_{1i}, M_{2i}, and N_{1i} are known mode-dependent matrices with appropriate dimensions, and $\Gamma_i(t)$ is the time-varying unknown matrix function with Lebesgue norm measurable elements satisfying $\Gamma_i^T(t)\Gamma_i(t)\leq I$.

Remark 1. It is always impossible to obtain the exact mathematical model of practical dynamics due to the complexity process, the environmental noises, and the difficulties of measuring various and uncertain parameters, and so forth; thus, the model of practical dynamics to be controlled almost contains some types of uncertainties. In general, the uncertainties $\Delta(*)$ in (1) satisfying the restraining conditions (4) and $\Gamma_i^T(t)\Gamma_i(t)\leq I$ are said to be admissible. The unknown mode-dependent matrix $\Gamma_i(t)$ can also be allowed to be state dependent; that is, $\Gamma_i(t)=\Gamma_i(t,x(t))$, as long as $\|\Gamma_i(t,x(t))\|\leq 1$ is satisfied.

We now consider the following resilient filter:

$$\begin{gathered}\dot{\hat{x}}(t)=\left(A_{fi}+\Delta A_{fi}\right)\hat{x}(t)+B_{fi}y(t),\\ \hat{z}(t)=\left(C_{fi}+\Delta C_{fi}\right)\hat{x}(t),\\ \hat{x}(t)=\hat{x}_0,\quad r_t=r_0,\quad t=0,\end{gathered}\tag{5}$$

where $\hat{x}(t)\in\mathfrak{R}^n$ is the filter state, $\hat{z}(t)\in\mathfrak{R}^q$ is the filer output, $\hat{x}_0$ is the initial estimation states, and the mode-dependent matrices A_{fi}, B_{fi}, and C_{fi} are unknown filter parameters to be designed for each value $i\in\mathbf{M}$. ΔA_{fi} and ΔC_{fi} are uncertain filter parameter matrices satisfying the following conditions:

$$\begin{bmatrix}\Delta A_{fi}\\ \Delta C_{fi}\end{bmatrix}=\begin{bmatrix}M_{3i}\\ M_{4i}\end{bmatrix}\Gamma_{fi}(t)N_{2i},\tag{6}$$

where M_{3i}, M_{4i}, N_{2i}, and $\Gamma_{fi}(t)$ are defined similarly as (4).

The objective of this paper is to design the resilient L_2-L_∞ filter of uncertain MJSs in (1) and obtain an estimate $\hat{z}(t)$ of the signal $z(t)$ such that the defined guaranteed performance criteria are minimized in an L_2-L_∞ estimation error sense. Define $e(t)=x(t)-\hat{x}(t)$ and $r(t)=z(t)-\hat{z}(t)$, such that the filtering error dynamic MJSs are given by

$$\begin{aligned}\dot{e}(t)=&\left(A_{fi}+\Delta A_{fi}\right)e(t)\\ &+\left[A_i+\Delta A_i-\left(A_{fi}+\Delta A_{fi}\right)-B_{fi}\left(C_i+\Delta C_i\right)\right]x(t)\\ &+\left(B_i-B_{fi}D_i\right)w(t),\\ r(t)=&\left(C_{fi}+\Delta C_{fi}\right)e(t)+\left[E_i-\left(C_{fi}+\Delta C_{fi}\right)\right]x(t).\end{aligned}\tag{7}$$

Let $\xi(t)=\left[\begin{smallmatrix}x(t)\\ e(t)\end{smallmatrix}\right]$, and we have

$$\begin{gathered}\dot{\xi}(t)=\overline{A}_i\xi(t)+\overline{B}_iw(t),\\ r(t)=\overline{C}_i\xi(t),\end{gathered}\tag{8}$$

where

$$\overline{A}_i = \begin{bmatrix} A_i + \Delta A_i & 0 \\ A_i + \Delta A_i - (A_{fi} + \Delta A_{fi}) - B_{fi}(C_i + \Delta C_i) & A_{fi} + \Delta A_{fi} \end{bmatrix},$$

$$\overline{B}_i = \begin{bmatrix} B_i \\ B_i - B_{fi} D_i \end{bmatrix},$$

$$\overline{C}_i = \begin{bmatrix} E_i - (C_{fi} + \Delta C_{fi}) & C_{fi} + \Delta C_{fi} \end{bmatrix}. \tag{9}$$

The external disturbance $w(t)$ is varying and satisfies the constraint condition with respect to the finite-time interval $[0 \quad T]$ as follows:

$$\int_0^T w^T(t) w(t) dt \le W, \tag{10}$$

where W is a positive scalar.

Definition 2. Given a time-constant $T > 0$, the filtering error dynamic MJSs (8) (setting $w(t) = 0$) are said to be stochastically finite-time stable (FTS) with respect to $(c_1 \quad c_2 \quad T \quad \widetilde{R}_i)$, if

$$\mathbf{E}\{\xi^T(0)\widetilde{R}_i\xi(0)\} \le c_1 \Longrightarrow \mathbf{E}\{\xi^T(t)\widetilde{R}_i\xi(t)\} < c_2, \quad t \in [0 \quad T], \tag{11}$$

where $c_1 > 0$, $c_2 > 0$, $\widetilde{R}_i = \operatorname{diag}[R_i \quad R_i] > 0$, and R_i is the weight coefficient matrix.

Definition 3. Given a time-constant $T > 0$, the filtering error dynamic MJSs (8) are stochastically finite-time bounded (FTB) with respect to $(c_1 \quad c_2 \quad T \quad \widetilde{R}_i \quad W)$, wherein $c_1 > 0$, $c_2 > 0$, $\widetilde{R}_i > 0$, if condition (10) holds.

Definition 4. For the filtering error dynamic MJSs (8), if there exist filter parameters A_{fi}, B_{fi}, and C_{fi}, as well as a positive scalar γ, such that the filtering error dynamic MJSs (8) are stochastically FTB and under the zero-valued initial condition, the system output error satisfies the following cost function inequality for $T > 0$ with attenuation $\gamma > 0$ and for all admissible $w(t)$ with the constraint condition (10):

$$J = \|r(t)\|_{E\infty}^T - \gamma\|w(t)\|_2^T < 0, \tag{12}$$

where $\|r(t)\|_{E\infty}^T = \sup_{t\in[0 \quad T]}\mathbf{E}[\|r(t)\|]$, $\|w(t)\|_2^T = \sqrt{\int_0^T w^T(t)w(t)\,dt}$.

Then, the resilient filter (5) is called the stochastic finite-time L_2-L_∞ filter of the uncertain dynamic MJSs (1) with γ-disturbance attenuation.

3. Main Results

In this section, we will study the robust stochastic finite-time resilient filtering problem for the filtering error dynamic MJSs (8) in an L_2-L_∞ estimation error sense. Before proceeding with the study, the following lemma is needed.

Lemma 5 (see [14]). *Let T, M, and N be real matrices with appropriate dimensions. Then, for all time-varying unknown matrix function $F(t)$ satisfying $F^T(t)F(t) \le I$, the following relation holds:*

$$T + MF(t)N + N^T F^T(t) M^T > 0, \tag{13}$$

if and only if there exists a positive scalar $\alpha > 0$, such that

$$T + \alpha^{-1}MM^T\alpha N^T N < 0. \tag{14}$$

Theorem 6. *For given $T > 0$, $\eta > 0$, $c_1 > 0$, and $\widetilde{R}_i > 0$, the filtering error dynamic MJSs (8) are stochastically FTB with respect to $(c_1 \quad c_2 \quad T \quad \widetilde{R}_i \quad W)$ and have a L_2-L_∞ performance level $\gamma > 0$, if there exist positive constants, $\gamma > 0$, $c_2 > 0$, and mode-dependent symmetric positive-definite matrices $\widetilde{P}_i$, such that*

$$\begin{bmatrix} \widetilde{P}_i\overline{A}_i + \overline{A}_i^T\widetilde{P}_i + \sum_{j=1}^N \pi_{ij}\widetilde{P}_j - \eta\widetilde{P}_i & \widetilde{P}_i\overline{B}_i \\ B_i^T\widetilde{P}_i & -e^{-\eta T} I \end{bmatrix} < 0, \tag{15}$$

$$\overline{C}_i^T\overline{C}_i < \gamma^2\widetilde{P}_i, \tag{16}$$

$$e^{\eta T}c_1\overline{\sigma}_{\widehat{P}} + \frac{W}{\eta}\left(1 - e^{-\eta T}\right) < c_2\underline{\sigma}_{\widehat{P}}, \tag{17}$$

where $\widehat{P}_i = \widetilde{R}_i^{-1/2}\widetilde{P}_i\widetilde{R}_i^{-1/2}$, $\overline{\sigma}_{\widehat{P}} = \max_{i\in\mathbf{M}}\sigma_{\max}(\widehat{P}_i)$, and $\underline{\sigma}_{\widehat{P}} = \min_{i\in\mathbf{M}}\sigma_{\min}(\widehat{P}_i)$.

Proof. Let the mode at time t be i; that is, $r_t = r \in \mathbf{M}$. Take the stochastic Lyapunov-Krasovskii functional $V(\xi(t), r_t, t > 0) : \mathfrak{R}^n \times \mathbf{M} \times \mathfrak{R}_+ \to \mathfrak{R}_+$ as

$$V(\xi(t), i) = \xi^T(t)\widetilde{P}_i\xi(t). \tag{18}$$

Then, we introduce a weak infinitesimal generator $\mathfrak{I}[*]$ (see [15, 16]), acting on $V(\xi(t), i)$, for all $i \in \mathbf{M}$, which is defined as

$$\mathfrak{I}V(\xi(t), i) = \lim_{\Delta t\to 0}\frac{1}{\Delta t}\left[\mathbf{E}\{V(\xi(t+\Delta t), r_{t+\Delta t}, t+\Delta t) \mid \xi(t), r_t = i\} - V(\xi(t), i, t)\right]. \tag{19}$$

The time derivative of $V(\xi(t), i)$ along the trajectories of the filtering error dynamic MJSs (8) is given by

$$\begin{aligned} \mathfrak{I}V(\xi(t), i) &= 2\xi^T(t)\widetilde{P}_i\dot{\xi}(t) + \xi^T(t)\sum_{j=1}^N \pi_{ij}\widetilde{P}_j\xi(t) \\ &= \xi^T(t)\left(\widetilde{P}_i\overline{A}_i + \overline{A}_i^T\widetilde{P}_i + \sum_{j=1}^N \pi_{ij}\widetilde{P}_j\right)\xi(t) \\ &\quad + 2\xi^T(t)\widetilde{P}_i\overline{B}_i w(t). \end{aligned} \tag{20}$$

Considering the L_2-L_∞ filtering performance for the dynamic filtering error system (8), we introduce the following cost function by Definition 4 with $t \ge 0$:

$$J(t) = \mathbf{E}[\mathfrak{I}V(\xi(t), i)] - \eta\mathbf{E}[V(\xi(t), i)] - e^{-\eta T}w^T(t)w(t). \tag{21}$$

It follows from relation (16) that $J(t) < 0$; that is,

$$\mathbf{E}\left[\Im V(\xi(t), i)\right] < \eta \mathbf{E}\left[V(\xi(t), i)\right] + e^{-\eta T} w^T(t) w(t). \quad (22)$$

Then, multiplying the previous inequality by $e^{-\eta t}$, we have

$$\mathbf{E}\left\{\Im\left[e^{-\eta t} V(\xi(t), i)\right]\right\} < e^{-\eta(T+t)} w^T(t) w(t). \quad (23)$$

Integrating the above inequality from 0 to T, we have

$$\begin{aligned} &e^{-\eta T}\mathbf{E}\{V(\xi(T), i)\} - \mathbf{E}\{V(\xi(0), r_0)\} \\ &\quad < e^{-\eta T}\int_0^T e^{-\eta s} w^T(s) w(s)\, ds. \end{aligned} \quad (24)$$

Considering $V(\xi(0), r_0) \geq 0$, as well as the zero initial condition; that is, $\xi(0) = 0$, for $t > 0$, then it follows that

$$\mathbf{E}\{V(\xi(T), i)\} < \int_0^T e^{-\eta s} w^T(s) w(s)\, ds. \quad (25)$$

Then, it can be verified from the defined Lyapunov-Krasovskii functional that

$$\mathbf{E}\left\{\xi^T(T)\widetilde{P}_i\xi(T)\right\} = \mathbf{E}\{V(\xi(T), i)\} < \int_0^T e^{-\eta s} w^T(s) w(s)\, ds. \quad (26)$$

By (15) and within the finite-time interval $[0 \quad T]$, we can also get

$$\begin{aligned} \mathbf{E}\left\{r^T(T) r(T)\right\} &= \mathbf{E}\left\{\xi^T(T)\overline{C}_i^T\overline{C}_i\xi(T)\right\} \\ &< \gamma^2\mathbf{E}\left\{\xi^T(T)\widetilde{P}_i\xi(T)\right\} \\ &= \gamma^2\mathbf{E}\{V(\xi(T), i)\} \\ &< \gamma^2\int_0^T e^{-\eta s} w^T(s) w(s)\, ds \\ &< \gamma^2\int_0^T w^T(s) w(s)\, ds. \end{aligned} \quad (27)$$

Since the previous inequality is always true for any $T > 0$, the following relation:

$$\sup_{t\in[0 \quad T]} \mathbf{E}\left[\|r(t)\|\right] < \gamma\sqrt{\int_0^T w^T(s) w(s)\, ds}. \quad (28)$$

It is easy to get the following result:

$$\begin{aligned} \|r(t)\|_{E\infty}^T &= \gamma\sum_{i=1}^N \pi_i\left\{\sup_{t\in[0 \quad T]}\mathbf{E}\left[\|r(t)\|\right]\right\} \\ &< \gamma\sum_{i=1}^N \pi_i\sqrt{\int_0^T w^T(s) w(s)\, ds}. \\ &= \gamma\sqrt{\int_0^T w^T(s) w(s)\, ds} = \gamma\|w(t)\|_2^T. \end{aligned} \quad (29)$$

Therefore, the cost function inequality (10) can be guaranteed, which implies $J = \|r(t)\|_{E\infty}^T - \gamma\|w(t)\|_2^T < 0$.

Denote that $\widehat{P}_i = \widetilde{R}_i^{-1/2}\widetilde{P}_i\widetilde{R}_i^{-1/2}$, $\overline{\sigma}_{\widehat{P}} = \max_{i\in\mathbf{M}}\sigma_{\max}(\widehat{P}_i)$, and $\underline{\sigma}_{\widehat{P}} = \min_{i\in\mathbf{M}}\sigma_{\min}(\widehat{P}_i)$. From equality (24), we have

$$\begin{aligned} \mathbf{E}\left\{\xi^T(t)\widetilde{P}_i\xi(t)\right\} &= \mathbf{E}\{V(\xi(t), i)\} < \int_0^t e^{-\eta s} w^T(s) w(s)\, ds \\ &\quad + e^{\eta t}\mathbf{E}\{V(\xi(0), r_0)\} < e^{\eta t}\mathbf{E}\{V(\xi(0), r_0)\} \\ &\quad + W\int_0^t e^{-\eta s} ds < e^{\eta t} c_1\overline{\sigma}_{\widehat{P}} \\ &\quad + \frac{W}{\eta}\left(1 - e^{-\eta t}\right) \leq e^{\eta T} c_1\overline{\sigma}_{\widehat{P}} \\ &\quad + \frac{W}{\eta}\left(1 - e^{-\eta T}\right). \end{aligned} \quad (30)$$

On the other hand, it results from the stochastic Lyapunov-Krasovskii functional that

$$\mathbf{E}\left\{\xi^T(t)\widetilde{P}_i\xi(t)\right\} \geq \underline{\sigma}_{\widehat{P}}\mathbf{E}\left\{\xi^T(t)\widetilde{R}_i\xi(t)\right\}. \quad (31)$$

Then, we can get

$$\mathbf{E}\left\{\xi^T(t)\widetilde{R}_i\xi(t)\right\} < \frac{e^{\eta T} c_1\overline{\sigma}_{\widehat{P}} + (W/\eta)\left(1 - e^{-\eta T}\right)}{\underline{\sigma}_{\widehat{P}}}. \quad (32)$$

It implies that for all $t \in [0 \quad T]$, we have $\mathbf{E}\{\xi^T(t)\widetilde{R}_i\xi(t)\} < c_2$ by condition (17). This completes the proof. □

Theorem 7. *For given $T > 0$, $\eta > 0$, $c_1 > 0$, and $R_i > 0$, the filtering error dynamic MJSs (8) are stochastically FTB with respect to $(c_1 \quad c_2 \quad T \quad R_i \quad W)$ with $R_i \in \Re^{n\times n} > 0$ and have a prescribed L_2-L_∞ performance level $\gamma > 0$, if there exist a set of mode-dependent symmetric positive-definite matrices P_i, a set of mode-dependent matrices X_i, Y_i, and a positive scalar σ_1 and mode-dependent sequences α_i, β_i, λ_i, satisfying the following matrix inequalities for all $i \in \mathbf{M}$:*

$$\begin{bmatrix} \Lambda_{1i} & * & P_iB_i & P_iM_{1i} & 0 \\ \Lambda_{2i} & \Lambda_{3i} & P_iB_i - Y_iD_i & P_iM_{1i} - Y_iM_{2i} & P_iM_{3i} \\ * & * & -e^{-\eta T}I & 0 & 0 \\ * & * & * & -\alpha_iI & 0 \\ * & * & * & * & -\beta_iI \end{bmatrix} < 0, \quad (33)$$

$$\begin{bmatrix} -P_i & 0 & -E_i^T + C_{fi}^T & N_{2i}^T \\ 0 & -P_i & -C_{fi}^T & -N_{2i}^T \\ * & * & -\gamma^2I + \lambda_iM_{4i}M_{4i}^T & 0 \\ * & * & * & -\lambda_iI \end{bmatrix} < 0, \quad (34)$$

$$R_i < P_i < \sigma_1R_i, \quad (35)$$

$$e^{\eta T}c_1\sigma_1 + \frac{W}{\eta}\left(1 - e^{-\eta T}\right) < c_2, \quad (36)$$

where $\Lambda_{1i} = P_iA_i + A_i^TP_i + \sum_{j=1}^{N}\pi_{ij}P_j - \eta P_i + \alpha_i N_{1i}^TN_{1i} + \beta_i N_{2i}^TN_{2i}$, $\Lambda_{2i} = P_iA_i - X_i - Y_iC_i - \beta_i N_{2i}^TN_{2i}$, $\Lambda_{2i} = X_i + X_i^T + \sum_{j=1}^{N}\pi_{ij}P_j - \eta P_i + \beta_i N_{2i}^TN_{2i}$.

Moreover, the suitable filter parameters can be given as

$$A_{fi} = P_i^{-1}X_i, \qquad B_{fi} = P_i^{-1}Y_i, \qquad C_{fi} = C_{fi}. \tag{37}$$

Proof. For convenience, we set $\widetilde{P}_i = \operatorname{diag}\{P_i, P_i\}$. Then, we can get the following relations according to matrix inequalities (15) and (16):

$$\Pi_i + \Delta\Pi_{1i} + \Delta\Pi_{2i} < 0, \tag{38}$$

$$\Sigma_i + \Delta\Sigma_i < 0, \tag{39}$$

where

$$\Pi_i = \begin{bmatrix} P_iA_i + A_i^TP_i + \sum_{j=1}^{N}\pi_{ij}P_j - \eta P_i & * & P_iB_i \\ P_iA_i - P_iA_{fi} - P_iB_{fi}C_i & P_iA_{fi} + A_{fi}^TP_i + \sum_{j=1}^{N}\pi_{ij}P_j - \eta P_i & P_iB_i - P_iB_{fi}D_i \\ * & * & -e^{-\eta T}I \end{bmatrix},$$

$$\Sigma_i = \begin{bmatrix} -P_i & 0 & -E_i^T + C_{fi}^T \\ 0 & -P_i & -C_{fi}^T \\ * & * & -\gamma^2 I \end{bmatrix},$$

$$\Delta\Pi_{1i} = \begin{bmatrix} P_i\Delta A_i + \Delta A_i^TP_i & * & 0 \\ P_i\Delta A_i - P_iB_{fi}\Delta C_i & 0 & 0 \\ * & * & 0 \end{bmatrix}, \tag{40}$$

$$\Delta\Pi_{2i} = \begin{bmatrix} 0 & * & 0 \\ -P_i\Delta A_{fi} & P_i\Delta A_{fi} + \Delta A_{fi}^TP_i & 0 \\ * & * & 0 \end{bmatrix},$$

$$\Delta\Sigma_i = \begin{bmatrix} 0 & 0 & \Delta C_{fi}^T \\ 0 & 0 & -\Delta C_{fi}^T \\ * & * & 0 \end{bmatrix}.$$

Referring to Lemma 5, $\Delta\Pi_{1i}$ and $\Delta\Pi_{2i}$ can be presented as the following form:

$$\begin{aligned} \Delta\Pi_{1i} &= L_{1i}\Gamma_i(t)L_{2i} + L_{2i}^T\Gamma_i^T(t)L_{1i}^T < \alpha_i^{-1}L_{1i}L_{1i}^T + \alpha_i L_{2i}^TL_{2i}, \\ \Delta\Pi_{2i} &= L_{3i}\Gamma_{fi}(t)L_{4i} + L_{4i}^T\Gamma_{fi}^T(t)L_{3i}^T < \beta_i^{-1}L_{3i}L_{3i}^T + \beta_i L_{4i}^TL_{4i}, \\ \Delta\Sigma_i &= L_{5i}\Gamma_{fi}(t)L_{6i} + L_{6i}^T\Gamma_{fi}^T(t)L_{5i}^T < \lambda_i L_{5i}L_{5i}^T + \lambda_i^{-1}L_{6i}^TL_{6i}, \end{aligned} \tag{41}$$

where $L_{1i} = \operatorname{col}[P_iM_{1i} \quad P_iM_{1i} - P_iB_{fi}M_{2i} \quad 0]$, $L_{2i} = [N_{1i} \quad 0 \quad 0]$, $L_{3i} = \operatorname{col}[0 \quad P_iM_{3i} \quad 0]$, $L_{4i} = [-N_{2i} \quad N_{2i} \quad 0]$, $L_{5i} = \operatorname{col}[0 \quad 0 \quad M_{4i}]$, and $L_{6i} = [N_{2i} \quad -N_{2i} \quad 0]$.

Then, inequalities (15) and (16) are equivalent to LMIs (38) and (39) by letting $X_i = P_iA_{fi}$ and $Y_i = P_iB_{fi}$.

On the other hand, we set $\widetilde{R}_i = \operatorname{diag}[R_i \quad R_i]$, and LMI (35) implies that

$$1 < \underline{\sigma}_{\widetilde{P}} = \min_{i\in\mathbf{M}} \sigma_{\min}\left(\widetilde{P}_i\right), \quad \overline{\sigma}_{\widetilde{P}} = \max_{i\in\mathbf{M}} \sigma_{\max}\left(\widetilde{P}_i\right) < \sigma_1. \tag{42}$$

Then, recalling condition (17), we can get LMI (36). This completes the proof. □

To obtain an optimal finite-time L_2-L_∞ filtering performance against unknown inputs, uncertainties, and model errors, the attenuation lever γ^2 can be reduced to the minimum possible value such that LMIs (33)–(36) are satisfied. The optimization problem can be described as follows:

$$\begin{gathered} \min_{P_i, X_i, Y_i, C_{fi}, D_{fi}, \alpha_i, \beta_i, \lambda_i, \rho} \rho \\ \text{s. t. LMIs (33)} - \text{(36) with } \rho = \gamma^2. \end{gathered} \tag{43}$$

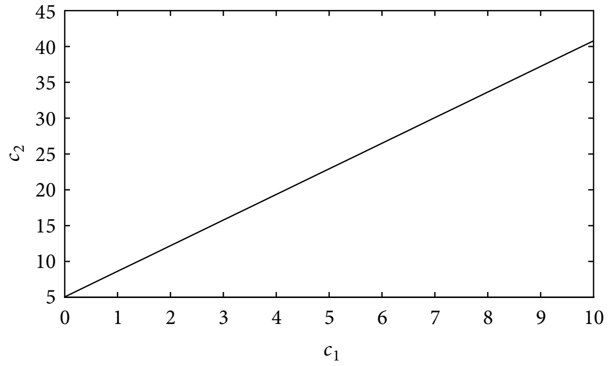

FIGURE 1: The optimal minimal upper bound c_2 with different initial c_1.

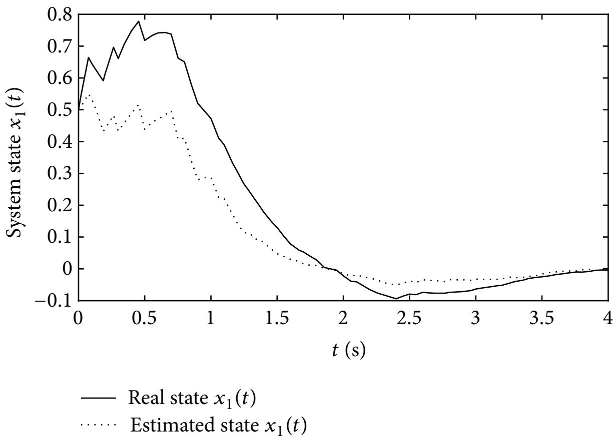

FIGURE 2: System response with state $x_1(t)$.

Remark 8. In Theorems 6 and 7, if c_2 is a variable to be solved, then (17) and (36) can be always satisfied as long as c_2 is sufficiently large. For these, we can also fix γ and look for the optimal admissible c_1 or c_2 guaranteeing the stochastic finite-time boundedness of desired filtering error dynamic properties.

4. Numeral Examples

Example 9. Consider a class of MJSs with two operation modes described as follows.

Mode 1:

$$A_1 = \begin{bmatrix} -2 & 1 \\ -1 & -1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 1 & 0.5 \end{bmatrix},$$
$$D_1 = 0.1, \qquad E_1 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, \qquad M_{11} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix},$$
$$M_{21} = -0.1, \qquad M_{31} = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}, \qquad M_{41} = 0.2,$$
$$N_{11} = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}, \qquad N_{21} = \begin{bmatrix} 0.1 & -0.2 \end{bmatrix}; \tag{44}$$

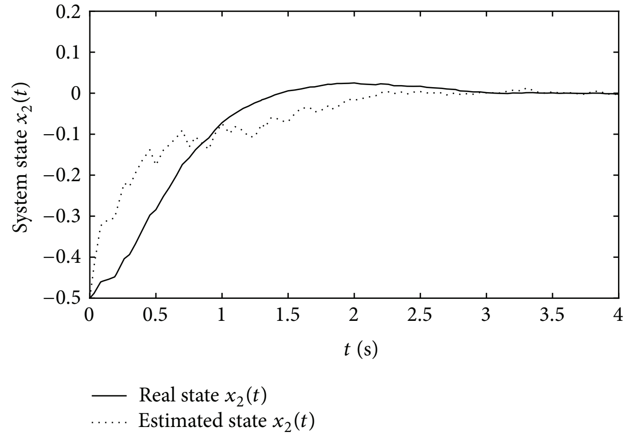

FIGURE 3: System response with state $x_2(t)$.

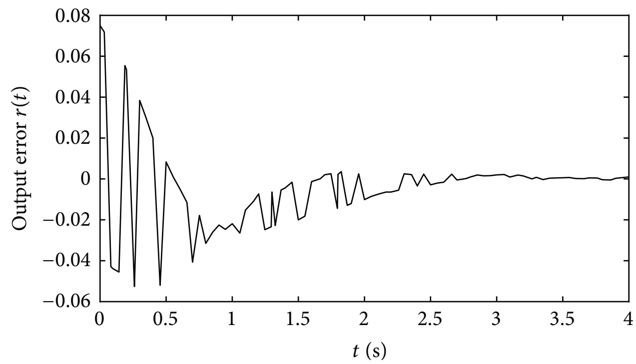

FIGURE 4: System response with output error $r(t)$.

Mode 2:

$$A_2 = \begin{bmatrix} 0 & 3 \\ -1 & -2 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 1 & 1 \end{bmatrix},$$
$$D_2 = -0.2, \qquad E_2 = \begin{bmatrix} 0.2 & -0.1 \end{bmatrix}, \qquad M_{12} = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix},$$
$$M_{22} = 0.2, \qquad M_{32} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \qquad M_{42} = -0.2,$$
$$N_{12} = \begin{bmatrix} -0.1 & 0.1 \end{bmatrix}, \qquad N_{22} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}. \tag{45}$$

The mode switching is governed by a Markov chain that has the following transition rate matrix:

$$\Pi = \begin{bmatrix} -0.3 & 0.3 \\ 0.5 & -0.5 \end{bmatrix}. \tag{46}$$

In this paper, we choose the initial values for $W = 2$, $T = 4$, $\eta = 0.25$, and $R_i = I_2$. Then, we fix $\gamma = 0.8$ and look for the optimal admissible c_2 with different c_1 which can guarantee the stochastic finite-time boundedness of desired filtering error dynamic properties. Figure 1 gives the optimal minimal admissible c_2 with different initial upper bound c_1.

For $c_1 = 1$, we solve LMIs (33)–(36) by Theorem 7 and the optimization algorithm (43) and get the following optimal resilient finite-time L_2-L_∞ filter:

$$\begin{gathered} A_{f1} = \begin{bmatrix} -7.7193 & -3.3359 \\ -6.9662 & -6.6546 \end{bmatrix}, \\ B_{f1} = \begin{bmatrix} 3.0189 \\ 6.7316 \end{bmatrix}, \qquad C_{f1} = \begin{bmatrix} 0.05 & 0.1 \end{bmatrix}; \\ A_{f2} = \begin{bmatrix} -1.3004 & 0.3608 \\ -3.0043 & -9.2166 \end{bmatrix}, \\ B_{f2} = \begin{bmatrix} 1.1415 \\ 2.0586 \end{bmatrix}, \qquad C_{f2} = \begin{bmatrix} 0.1 & -0.05 \end{bmatrix}. \end{gathered} \tag{47}$$

And then, we can also get the attenuation lever as $\gamma = 0.0777$ and the relevant upper bound $c_2 = 592.45$.

To show the effectiveness of the designed optimal resilient finite-time L_2-L_∞ filter, we assume that the unknown inputs are unknown white noise with noise power 0.05 over a finite-time interval $t \in [0 \quad 4]$. For the selected initial conditions $x(0) = \left[\begin{smallmatrix} 0.5 \\ -0.5 \end{smallmatrix} \right]$ and $r_0 = 2$, the simulation results of the jumping modes and the response of system states are shown in Figures 2, 3, and 4. It is clear from the simulation figures that the estimated states can track the real states smoothly.

5. Conclusion

The resilient finite-time L_2-L_∞ filtering problems for a class of stochastic MJSs with uncertain parameters have been studied. By using the Lyapunov-Krasovskii functional approach and LMIs optimization techniques, a sufficient condition is derived such that the filtering dynamic error MJSs are finite-time bounded and satisfy a prescribed level of L_2-L_∞ disturbance attenuation in a finite time-interval. The robust resilient filter gains can be solved directly by using the existing LMIs. Simulation results illustrate the effectiveness of the proposed a pproach.

Acknowledgments

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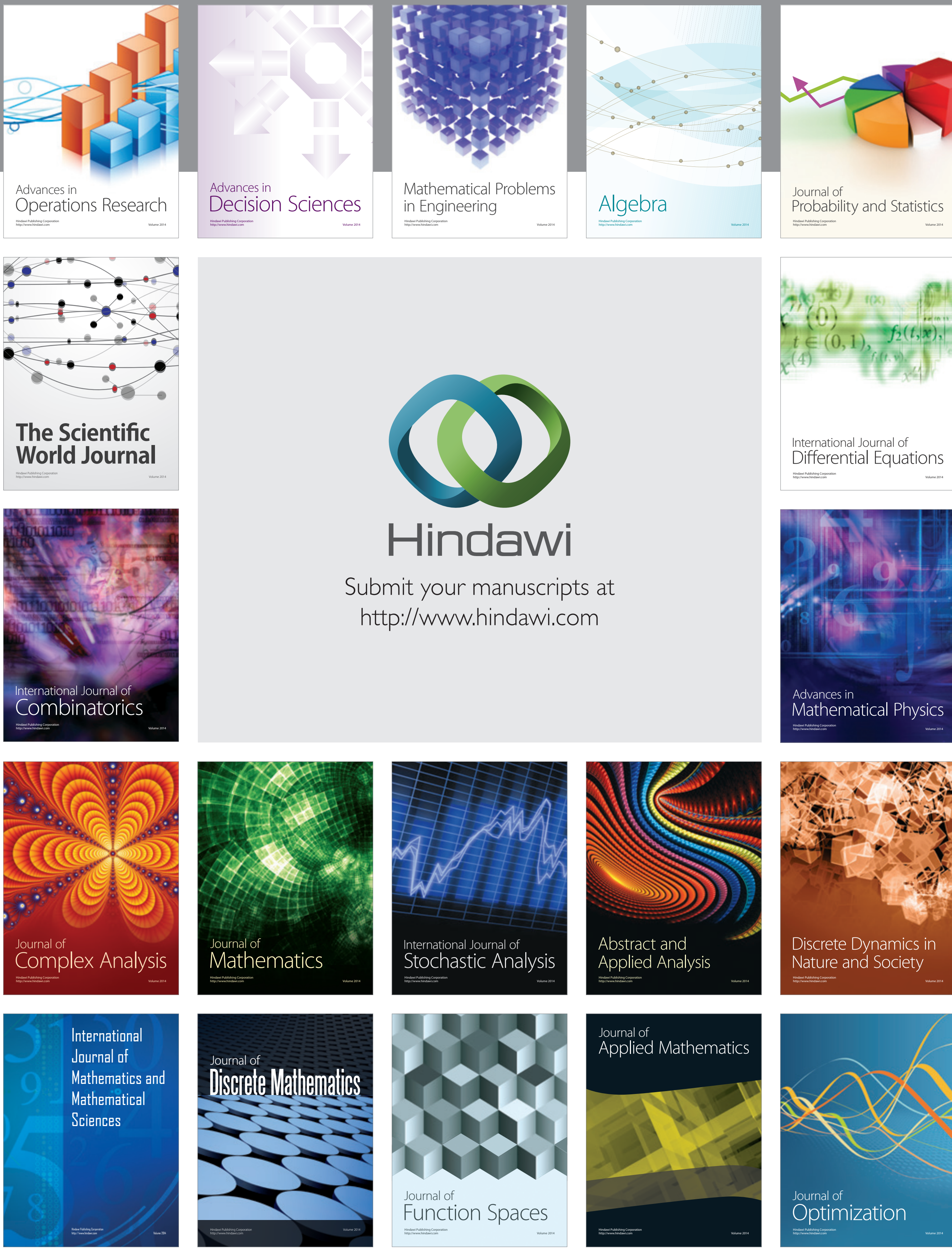
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