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Research Article

μ-Stability of Impulsive Neural Networks with Unbounded Time-Varying Delays and Continuously Distributed Delays

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This paper is concerned with the problem of μ-stability of impulsive neural systems with unbounded time-varying delays and continuously distributed delays. Some μ-stability criteria are derived by using the Lyapunov-Krasovskii functional method. Those criteria are expressed in the form of linear matrix inequalities (LMIs), and they can easily be checked. A numerical example is provided to demonstrate the effectiveness of the obtained results.

1. Introduction

In recent years, the dynamics of neural networks have been extensively studied because of their application in many areas, such as associative memory, pattern recognition, and optimization [1–4]. Many researchers have a lot of contributions to these subjects. Stability is a basic knowledge for dynamical systems and is useful to the real-life systems. The time delays happen frequently in various engineering, biological, and economical systems, and they may cause instability and poor performance of practical systems. Therefore, the stability analysis for neural networks with time-delay has attracted a large amount of research interest, and many sufficient conditions have been proposed to guarantee the stability of neural networks with various type of time delays, see for example [5–20] and the references therein. However, most of the results are obtained based on the assumption that *the time delay is bounded*. As we know, time delays occur and vary frequently and irregularly in many engineering systems, and sometimes they depend on the histories heavily and may be unbounded [21, 22]. In such case, those existing results in [5–20] are all invalid.

How to guarantee the desirable stability if the time delays are unbounded? Recently, Chen et al. [23, 24] proposed a new concept of μ-stability and established some sufficient conditions to guarantee the global μ-stability of delayed neural networks with or without uncertainties via different approaches. Those results can be applied to neural networks with *unbounded time-varying delays*. Moreover, few results have been reported in the literature concerning the problem of μ-stability of impulsive neural networks with unbounded time-varying delays and continuously distributed delays. As we know, the impulse phenomenon as well as time delays are ubiquitous in the real world [25–27]. The systems with impulses and time delays can describe the real world well and truly. This inspire our interests.

In this paper, we investigate the problem of μ-stability for a class of impulsive neural networks with unbounded time-varying delays and continuously distributed delays. Based on Lyapunov-Krasovskii functional and some analysis techniques, several sufficient conditions that ensure the μ-stability of the addressed systems are derived in terms of LMIs, which can easily be checked by resorting to available software packages. The organization of this paper is as follows. The problems investigated in the paper are formulated, and some preliminaries are presented, in Section 2. In Section 3, we state and prove our main results. Then, a numerical example is given to demonstrate the effectiveness of the obtained results in Section 4. Finally, concluding remarks are made in Section 5.

2. Preliminaries

Notations

Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{Z}_+$ denote the set of positive integers, and $\mathbb{R}^n$ denote the n-dimensional real spaces equipped with the Euclidean norm $|\cdot|$. Let $\mathcal{A} \geq 0$ or $\mathcal{A} \leq 0$ denote that the matrix $\mathcal{A}$ is a symmetric and positive semidefinite or negative semidefinite matrix. The notations $\mathcal{A}^T$ and $\mathcal{A}^{-1}$ mean the transpose of $\mathcal{A}$ and the inverse of a square matrix. $\lambda_{\max}(\mathcal{A})$ or $\lambda_{\min}(\mathcal{A})$ denote the maximum eigenvalue or the minimum eigenvalue of matrix $\mathcal{A}$. I denotes the identity matrix with appropriate dimensions and $\Lambda = \{1, 2, \dots, n\}$. In addition, the notation $\star$ always denotes the symmetric block in one symmetric matrix.

Consider the following impulsive neural networks with time delays:

$$\begin{aligned}
\dot{x}(t) &= -Cx(t) + Af(x(t)) + Bf(x(t-\tau(t))) \\
&\quad + W\int_0^{\infty} h(s) f(x(t-s))ds + J, \quad t \neq t_k,\ t > 0, \\
\Delta x(t_k) &= x(t_k) - x\left(t_k^-\right) = J_k\left(x\left(t_k^-\right)\right), \quad k \in \mathbb{Z}_+,
\end{aligned} \tag{2.1}$$

where the impulse times t_k satisfy $0 = t_0 < t_1 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = +\infty$; $x(t) = (x_1(t), \dots, x_n(t))^T$ is the neuron state vector of the neural network; $C = \operatorname{diag}(c_1, \dots, c_n)$ is a diagonal matrix with $c_i > 0$, $i = 1, \dots, n$; A, B, W are the connection weight matrix, the delayed weight matrix, and the distributively delayed connection weight matrix, respectively; J is an input constant vector; $\tau(t)$ is the transmission delay of the neural networks; $f(x(\cdot)) = (f_1(x_1(\cdot)), \dots, f_n(x_n(\cdot)))^T$ represents the neuron activation function; $h(\cdot) = \operatorname{diag}(h_1(\cdot), \dots, h_n(\cdot))$ is the delay kernel function and J_k is the impulsive function.

Throughout this paper, the following assumptions are needed.

(H_1) The neuron activation functions $f_j(\cdot)$, $j \in \Lambda$, are bounded and satisfy

$$\delta_j^- \leq \frac{f_j(u) - f_j(v)}{u - v} \leq \delta_j^+, \quad j \in \Lambda, \tag{2.2}$$

for any $u, v \in \mathbb{R}$, $u \neq v$. Moreover, we define

$$\Sigma_1 = \operatorname{diag}\left(\delta_1^- \delta_1^+, \ldots, \delta_n^- \delta_n^+\right), \qquad \Sigma_2 = \operatorname{diag}\left(\frac{\delta_1^- + \delta_1^+}{2}, \ldots, \frac{\delta_n^- + \delta_n^+}{2}\right), \tag{2.3}$$

where $\delta_j^-, \delta_j^+, j \in \Lambda$ are some real constants and they may be positive, zero, or negative.

(H_2) The delay kernels $h_j, j \in \Lambda$, are some real value nonnegative continuous functions defined in $[0, \infty)$ and satisfy

$$\int_0^\infty h_j(s) ds = 1. \tag{2.4}$$

(H_3) $\tau(t)$ is a nonnegative and continuously differentiable time-varying delay and satisfies $\dot{\tau}(t) \leq \rho < 1$, where ρ is a positive constant.

If the function f_j satisfies the hypotheses (H_1) above, there exists an equilibrium point for system (2.1), see [28]. Assume that $x^* = (x_1^*, \ldots, x_n^*)^T$ is an equilibrium of system (2.1) and the impulsive function in system (2.1) characterized by $J_k(x(t_k^-)) = -D_k(x(t_k^-) - x^*)$, where D_k is a real matrix. Then, one can derive from (2.1) that the transformation $y = x - x^*$ transforms system (2.1) into the following system:

$$\begin{aligned} \dot{y}(t) &= -Cy(t) + Ag\left(y(t)\right) + Bg\left(y(t - \tau(t))\right) \\ &\quad + W \int_0^\infty h(s) g\left(y(t-s)\right) ds, \quad t \neq t_k, \ t > 0, \\ \Delta y(t_k) &= \mathrm{y}(t_k) - y\left(t_k^-\right) = -D_k y\left(t_k^-\right), \qquad k \in \mathbb{Z}_+, \end{aligned} \tag{2.5}$$

where $g(y(\cdot)) = f(y(\cdot) + x^*) - f(x^*)$.

Obviously, the μ-stability analysis of the equilibrium point x^* of system (2.1) can be transformed to the μ-stability analysis of the trivial solution $y = 0$ of system (2.5). For completeness, we first give the following definition and lemmas.

Definition 2.1 (see [23]). Suppose that $\mu(t)$ is a nonnegative continuous function and satisfies $\mu(t) \to \infty$ as $t \to \infty$. If there exists a scalar $M > 0$ such that

$$\|x\| \leq \frac{M}{\mu(t)}, \quad t \geq 0, \tag{2.6}$$

then the system (2.1) is said to be μ-stable.

Obviously, the definition of μ-stable includes the global asymptotical and the global exponential stability.

Lemma 2.2 (see [29]). *For a given matrix*

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} > 0, \tag{2.7}$$

where $S_{11}^T = S_{11},\ S_{22}^T = S_{22}$, *is equivalent to any one of the following conditions:*

(1) $S_{22} > 0,\ S_{11} - S_{12}S_{22}^{-1}S_{12}^T > 0$;

(2) $S_{11} > 0,\ S_{22} - S_{12}^T S_{11}^{-1} S_{12} > 0$.

3. Main Results

Theorem 3.1. *Assume that assumptions* (H_1), (H_2), *and* (H_3) *hold. Then, the zero solution of system* (2.5) *is* μ*-stable if there exist some constants* $\beta_1 \geq 0,\ \beta_2 > 0,\ \beta_3 > 0$, *two* $n\times n$ *matrices* $P > 0,\ Q > 0$, *two diagonal positive definite* $n \times n$ *matrices* $M = \operatorname{diag}(m_1, \ldots, m_n),\ U$, *a nonnegative continuous differential function* $\mu(t)$ *defined on* $[0, \infty)$, *and a constant* $T > 0$ *such that, for* $t \geq T$

$$\frac{\dot{\mu}(t)}{\mu(t)} \leq \beta_1, \qquad \frac{\mu(t-\tau(t))}{\mu(t)} \geq \beta_2, \qquad \frac{\int_0^\infty h_j(s)\mu(s+t)ds}{\mu(t)} \leq \beta_3, \quad j \in \Lambda, \tag{3.1}$$

and the following LMIs hold:

$$\begin{gathered} \begin{bmatrix} \Sigma & PA + U\Sigma_2 & PB & PW \\ \star & Q + N - U & 0 & 0 \\ \star & \star & -\beta_2 Q(1-\rho) & 0 \\ \star & \star & \star & -M \end{bmatrix} \leq 0, \\ \begin{bmatrix} P & (I - D_k)P \\ \star & P \end{bmatrix} \geq 0, \end{gathered} \tag{3.2}$$

where $\Sigma = \beta_1 P - PC - CP - U\Sigma_1,\ N = \operatorname{diag}(m_1\beta_3, \ldots, m_n\beta_3)$.

Proof. Consider the Lyapunov-Krasovskii functional:

$$\begin{aligned} V(t) &= \mu(t)y^T(t)Py(t) + \int_{t-\tau(t)}^{t} \mu(s)g^T(y(s))Qg(y(s))ds \\ &\quad + \sum_{j=1}^{n} m_j \int_0^\infty h_j(\sigma)\int_{t-\sigma}^{t} \mu(s+\sigma)g_j^2(y_j(s))ds\,d\sigma. \end{aligned} \tag{3.3}$$

The time derivative of V along the trajectories of system (2.5) can be derived as

$$
\begin{aligned}
D^{+}V &= \dot{\mu}(t)y^{T}(t)Py(t) + 2\mu(t)y^{T}(t)P\dot{y}(t) + \mu(t)g^{T}(y(t))Qg(y(t)) \\
&\quad - \mu(t-\tau(t))g^{T}(y(t-\tau(t)))Qg(y(t-\tau(t)))[1-\dot{\tau}(t)] \\
&\quad + \sum_{j=1}^{n} m_j g_j^2(y_j(t)) \int_0^{\infty} \mu(\sigma+t)h_j(\sigma)d\sigma \\
&\quad - \mu(t)\sum_{j=1}^{\infty} m_j \int_0^{\infty} h_j(\sigma)g_j^2(y_j(t-\sigma))d\sigma \le \dot{\mu}(t)y^{T}(t)Py(t) + 2\mu(t)y^{T}(t)P \\
&\quad \times \left[-Cy(t) + Ag(y(t)) + Bg(y(t-\tau(t))) \ + W\int_0^{\infty} h(s)g(y(t-s))ds \right] \\
&\quad + \mu(t)g^{T}(y(t))Qg(y(t)) \\
&\quad - \mu(t-\tau(t))g^{T}(y(t-\tau(t)))Qg(y(t-\tau(t)))[1-\rho] \\
&\quad + \mu(t)\sum_{j=1}^{n} m_j g_j^2(y_j(t)) \frac{\int_0^{\infty} \mu(\sigma+t)h_j(\sigma)d\sigma}{\mu(t)} \\
&\quad - \mu(t)\sum_{j=1}^{n} m_j \int_0^{\infty} h_j(\sigma)g_j^2(y_j(t-\sigma))d\sigma.
\end{aligned}
\tag{3.4}
$$

It follows from the assumption (3.1) that

$$
\sum_{j=1}^{n} m_j g_j^2(y_j(t)) \frac{\int_0^{\infty} \mu(\sigma+t)h_j(\sigma)d\sigma}{\mu(t)} \le \sum_{j=1}^{n} m_j \beta_3 g_j^2(y_j(t)) = g^{T}(y(t))Ng(y(t)). \tag{3.5}
$$

We use the assumption (H_2) and Cauchy's inequality $(\int p(s)q(s))^2 \le (\int p^2(s)ds)(\int q^2(s)ds)$ and get

$$
\begin{aligned}
\sum_{j=1}^{n} m_j \int_0^{\infty} h_j(\sigma)g_j^2(y_j(t-\sigma))d\sigma &= \sum_{j=1}^{n} m_j \int_0^{\infty} h_j(\sigma)d\sigma \int_0^{\infty} h_j(\sigma)g_j^2(y_j(t-\sigma))d\sigma \\
&\ge \sum_{j=1}^{n} m_j \left[\int_0^{\infty} h_j(\sigma)g_j(y_j(t-\sigma))d\sigma\right]^2 \\
&= \left[\int_0^{\infty} h(\sigma)g(y(t-\sigma))d\sigma\right]^{T} \\
&\quad \times M\left[\int_0^{\infty} h(\sigma)g(y(t-\sigma))d\sigma\right].
\end{aligned}
\tag{3.6}
$$

Note that, for any $n \times n$ diagonal matrix $U > 0$ it follows that

$$\mu(t)\begin{pmatrix} y(t) \\ g(y(t)) \end{pmatrix}^T \begin{pmatrix} -U\Sigma_1 & U\Sigma_2 \\ \star & -U \end{pmatrix}\begin{pmatrix} y(t) \\ g(y(t)) \end{pmatrix} \geq 0. \tag{3.7}$$

Substituting (3.5), (3.6) and (3.7), to (3.4), we get, for $t \geq T$,

$$\begin{aligned}
D^+V \leq\ & \mu(t)y^T(t)\left[\frac{\dot{\mu}(t)}{\mu(t)}P - PC - CP - U\Sigma_1\right]y(t) \\
& + 2\mu(t)y^T(t)[PA + U\Sigma_2]g(y(t)) + 2\mu(t)y^T(t)PBg(y(t-\tau(t))) \\
& + 2\mu(t)y^T(t)PW\int_0^\infty h(\sigma)g(y(t-\sigma))d\sigma \\
& - \mu(t-\tau(t))g^T(y(t-\tau(t)))Qg(y(t-\tau(t)))[1-\rho] \\
& + \mu(t)g^T(y(t))[N + Q - U]g(y(t)) \\
& - \mu(t)\left[\int_0^\infty h(\sigma)g(y(t-\sigma))d\sigma\right]^T M\left[\int_0^\infty h(\sigma)g(y(t-\sigma))d\sigma\right] \\
=\ & \mu(t)\cdot\begin{bmatrix} y(t) \\ g(y(t)) \\ g(y(t-\tau(t))) \\ \int_0^\infty h(s)g(y(t-s))ds \end{bmatrix}^T \Xi \begin{bmatrix} y(t) \\ g(y(t)) \\ g(y(t-\tau(t))) \\ \int_0^\infty h(s)g(y(t-s))ds \end{bmatrix},
\end{aligned} \tag{3.8}$$

where

$$\Xi = \begin{bmatrix} \Sigma & PA + U\Sigma_2 & PB & PW \\ \star & Q + N - U & 0 & 0 \\ \star & \star & -\beta_2 Q(1-\rho) & 0 \\ \star & \star & \star & -M \end{bmatrix}. \tag{3.9}$$

So, by assumption (3.2) and (3.8), we have

$$D^+V \leq 0 \quad \text{for } t \in [t_{k-1}, t_k) \cap [T, \infty),\ k \in \mathbb{Z}_+. \tag{3.10}$$

In addition, we note that

$$\begin{aligned}
&\begin{bmatrix} P & (I-D_k)P \\ \star & P \end{bmatrix} \geq 0 \\
&\Longleftrightarrow \begin{bmatrix} I & 0 \\ 0 & P^{-1} \end{bmatrix} \begin{bmatrix} P & (I-D_k)P \\ \star & P \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & P^{-1} \end{bmatrix} \geq 0 \\
&\Longleftrightarrow \begin{bmatrix} P & (I-D_k) \\ \star & P^{-1} \end{bmatrix} \geq 0,
\end{aligned} \tag{3.11}$$

which, together with assumption (3.2) and Lemma 2.2, implies that

$$P-(I-D_k)^T P(I-D_k) \geq 0. \tag{3.12}$$

Thus, it yields

$$\begin{aligned}
V(t_k) &= \mu(t_k)y^T(t_k)Py(t_k) + \int_{t_k-\tau(t_k)}^{t_k} \mu(s)g^T(y(s))Qg(y(s))ds \\
&\quad + \sum_{j=1}^{n} m_j \int_0^{\infty} h_j(\sigma) \int_{t_k-\sigma}^{t_k} \mu(s+\sigma)g_j^2(y_j(s))ds\,d\sigma \\
&= \mu(t_k^-)y^T(t_k^-)(I-D_k)^T P(I-D_k)y(t_k^-) \\
&\quad + \int_{t_k^- -\tau(t_k^-)}^{t_k^-} \mu(s)g^T(y(s))Qg(y(s))ds \\
&\quad + \sum_{j=1}^{n} m_j \int_0^{\infty} h_j(\sigma) \int_{t_k^- -\sigma}^{t_k^-} \mu(s+\sigma)g_j^2(y_j(s))ds\,d\sigma \\
&\leq \mu(t_k^-)y^T(t_k^-)Py(t_k^-) + \int_{t_k^- -\tau(t_k^-)}^{t_k^-} \mu(s)g^T(y(s))Qg(y(s))ds \\
&\quad + \sum_{j=1}^{n} m_j \int_0^{\infty} h_j(\sigma) \int_{t_k^- -\sigma}^{t_k^-} \mu(s+\sigma)g_j^2(y_j(s))ds\,d\sigma \\
&\leq V(t_k^-).
\end{aligned} \tag{3.13}$$

Hence, we can deduce that

$$V(t_k) \leq V(t_k^-), \quad k \in \mathbb{Z}_+. \tag{3.14}$$

By (3.10) and (3.14), we know that V is monotonically nonincreasing for $t \in [T, \infty)$, which implies that

$$V(t) \leq V(T), \quad t \geq T. \tag{3.15}$$

It follows from the definition of V that

$$\mu(t)\lambda_{\min}(P)\|y(t)\|^2 \leq \mu(t)y^T(t)Py(t) \leq V(t) \leq V_0 < \infty, \quad t \geq 0, \tag{3.16}$$

where $V_0 = \max_{0\leq s\leq T} V(s)$.

It implies that

$$\|y(t)\|^2 \leq \frac{V_0}{\mu(t)\lambda_{\min}(P)}, \quad t \geq 0. \tag{3.17}$$

This completes the proof of Theorem 3.1. □

Remark 3.2. Theorem 3.1 provides a μ-stability criterion for an impulsive differential system (2.5). It should be noted that the conditions in the theorem are dependent on the upper bound of the derivative of time-varying delay and the delay kernels $h_j, j \in \Lambda$, and independent of the range of time-varying delay. Thus, it can be applied to impulsive neural networks with unbounded time-varying and continuously distributed delays.

Remark 3.3. In [23, 24], the authors have studied μ-stability for neural networks with unbounded time-varying delays and continuously distributed delays via different approaches. However, the impulsive effect is not taken into account. Hence, our developed result in this paper complements and improves those reported in [23, 24]. In particular, if we take $D_k = \operatorname{diag}(d_1^{(k)}, \ldots, d_n^{(k)})$, $d_i^{(k)} \in [0,2], i \in \Lambda$, $k \in \mathbb{Z}_+$, then the following result can be obtained.

Corollary 3.4. *Assume that assumptions (H_1), (H_2) and (H_3) hold. Then, the zero solution of system* (2.5) *is μ-stable if there exist some constants $\beta_1 \geq 0$, $\beta_2 > 0$, $\beta_3 > 0$, $j \in \Lambda$, two $n \times n$ matrices $P > 0$, $Q > 0$, two diagonal positive definite $n \times n$ matrices $M = \operatorname{diag}(m_1, \ldots, m_n)$, U,* *a nonnegative continuous differential function $\mu(t)$ defined on $[0,\infty)$, and a constant $T > 0$ such that, for $t \geq T$*

$$\frac{\dot{\mu}(t)}{\mu(t)} \leq \beta_1, \quad \frac{\mu(t-\tau(t))}{\mu(t)} \geq \beta_2, \quad \frac{\int_0^{\infty} h_j(s)\mu(s+t)ds}{\mu(t)} \leq \beta_3, \quad j \in \Lambda, \tag{3.18}$$

and the following LMIs hold:

$$\begin{bmatrix} \Sigma & PA+U\Sigma_2 & PB & PW \\ \star & Q+N-U & 0 & 0 \\ \star & \star & -\beta_2 Q(1-\rho) & 0 \\ \star & \star & \star & -M \end{bmatrix} \leq 0, \tag{3.19}$$

where $\Sigma = \beta_1 P - PC - CP - U\Sigma_1$, $N = \mathrm{diag}(m_1\beta_3,\ldots,m_n\beta_3)$.

If we take $\mu(t) = \mu$ (μ denotes a constant), then the following global bounded result can be obtained.

Corollary 3.5. *Assume that assumptions (H_1), (H_2), and (H_3) hold. Then, the all solutions of system* (2.5) *have global boundedness if there exist two $n \times n$ matrices $P > 0$, $Q > 0$, two diagonal positive definite $n \times n$ matrices $M = \mathrm{diag}(m_1,\ldots,m_n)$,$U$, such that, the following LMIs hold:*

$$\begin{gathered} \begin{bmatrix} \Sigma & PA+U\Sigma_2 & PB & PW \\ \star & Q+M-U & 0 & 0 \\ \star & \star & -Q(1-\rho) & 0 \\ \star & \star & \star & -M \end{bmatrix} \leq 0, \\ \begin{bmatrix} P & (I-D_k)P \\ \star & P \end{bmatrix} \geq 0, \end{gathered} \tag{3.20}$$

where $\Sigma = -PC - CP - U$.

Remark 3.6. Notice that $\beta_1 = 0$, $\beta_2 = 1$, $\beta_3 = 1$, $j \in \Lambda$, and using the similar proof of Theorem 3.1, we can obtain the result easily.

4. A Numerical Example

In the following, we give an example to illustrate the validity of our method.

Example 4.1. Consider a two-dimensional impulsive neural network with unbounded time-varying delays and continuously distributed delays:

$$\begin{aligned}
\begin{pmatrix}\dot{y}_1(t)\\ \dot{y}_2(t)\end{pmatrix} &= -\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}\begin{pmatrix}y_1(t)\\ y_2(t)\end{pmatrix} + \begin{pmatrix}0.1 & 0.1\\ 0.1 & 0.1\end{pmatrix}\begin{pmatrix}\tanh(y_1(t))\\ \tanh(y_2(t))\end{pmatrix}\\
&\quad + \begin{pmatrix}0.1 & 0.1\\ 0.5 & -0.1\end{pmatrix}\begin{pmatrix}\tanh(y_1(t-0.5t))\\ \tanh(y_2(t-0.5t))\end{pmatrix}\\
&\quad + \begin{pmatrix}0.5 & 0.5\\ 0.5 & -0.5\end{pmatrix}\begin{pmatrix}\displaystyle\int_0^{\infty} e^{-s}\tanh(y_1(t-s))\,ds\\ \displaystyle\int_0^{\infty} e^{-s}\tanh(y_2(t-s))\,ds\end{pmatrix}, \quad t \neq t_k,\ t > 0,\\
\begin{pmatrix}\Delta y_1(t_k)\\ \Delta y_2(t_k)\end{pmatrix} &= -\begin{pmatrix}1.5 & 0\\ 0 & 1.5\end{pmatrix}\begin{pmatrix}y_1(t_k^-)\\ y_2(t_k^-)\end{pmatrix}, \quad t_k = k,\ k \in \mathbb{Z}_+.
\end{aligned} \tag{4.1}$$

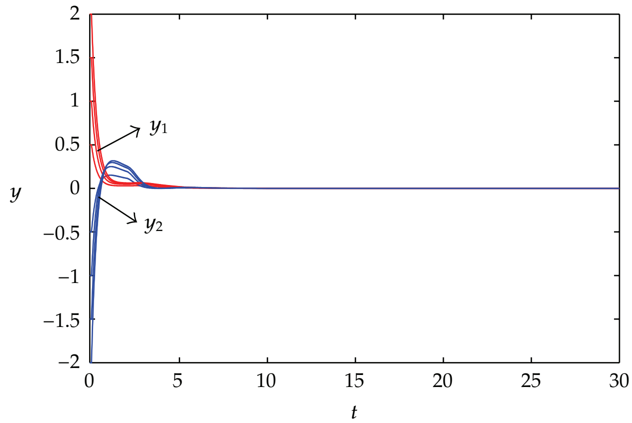
Then, $\tau(t) = 0.5t$, $h_j(s) = e^{-s}$, $\Sigma_1 = \mathrm{diag}(0,0)$, $\Sigma_2 = \mathrm{diag}(0.5, 0.5)$, and $\rho = 0.5$. It is obvious that $(0,0)^T$ is an equilibrium point of system (4.1). Let $\mu(t) = t$ and choose $\beta_1 = 0.1$, $\beta_2 = 0.5$, $\beta_3 = 1.2$, then the LMIs in Theorem 3.1 have the following feasible solution via MATLAB LMI toolbox:

$$\begin{gathered}
P = \begin{pmatrix}4.4469 & -0.0230\\ -0.0230 & 4.3377\end{pmatrix}, \qquad Q = \begin{pmatrix}5.6557 & -0.2109\\ -0.2109 & 5.5839\end{pmatrix},\\
M = \begin{pmatrix}5.5189 & 0\\ 0 & 5.5189\end{pmatrix}, \qquad U = \begin{pmatrix}20.5095 & 0\\ 0 & 20.5095\end{pmatrix}.
\end{gathered} \tag{4.2}$$

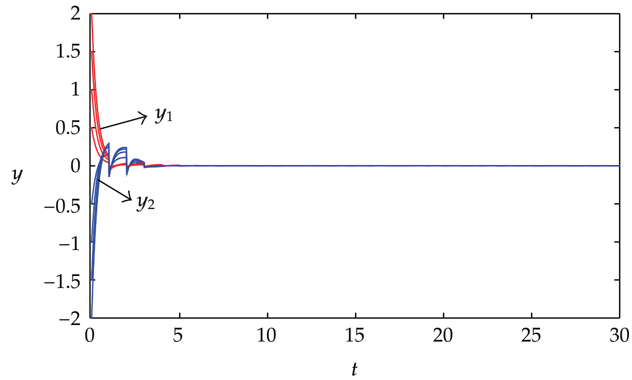
The above results shows that all the conditions stated in Theorem 3.1 have been satisfied and hence system (4.1) with unbounded time-varying delay and continuously distributed delay is μ-stable. The numerical simulations are shown in Figure 1.

5. Conclusion

In this paper, some sufficient conditions for μ-stability of impulsive neural networks with unbounded time-varying delays and continuously distributed delays are derived. The results are described in terms of LMIs, which can be easily checked by resorting to available software packages. A numerical example has been given to demonstrate the effectiveness of the results obtained.



(a)



(b)

Figure 1: (a) State trajectories of system (4.1) without impulsive effects. (b) State trajectories of system (4.1) under impulsive effects.

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