Hindawi Publishing Corporation
Advances in Operations Research
Volume 2011, Article ID 521351, 17 pages
doi:10.1155/2011/521351

Research Article

Geometric Programming Approach to an Interactive Fuzzy Inventory Problem

Nirmal Kumar Mandal

Department of Mathematics, Silda Chandrasekhar College, Silda, West Bengal, Paschim Medinipur 721515, India

Correspondence should be addressed to Nirmal Kumar Mandal, mandal_n_k@yahoo.co.in

Received 10 April 2011; Accepted 20 July 2011

Academic Editor: Lars Mönch

An interactive multiobjective fuzzy inventory problem with two resource constraints is presented in this paper. The cost parameters and index parameters, the storage space, the budgetary cost, and the objective and constraint goals are imprecise in nature. These parameters and objective goals are quantified by linear/nonlinear membership functions. A compromise solution is obtained by geometric programming method. If the decision maker is not satisfied with this result, he/she may try to update the current solution to his/her satisfactory solution. In this way we implement man-machine interactive procedure to solve the problem through geometric programming method.

1. Introduction

In formulating an inventory problem, various parameters involve in the objective functions and constraints which are assigned by the decision maker (DM) from past experiences. But in real world situation, it is observed that the possible values of the parameters are often imprecise and ambiguous to the DM. In different situations, different circumstances, it takes different values. So, it is difficult to assign the precise values of the parameters. With this observation, it would be certainly more appropriate to interpret the DM's understanding of the parameters as fuzzy numerical data which can be represented by fuzzy numbers. In the conventional approaches the objective goals are taken as deterministic. The objective goals, however, may not be exactly known. The target may vary to some extent, which is then represented by the tolerance value. Due to inexactness of the objective goals, the objective functions may be characterized by different types of membership functions.

In a multiobjective nonlinear programming (MONLP) problem DM plays an important role to achieve his/her optimum goal. He/she has every right to choose or rechoose the suitable set of membership functions for different objective functions. He/she decides the types of fuzzy parameters and also free to assign the reference values of each objective functions/membership functions for the desired optimal solutions. In this way an interactive procedure can be established with the DM. Sakawa and Yano [1, 2] proposed a new technique to solve such type of problems.

Zadeh [3] first introduced the concept of fuzzy set theory. Later on, Bellman and Zadeh [4] used the fuzzy set theory to the decision-making problem. Fuzzy set theory now has made an entry into the inventory control systems. Park [5] examined the EOQ formula in the fuzzy set theoretic perspective associating the fuzziness with the cost data. Roy and Maiti [6] solved a single objective fuzzy EOQ model using GP technique. Ishibuchi and Tanaka [7] developed a concept for optimization of multiobjective-programming problem with interval objective function.

Such type of nonlinear programming (NLP) problem can be solved by geometric programming (GP) problem. It has a very popular and effective use to solve many real-life decision-making problem. Duffin et al. [8] first developed the idea on GP method. Kotchenberger [9] was the first to use it in an inventory problem. Later on, Worral and Hall [10] analysed a multi-item inventory problem with several constraints. Abou-El-Ata et al. [11] and Jung and Klein [12] developed single item inventory problems and solved by GP method. Mandal et al. [13] used GP technique in multi-item inventory problem and compared with the nonlinear programming (NLP) problem. Recently, inventory problems in fuzzy environment were formulated and solved by GP technique by Liu [14] and Sadjadi et al. [15].

In this paper, we formulate a multiobjective inventory problem. The manufacturer wants to minimize the total average cost which includes the production cost, set up cost and holding cost. He/she also wants to minimize the number of orders to supply the finished goods to different shops/wholesalers. The problem is formulated under total budgetary cost and total storage space capacity restrictions. The parameters involve in the model are taken as different types of fuzzy numbers. The fuzzy numbers are described by linear/nonlinear types of membership functions which will be selected by the DM. Interactive min-max procedure is followed up by two different ways. First, we solve a multiobjective interactive programming with crisp goal by GP method. Next, we proceed with fuzzy goal through GP method. The DM may update the solution until his/her satisfaction.

2. Definitions and Basic Concepts

2.1. Fuzzy Number and Its Membership Function

A fuzzy number $\widetilde{A}$ is a fuzzy set of the real line $\widetilde{A}$ whose membership function $\mu_{\widetilde{A}}(x)$ has the following characteristics with $\infty < a_1 \leq a_2 \leq a_3 < \infty$:

$$\mu_{\widetilde{A}}(x) = \begin{cases} \mu^L_{\widetilde{A}}(x), & \text{for } a_1 \leq x \leq a_2, \\ 1, & \text{for } x = a_2, \\ \mu^R_{\widetilde{A}}(x), & \text{for } a_2 \leq x \leq a_3, \\ 0, & \text{for otherwise}, \end{cases} \tag{2.1}$$

where $\mu^L_{\widetilde{A}}(x) : [a_1, a_2] \to [0, 1]$ is continuous and strictly increasing; $\mu^R_{\widetilde{A}}(x) : [a_2, a_3] \to [0, 1]$ is continuous and strictly decreasing.

The general shape of a fuzzy number following the above definition is known as triangular-shaped fuzzy number (TiFN) (Buckley and Eslami [16]).

2.2. α-Level Set

The α-level of a fuzzy number $\widetilde{A}$ is defined as a crisp set $A_\alpha = [x : \mu_{\widetilde{A}}(x) \geq \alpha, \ x \in X]$, where $\alpha \in [0,1]$. A_α is a nonempty bounded closed interval contained in X and it can be denoted by $A_\alpha = [A_\alpha^L, A_\alpha^R]$. A_α^L and A_α^R are the lower and upper bounds of the closed interval, respectively.

2.3. Multiobjective Nonlinear Programming (MONLP)

The MONLP problem is represented as the following vector minimization problem (Sakawa [17]):

$$\text{Min} \quad f(x;C) = \left(f_1(x;C_1), f_2(x;C_2), \ldots, f_m(x;C_m)\right)^T, \tag{2.2}$$

subject to $x \in X(A,B) = \{x \in R^n \mid g_j(x;A_j) \leq B_j, j = 1,2,\ldots,k\}$, where $m(\geq 2)$ objective functions $f_r : R^n \to R$ and $x \in R^n$.

Note. When $m = 1$ problem (2.2) reduces to a single objective nonlinear programming problem.

In general, the parameters in objectives and constraints are considered as crisp numbers. But there is some ambiguity to express the parameters precisely. So, it will be more realistic, if the parameters are considered as fuzzy numbers. The multiobjective nonlinear programming with fuzzy parameters (MONLP-FP) is described as

$$\text{Min} \quad f\left(x;\widetilde{C}\right) = \left(f_1\left(x;\widetilde{C}_1\right), f_2\left(x;\widetilde{C}_2\right), \ldots, f_m\left(x;\widetilde{C}_m\right)\right)^T, \tag{2.3}$$

subject to $x \in X(\widetilde{A},\widetilde{B}) = \{x \in R^n \mid g_j(x,\widetilde{A}_j) \leq \widetilde{B}_j, j = 1,2,\ldots,k\}$, where $\widetilde{C}_r = (\widetilde{C}_{r1}, \widetilde{C}_{r2}, \ldots, \widetilde{C}_{rp_r})$, $\widetilde{A}_j = (\widetilde{A}_{j1}, \widetilde{A}_{j2}, \ldots, \widetilde{A}_{jq_j})$ and $\widetilde{B}_j$ represent, respectively, fuzzy parameters involved in the objective functions $f_r(x,\widetilde{C}_r)$ $(r = 1,2,\ldots,m)$ and the constraint functions $g_j(x,\widetilde{A}_j)$ $(j = 1,2,\ldots,k)$. These fuzzy parameters, which reflect the expert's ambiguous understanding of the nature of the parameters in the problem formulation process are assumed to be characterized as fuzzy numbers.

We now assume that $\widetilde{C}_{rt}$, $\widetilde{A}_{js}$, and $\widetilde{B}_j$ in the MONLP-FP (2.3) are fuzzy numbers whose membership functions are $\mu_{\widetilde{C}_{rt}}(c_{rt})$, $\mu_{\widetilde{A}_{js}}(a_{js})$, and $\mu_{\widetilde{B}_j}(b_j)$ $(r = 1,2,\ldots,m;\ t = 1,2,\ldots,p_r;\ j = 1,2,\ldots,k;\ s = 1,2,\ldots,q_j)$, respectively.

Table 1: α-level interval of coefficient parameters.

Br	$\mu^L_{\widetilde{M}}(x)$	Br	$\mu^R_{\widetilde{M}}(x)$	$(\widetilde{M}_\alpha) = [\widetilde{M}^L_\alpha,\ \widetilde{M}^R_\alpha]$
L:	$1-(M_2-x)/(M_2-M_1)$	L:	$1-(x-M_2)/(M_3-M_2)$	$[M_1+\alpha(M_2-M_1), M_3-\alpha(M_3-M_2)]$
L:	$1-(M_2-x)/(M_2-M_1)$	P:	$1-((x-M_2)/(M_3-M_2))^2$	$[M_1+\alpha(M_2-M_1),\ M_2+\sqrt{1-\alpha}(M_3-M_2)]$
L:	$1-(M_2-x)/(M_2-M_1)$	E:	$\alpha_2(1-e^{-\beta_2((M_3-x)/(M_3-M_2))})$	$[M_1+\alpha(M_2-M_1),\ M_3+((M_3-M_2)/\beta_2)\log(1-\alpha/\alpha_2)]$
P:	$1-((M_2-x)/(M_2-M_1))^2$	L:	$1-(x-M_2)/(M_3-M_2)$	$[M_2-\sqrt{1-\alpha}(M_2-M_1), M_3-\alpha(M_3-M_2)]$
P:	$1-((M_2-x)/(M_2-M_1))^2$	P:	$1-((x-M_2)/(M_3-M_2))^2$	$[M_2-\sqrt{1-\alpha}(M_2-M_1),\ M_2+\sqrt{1-\alpha}(M_3-M_2)]$
P:	$1-((M_2-x)/(M_2-M_1))^2$	E:	$\alpha_2(1-e^{-\beta_2((M_3-x)/(M_3-M_2))})$	$[M_2-\sqrt{1-\alpha}(M_2-M_1), M_3+((M_3-M_2)/\beta_2)\log(1-\alpha/\alpha_2)]$
E:	$\alpha_1(1-e^{-\beta_1((x-M_1)/((M_2-M_1))})$	L:	$1-(x-M_2)/(M_3-M_2)$	$[M_1-((M_2-M_1)/\beta_1)\log(1-\alpha/\alpha_1), M_3-\alpha(M_3-M_2)]$
E:	$\alpha_1(1-e^{-\beta_1((x-M_1)/(M_2-M_1))})$	P:	$1-((x-M_2)/(M_3-M_2))^2$	$[M_1-((M_2-M_1)/\beta_1)\log(1-\alpha/\alpha_1), M_2+\sqrt{1-\alpha}(M_3-M_2)]$
E:	$\alpha_1(1-e^{-\beta_1((x-M_1)/(M_2-M_1))})$	E:	$\alpha_2(1-e^{-\beta_2((M_3-x)/(M_3-M_2))})$	$[M_1-((M_2-M_1)/\beta_1)\log(1-\alpha/\alpha_1), M_3+((M_3-M_2)/\beta_2)\log(1-\alpha/\alpha_2)]$

L, P, and E stand for linear, parabolic, and exponential membership functions, respectively, $\alpha_1,\ \alpha_2 > 1;\ \beta_1,\ \beta_2 > 0;\ \ 0 < \alpha < 1$.

The α-level set of the fuzzy numbers $\widetilde{C}_{rt},\ (r = 1,2,\ldots,m;\ t = 1,2,\ldots,p_r)$ and $\widetilde{A}_{js},\ (j = 1,2,\ldots,k;\ s = 1,2,\ldots,q_j)$ are defined as the ordinary set $(\widetilde{A},\widetilde{B},\widetilde{C})_\alpha$ for which the degree of their membership functions exceeds the level α:

$$\begin{aligned}\left(\widetilde{A},\widetilde{B},\widetilde{C}\right)_\alpha = \Big\{(a,b,c) \mid \mu_{\widetilde{A}_{rt}}(a_{rt}) \geq \alpha,\ r = 1,2,\ldots,m;\ t = 1,2,\ldots,p_r;\\ \mu_{\widetilde{B}_j}(b_j) \geq \alpha,\quad j = 1,2,\ldots,k;\\ \mu_{\widetilde{C}_{js}}(c_{js}) \geq \alpha,\ j = 1,2,\ldots,k,\ s = 1,2,\ldots,q_j\Big\}.\end{aligned} \tag{2.4}$$

The α-level sets have the following property:

$$\alpha_1 \leq \alpha_2 \Leftrightarrow \left(\widetilde{A},\widetilde{B},\widetilde{C}\right)_{\alpha_1} \supseteq \left(\widetilde{A},\widetilde{B},\widetilde{C}\right)_{\alpha_2}. \tag{2.5}$$

$(\widetilde{A},\widetilde{B},\widetilde{C})_\alpha$ are the nonempty bounded closed intervals contained in X and it can be defined as $(\widetilde{A},\widetilde{B},\widetilde{C})_\alpha = [(\widetilde{A},\widetilde{B},\widetilde{C})^L_\alpha,\ (\widetilde{A},\widetilde{B},\widetilde{C})^R_\alpha]$, where $(\widetilde{A},\widetilde{B},\widetilde{C})^L_\alpha$ and $(\widetilde{A},\widetilde{B},\widetilde{C})^R_\alpha$ are the lower and upper bounds of $(\widetilde{A},\widetilde{B},\widetilde{C})_\alpha$ and can be obtained from the left branch and right branch of the membership functions $\mu_{\widetilde{A}_{js}}(a_{js}),\ \mu_{\widetilde{B}_j}(b_j)$, and $\mu_{\widetilde{C}_{rt}}(c_{rt})$.

$\widetilde{A}_{js},\ \widetilde{C}_{rt}$ are TiFNs with different types of left and right branch of the membership functions. They may be of linear, parabolic, exponential, and so forth, type membership functions (Table 1).

The constraint goals (B_j), $j = 1, 2, \ldots, k$ may be more realistic if it can be taken a TiFN with only right membership functions called right TiFN such as $\widetilde{B}_j^{r_1} = (B_{j1}, B_{j1}, B_{j2})$. The corresponding membership function is

$$\mu_{\widetilde{B_j}}(x) = \begin{cases} 1, & \text{for } x \le B_{j1}, \\ \mu_{B_j}^R(x), & \text{for } B_{j1} \le x \le B_{j2}, \\ 0, & \text{for } x \ge B_{j2}. \end{cases} \tag{2.6}$$

The right branch $\mu_{B_j}^R(x)$ is monotone decreasing continuous function in $x \in [B_{j1}, B_{j2}]$ which may be linear, parabolic, or exponential type membership functions. The corresponding α-level interval is

L: $[B_{j1},\ B_{j2} - \alpha(B_{j2} - B_{j1})]$,

P: $[B_{j1},\ B_{j1} + \sqrt{1-\alpha}(B_{j2} - B_{j1})]$,

E: $[B_{j1},\ B_{j2} + ((B_{j2} - B_{j1})/\beta_2)\log(1 - (\alpha/\alpha_2))]$.

Now suppose that the DM decides that the degree of all of the membership functions of the fuzzy numbers involved in the MONLP-FP should be greater than or equal to some value of α. Then for such a degree α, the α-MONLP-FP can be interpreted as the following crisp multiobjective linear programming problem which depends on the coefficient vector $(a, b, c) \in (A, B, C)_\alpha$:

$$\begin{aligned} \text{Min} \quad & \left(f_1(x, c_1), f_2(x, c_2), \ldots, f_m(x, c_m)\right)^T, \\ \text{subject to} \quad & x \in X(a, b) = \left\{x \in R^n \mid g_j\left(x, a_j\right) \le b_j,\ j = 1, 2, \ldots, k;\ x \ge 0\right\}. \end{aligned} \tag{2.7}$$

Observe that there exists an infinite number of such problem (2.7) depending on the coefficient vector $(a, b, c) \in (\widetilde{A}, \widetilde{B}, \widetilde{C})_\alpha$, and the values of (a, b, c) are arbitrary for any $(a, b, c) \in (\widetilde{A}, \widetilde{B}, \widetilde{C})_\alpha$ in the sense that the degree of all of the membership functions in the problem (2.7) exceeds the level $\alpha \in [0, 1]$. However, if possible, it would be desirable for the DM to choose $(a, b, c) \in (A, B, C)_\alpha$ in the problem (2.7) to minimize the objective functions under the constraints. From such a point of view, for a certain degree α, it seems to be quite natural to have the α-MONLP-FP as the following MONLP problem (2.7):

$$\begin{aligned} \text{Min} \quad & \left(f_1(x, c_1), f_2(x, c_2), \ldots, f_m(x, c_m)\right)^T, \\ \text{subject to} \quad & x \in X(a, b) = \left\{x \in R^n \mid g_j\left(x, a_j\right) \le b_j,\ j = 1, 2, \ldots, k; x \ge 0\right\}, \\ & (a, b, c) \in \left(\widetilde{A}, \widetilde{B}, \widetilde{C}\right)_\alpha. \end{aligned} \tag{2.8}$$

On the basis of the α-level sets of the fuzzy numbers, we can introduce the concept of an Pareto optimal solution to the α-MONLP.

3. Interactive Min-Max Method

3.1. Interactive Nonlinear Programming with Fuzzy Parameter

To obtain the optimal solution, the DM is asked to specify the degree α of the α-level set and the reference levels of achievement of the objective functions. For the DM's degree α and reference levels $\overline{f}_r,\ r = 1,2,\ldots,m$ the corresponding optimal solution, which is, in the min-max sense, nearest to the requirement (or better than that of the reference levels) are attainable, is obtained by solving the following min-max problem:

$$\begin{aligned} \text{Min} \quad & \max_{r=1,2,\ldots,m} \left(f_r(x, c_r) - \overline{f}_r \right), \\ \text{subject to} \quad & (a,b,c) \in (A,B,C)_\alpha,\ x \in X(a,b), \end{aligned} \tag{3.1}$$

or equivalently

$$\begin{aligned} \text{Min} \quad & v, \\ \text{subject to} \quad & f_r(x, c_r) - \overline{f}_r \le v, \quad r = 1,2,\ldots,m, \\ & (a,b,c) \in (A,B,C)_\alpha,\ x \in X(a,b). \end{aligned} \tag{3.2}$$

3.2. Interactive Fuzzy Nonlinear Programming with Fuzzy Goals

Considering the imprecise nature of the DM's judgements, it is quite natural to assume that the DM may have imprecise (or fuzzy) goals for each of the objective functions in the α-MONLP. In a minimization problem, a fuzzy goal stated by the DM may have to achieve "substantially less than or equal to some value specified." This type of statement can be quantified by eliciting a corresponding membership function.

In order to elicit a membership function $\mu_r(f_r(x, c_r))$ from the DM for each of the objective functions $f_r(x, c_r)$ in the α-MONLP, we first calculate the individual minimum $f_r^{\min}$ and maximum $f_r^{\max}$ of each objective function $f_r(x, c_r)$ under the given constraints for $\alpha = 0$ and $\alpha = 1$. By taking into account the calculated individual minimum and maximum of each objective function for $\alpha = 0$ and $\alpha = 1$ together with the rate of increase of membership satisfaction, the DM may be able to determine a subjective membership function $\mu_r(f_r(x, c_r))$ which is a strictly monotone decreasing function with respect to $f_r(x, c_r)$. For example, two nonlinear membership functions are shown below.

3.2.1. Parabolic Membership Function (Type 1)

For each of the objective functions, the corresponding quadratic membership functions are

$$\mu_{f_r}(f_r(x,c_r)) = \begin{cases} 1 & \text{for } f_r(x,c_r) < f_r^1, \\ m_r(x,c_r) = 1 - \left(\dfrac{f_r(x,c_r) - f_r^1}{p_r}\right)^2, & \text{for } f_r^1 \le f_r(x,c_r) \le f_r^0, \\ 0 & \text{for } f_r(x,c_r) > f_r^0, \end{cases} \quad \text{for } r = 1,2,\ldots,m, \tag{3.3}$$

where f_r^1 and f_r^0 are to be chosen such that $f_r^{\min} \le f_r^1 \le f_r^0 \le f_r^{\max}$ and $p_r(= f_r^0 - f_r^1)$ is the tolerance of the r-th objective function $f_r(x,c_r)$.

3.2.2. Exponential Membership Function (Type 2)

For each objective function, the corresponding exponential membership function is as follows:

$$\mu_{f_r}(f_r(x,c_r)) = \begin{cases} 1 & \text{for } f_r(x,c_r) < f_r^1, \\ m_r(x,c_r) = \alpha_r\left[1 - e^{-\beta_r((f_r(x,c_r)-f_r^1)/p_r)}\right], & \text{for } f_r^1 \le f_r(x,c_r) \le f_r^0, \\ 0 & \text{for } f_r(x,c_r) > f_r^0, \end{cases} \quad \text{for } r = 1,2,\ldots,m, \tag{3.4}$$

The constants $\alpha_r > 1$, $\beta_r > 0$ can be determined by asking the DM to specify the three points f_r^1, $f_r^{0.5}$, and f_r^0 such that $f_r^{\min} \le f_r^1 \le f_r^0 \le f_r^{\max}$ and $p_r(= f_r^0 - f_r^1)$ are the tolerance of the r-th objective function $f_r(x,c_r)$.

In a minimization problem, DM has a target goal $f_r^1(x,c_r)$ with a flexibility p_r. Having determined the membership functions for each of the objective functions, to generate a candidate for the satisficing solution which is also Pareto optimal, the DM is asked to specify the degree α of the α-level set and the reference levels of achievement of the membership functions called the reference membership values. Observe that the idea of the reference membership values (e.g., Sakawa and Yano [1, 2]) can be viewed as an obvious extension of the idea of the reference point. For the DM's degree α and the reference membership values $\overline{\mu}_r$, $(r = 1,2,\ldots,m)$ the following min-max problem is solved to generate the Pareto optimal solution, which is, in the min-max sense, nearest to the requirement or better than that if the reference membership values are attainable

$$\begin{aligned} &\text{Min} \quad \max_{r=1,2,\ldots,m} \left(\overline{\mu}_r - \mu_r(f_r(x,c_r))\right), \\ &\text{subject to} \quad (a,b,c) \in (A,B,C)_\alpha, \ x \in \mathrm{X}(a,b), \end{aligned} \tag{3.5}$$

which is equivalent to

$$\begin{aligned} \text{Min} \quad & \nu, \\ \text{subject to} \quad & \overline{\mu}_r - \mu_r(f_r(x, c_r)) \leq \nu, \quad r = 1, 2, \dots, m, \\ & (a, b, c) \in (A, B, C)_\alpha, \ x \in X(a, b). \end{aligned} \tag{3.6}$$

The DM will select the membership functions for the corresponding objective functions from Type 1 and Type 2 membership functions. Then the above primal function can be solved by GP method as it has been expressed in signomial form and obtain optimal solution of ν says ν^*:

Now the DM selects his most important objective function from among the objective functions $f_j(x_j)$, $(j = 1, 2, \dots, m)$. If it is the j-th objective, the following posynomial programming problem is solved by GP for $\nu = \nu^*$:

$$\begin{aligned} \text{Min} \quad & f_j(x, c_j), \\ \text{subject to} \quad & f_r(x, c_r) \leq m_r^{-1}(\overline{\mu}_r - \nu), \quad r, j = 1, 2, \dots, m \ (r \neq j), \\ & (a, b, c) \in (A, B, C)_\alpha, \ x \in X(a, b), \end{aligned} \tag{3.7}$$

The problem is now solved by GP method and optimal solution is then examined by following Pareto optimality test by Wendell and Lee [18].

Pareto Optimality Test

Let x^* be the optimal decision vector which is obtained from (3.7),
solve the problem

$$\begin{aligned} \text{Min} \quad & V = \sum_{r=1}^{m} pt f_r(x, c_r), \\ \text{subject to} \quad & f_r(x, c_r) \leq f_r^*(x^*, c_r), \quad \forall r = 1, 2, \dots, m, \\ & (a, b, c) \in (A, B, C)_\alpha, \ x \in X(a, b). \end{aligned} \tag{3.8}$$

4. Interactive Min-Max Method in Inventory Problem

The following notations and assumptions are used in developing a multiobjective multi-item inventory model.

Notations

For the $i(= 1, 2, \dots, n)$th item,

D_i = demand per unit item,

Q_i = order quantity (decision variable), $(Q \equiv (Q_1, Q_2, \dots, Q_n)^T)$,

C_{0i} = unit cost of production (decision variable), $(C_0 \equiv (C_{01}, C_{02}, \ldots, C_{0n})^T)$,

c_{1i} = inventory carrying cost per item,

c_{3i} = set up cost per cycle,

w_i = storage area per each item,

C = total budgetary cost,

W = total available storage area,

$\text{TC}(C_0, Q)$ = total average inventory cost function,

$\text{NO}(C_0, Q)$ = total number of order function,

$\text{SS}(Q)$ = storage space function,

$\text{BC}(C_0, Q)$ = budgetary cost function.

Assumptions. (1) Production is instantaneous with zero lead-time,

(2) when the demand of an item increases then the total purchasing cost spread all over the items and hence the demand of an item is inversely proportional to unit cost of production, that is, $D_i = a_i C_{0i}^{-b_i}$ since the purchasing cost and the demand of an item are non-negative. We also require that the scaling constant $a_i > 0$, and index parameter $b_i < 1$ as C_{0i} and D_i are inversely related to each other.

4.1. Problem Formulation

A wholesaling organisation purchase and stocks some commodities in his/her godown. He/she then supplies that commodities to some retailers. In such environment, the wholesaler always tries to minimize the total average cost which includes the purchasing cost, set-up and cost, and holding cost. His/her aim is also to minimize the total numbers of order supply to the retailer.

For the $i(= 1, 2, \ldots, n)$th item, the inventory costs over the time cycle $T_i = Q_i/D_i$ is purchasing cost = $C_{0i}Q_i$, set-upcost = c_{3i}, holding cost = $c_{1i}(Q_i/2)T_i$.

Total average inventory cost $\text{TC}(C_0, Q)$ = average purchasing cost + average set-up cost + average holding cost

$$
\begin{aligned}
&= \sum_{i=1}^{n} pt \frac{[C_{0i}Q_i + c_{3i} + c_{1i}(Q_i/2)T_i]}{T_i} \\
&= \sum_{i=1}^{n} pt \left[a_i C_{0i}^{1-b_i} + \frac{a_i c_{3i} C_{0i}^{-b_i}}{Q_i} + \frac{c_{1i} Q_i}{2} \right].
\end{aligned}
\tag{4.1}
$$

Total number of orders $\text{NO}(C_0, Q)$ = sum of orders of all items

$$
= \sum_{i=1}^{n} pt \frac{D_i}{Q_i} = \sum_{i=1}^{n} pt \frac{a_i C_{0i}^{-b_i}}{Q_i}. \tag{4.2}
$$

Total budgetary cost $\mathrm{BC}(C_0, Q)$ = sum of purchasing cost of all items

$$= \sum_{i=1}^{n} ptC_{0i}Q_i. \tag{4.3}$$

Total storage space $\mathrm{SS}(Q)$ = sum of storage space of all items

$$= \sum_{i=1}^{n} ptw_iQ_i. \tag{4.4}$$

In formulating the inventory models, the effect of constraints like total budgetary cost and total storage space cannot be unlimited, they must have restrictions.

Crisp Model

Under these circumstances the multiobjective inventory problem is then written as

$$\begin{aligned}
\text{Min} \quad & \mathrm{TC}(C_0, Q) = \sum_{i=1}^{n} pt\left[a_iC_{0i}^{1-b_i} + \frac{a_ic_{3i}C_{0i}^{-b_i}}{Q_i} + \frac{c_{1i}Q_i}{2}\right], \\
\text{Min} \quad & \mathrm{NO}(C_0, Q) = \sum_{i=1}^{n} pt\frac{a_iC_{0i}^{-b_i}}{Q_i}, \\
\text{subject to} \quad & \mathrm{BC}(C_0, Q) \le C, \\
& \mathrm{SS}(Q) \le W, \\
\text{and boundary conditions} \quad & C_{0i}^{l} \le C_{0i} \le C_{0i}^{u}, \quad Q_i^{l} \le Q_i \le Q_i^{u}, \\
& i = 1, 2, \ldots, n.
\end{aligned} \tag{4.5}$$

Fuzzy Model

In reality, the inventory costs such as carrying cost (c_{1i}), set-up cost (c_{3i}), the index parameter (b_i), storage area per item w_i, total budgetary cost (C), and total available storage area (W) are not exactly known previously. They may fluctuate within some range and can be expressed as a fuzzy number

$$\begin{aligned}
\text{Min} \quad & \widetilde{\mathrm{TC}}(C_0, Q) = \sum_{i=1}^{n} pt\left[a_iC_{0i}^{1-\tilde{b}_i} + \frac{a_i\tilde{c}_{3i}C_{0i}^{-\tilde{b}_i}}{Q_i} + \frac{\tilde{c}_{1i}Q_i}{2}\right], \\
\text{Min} \quad & \widetilde{\mathrm{NO}}(C_0, Q) = \sum_{i=1}^{n} pt\frac{a_iC_{0i}^{-\tilde{b}_i}}{Q_i},
\end{aligned}$$

$$\begin{aligned}
\text{subject to}\quad & \widetilde{\mathrm{BC}}(C_0,Q)=\sum_{i=1}^{n} ptC_{0i}Q_i \le \widetilde{C},\\
& \widetilde{\mathrm{SS}}(Q)=\sum_{i=1}^{n} pt\widetilde{w}_iQ_i \le \widetilde{W},\\
\text{and boundary conditions}\quad & C_{0i}^{l}\le C_{0i}\le C_{0i}^{u},\quad Q_i^{l}\le Q_i\le Q_i^{u},\\
& i=1,2,\ldots,n,
\end{aligned} \tag{4.6}$$

where $\widetilde{c}_{1i} = (c_{11i},\ c_{12i},\ c_{13i})$, $\widetilde{c}_{3i} = (c_{31i},\ c_{32i},\ c_{33i})$, $\widetilde{b}_i = (b_{1i},\ b_{2i},\ b_{3i})$, $\widetilde{C}^{r_1} = (C_1,\ C_1,\ C_2)$, $\widetilde{w}_i = (w_{1i},\ w_{2i},\ w_{3i})$, $\widetilde{W}^{r_1} = (W_1,\ W_1,\ W_2)$.

The α-level interval of these fuzzy numbers are represented by

$(\widetilde{c}_{1i})_\alpha = [c_{1i\alpha}^{L},\ c_{1i\alpha}^{R}]$, $(\widetilde{c}_{3i})_\alpha = [c_{3i\alpha}^{L},\ c_{3i\alpha}^{R}]$, $(\widetilde{b}_i)_\alpha = [b_{i\alpha}^{L},\ b_{i\alpha}^{R}]$, $(\widetilde{C}^{r_1})_\alpha = [C_\alpha^{L},\ C_\alpha^{R}]$,$(\widetilde{w}_i)_\alpha = [w_{i\alpha}^{L},\ w_{i\alpha}^{R}]$, and $(\widetilde{W}^{r_1})_\alpha = [W_\alpha^{L},\ W_\alpha^{R}]$.

For any given $\alpha \in [0,1]$, the problem (4.6) is then reduced to

$$\begin{aligned}
\text{Min}\quad & \mathrm{TC}_\alpha^{L}(C_0,Q)=\sum_{i=1}^{n} pt\left[a_iC_{0i}^{1-b_{i\alpha}^{R}}+\frac{a_ic_{3i\alpha}^{L}C_{0i}^{-b_{i\alpha}^{R}}}{Q_i}+\frac{c_{1i\alpha}^{L}Q_i}{2}\right],\\
\text{Min}\quad & \mathrm{NO}_\alpha^{L}(C_0,Q)=\sum_{i=1}^{n} pt\frac{a_iC_{0i}^{-b_{i\alpha}^{R}}}{Q_i},\\
\text{subject to}\quad & \sum_{i=1}^{n} ptC_{0i}Q_i \le C_\alpha^{r_1R},\\
& \sum_{i=1}^{n} ptw_{i\alpha}^{L}Q_i \le W_\alpha^{r_1R},\\
& C_{0i}^{l}\le C_{0i}\le C_{0i}^{u},\quad Q_i^{l}\le Q_i\le Q_i^{u},\quad (\text{for}\ \ i=1,2,\ldots,n).
\end{aligned} \tag{4.7}$$

4.2. Interactive Geometric Programming (IGP) Technique with Fuzzy Parameters

Following Section 3, the problem (3.2) can be written as

$$\begin{aligned}
\text{Min}\quad & v,\\
\text{subject to}\quad & \mathrm{TC}_\alpha^{L}(C_0,Q)-\overline{\mathrm{TC}}\le v,\\
& \mathrm{NO}_\alpha^{L}(C_0,Q)-\overline{\mathrm{NO}}\le v,\\
& \sum_{i=1}^{n} ptC_{0i}Q_i \le C_\alpha^{r_1R},
\end{aligned}$$

$$\sum_{i=1}^{n} ptw_{i\alpha}^{L} Q_i \le W_{\alpha}^{r_1 R},$$

$$C_{0i}^{l} \le C_{0i} \le C_{0i}^{u}, \quad Q_i^{l} \le Q_i \le Q_i^{u}, \quad i = 1, 2, \ldots, n,$$

$$v > 0. \tag{4.8}$$

where $\overline{\mathrm{TC}}$ and $\overline{\mathrm{NO}}$ are the reference values of $\mathrm{TC}_{\alpha}^{L}(C_0,\ Q)$ and $\mathrm{NO}_{\alpha}^{L}(C_0,\ Q)$, respectively.

For any given $\alpha \in [0, 1]$, the problem (4.8) is equivalent to the standard form of primal GP problem

$$\begin{aligned}
\text{Min} \quad & v, \\
\text{subject to} \quad & \frac{1}{\overline{\mathrm{TC}}}\left[\sum_{i=1}^{n} pt\left(a_i C_{0i}^{1-b_{i\alpha}^{R}} + \frac{a_i c_{3i\alpha}^{L} C_{0i}^{-b_{i\alpha}^{R}}}{Q_i} + \frac{c_{1i\alpha}^{L} Q_i}{2}\right) - v\right] \le 1, \\
& \frac{1}{\overline{\mathrm{NO}}}\left[\sum_{i=1}^{n} pt \frac{a_i C_{0i}^{-b_{i\alpha}^{R}}}{Q_i} - v\right] \le 1, \\
& \frac{1}{C_{\alpha}^{r_1 R}} \sum_{i=1}^{n} pt C_{0i} Q_i \le 1, \\
& \frac{1}{W_{\alpha}^{r_1 R}} \sum_{i=1}^{n} ptw_{i\alpha}^{L} Q_i \le 1, \\
& C_{0i}^{l} \le C_{0i} \le C_{0i}^{u}, \quad Q_i^{l} \le Q_i \le Q_i^{u}, \quad (\text{for } \ i = 1, 2, \ldots, n), \\
& v > 0.
\end{aligned} \tag{4.9}$$

Problem (4.9) is a constrained signomial problem with $6n + 3 - (2n + 1) = 4n + 2$ degree of difficulty. Following Kuester and Mize [19], the problem is solved to obtain the Pareto-optimal solutions for different choices of α and membership functions of fuzzy parameters by DM.

4.3. Interactive Fuzzy Geometric Programming (IFGP) Technique with Fuzzy Parameters and Fuzzy Goals

After determining the different linear/nonlinear membership functions for each of the objective functions proposed by Bellman and Zadeh [4] and following Zimmermann [20] the given problem can be formulated as

$$\begin{aligned}
\text{Min} \quad & \nu, \\
\text{subject to} \quad & \overline{\mu}_{\mathrm{TC}} - \mu_{\mathrm{TC}}\left(\mathrm{TC}_{\alpha}^{L}(C_0, Q)\right) \le \nu,
\end{aligned}$$

$$\begin{aligned}
&\overline{\mu}_{\mathrm{NO}} - \mu_{\mathrm{NO}}\left(\mathrm{NO}_{\alpha}^{L}(C_0, Q)\right) \le \nu, \\
&\sum_{i=1}^{n} ptC_{0i}Q_i \le C_{\alpha}^{r_1R}, \\
&\sum_{i=1}^{n} ptw_{i\alpha}^{L}Q_i \le W_{\alpha}^{r_1R}, \\
&C_{0i}^{l} \le C_{0i} \le C_{0i}^{u}, \quad Q_i^{l} \le Q_i \le Q_i^{u}, \quad \nu > 0.
\end{aligned} \tag{4.10}$$

For any given $\alpha \in [0, 1]$ the problem (4.10) is equivalent to the standard form of primal GP problem

$$\begin{aligned}
\text{Min} \quad & \nu, \\
\text{subject to} \quad & \frac{1}{\nu}\overline{\mu}_{\mathrm{TC}} - \frac{1}{\nu}\mu_{\mathrm{TC}}\left(\mathrm{TC}_{\alpha}^{L}(C_0, Q)\right) \le 1, \\
& \frac{1}{\nu}\overline{\mu}_{\mathrm{NO}} - \frac{1}{\nu}\mu_{\mathrm{NO}}\left(\mathrm{NO}_{\alpha}^{L}(C_0, Q)\right) \le 1, \\
& \frac{1}{C_{\alpha}^{r_1R}}\sum_{i=1}^{n} ptC_{0i}Q_i \le 1, \\
& \frac{1}{W_{\alpha}^{r_1R}}\sum_{i=1}^{n} ptw_{i\alpha}^{L}Q_i \le 1, \\
& C_{0i}^{l} \le C_{0i} \le C_{0i}^{u}, \quad Q_i^{l} \le Q_i \le Q_i^{u}, \quad \nu > 0.
\end{aligned} \tag{4.11}$$

Primal GP (4.11) may be solved by Fortran 77 with software code (Kuester and Mize [19]). Following (3.7) and (3.8), we get the Pareto optimal solution.

5. Numerical Example

A contractor undertakes to supply two types of goods to different distributors. The minimum storage space requirement for the goods are 600 m^2. He can also arrange up to 640 m^2 storage space for the goods if necessary. The contractor invests \$220 for his business with a maximum limit up to \$250. From the past experience it was found that the holding cost of item-I is near about \$1.5 but never less than \$1.2 and above \$2 (i.e., $\tilde{c}_{11} \equiv \$(1.2, 1.5, 2)$). Similarly, holding cost of item-II is $\tilde{c}_{12} \equiv \$(1.5, 1.8, 2.3)$. The set-up cost and the index parameter of purchasing cost of each item are $\tilde{c}_{31} \equiv \$(100, 115, 130)$, $\tilde{c}_{32} \equiv \$(130, 145, 160)$; $\tilde{b}_1 \equiv (0.2, 0.3, 0.45)$ and $\tilde{b}_2 \equiv (0.5, 0.65, 0.9)$, respectively. The storage spaces of each item are $w_1 \equiv (1.4, 1.8, 2.2)\,\mathrm{m}^2$ and $w_2 \equiv (2.6, 3, 3.5)\,\mathrm{m}^2$, respectively. It is also recorded from past that the scaling constant of the purchasing cost of each item are $1000 (= a_1)$ and $1120 (= a_2)$, respectively.

The contractor wants to find the purchasing cost and inventory level of each item so as to minimize the total average cost and total number of order supply to the distributors.

The boundary level of purchasing cost and inventory level are given in Table 2.

Table 2: Boundary level of decision variables (C_0, Q).

i	C_{0i} (\$)		Q_i	
	Lower limit	Upper limit	Lower limit	Upper limit
1	0.2	1.5	80	250
2	0.6	2.5	80	180

INPUT THE VALUE OF α

= 0.7

DO YOU WANT LIST OF MEMBERSHIP FUNCTIONS FOR FUZZY PARAMETERS?

= YES

(1) LINEAR (L) (2) PARABOLIC (P) (3) EXPONENTIAL (E)

INPUT THE LIST OF MEMBERSHIP FUNCTIONS FOR LEFT BRANCH AND RIGHT BRANCH OF THE FUZZY PARAMETERS:

(LEFT AND RIGHT BRANCH OF FUZZY PARAMETERS)

BRANCH	c_{11}	c_{12}	c_{31}	c_{32}	b_1	b_2	w_1	w_2	C	W
LEFT	P	E	E	P	L	L	L	E	P	E
RIGHT	P	P	L	E	P	L	E	E	L	P

INPUT THE VALUES OF (α_1, β_1) FOR c_{12}, c_{31}, w_2, W

	c_{12}	c_{31}	w_2	W
(α_1, β_1)	(1.4, 0.6)	(1.2, 1.6)	(2.2, 1.2)	(1.3, 0.6)

INPUT THE VALUES OF (α_2, β_2) FOR c_{32}, w_1, w_2

	c_{32}	w_1	w_2
(α_2, β_2)	(1.5, 0.2)	(1.9, 0.7)	(2.4, 1.3)

Solution with IGP

CALCULATION OF MAXIMUM AND MINIMUM VALUES OF OBJECTIVE FUNCTIONS

(INDIVIDUAL MINIMUM AND MAXIMUM)

OBJECTIVE FUNCTIONS	MINIMUM	MAXIMUM
TC(\$)	3861.849	6316.976
NO	15.17298	43.39745

CHOICE THE REFERENCE VALUE OF TC

= 3860

CHOICE THE REFERENCE VALUE OF NO

= 13

CALCULATION OF OPTIMAL SOLUTION

i	$C_{0i}^{*}(\$)$	Q_i^{*}	$TC^{*}(\$)$	NO^{*}	v^{*}
1	0.2436070	208.4311	3862.605	15.60456	2.604558
2	1.788949	99.62540			

ARE YOU SATISFIED WITH THE CURRENT OPTIMAL SOLUTION (OTHERWISE RECHOICE THE VALUE α OR MEMBERSHIP FUNCTIONS OF FUZZY PARAMETERS)?

= YES SATISFIED.

Solution with IFGP

DO YOU WANT LIST OF MEMBERSHIP FUNCTIONS?

= YES

LIST OF MEMBERSHIP FUNCTIONS

(1) PARABOLIC (2) EXPONENTIAL

INPUT MEMBERSHIP FUNCTION TYPE FOR 1ST OBJECTIVE:

= 2

INPUT TWO POINTS $TC^1,\ TC^0$

= 3861.849, 6316.976

INPUT MEMBERSHIP FUNCTION TYPE FOR 2ND OBJECTIVE:

= 1

INPUT TWO POINTS $NO^1,\ NO^0$

= 15.17298, 43.39745

INPUT $\overline{\mu}_{TC}\ \overline{\mu}_{NO}$

= 0.99, 0.988

ν-MAX CALCULATION:

OPTIMAL VALUE OF ν = 0.004781238

CHOICE MOST IMPORTANT OBJECTIVE FUNCTION $TC(C_0, Q),\ NO(C_0, Q)$

= $TC(C_0, Q)$

MINIMIZE $TC(C_0, Q)$

i	$C_{0i}^{*}(\$)$	Q_i^{*}	$TC^{*}(\$)$	NO^{*}
1	0.2350434	205.9959	3861.849	15.71666
2	1.785726	101.1253		

PARETO OPTIMALITY TEST

i	$C_{0i}^{*}(\$)$	Q_i^{*}	$TC^{*}(\$)$	NO^{*}
1	0.2351641	206.0291	3861.849	15.71500
2	1.7857650	101.1048		

ARE YOU SATISFIED WITH THE CURRENT PARETO OPTIMAL SOLUTION (OTHERWISE RECHOICE THE VALUE OF α OR THE MEMBERSHIP FUNCTIONS OF FUZZY PARAMETERS OR OBJECTIVE FUNCTIONS AND PROCEED AS BEFORE)?

= YES SATISFIED.

6. Conclusion

In a real-life problem, it is not always possible to achieve the optimum goal set by a DM. Depending upon the constraints and unavoidable, unthinkable and compelling conditions prevailed at that particular time, a DM has to comprise and to be satisfied with a near optimum (Pareto-optimal) solution for the decision. This phenomenon is more prevalent when there is more than one objective goal for a DM. But, the usual mathematical programming methods in both crisp and fuzzy environments evaluate only one best possible solution against a problem. Moreover, GP method is the most appropriate method applied to engineering design problems. Nowadays, it is also applied to solve the inventory control problems.

In this paper, for the first time GP methods in an imprecise environment have been used to obtain a Pareto-optimal solution for most suitable choice of the DM. In this connection, we introduce here a man-machine interaction for the DM. This may be easily applied to other inventory models with dynamic demand, quantity discount, and so forth. The method can be easily expanded to stochastic, fuzzy-stochastic environments of inventory models.

References

[1] M. Sakawa and H. Yano, "An interactive fuzzy satisficing method for generalized multiobjective linear programming problems with fuzzy parameters," *Fuzzy Sets and Systems*, vol. 35, no. 2, pp. 125–142, 1990.

[2] M. Sakawa and H. Yano, "Interactive fuzzy decision making for multi-objective non-linear programming using augmented minimax problems," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 31–41, 1986.

[3] L. A. Zadeh, "Fuzzy sets," *Information and Computation*, vol. 8, pp. 338–353, 1965.

[4] R. E. Bellman and L. A. Zadeh, "Decision-making in a fuzzy environment," *Management Science*, vol. 17, pp. 141–164, 1970.

[5] K. S. Park, "Fuzzy set theoretic interpretation of economic order quantity," *IEEE Transactions on Systems, Man and Cybernetics*, vol. 17, no. 6, pp. 1082–1084, 1987.

[6] T. K. Roy and M. Maiti, "A fuzzy EOQ model with demand-dependent unit cost under limited storage capacity," *European Journal of Operational Research*, vol. 99, no. 2, pp. 425–432, 1997.

[7] H. Ishibuchi and H. Tanaka, "Multiobjective programming in optimization of the interval objective function," *European Journal of Operational Research*, vol. 48, no. 2, pp. 219–225, 1990.

[8] R. J. Duffin, E. L. Peterson, and C. Zener, *Geometric Programming: Theory and Application*, John Wiley & Sons, New York, NY, USA, 1967.

[9] G. A. Kotchenberger, "Inventory models: optimization by geometric programming," *Decision Sciences*, vol. 2, pp. 193–205, 1971.

[10] B. M. Worral and M. A. Hall, "The analysis of an inventory control model using Posynomial geometric programming," *International Journal of Production Research*, vol. 20, pp. 657–667, 1982.

[11] M. O. Abuo-El-Ata, H. A. Fergany, and M. F. El-Wakeel, "Probabilistic multi-item inventory model with varying order cost under two restrictions: a geometric programming approach," *International Journal of Production Economics*, vol. 83, no. 3, pp. 223–231, 2003.

[12] H. Jung and C. M. Klein, "Optimal inventory policies for an economic order quantity model with decreasing cost functions," *European Journal of Operational Research*, vol. 165, no. 1, pp. 108–126, 2005.

[13] N. K. Mandal, T. K. Roy, and M. Maiti, "Multi-objective fuzzy inventory model with three constraints: a geometric programming approach," *Fuzzy Sets and Systems*, vol. 150, no. 1, pp. 87–106, 2005.

[14] S. T. Liu, "Using geometric programming to profit maximization with interval coefficients and quantity discount," *Applied Mathematics and Computation*, vol. 209, no. 2, pp. 259–265, 2009.

[15] S. J. Sadjadi, M. Ghazanfari, and A. Yousefli, "Fuzzy pricing and marketing planning model: a possibilistic geometric programming approach," *Expert Systems with Applications*, vol. 37, no. 4, pp. 3392–3397, 2010.

[16] J. J. Buckley and E. Eslami, *An Introduction to Fuzzy Logic and Fuzzy Sets*, Fhysica, 2002.

[17] M. Sakawa, *Fuzzy Sets and Interactive Multiobjective Optimization*, Plenum Press, New York, USA, 1993.
[18] R. E. Wendell and D. N. Lee, "Efficiency in multiple objective optimization problems," *Mathematical Programming*, vol. 12, no. 3, pp. 406–414, 1977.
[19] J. L. Kuester and J. H. Mize, *Optimization Techniques with Fortran*, McGraw-Hill, New York, NY, USA, 1973.
[20] H. J. Zimmermann, "Fuzzy programming and linear programming with several objective functions," *Fuzzy Sets and Systems*, vol. 1, no. 1, pp. 46–55, 1978.