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Research Article

Multiple Positive Solutions for Singular Quasilinear Multipoint BVPs with the First-Order Derivative

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The existence of at least three positive solutions for differential equation $(\phi_p(u'(t)))' + g(t)f(t, u(t), u'(t)) = 0$, under one of the following boundary conditions: $u(0) = \sum_{i=1}^{m-2} a_i u(\xi_i)$, $\varphi_p(u'(1)) = \sum_{i=1}^{m-2} b_i \varphi_p(u'(\xi_i))$ or $\varphi_p(u'(0)) = \sum_{i=1}^{m-2} a_i \varphi_p(u'(\xi_i))$, $u(1) = \sum_{i=1}^{m-2} b_i u(\xi_i)$ is obtained by using the H. Amann fixed point theorem, where $\varphi_p(s) = |s|^{p-2}s$, $p > 1$, $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < 1$, $a_i > 0$, $b_i > 0$, $0 < \sum_{i=1}^{m-2} a_i < 1$, $0 < \sum_{i=1}^{m-2} b_i < 1$. The interesting thing is that $g(t)$ may be singular at any point of [0,1] and f may be noncontinuous.

1. Introduction

In this paper, of multiple positive solutions for differential equation

$$\left(\varphi_p\left(u'(t)\right)\right)' + g(t)f\left(t, u(t), u'(t)\right) = 0, \quad \text{a.e. } t \in (0,1), \tag{1.1}$$

subject to boundary conditions:

$$u(0) = \sum_{i=1}^{m-2} a_i u\left(\xi_i\right), \qquad \varphi_p\left(u'(1)\right) = \sum_{i=1}^{m-2} b_i \varphi\left(u'\left(\xi_i\right)\right), \tag{1.2}$$

$$\varphi_p\left(u'(0)\right) = \sum_{i=1}^{m-2} a_i \varphi_p\left(u'\left(\xi_i\right)\right), \qquad u(1) = \sum_{i=1}^{m-2} b_i u\left(\xi_i\right), \tag{1.3}$$

respectively, where $\varphi_p(s) = |s|^{p-2}s$, $p > 1$, $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < 1$, $a_i > 0$, $b_i > 0$, $0 < \sum_{i=1}^{m-2} a_i < 1$, $0 < \sum_{i=1}^{m-2} b_i < 1$, $g(t)$ may be singular at any point of [0,1].

The multipoint boundary value problems for ordinary differential equations arise in a variety of different areas of applied mathematics and physics. The study of the multipoint boundary value problems for linear second-order ordinary differential equations was initiated by Il'in and Moiseev [1, 2]. Since then, nonlinear second-order multipoint boundary value problems have been studied by several authors. We refer the reader to [3–9] and references cited therein. Recently, in [10], Liang and Zhang studied the existence of positive solutions for differential equation

$$(\varphi(u'))' + a(t)f(u(t)) = 0, \quad 0 < t < 1, \tag{1.4}$$

under the boundary conditions (1.2) by using the fixed point index theory. Wang and Hou [11] investigated the multiplicity of solutions for the differential equation

$$(\varphi_p(u'(t)))' + f(t, u(t)) = 0, \quad t \in (0,1), \tag{1.5}$$

under the boundary conditions (1.3) by utilizing the fixed point theorem for operators on a cone. Guo et al. [12] proved the existence of at least three positive solutions for differential equation

$$(\varphi_p(u'(t)))' + a(t)f(t, u(t)) = 0, \quad t \in (0,1), \tag{1.6}$$

subject to (1.2) and (1.3), respectively, by using the five-functional-fixed-point theorem, where $a(t)$ and $f(t,u)$ are continuous.

All of the above work was done under the assumption that f is allowed to depend just on u, while the first-order derivative $u'(t)$ is not involved explicitly in the nonlinear term f.

In [13, 14], Wang and Ge and Sun et al. proved the existence of multiple positive solutions for (1.1) subject to boundary conditions:

$$\begin{aligned} u(0) &= \sum_{i=1}^{n-2} \alpha_i u(\xi_i), \qquad u'(1) = \sum_{i=1}^{n-2} \beta_i u'(\xi_i), \\ u'(0) &= \sum_{i=1}^{n} \alpha_i u'(\xi_i), \qquad u(1) = \sum_{i=1}^{n} \beta_i u(\xi_i), \end{aligned} \tag{1.7}$$

respectively, where g and f are continuous.

However, in the existing literature, few people considered the case where the nonlinear term is not only involved in the first-order derivative but also noncontinuous. Our paper will fill this gap in the literature. The purpose of this paper is to improve and generalize the results in the above mentioned references. We will prove that the problem (1.1), (1.2) and the problem (1.1), (1.3) have at least three positive solutions by using the H. Amann fixed point theorem, where $g(t)$ may be singular at any point of [0,1] and $f(t,u,v)$ may be noncontinuous.

In this paper, we always suppose the following conditions are satisfied:

(H_1) $0 = \xi_0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < \xi_{m-1} = 1$, $a_i > 0$, $b_i > 0$, $0 < \sum_{i=1}^{m-2} a_i < 1$, $0 < \sum_{i=1}^{m-2} b_i < 1$;

(H_2) $g(t) \in L^1[0,1]$, $g(t) > 0$, a.e. $t \in [0,1]$;

(H_3) $f(t,u,v) : [0,1]\times R\times R \to [0,\infty)$ is bounded in a bounded subset of $[0,1]\times R\times R$, $f(t,u(t),v(t))$ is measurable for $u(t),v(t)\in C[0,1]$, and $f(t,\cdot,\cdot)$ is continuous for a.e. $t\in[0,1]$.

Sometimes, we will make use of the following conditions.

(H_4) $f(t,\cdot,v)$ and $f(t,u,\cdot)$ are strictly increasing in $[0,\infty)$.

(H_5) $f(t,\cdot,v)$ is strictly increasing in $[0,\infty)$ and $f(t,u,\cdot)$ is strictly decreasing in $(-\infty,0]$.

2. Preliminaries

Definition 2.1. Let X be a real Banach space and $P\subset X$ be a cone. For $u,v\in X$, we denote

$$\begin{gathered} u \preccurlyeq v \Longleftrightarrow u-v\in P, \\ u \prec v \Longleftrightarrow u-v\in P, \quad \text{but } u\neq v, \\ u \prec\prec v \Longleftrightarrow u-v\in \operatorname{int} P. \end{gathered} \tag{2.1}$$

If P has nonempty interior, then it is called a solid cone. If every ordered interval is bounded, then P is called a normal cone.

Definition 2.2. An operator $T : P\to X$ is called order preserving if $u \preccurlyeq v \Rightarrow Tu \preccurlyeq Tv$, strictly order preserving if $u\prec v\Rightarrow Tu\prec Tv$, and strongly order preserving if $u\prec\prec v\Rightarrow Tu\prec\prec Tv$.

Lemma 2.3 (see [15, 16]). *Let P be a normal solid cone of a real Banach space X and suppose there exist $y_1,z_1,y_2,z_2\in X$ such that $y_1\prec z_1\prec y_2\prec z_2$. In addition, suppose that $T : [y_1,z_2]\to X$ is a completely continuous and strongly order preserving operator such that*

$$y_1\preccurlyeq Ty_1,\quad Tz_1\prec z_1,\quad y_2\prec Ty_2,\quad Tz_2\preccurlyeq z_2. \tag{2.2}$$

Then T has at least three fixed points x_1, x_2 and x_3 satisfying

$$y_1\preccurlyeq x_1\prec\prec z_1,\quad y_2\prec\prec x_2\preccurlyeq z_2,\quad y_2\not\preccurlyeq x_3\not\preccurlyeq z_1. \tag{2.3}$$

3. The positive solutions for the problem (1.1), (1.2)

Let $X=C^1[0,1]$ with norm $\|u\|=\max\{\max_{t\in[0,1]}|u(t)|,\max_{t\in[0,1]}|u'(t)|\}$. Define $P\subset X$ by

$$P=\{u\in X \mid u(t)\geq 0,\ u'(t)\geq 0,\ t\in[0,1]\}. \tag{3.1}$$

Obviously, X is a Banach space and P is a normal solid cone of X.

We can easily get the following lemmas.

Lemma 3.1. *The boundary value problem* (1.1), (1.2) *has a solution $u(t)$ if and only if $u(t)$ satisfies the equation*

$$u(t)=\int_0^t\varphi_q\left(\int_s^1 g(\tau)f(\tau,u(\tau),u'(\tau))d\tau+Bf\right)ds+Af, \tag{3.2}$$

where φ_q *is the inverse function of* φ_p, *that is,* $\varphi_q = \varphi_p^{-1}$, $1/p + 1/q = 1$, *and*

$$Bu = \frac{1}{1 - \sum_{i=1}^{m-2} b_i} \sum_{i=1}^{m-2} b_i \int_{\xi_i}^{1} g(\tau)u(\tau)d\tau, \quad u \in L^\infty[0,1],$$
$$Au = \frac{1}{1 - \sum_{i=1}^{m-2} a_i} \sum_{i=1}^{m-2} a_i \int_{0}^{\xi_i} \varphi_q\left(\int_s^1 g(\tau)u(\tau)d\tau + Bu\right)ds, \quad u \in L^\infty[0,1]. \tag{3.3}$$

Define an operator T *in* P *by*

$$Tu(t) = \int_0^t \varphi_q\left(\int_s^1 g(\tau)f(\tau, u(\tau), u'(\tau))d\tau + Bf\right)ds + Af. \tag{3.4}$$

Evidently, $u(t)$ *is a fixed point of* T *if and only if it is a solution of the problem* (1.1), (1.2).

Lemma 3.2. *Suppose* (H_1)–(H_4) *hold,* $u(t) \in P$, *then* $Tu(t) \geq 0$, $(Tu)'(t) \geq 0$, $t \in [0,1]$.

For convenience, we denote

$$Lu(t) = \int_0^t \varphi_q\left(\int_s^1 g(\tau)u(\tau)d\tau + Bu\right)ds + Au, \quad u \in L^\infty[0,1]. \tag{3.5}$$

Obviously, we have $Tu(t) = Lf(t, u(t), u'(t))$.

Lemma 3.3. *Suppose* (H_2) *holds. If* $u_1(t), u_2(t) \in L^\infty[0,1]$ *and* $u_1(t) \leq (<) u_2(t)$, a.e. $t \in [0,1]$, *then* $L_1u_1(t) \leq (<) L_1u_2(t)$, $t \in [0,1]$.

Assume (H_1)–(H_4) hold. By Lemmas 3.2 and 3.3, the absolute continuity of integral, Ascoli-Arzela theorem and Lebesgue-Dominated-Convergence-theorem, we obtain that $T : P \to P$ is completely continuous and T is strongly order preserving.

Theorem 3.4. *Suppose* (H_1)–(H_4) *hold. In addition, suppose there exist constants* $0 < a < c < b < d$ *and* $u_0(t) \in P \setminus \{0\}$ *satisfying* $cLu_0(1) < bAu_0$ *such that*

(A_1) $f(t, aAu_0, 0) \geq \varphi_p(a)u_0(t)$, a.e. $t \in [0,1]$;

(A_2) $f(t, cLu_0(1), c(Lu_0)'(0)) < \varphi_p(c)u_0(t)$, a.e. $t \in [0,1]$;

(A_3) $f(t, bAu_0, 0) > \varphi_p(b)u_0(t)$, a.e. $t \in [0,1]$;

(A_4) $f(t, dLu_0(1), d(Lu_0)'(0)) \leq \varphi_p(d)u_0(t)$, a.e. $t \in [0,1]$.

Then the problem (1.1), (1.2) *has at least three positive solutions* $u_1(t), u_2(t)$, *and* $u_3(t)$ *satisfying*

$$aLu_0(t) \preccurlyeq u_1(t) \prec\prec cLu_0(t), \qquad bLu_0(t) \prec\prec u_2(t) \preccurlyeq dLu_0(t),$$
$$bLu_0(t) \not\preceq u_3(t) \not\preceq cLu_0(t). \tag{3.6}$$

Proof. Let $y_1(t) = aLu_0(t)$, $z_1(t) = cLu_0(t)$, $y_2(t) = bLu_0(t)$, $z_2(t) = dLu_0(t)$. Obviously, we have

$$y_1(t) \prec z_1(t) \prec y_2(t) \prec z_2(t). \tag{3.7}$$

Firstly, we will show $y_1(t) \preccurlyeq Ty_1(t)$.

By $y_1(t) \geq aAu_0$, $y_1'(t) \geq 0$, (A_1) and Lemma 3.3, we have

$$\begin{aligned} Ty_1(t) &= Lf\big(t, y_1(t), y_1'(t)\big) \geq Lf\big(t, aAu_0, 0\big) \geq aLu_0(t) = y_1(t), \\ \big(Ty_1\big)'(t) &= \varphi_q\bigg(\int_t^1 g(\tau)f\big(\tau, y_1(\tau), y_1'(\tau)\big)d\tau + Bf\bigg) \geq a\bigg[\varphi_q\int_t^1 g(\tau)u_0(\tau)d\tau + Bu_0\bigg] = y_1'(t). \end{aligned} \tag{3.8}$$

So, we get $y_1 \preccurlyeq Ty_1$.

Similarly, by (A_3) and Lemma 3.3, we get $y_2 \prec Ty_2$.

Next, we prove $Tz_1 \prec z_1$.

From $z_1(t) \leq cLu_0(1)$, $z_1'(t) \leq c(Lu_0)'(0)$, (A_2), and Lemma 3.3, we have

$$\begin{aligned} Tz_1(t) &= Lf\big(t, z_1(t), z_1'(t)\big) \leq Lf\big(t, cLu_0(1), c\big(Lu_0\big)'(0)\big) < cLu_0(t) = z_1(t), \\ \big(Tz_1\big)'(t) &= \varphi_q\bigg(\int_t^1 g(\tau)f\big(\tau, z_1(\tau), z_1'(\tau)\big)d\tau + Bf\bigg) \leq c\bigg[\varphi_q\int_t^1 g(\tau)u_0(\tau)d\tau + Bu_0\bigg] = z_1'(t). \end{aligned} \tag{3.9}$$

So, we get $Tz_1 \prec z_1$.

Similarly, by (A_4) and Lemma 3.3, we get $Tz_2 \preccurlyeq z_2$.

By Lemma 2.3, we get that the operator T has at least three fixed points $u_1(t)$, $u_2(t)$, and $u_3(t)$ satisfying

$$\begin{gathered} aLu_0(t) \preccurlyeq u_1(t) \prec\prec cLu_0(t), \qquad bLu_0(t) \prec\prec u_2(t) \preccurlyeq dLu_0(t), \\ bLu_0(t) \not\preccurlyeq u_3(t) \not\preccurlyeq cLu_0(t). \end{gathered} \tag{3.10}$$

The proof is completed. □

4. The positive solutions for the problem (1.1), (1.3)

Let X be the same as the one in Section 3. Define $P_1 \subset X$ by

$$P_1 = \big\{u \in X \mid u(t) \geq 0,\ u'(t) \leq 0,\ t \in [0,1]\big\}. \tag{4.1}$$

Evidently, P_1 is a normal solid cone of X.

We can easily get the following lemmas.

Lemma 4.1. *The boundary value problem* (1.1), (1.3) *has a solution* $u(t)$ *if and only if* $u(t)$ *satisfies the equation*

$$u(t) = -\int_0^t \varphi_q\bigg(\int_0^s g(\tau)f\big(\tau, u(\tau), u'(\tau)\big)d\tau + \widetilde{B}f\bigg)ds + \widetilde{A}f, \tag{4.2}$$

where φ_q is the same as the one in Lemma 3.1, and

$$\widetilde{B}u = \frac{1}{1-\sum_{i=1}^{m-2}a_i}\sum_{i=1}^{m-2}a_i\int_0^{\xi_i} g(\tau)u(\tau)d\tau, \quad u \in L^\infty[0,1],$$

$$\widetilde{A}u = \frac{1}{1-\sum_{i=1}^{m-2}b_i}\left[\int_0^1 \varphi_q\left(\int_0^s g(\tau)u(\tau)d\tau + \widetilde{B}u\right)ds - \sum_{i=1}^{m-2}b_i\int_0^{\xi_i}\varphi_q\left(\int_0^s g(\tau)u(\tau)d\tau + \widetilde{B}u\right)ds\right], \quad u \in L^\infty[0,1]. \tag{4.3}$$

Define an operator T_1 in P_1 by

$$T_1u(t) = -\int_0^t \varphi_q\left(\int_0^s g(\tau)f(\tau,u(\tau),u'(\tau))d\tau + \widetilde{B}f\right)ds + \widetilde{A}f. \tag{4.4}$$

Obviously, $u(t) \in P_1$ is a fixed point of the operator T_1 if and only if it is a positive solution of the problem (1.1), (1.3).

Lemma 4.2. *Suppose (H_1)–(H_3), (H_5) hold, and $u(t) \in P_1$, then $T_1u(t) \ge 0$, $(T_1u)'(t) \le 0$, $t \in [0,1]$.*

For convenience, we denote

$$L_1u(t) = -\int_0^t \varphi_q\left(\int_0^s g(\tau)u(\tau)d\tau + \widetilde{B}u\right)ds + \widetilde{A}u. \tag{4.5}$$

Clearly, $T_1u(t) = L_1f(t,u(t),u'(t))$.

Lemma 4.3. *Suppose (H_2) holds. If $u_1(t)$, $u_2(t) \in L^\infty[0,1]$, and $u_1(t) \le (<)u_2(t)$, a.e. $t \in [0,1]$, then $L_1u_1(t) \le (<)L_1u_2(t)$, $t \in [0,1]$.*

Assume (H_1)–(H_3) and (H_5) hold, by Lemmas 4.2 and 4.3, the absolute continuity of integral, Ascoli-Arzela theorem, and Lebesgue-Dominated-Convergence-theorem, we obtain that $T_1 : P_1 \to P_1$ is completely continuous, and T_1 is strongly order preserving.

Theorem 4.4. *Suppose (H_1)–(H_3) and (H_5) hold. In addition, suppose there exist constants $0 < \widetilde{a} < \widetilde{c} < \widetilde{b} < \widetilde{d}$ and function $u_0(t) \in P_1 \setminus \{0\}$ satisfying $\widetilde{c}\widetilde{A}u_0 < \widetilde{b}L_1u_0(1)$ such that*

(C_1) $f(t,\widetilde{a}L_1u_0(1),0) \ge \varphi_p(\widetilde{a})u_0(t)$, a.e. $t \in [0,1]$;

(C_2) $f(t,\widetilde{c}\widetilde{A}u_0,\widetilde{c}(L_1u_0)'(1)) < \varphi_p(\widetilde{c})u_0(t)$, a.e. $t \in [0,1]$;

(C_3) $f(t,\widetilde{b}L_1u_0(1),0) > \varphi_p(\widetilde{b})u_0(t)$, a.e. $t \in [0,1]$;

(C_4) $f(t,\widetilde{d}\widetilde{A}u_0,\widetilde{d}(L_1u_0)'(1)) \le \varphi_p(\widetilde{d})u_0(t)$, a.e. $t \in [0,1]$.

Then the problem (1.1), (1.3) has at least three positive solutions $v_1(t), v_2(t)$, and $v_3(t)$ satisfying

$$\widetilde{a}L_1u_0(t) \preccurlyeq v_1(t) \prec\prec \widetilde{c}L_1u_0(t), \qquad \widetilde{b}L_1u_0(t) \prec\prec v_2(t) \preccurlyeq \widetilde{d}L_1u_0(t), \qquad \widetilde{b}L_1u_0(t) \not\preccurlyeq v_3(t) \not\preccurlyeq \widetilde{c}L_1u_0(t). \tag{4.6}$$

Proof. Let $y_1(t) = \tilde{a}L_1u_0(t)$, $z_1(t) = \tilde{c}L_1u_0(t)$, $y_2(t) = \tilde{b}L_1u_0(t)$, $z_2(t) = \tilde{d}L_1u_0(t)$. Obviously, we have

$$y_1(t) \prec z_1(t) \prec y_2(t) \prec z_2(t). \tag{4.7}$$

Firstly, we will show $y_1(t) \preccurlyeq T_1y_1(t)$.

By $y_1(t) \geq \tilde{a}L_1u_0(1)$, $y_1'(t) \leq 0$, (C_1), and Lemma 4.3, we have

$$\begin{aligned} T_1y_1(t) &= L_1f\left(t, y_1(t), y_1'(t)\right) \geq L_1f\left(t, \tilde{a}L_1u_0(1), 0\right) \geq \tilde{a}L_1u_0(t) = y_1(t), \\ (T_1y_1)'(t) &= -\varphi_q\left(\int_0^t g(\tau)f\left(\tau, y_1(\tau), y_1'(\tau)\right)d\tau + \widetilde{B}f\right) \\ &\leq -\tilde{a}\varphi_q\left[\int_0^t g(\tau)u_0(\tau)d\tau + \widetilde{B}u_0\right] = y_1'(t). \end{aligned} \tag{4.8}$$

So, we get $y_1 \preccurlyeq T_1y_1$.

Similarly, by (C_3) and Lemma 4.3, we get $y_2 \prec T_1y_2$.

Next, we will prove $T_1z_1 \prec z_1$.

From $z_1(t) \leq \tilde{c}\widetilde{A}u_0$, $z_1'(t) \geq \tilde{c}(L_1u_0)'(1)$, ($C_2$), and Lemma 4.3, we have

$$\begin{aligned} T_1z_1(t) &= L_1f\left(t, z_1(t), z_1'(t)\right) \leq L_1f\left(t, \tilde{c}\widetilde{A}u_0, \tilde{c}(L_1u_0)'(1)\right) < \tilde{c}L_1u_0(t) = z_1(t), \\ (T_1z_1)'(t) &= -\varphi_q\left(\int_0^t g(\tau)f\left(\tau, z_1(\tau), z_1'(\tau)\right)d\tau + \widetilde{B}f\right) \\ &\geq -\tilde{c}\varphi_q\left[\int_0^t g(\tau)u_0(\tau)d\tau + \widetilde{B}u_0\right] = z_1'(t). \end{aligned} \tag{4.9}$$

So, we get $T_1z_1 \prec z_1$.

Similarly, by (C_4) and Lemma 4.3, we get $T_1z_2 \preccurlyeq z_2$.

By Lemma 2.3, we get that the operator T_1 has at least three fixed points $v_1(t)$, $v_2(t)$, and $v_3(t)$ satisfying

$$\begin{gathered} \tilde{a}L_1u_0(t) \preccurlyeq v_1(t) \prec\prec \tilde{c}L_1u_0(t), \qquad \tilde{b}L_1u_0(t) \prec\prec v_2(t) \preccurlyeq \tilde{d}L_1u_0(t), \\ \tilde{b}L_1u_0(t) \not\preccurlyeq v_3(t) \not\preccurlyeq \tilde{c}L_1u_0(t). \end{gathered} \tag{4.10}$$

The proof is completed. □

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