Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2008, Article ID 186539, 19 pages
doi:10.1155/2008/186539

Research Article

Dynamics Behaviors of a Discrete Ratio-Dependent Predator-Prey System with Holling Type III Functional Response and Feedback Controls

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Received 24 April 2008; Revised 16 August 2008; Accepted 2 October 2008

Recommended by Juan Jose Nieto

A ratio-dependent predator-prey system with Holling type III functional response and feedback controls is proposed. By constructing a suitable Lyapunov function and using the comparison theorem of difference equation, sufficient conditions which ensure the permanence and global attractivity of the system are obtained. After that, under some suitable conditions, we show that the predator species y will be driven to extinction. Examples together with their numerical simulations show that the main results are verifiable.

1. Introduction

Wang and Li [1, 2] established verifiable criteria for the existence of globally attractive positive periodic solutions of the following delayed predator-prey model with Holling type III functional response:

$$\begin{aligned} N_1'(t) &= N_1(t)\left(b_1(t) - a_1(t)N_1(t-\tau_1(t)) - \frac{\alpha_1(t)N_1(t)}{1+mN_1^2(t)}N_2(t-\sigma(t))\right),\\ N_2'(t) &= N_2(t)\left(-b_2(t) - a_2(t)N_2(t) + \frac{\alpha_2(t)N_1^2(t-\tau_2(t))}{1+mN_1^2(t-\tau_2(t))}\right), \end{aligned} \tag{1.1}$$

where $N_1(t)$, $N_2(t)$ are the densities of the prey population and predator population at time t, respectively; $b_i : \mathbb{R} \to \mathbb{R}$, $a_i, \tau_i, \sigma, \alpha_i : \mathbb{R} \to [0, +\infty)$ $(i = 1, 2)$ are continuous functions of period T and $\int_0^T b_i(t)dt > 0$, $\alpha_i(t) \neq 0$; m is a nonnegative constant. For more works on the predator-prey system with Holling type functional response, one could refer to [3–11] and the references cited therein. But, recently, lots of scholars found that when predators have to search for food (and, therefore, have to share or compete for food), a more suitable general predator-prey theory should be based on the so-called ratio-dependent theory, which can be roughly stated as that the per capita predator growth rate should be the so-called ratio-dependent functional response. This is strongly supported by numerous fields and laboratory experiments and observations [12, 13]. In [14], Wang and Li proposed the following ratio-dependent predator-prey system with Holling type III functional response:

$$\begin{aligned} x'(t) &= x(t)\left(a(t) - b(t)\int_{-\infty}^{t} k(t-s)x(s)ds\right) - \frac{c(t)x^2(t)y(t)}{m^2y^2(t) + x^2(t)}, \\ y'(t) &= y(t)\left(\frac{e(t)x^2(t-\tau)}{m^2y^2(t-\tau) + x^2(t-\tau)} - d(t)\right), \end{aligned} \tag{1.2}$$

where $x(t),\ y(t)$ are the densities of the prey population and predator population at time t, respectively, $a(t),\ b(t),\ c(t),\ d(t)$, and $e(t)$ are all positive periodic continuous functions, and $m > 0,\ \tau \geq 0$ are real constants. They found that the criteria for the permanence are exactly the same as those for the existence of positive periodic solution of (1.2). For more works on the ratio-dependent predator-prey system, one could refer to [15, 16] and the references cited therein.

On the other hand, when the size of the population is rarely small or the population has nonoverlapping generations, the discrete time models are more appropriate than the continuous ones [17, 18]. For the discrete ratio-dependent predator-prey model with Holling type III functional response, Fan and Li [19] considered the following system:

$$\begin{aligned} N_1(k+1) &= N_1(k)\exp\left\{b_1(k) - a_1(k)N_1(k-[\tau_1]) - \frac{a_1(k)N_1(k)N_2(k)}{N_1^2(k) + m^2(k)N_2^2(K)}\right\}, \\ N_2(k+1) &= N_2(k)\exp\left\{-b_2(k) + \frac{a_2(k)N_1^2(k-[\tau_2])}{N_1^2(k-[\tau_1]) + m^2(k)N_2^2(k-[\tau_2])}\right\}, \end{aligned} \tag{1.3}$$

sufficient conditions which ensure the permanence of system (1.3) are obtained.

However, as pointed out by Huo and Li [20], "ecosystem in the real world is continuously disturbed by unpredictable forces which can result in changes in the biological parameters such as survival rates. The question of whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of time is of practical interest in ecology . In the language of control variables, we call the disturbance functions as control variables." For this direction, in [15], Chen and Ji proposed the following system:

$$\begin{aligned} \dot{x}_1 &= x_1\left[a_1(t) - b(t)x - \frac{c_1(t)}{x_1 + \alpha(t)x_2} - e_1(t)u_1\right], \\ \dot{x}_2 &= x_2\left[-a_2(t) + \frac{c_2(t)}{x_1 + \alpha(t)x_2} + e_2(t)u_2\right], \\ \dot{u}_1 &= h_1(t) - d_1(t)u_1 + f_1(t)x_1, \qquad \dot{u}_2 = h_2(t) - d_2(t)u_2 + f_2(t)x_2, \end{aligned} \tag{1.4}$$

where $x_i(t)$ stand for the densities of the prey and predator, respectively, $u_i(t)$ $(i = 1,2)$ is the control variable, $\alpha(t)$, $b(t)$, $a_i(t)$, $c_i(t)$, $h_i(t)$, $d_i(t)$, $f_i(t)$, and $e_i(t)$ $(i = 1,2)$ are continuous and strictly positive functions. In this paper, they considered the almost periodic solution of system (1.4). For more work on this direction, one could refer to [15, 21–28].

To the best of the author's knowledge, so far no scholar has considered system (1.3) with feedback controls. This motivates us to propose and study the following discrete ratio-dependent predator-prey system with Holling type III and feedback controls:

$$\begin{aligned}
x(n+1) &= x(n)\exp\left\{a(n) - b(n)x(n) - \frac{c(n)x(n)y(n)}{x^2(n)+m^2(n)y^2(n)} - e_1(n)u_1(n)\right\},\\
y(n+1) &= y(n)\exp\left\{-d(n) + \frac{f(n)x^2(n)}{x^2(n)+m^2(n)y^2(n)} - e_2(n)u_2(n)\right\},\\
\Delta u_1(n) &= -\eta_1(n)u_1(n) + q_1(n)x(n), \qquad \Delta u_2(n) = -\eta_2(n)u_2(n) + q_2(n)y(n),
\end{aligned} \tag{1.5}$$

where $x(t)$, $y(t)$ are the densities of the prey population and predator population at time t, respectively, for $i = 1,2$, $\{a(n)\}$, $\{b(n)\}$, $\{c(n)\}$, $\{d(n)\}$, $\{f(n)\}$, $\{m(n)\}$, $\{e_i(n)\}$, $\{\eta_i(n)\}$, and $\{q_i(n)\}$ are all bounded nonnegative sequences such that

$$\begin{gathered}
0 < a^L \le a(n) \le a^U, \qquad 0 < b^L \le b(n) \le b^U,\\
0 < c^L \le c(n) \le c^U, \qquad 0 < d^L \le d(n) \le d^U,\\
0 < f^L \le f(n) \le f^U, \qquad 0 < m^L \le m(n) \le m^U,\\
0 < e_i^L \le e_i(n) \le e_i^U, \qquad 0 < \eta_i^L \le \eta_i(n) \le \eta_i^U < 1,\\
0 < q_i^L \le q_i(n) \le q_i^U < 1.
\end{gathered} \tag{H_0}$$

Here, for any bounded sequence $\{a(n)\}$, $a^L = \inf_{n\in\mathbb{N}}\{a(n)\}$, $a^U = \sup_{n\in\mathbb{N}}\{a(n)\}$.

By the biological meaning, we will focus our discussion on the positive solutions of system (1.5). So, it is assumed that the initial conditions of system (1.5) are of the form

$$x(0) > 0, \qquad y(0) > 0, \qquad u_i(0) > 0, \quad i = 1,2. \tag{1.6}$$

It is not difficult to see that solutions of (1.5) and (1.6) are well defined and satisfy

$$x(n) > 0, \quad y(n) > 0, \quad u_i(n) > 0, \quad \text{for } k \in \mathbb{N}^+. \tag{1.7}$$

The main purpose of this paper is to derive sufficient conditions for the permanence, global attractivity, and extinction of system (1.5).

The organization of this paper is as follows. In Section 2, we introduce some useful lemmas. In Section 3, we will study the permanence and global attractivity of system (1.5). In Section 4, we will study the extinction of the predator species y. In the last section, numerical simulation is presented to illustrate the feasibility of our main results.

2. Preliminaries

Now, let us state several lemmas which will be useful in proving the main results.

First, let us consider the first-order difference equation

$$y(k+1) = Ay(k) + B, \quad k = 1, 2 \ldots, \tag{2.1}$$

where $A,\ B$ are positive constants. The following Lemma 2.1 is a direct corollary of Theorem 6.2 of L. Wang and M. Q. Wang [29, page 125].

Lemma 2.1. *Assume that $|A| < 1$. For any initial $y(0)$, there exists a unique solution $y(k)$ of (2.1), which can be expressed as follows:*

$$y(k) = A^k (y(0) - y^*) + y^*, \tag{2.2}$$

where $y^* = B/(1-A)$. Thus, for any solutions $y(k)$ of system (2.1),

$$\lim_{k \to +\infty} y(k) = y^*. \tag{2.3}$$

The following comparison theorem for difference equation is Theorem 2.1 of [29, page 241].

Lemma 2.2. *Let $k \in \mathbb{N}^+_{k_0} = \{k_0, k_0+1, \ldots, k_0+l, \ldots\}$ and $r \geq 0$. For any fixed $k,\ g(k,r)$ is a nondecreasing function with respect to r, and for $k \geq k_0$, the following inequalities hold:*

$$y(k+1) \leq g(k, y(k)), \qquad u(k+1) \geq g(k, u(k)). \tag{2.4}$$

If $y(k_0) \leq u(k_0)$, then $y(k) \leq u(k)$ for all $k \geq k_0$.

The following Lemmas 2.3 and 2.4 can be found in [21].

Lemma 2.3 (see [21]). *Assume that $x(n)$ satisfies $x(n) > 0$ and*

$$x(n+1) \leq x(n) \exp(r(n) - a(n)x(n)) \tag{2.5}$$

for $n \in \mathbb{N}$, where $r(n)$ and $a(n)$ are nonnegative sequences bounded above and below by positive constants. Then

$$\limsup_{n \to +\infty} x(n) \leq \frac{1}{a^L} \exp(r^U - 1). \tag{2.6}$$

Lemma 2.4 (see [21]). *Assume that $\{x(n)\}$ satisfies*

$$x(n+1) \geq x(n) \exp(r(n) - a(n)x(n)), \quad k \geq N_0, \tag{2.7}$$

$\limsup_{n\to+\infty} x(n) \le x^*$ *and* $x(N_0) > 0$, *where* $r(n)$ *and* $a(n)$ *are nonnegative sequences bounded above and below by positive constants and* $N_0 \in \mathbb{N}$. *Then*

$$\liminf_{n\to+\infty} x(n) \ge \frac{r^L \exp\{r^L - a^U x^*\}}{a^U}. \tag{2.8}$$

3. Permanence and global attractivity

Now, we investigate the permanence and global attractivity of system (1.5).

Theorem 3.1. *Assume that*

$$a^L - \frac{c^U}{2m^L} - e_1^U W_1 > 0, \tag{H_1}$$

$$f^L - d^U - e_2^U W_2 > 0 \tag{H_2}$$

hold, then the species x *and* y *are permanent, that is, for any positive solution* $(x(n), y(n), u_1(n), u_2(n))$ *of system* (1.5) *with the initial conditions* (1.6),

$$\begin{aligned} m_1 &\le \liminf_{n\to+\infty} x(n) \le \limsup_{n\to+\infty} x(n) \le M_1, \\ m_2 &\le \liminf_{n\to+\infty} y(n) \le \limsup_{n\to+\infty} y(n) \le M_2, \\ w_1 &\le \liminf_{n\to+\infty} u_1(n) \le \limsup_{n\to+\infty} u_1(n) \le W_1, \\ w_2 &\le \liminf_{n\to+\infty} u_2(n) \le \limsup_{n\to+\infty} u_2(n) \le W_2, \end{aligned} \tag{3.1}$$

where

$$\begin{aligned} m_1 &= \frac{a^L - c^U/2m^L - e_1^U W_1}{b^U} \exp\left\{a^L - b^U M_1 - \frac{c^U}{2m^L} - e_1^U W_1\right\}, \\ m_2 &= \frac{2m_1(f^L - d^U - e_2^U W_2)}{f^L m^U} \exp\{-d^U - e_2 W_2\}, \\ w_1 &= \frac{q_1^L m_1}{\eta_1^U}, \qquad w_2 = \frac{q_2^L m_2}{\eta_2^U}, \\ M_1 &= \frac{1}{b^L}\exp\{a^U - 1\}, \qquad M_2 = \frac{f^U M_1}{2d^L m^L}\exp\{f^U - d^L\}, \\ W_1 &= \frac{q_1^U M_1}{\eta_1^L}, \qquad W_2 = \frac{q_2^U M_2}{\eta_2^L}. \end{aligned} \tag{3.2}$$

Proof. We divided the proof into five steps.

Step 1. We show

$$\limsup_{n \to +\infty} x(n) \le M_1. \tag{3.3}$$

From the first equation of system (1.5),

$$x(n+1) \le x(n) \exp\{a(n) - b(n)x(n)\}. \tag{3.4}$$

Then (3.3) follows immediately from Lemma 2.3. Thus, for any positive ε, there exists an $n_0 > 0$ such that

$$x(n) \le M_1 + \varepsilon \quad \forall n > n_0. \tag{3.5}$$

Step 2. We prove $\limsup_{n \to +\infty} y(n) \le M_2$ by distinguishing two cases.

Case 1. There exists an $n_1 \ge n_0$ such that $y(n_1+1) \ge y(n_1)$. Then, by the second equation of system (1.5), we have

$$-d(n_1) + \frac{f(n_1)x^2(n_1)}{x^2(n_1) + m^2(n_1)y^2(n_1)} - e_2(n_1)u_2(n_1) \ge 0, \tag{3.6}$$

which implies

$$-d^L + \frac{f^U x(n_1)}{2m^L y(n_1)} \ge 0. \tag{3.7}$$

The above inequality combined with (3.5) leads to $y(n_1) \le f^U(M_1+\varepsilon)/2m^L d^L$. Thus from the second equation of system (1.5) again we have

$$\begin{aligned} y(n_1+1) &\le y(n_1) \exp\left\{ - d(n_1) + \frac{f(n_1)x^2(n_1)}{x^2(n_1) + y^2(n_1)} \right\} \\ &\le y(n_1) \exp\left\{ - d^L + f^U \right\} \\ &\le \frac{f^U(M_1+\varepsilon)}{2m^L d^L} \exp\left\{ - d^L + f^U \right\} \stackrel{\text{def}}{=} M_2^{\varepsilon}. \end{aligned} \tag{3.8}$$

We claim that $y(n) \le M_2^{\varepsilon}$ for all $n \ge n_1$. In fact, suppose there exists $q \ge n_1 + 2$ such that $y(q) > M_2^{\varepsilon}$. Let q_0 be the smallest integer between n_1 and q such that $y(q_0) > M_2^{\varepsilon}$ and $y(q_0 - 1) \le M_2^{\varepsilon}$. Then $y(q_0) > y(q_0 - 1)$ implies $y(q_0) \le M_2^{\varepsilon}$, a contradiction. This proves the claim. Setting $\varepsilon \to 0$ in (3.8) leads to $\limsup_{n \to +\infty} y(n) \le M_2$.

Case 2. Suppose $y(n+1) \leq y(n)$ for all $n \geq n_0$. In this case, $\lim_{n\to+\infty} y(n)$ exists, denoted by $\overline{y}$. We claim that $\overline{y} \leq f^U M_1/2m^L d^L$. If not, suppose $\overline{y} > f^U M_1/2m^L d^L$. Choose $\sigma > 0$ such that $\sigma < \overline{y} - f^U M_1/2m^L d^L$. Taking limit in the second equation of system (1.5) produces

$$\lim_{n\to+\infty}\left(- d(n) + \frac{f(n)x^2(n)}{x^2(n) + m^2(n)y^2(n)} - e_2(n)u_2(n)\right) = 0, \tag{3.9}$$

which is impossible as

$$-d(n) + \frac{f(n)x^2(n)}{x^2(n) + m^2(n)y^2(n)} - e_2(n)u_2(n) \leq -d^L + \frac{f^U x(n)}{2m^L y(n)} < -d^L + \frac{f^U M_1}{2m^L(\overline{y} - \sigma)} < 0 \tag{3.10}$$

for sufficiently large n. This proves the claim. Since (H_2) implies $f^U M_1/2m^L d^L \leq M_2$, we have proved $\limsup_{n\to+\infty} y(n) \leq M_2$.

Step 3. We verify

$$\limsup_{n\to+\infty} u_1(n) \leq W_1, \qquad \limsup_{n\to+\infty} u_2(n) \leq W_2. \tag{3.11}$$

For any positive ε, there exists n_2 such that

$$x(n) \leq M_1 + \varepsilon, \quad y(n) \leq M_2 + \varepsilon \quad \text{for } n \geq n_2. \tag{3.12}$$

For $n \geq n_2$, (3.12) combined with the third equation of (1.5) gives

$$\Delta u_1(n) \leq -\eta_1(n)u_1(n) + q_1(n)(M_1 + \varepsilon). \tag{3.13}$$

Thus

$$u_1(n+1) \leq (1 - \eta_1^L)u_1(n) + q_1^U(M_1 + \varepsilon) \quad \text{for } n \geq n_2. \tag{3.14}$$

With the help of Lemmas 2.1 and 2.2, we obtain

$$\limsup_{n\to+\infty} u_1(n) \leq \frac{q_1^U(M_1 + \varepsilon)}{\eta_1^L}. \tag{3.15}$$

Letting $\varepsilon \to 0$, we immediately get

$$\limsup_{n\to+\infty} u_1(n) \leq W_1. \tag{3.16}$$

Similarly, one can show $\limsup_{n\to+\infty} u_2(n) \leq W_2$.

Step 4. We check

$$\liminf_{n\to+\infty} x(n) \geq m_1, \qquad \limsup_{n\to+\infty} y(n) \geq m_2. \tag{3.17}$$

Conditions (H_1) and (H_2) imply that

$$a^L - \frac{c^U}{2m^L} - e_1^U(W_1+\varepsilon) > 0, \qquad f^L - d^U - e_2^U(W_1+\varepsilon) > 0 \tag{3.18}$$

hold for all small enough positive constant ε. For any such ε, there exists n_3 such that

$$x(n) \leq M_1 + \varepsilon, \quad u_1(n) \leq W_1 + \varepsilon \quad \text{for } n \geq n_3. \tag{3.19}$$

Then, for $n \geq n_3$, it follows from (3.19) and the first equation of system (1.5) that

$$x(n+1) \geq x(n)\exp\left\{a^L - \frac{c^U}{2m^L} - e_1^U(W_1+\varepsilon) - b^U x(n)\right\}. \tag{3.20}$$

According to Lemma 2.4, one has

$$\liminf_{n\to+\infty} x(n) \geq \frac{a^L - c^U/2m^L - e_1^U(W_1+\varepsilon)}{b^U}\exp\left\{a^L - \frac{c^U}{2m^L} - e_1^U(W_1+\varepsilon) - b^U(M_1+\varepsilon)\right\}. \tag{3.21}$$

Letting $\varepsilon \to 0$ leads to

$$\liminf_{n\to+\infty} x(n) \geq m_1. \tag{3.22}$$

Now, for any small positive $\varepsilon_1 < m_1/2$, it follows from (3.19) and (3.22) that there exists $n_4 > 0$ such that

$$x(n) \geq m_1 - \varepsilon_1, \quad y(n) \leq M_2 + \varepsilon_1, \quad u_2(n) \leq W_2 + \varepsilon_1 \tag{3.23}$$

for $n \geq n_4$. This, combined with the second equation of system (1.5), gives

$$y(n+1) \geq y(n)\exp\left\{-d^U + f^L - e_2^U(W_2+\varepsilon_1) - \frac{f^L m^U y(n)}{2(m_1-\varepsilon_1)}\right\}. \tag{3.24}$$

Applying Lemma 2.4, one easily obtains

$$\liminf_{n\to+\infty} y(n) \geq \frac{2(m_1-\varepsilon_1)(f^L-d^U-e_2^U(W_2+\varepsilon_1))}{f^L m^U} \exp\left\{f^L-d^U-e_2^U(W_2+\varepsilon_1)-\frac{f^L m^U(M_2+\varepsilon_1)}{2(m_1-\varepsilon_1)}\right\}. \tag{3.25}$$

Because of the arbitrariness of ε, it is not difficult to see that $\liminf_{n\to+\infty} y(n) \geq m_2$.

Step 5. Finally, we only show $\liminf_{n\to+\infty} u_1(n) \geq w_1$ as the proof of $\liminf_{n\to+\infty} u_2(n) \geq w_2$ is similar. For any $\varepsilon_2 > 0$ such that $\varepsilon_2 < m_1$, there exists n_5 such that

$$x(n) \geq m_1 - \varepsilon_2 \quad \text{for } n \geq n_5. \tag{3.26}$$

This and the third equation of system (1.5) imply that

$$\Delta u_1(n) \geq -\eta_1(n)u_1(n) + q_1^L(m_1-\varepsilon_2) \tag{3.27}$$

for $n \geq n_5$. Then, for $n \geq n_5$,

$$u_1(n+1) \geq (1-\eta_1^U)u_1(n) + q_1^L(m_1-\varepsilon_2). \tag{3.28}$$

It follows from Lemmas 2.1 and 2.2 immediately that

$$\liminf_{n\to+\infty} u_1(n) \geq \frac{q_1^L(m_1-\varepsilon_2)}{\eta_1^U}. \tag{3.29}$$

Letting $\varepsilon_2 \to 0$ gives $\liminf_{n\to+\infty} u_1(n) \geq w_1$. This completes the proof of the theorem. □

Theorem 3.2. *Assume that* (H_1) *and* (H_2) *hold. Assume further that there exist positive constants* α, β, γ, ζ, *and* δ *such that*

$$\alpha \min\left\{b^L, \frac{2}{M_1}-b^U\right\} - \alpha\frac{c^U}{4(m^L)^2 m_2} - \alpha\frac{c^U M_1}{4{m_1}^2} - \beta\frac{f^U M_2}{2m_2 m_1} - \gamma q_1^U > \delta, \tag{H_3}$$

$$\beta \min\left\{\frac{2f^L(m^L)^2 m_1^2 m_2}{(M_1^2+(m^U)^2M_2^2)^2}, \frac{2}{M_2}-\frac{f^U M_1}{2m_1 m_2}\right\} - \alpha\frac{c^U M_1}{4(m^L)^2 m_2^2} - \alpha\frac{c^U}{4m_1} - \zeta q_2^U > \delta, \tag{H_4}$$

$$\gamma \eta_1^L - \alpha e_1^U > \delta, \tag{H_5}$$

$$\zeta \eta_2^L - \beta e_2^U > \delta. \tag{H_6}$$

Then the species x and y are globally attractive, that is, for any positive solutions $(x_1(n), y_1(n), u_1(n), u_2(n))$ and $(x_2(n), y_2(n), \widetilde{u_1}(n), \widetilde{u_2}(n))$ of system (1.5) *with the initial conditions* (1.6),

$$\lim_{n\to+\infty}\left|x_1(n)-x_2(n)\right|=0, \qquad \lim_{n\to+\infty}\left|y_1(n)-y_2(n)\right|=0,$$
$$\lim_{n\to+\infty}\left|u_1(n)-\widetilde{u_1}(n)\right|=0, \qquad \lim_{n\to+\infty}\left|u_2(n)-\widetilde{u_2}(n)\right|=0. \tag{3.30}$$

Proof. From conditions (H_3)–(H_6), there exists small enough positive constant $\varepsilon < \min\{m_1/2, m_2/2\}$ such that

$$\alpha\min\left\{b^L, \frac{2}{(M_1+\varepsilon)}-b^U\right\}-\alpha\frac{c^U}{4(m^L)^2(m_2-\varepsilon)}-\alpha\frac{c^U(M_1+\varepsilon)}{4(m_1-\varepsilon)^2}-\beta\frac{f^U(M_2+\varepsilon)}{2(m_1-\varepsilon)(m_2-\varepsilon)}-\gamma q_1^U>\delta,$$
$$\beta\min\left\{\frac{2f^L(m^L)^2(m_1-\varepsilon)^2(m_2-\varepsilon)}{\left((M_1+\varepsilon)^2+(m^U)^2(M_2+\varepsilon)^2\right)^2}, \frac{2}{(M_2+\varepsilon)}-\frac{f^U(M_1+\varepsilon)}{2(m_1-\varepsilon)(m_2-\varepsilon)}\right\}$$
$$-\alpha\frac{c^U(M_1+\varepsilon)}{4(m^L)^2(m_2-\varepsilon)^2}-\alpha\frac{c^U}{4(m_1-\varepsilon)}-\zeta q_2^U>\delta,$$
$$\gamma\eta_1^L-\alpha e_1^U>\delta,$$
$$\zeta\eta_2^L-\beta e_2^U>\delta. \tag{3.31}$$

Since (H_1) and (H_2) hold, for any positive solutions $(x_1(n), y_1(n), u_1(n), u_2(n))$ and $(x_2(n), y_2(n), \widetilde{u_1}(n), \widetilde{u_2}(n))$ of system (1.5) with the initial conditions (1.6), it follows from Theorem 3.1 that

$$m_1\le\liminf_{n\to+\infty}x_i(n)\le\limsup_{n\to+\infty}x_i(n)\le M_1,$$
$$m_2\le\liminf_{n\to+\infty}y_i(n)\le\limsup_{n\to+\infty}y_i(n)\le M_2,$$
$$w_i\le\liminf_{n\to+\infty}u_i(n)\le\limsup_{n\to+\infty}u_i(n)\le W_i,$$
$$w_i\le\liminf_{n\to+\infty}\tilde{u}_i(n)\le\limsup_{n\to+\infty}\tilde{u}_i(n)\le W_i, \quad i=1,2. \tag{3.32}$$

Then, there exists an $n_0>0$ such that, for all $n>n_0$,

$$m_1-\varepsilon\le x_i(n)\le M_1+\varepsilon, \qquad m_2-\varepsilon\le y_i(n)\le M_2+\varepsilon,$$
$$w_i-\varepsilon\le u_i(n)\le W_i+\varepsilon, \quad w_i-\varepsilon\le\tilde{u}_i(n)\le W_i+\varepsilon, \quad i=1,2. \tag{3.33}$$

Let

$$V_1(n) = |\ln x_1(n) - \ln x_2(n)|. \tag{3.34}$$

Then, from the first equation of system (1.5), we have

$$\begin{aligned}
V_1(n+1) &= |\ln x_1(n+1) - \ln x_2(n+1)| \\
&\le |\ln x_1(n) - \ln x_2(n) - b(n)(x_1(n) - x_2(n))| \\
&\quad + c(n)\left|\frac{x_1(n)y_1(n)}{x_1^2(n) + m^2(n)y_1^2(n)} - \frac{x_2(n)y_2(n)}{x_2^2(n) + m^2(n)y_2^2(n)}\right| \\
&\quad + e_1(n)|u_1(n) - \widetilde{u_1}(n)|.
\end{aligned} \tag{3.35}$$

Using the mean value theorem, we get

$$x_1(n) - x_2(n) = \exp(\ln x_1(n)) - \exp(\ln x_2(n)) = \xi_1(n)(\ln x_1(n) - \ln x_2(n)), \tag{3.36}$$

where $\xi_1(n)$ lies between $x_1(n)$ and $x_2(n)$.

It follows from (3.35) and (3.36) that

$$\begin{aligned}
V_1(n+1) \le{}& |\ln x_1(n) - \ln x_2(n)| - \left(\frac{1}{\xi_1(n)} - \left|\frac{1}{\xi_1(n)} - b(n)\right|\right)|x_1(n) - x_2(n)| \\
&+ \left|\frac{c(n)x_1(n)y_1(n)x_2(n)}{(x_1^2(n) + m^2(n)y_1^2(n))(x_2^2(n) + m^2(n)y_2^2(n))}\right||x_1(n) - x_2(n))| \\
&+ \left|\frac{c(n)m^2(n)y_1^2(n)y_2(n)}{(x_1^2(n) + m^2(n)y_1^2(n))(x_2^2(n) + m^2(n)y_2^2(n))}\right||x_1(n) - x_2(n))| \\
&+ \left|\frac{c(n)x_1^2(n)x_2(n)}{(x_1^2(n) + m^2(n)y_1^2(n))(x_2^2(n) + m^2(n)y_2^2(n))}\right||y_1(n) - y_2(n))| \\
&+ \left|\frac{c(n)m^2(n)y_1(n)y_2(n)x_1(n)}{(x_1^2(n) + m^2(n)y_1^2(n))(x_2^2(n) + m^2(n)y_2^2(n))}\right||y_1(n) - y_2(n))| \\
&+ e_1(n)|u_1(n) - \widetilde{u_1}(n)|.
\end{aligned} \tag{3.37}$$

So, for $n \geq n_0$,

$$\begin{aligned}\Delta V_1(n) \leq & -\min\left\{b^L, \frac{2}{M_1+\varepsilon} - b^U\right\}|x_1(n)-x_2(n)| + \frac{c^U}{4(m^L)^2(m_2-\varepsilon)}|x_1(n)-x_2(n)| \\ & + \frac{c^U(M_1+\varepsilon)}{4(m_1-\varepsilon)^2}|x_1(n)-x_2(n)| + \frac{c^U(M_1+\varepsilon)}{4(m^L)^2(m_2-\varepsilon)^2}|y_1(n)-y_2(n)| \\ & + \frac{c^U}{4(m_1-\varepsilon)}|y_1(n)-y_2(n)| + e_1^U|u_1(n)-\widetilde{u_1}(n)|.\end{aligned} \tag{3.38}$$

Let

$$V_2(n) = |\ln y_1(n) - \ln y_2(n)|. \tag{3.39}$$

Then, from the second equation of system (1.5), we have

$$\begin{aligned}& V_2(n+1) \\ & = |\ln y_1(n+1) - \ln y_2(n+1)| \\ & = \left|\ln y_1(n) - \ln y_2(n) + f(n)\left(\frac{x_1^2(n)}{x_1^2+m^2(n)y_1^2(n)} - \frac{x_2^2(n)}{x_2^2(n)+m^2(n)y_2^2(n)}\right) - e_2(n)(u_2(n)-\widetilde{u_2}(n))\right| \\ & \leq \left|\ln y_1(n) - \ln y_2(n) - \frac{f(n)m^2(n)x_1(n)(x_1(n)y_2(n)+y_1(n)x_2(n))}{(x_1^2(n)+m^2(n)y_1^2(n))(x_2^2(n)+m^2(n)y_2^2(n))}\right| \\ & \quad + \frac{f(n)m^2(n)y_1(n)(x_1(n)y_2(n)+y_1(n)x_2(n))}{(x_1^2(n)+m^2(n)y_1^2(n))(x_2^2(n)+m^2(n)y_2^2(n))} \\ & \quad \times |x_1(n)-x_2(n)| + e_2(n)|u_1(n)-\widetilde{u_1}(n)|.\end{aligned} \tag{3.40}$$

Using the mean value theorem, we get

$$y_1(n) - y_2(n) = \exp(\ln y_1(n)) - \exp(\ln y_2(n)) = \xi_2(n)(\ln y_1(n) - \ln y_2(n)), \tag{3.41}$$

where $\xi_2(n)$ lies between $y_1(n)$ and $y_2(n)$.

Now, it follows from (3.40) and (3.41) that, for $n > n_0$,

$$\begin{aligned}
&\Delta V_2(n) \\
&\le -\left(\frac{1}{\xi_2(n)} - \left|\frac{1}{\xi_2(n)} - \frac{f(n)m^2(n)x_1(n)(x_1(n)y_2(n)+y_1(n)x_2(n))}{(x_1^2(n)+m^2(n)y_1^2(n))(x_2^2(n)+m^2(n)y_2^2(n))}\right|\right)|y_1(n)-y_2(n)| \\
&\quad + \frac{f(n)m^2(n)y_1(n)(x_1(n)y_2(n)+y_1(n)x_2(n))}{(x_1^2(n)+m^2(n)y_1^2(n))(x_2^2(n)+m^2(n)y_2^2(n))}|x_1(n)-x_2(n)| + e_2(n)|u_2(n)-\widetilde{u_2}(n)| \\
&\le -\min\left\{\frac{2f^L(m^L)^2(m_1-\varepsilon)^2(m_2-\varepsilon)}{((M_1+\varepsilon)^2+(m^U)^2(M_2+\varepsilon))^2}, \frac{2}{(M_2+\varepsilon)} - \frac{f^U(M_1+\varepsilon)}{2(m_1-\varepsilon)(m_2-\varepsilon)}\right\} \\
&\quad \times |y_1(n)-y_2(n)| + \frac{f^U(M_2+\varepsilon)}{2(m_1-\varepsilon)(m_2-\varepsilon)}|x_1(n)-x_2(n)| + e_2^U|u_2(n)-\widetilde{u_2}(n)|.
\end{aligned} \tag{3.42}$$

Let

$$V_3(n) = \left|u_1(n) - \widetilde{u_1}(n)\right|. \tag{3.43}$$

Then, from the third equation of system (1.5), one can easily obtain that

$$\begin{aligned}
V_3(n+1) &= \left|u_1(n+1) - \widetilde{u_1}(n+1)\right| \\
&\le (1-\eta_1(n))\left|u_1(n) - \widetilde{u_1}(n)\right| + q_1(n)|x_1(n)-x_2(n)|.
\end{aligned} \tag{3.44}$$

It follows from (3.44) that, for $n > n_0$,

$$\Delta V_3(n) \le -\eta_1^L\left|u_1(n) - \widetilde{u_1}(n)\right| + q_1^U\left|x_1(n)-x_2(n)\right|. \tag{3.45}$$

Let

$$V_4(n) = \left|u_2(n) - \widetilde{u_2}(n)\right|. \tag{3.46}$$

Similar to the analysis of (3.44) and (3.45), we can get

$$\Delta V_4(n) \le -\eta_2^L\left|u_2(n) - \widetilde{u_2}(n)\right| + q_2^U\left|y_1(n)-y_2(n)\right|. \tag{3.47}$$

Now, we define a Lyapunov function as follows:

$$V(n) = \alpha V_1(n) + \beta V_2(n) + \gamma V_3(n) + \zeta V_4(n). \tag{3.48}$$

For $n > n_0$, it follows from (3.38), (3.42), (3.45), and (3.47) that

$$\begin{aligned}\Delta V(n) \le &-\left[\alpha \min\left\{b^L, \frac{2}{(M_1+\varepsilon)} - b^U\right\} - \alpha\frac{c^U}{4(m^L)^2(m_2-\varepsilon)} - \alpha\frac{c^U(M_1+\varepsilon)}{4(m_1-\varepsilon)^2}\right.\\
&\left.\quad - \beta\frac{f^U(M_2+\varepsilon)}{2(m_1-\varepsilon)(m_2-\varepsilon)} - \gamma q_1^U\right] \times |x_1(n) - x_2(n)|\\
&- \left[\beta \min\left\{\frac{2f^L(m^L)^2(m_1-\varepsilon)^2(m_2-\varepsilon)}{\left((M_1+\varepsilon)^2 + (m^U)^2(M_2+\varepsilon)^2\right)^2}, \frac{2}{(M_2+\varepsilon)} - \frac{2f^U(M_1+\varepsilon)}{2(m_1-\varepsilon)(m_2-\varepsilon)}\right\}\right.\\
&\left.\quad - \alpha\frac{c^U(M_1+\varepsilon)}{4(m^L)^2(m_2-\varepsilon)^2} - \alpha\frac{c^U}{4(m_1-\varepsilon)} - \zeta q_2^U\right] \times |y_1(n) - y_2(n)|\\
&- (\gamma\eta_1^L - \alpha e_1^u) \times \left|u_1(n) - \widetilde{u_1}(n)\right| - (\zeta\eta_2^L - \beta e_2^U) \times \left|u_2(n) - \widetilde{u_2}(n)\right|\\
\le &-\delta(|x_1(n) - x_2(n)| + |y_1(n) - y_2(n)| + \left|u_1(n) - \widetilde{u_1}(n)\right| + |u_2(n) - \widetilde{u_2}(n)|).\end{aligned} \tag{3.49}$$

Summating both sides of the above inequalities from n_0 to n, we have

$$\begin{aligned}&\sum_{p=n_0}^{n}(V(p+1) - V(p))\\
&\quad\le -\delta\sum_{p=n_0}^{n}\left(|x_1(p) - x_2(p)| + |y_1(p) - y_2(p)| + \left|u_1(p) - \widetilde{u_1}(p)\right| + \left|u_2(p) - \widetilde{u_2}(p)\right|\right),\end{aligned} \tag{3.50}$$

which implies

$$V(n_0) \ge \delta\sum_{p=n_0}^{n}\left(|x_1(p) - x_2(p)| + |y_1(p) - y_2(p)| + \left|u_1(p) - \widetilde{u_1}(p)\right| + \left|u_2(p) - \widetilde{u_2}(p)\right|\right) + V(n+1). \tag{3.51}$$

It follows that

$$\frac{V(n_0)}{\delta} \ge \sum_{p=n_0}^{n}\left(|x_1(p) - x_2(p)| + |y_1(p) - y_2(p)| + \left|u_1(p) - \widetilde{u_1}(p)\right| + \left|u_2(p) - \widetilde{u_2}(p)\right|\right). \tag{3.52}$$

Letting $n \to \infty$ gives

$$\frac{V(n_0)}{\delta} \geq \sum_{p=n_0}^{+\infty} \left(\left| x_1(p) - x_2(p) \right| + \left| y_1(p) - y_2(p) \right| + \left| u_1(p) - \widetilde{u_1}(p) \right| + \left| u_2(p) - \widetilde{u_2}(p) \right| \right), \tag{3.53}$$

which implies that

$$\lim_{n \to \infty} \left(\left| x_1(n) - x_2(n) \right| + \left| y_1(n) - y_2(n) \right| + \left| u_1(n) - \widetilde{u_1}(n) \right| + \left| u_2(n) - \widetilde{u_2}(n) \right| \right) = 0, \tag{3.54}$$

that is,

$$\begin{aligned} &\lim_{n \to \infty} \left| x_1(n) - x_2(n) \right| = 0, \qquad \lim_{n \to \infty} \left| y_1(n) - y_2(n) \right| = 0, \\ &\lim_{n \to \infty} \left| u_1(n) - \widetilde{u_1}(n) \right| = 0, \qquad \lim_{n \to \infty} \left| u_1(n) - \widetilde{u_1}(n) \right| = 0. \end{aligned} \tag{3.55}$$

This completes the proof of Theorem 3.2. □

4. Extinction of the predator species

This section is devoted to studying the extinction of the predator species y.

Theorem 4.1. *Assume that*

$$-d^L + f^U < 0. \tag{H_7}$$

Then, for any solution $(x(n), y(n), u_1(n), u_2(n))$ *of system* (1.5),

$$\lim_{n \to +\infty} y(n) = 0. \tag{4.1}$$

Proof. From condition (H_7), there exists small enough positive constant $\gamma > 0$ such that

$$-d^L + f^U < -\gamma < 0. \tag{4.2}$$

For all $n \in \mathbb{N}$, from (4.2) and the second equation of system (1.5), one can easily obtain that

$$\begin{aligned} y(n+1) &= y(n) \exp\left\{ -d(n) + \frac{f(n)x^2(n)}{x^2(n) + m^2(n)y^2(n)} - e_2(n)u_2(n) \right\} \\ &< y(n) \exp\{-d^L + f^U\} \\ &< y(n) \exp\{-\gamma\}. \end{aligned} \tag{4.3}$$

Therefore,

$$y(n+1) < y(0) \exp\{-n\gamma\}, \tag{4.4}$$

which yields

$$\lim_{n\to+\infty} y(n) = 0. \tag{4.5}$$

The proof of Theorem 4.1 is complete. □

5. Examples

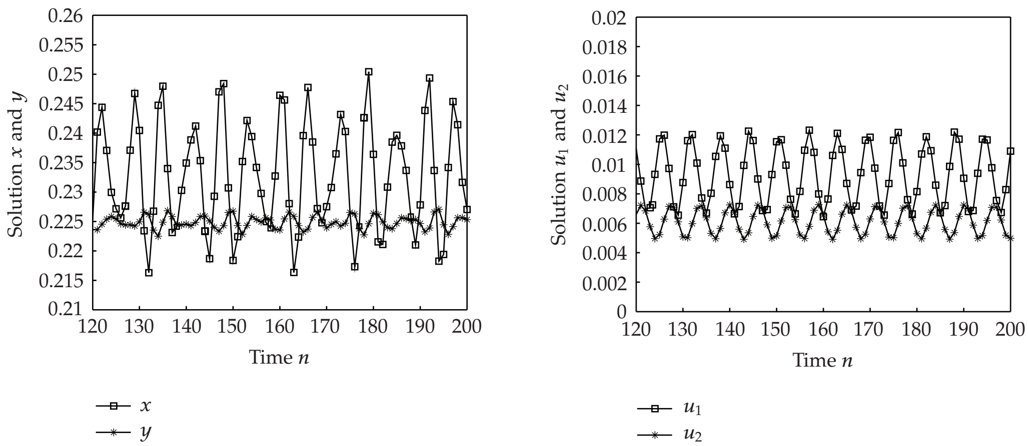
The following two examples show the feasibility of the main results.

Example 5.1. Consider the following system:

$$\begin{aligned}
x(n+1) &= x(n)\exp\Bigg\{1.225 + 0.025\sin\left(\sqrt{2}n\right) - (5.25 + 0.25\cos(n))x(n) \\
&\quad - \frac{((0.0075 + 0.0025\sin(n))y(n)x(n)}{x^2(n) + \left(0.885 + 0.005\cos\left(\sqrt{3}n\right)\right)^2 y^2(n)} \\
&\quad - \left(0.002 + 0.001\cos\left(\sqrt{3}n\right)\right)u_1(n)\Bigg\}, \\
y(n+1) &= y(n)\exp\Bigg\{ - \left(0.1125 + 0.0025\cos\left(\sqrt{2}n\right)\right) \\
&\quad + \frac{\left(0.195 + 0.005\sin\left(\sqrt{3}n\right)\right)x^2(n)}{x^2(n) + \left(0.885 + 0.005\cos\left(\sqrt{3}n\right)\right)^2 y^2(n)} \\
&\quad - (0.0015 + 0.0005\sin(n))u_2(n)\Bigg\}, \\
\Delta u_1(n) &= -(0.925 + 0.025\sin(n))u_1(n) + (0.0375 + 0.0275\cos(n))x(n), \\
\Delta u_2(n) &= -(0.925 + 0.025\cos(n))u_2(n) + (0.025 + 0.005\sin(n))y(n).
\end{aligned} \tag{5.1}$$

One could easily see that there exist $\alpha = 0.06$, $\beta = 0.05$, $\gamma = 0.01$, $\zeta = 0.001$, and $\delta = 0.00001$ such that

$$\begin{gathered}
a^L - \frac{c^U}{2m^L} - e_1^U W_1 \approx 1.2 > 0, \\
f^L - d^U - e_2^U W_2 \approx 0.04 > 0, \\
\alpha\min\left\{b^L, \frac{2}{M_1} - b^U\right\} - \alpha\frac{c^U}{4(m^L)^2 m_2} - \alpha\frac{c^U M_1}{4m_1^2} - \beta\frac{f^U M_2}{2m_1 m_2} - \gamma q_1^U \approx 0.04512 > \delta, \\
\beta\min\left\{\frac{2f^L(m^L)^2 m_1^2 m_2}{M_1^2 + (m^U)^2 M_2^2}, \frac{2}{M_2} - \frac{f^U M_1}{2m_1 m_2}\right\} - \alpha\frac{c^U M_1}{4(m^L)^2 m_2} - \alpha\frac{c^U}{4m_1} - \zeta q_2^U \approx 0.000013 > \delta, \\
\gamma\eta_1^L - \alpha e_1^U \approx 0.0072 > \delta, \\
\zeta\eta_2^L - \beta e_2^U \approx 0.00087 > \delta.
\end{gathered} \tag{5.2}$$



(a) Species $x,\ y$

(b) Feedback controls $u_1,\ u_2$

Figure 1: Dynamics behaviors of system (5.1) with initial conditions $(x(0), y(0), u_1(0), u_2(0))^T = (0.31, 0.26, 0.02, 0.08)^T$ and $(0.18, 0.18, 0.01, 0.1)^T$, respectively.

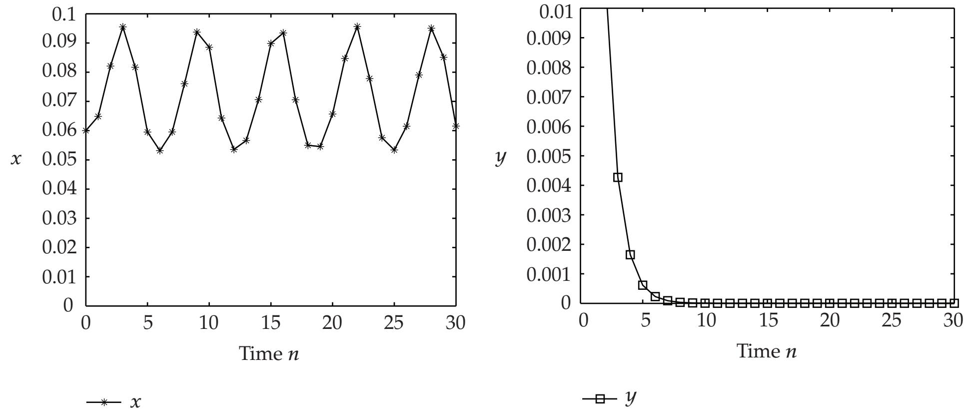
Clearly, conditions (H_1)–(H_6) are satisfied. From Theorems 3.1 and 3.2, the system is permanent and globally attractive. Numeric simulation (Figure 1) strongly supports our results.

Example 5.2. Consider the following system:

$$\begin{aligned}
x(n+1) &= x(n)\exp\Bigg\{0.8+0.1\sin(n)-(11.5+3.5\cos(n))x(n) \\
&\qquad -\frac{(0.035+0.025\sin(n))y(n)x(n)}{x^2(n)+\left(5.75+0.35\cos\left(\sqrt{2}n\right)\right)^2 y^2(n)}-(0.025+0.005\cos(n))u_1(n)\Bigg\}, \\
y(n+1) &= y(n)\exp\Bigg\{-\left(0.975+0.025\cos\left(\sqrt{2}n\right)\right)+\frac{(0.195+0.005\sin(\sqrt{3}n))x^2(n)}{x^2(n)+(5.75+0.35\cos(\sqrt{2}n))^2 y^2(n)} \\
&\qquad -\left(0.0015+0.0005\cos\left(\sqrt{3}n\right)\right)u_2(n)\Bigg\}, \\
\Delta u_1(n) &= -(0.925+0.025\sin(n))u_1(n)+(0.0375+0.0275\cos(n))x(n), \\
\Delta u_2(n) &= -(0.925-0.025\cos(n))u_2(n)+(0.025+0.005\sin(n))y(n).
\end{aligned} \tag{5.3}$$

By simple computation, we can easily have

$$-d^L+f^U=-0.8<0. \tag{5.4}$$



(a) Species x

(b) Species y

Figure 2: Dynamics behaviors of system (5.2) with initial conditions $(x(0), y(0), u_1(0), u_2(0))^T = (0.06, 0.08, 0.04, 0.02)^T$.

Thus, condition (H_7) is satisfied; from Theorem 4.1, it follows that $\lim_{n\to+\infty} y(n) = 0$. Numeric simulation (Figure 2) strongly supports our result.

Acknowledgment

This work was supported by the Program for New Century Excellent Talents in Fujian Province University (0330-003383).

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