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Research Article

Existence of Monotone Solutions of a Difference Equation

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We consider the nonlinear difference equation $x_{n+1} = f(x_{n-k}, x_{n-k+1}, \ldots, x_n)$, $n = 0, 1, \ldots,$ where $k \in \{1, 2, \ldots\}$ and the initial values $x_{-k}, x_{-k+1}, \ldots, x_0 \in (0, +\infty)$. We give sufficient conditions under which this equation has monotone positive solutions which converge to the equilibrium, extending and including in this way some results of the literature.

1. Introduction

In this paper, we study the existence of monotone positive solutions which converge to the equilibrium of a nonlinear difference equation. Recently, there has been a lot of interest in studying such solutions and the existence of some specific solutions (see [1–20]). In [8], Karakostas and Stević studied the boundedness, global attractivity, and oscillatory and asymptotic periodicity of the nonnegative solutions of the difference equation

$$x_{n+1} = B + \frac{x_{n-k}}{a_0 x_n + \cdots + a_{k-1} x_{n-k+1} + \gamma}, \quad n = 0, 1, \ldots, \tag{E1}$$

where $B \geq 0$, $\gamma > 0$, $k \in \{1, 2, \ldots\}$, and $a_i \geq 0$ for every $i \in \{0, \ldots, k-1\}$ with $\sum_{i=0}^{k-1} a_i > 0$ and the initial conditions $x_{-k}, \ldots, x_0 \in (0, +\infty)$. They proposed the following open problem.

Open problem A. Let $\gamma = 1$, $B = 0$, and $k \geq 2$. Is there a positive solution $\{x_n\}$ of (E1) such that $x_n \to 0$ as $n \to \infty$?

In [5], Devault et al. studied the boundedness, global stability, and periodic character of positive solutions of the difference equation

$$x_{n+1} = p + \frac{x_{n-k}}{x_n}, \quad n = 0, 1, \ldots, \tag{E2}$$

where $k \in \{1, 2, \ldots\}$, $p \in (0, +\infty)$, and the initial conditions $x_{-k}, \ldots, x_0 \in (0, +\infty)$. They proposed the following Open problem B (which has been solved in [1, 18] by quite different methods).

Open problem B. Do there exist nonoscillatory solutions of (E2)?

Recently, Stević [12] studied the following difference equations:

$$x_{n+1} = p + \frac{x_{n-k}}{\alpha_0 x_n + \cdots + \alpha_{k-1} x_{n-k+1}}, \quad n = 0, 1, \ldots, \tag{E3}$$

$$x_{n+1} = \frac{1 + x_{n-k}}{\alpha_0 x_n + \cdots + \alpha_{k-1} x_{n-k+1}}, \quad n = 0, 1, \ldots, \tag{E4}$$

$$x_{n+1} = \frac{\alpha + x_{n-k}}{1 + \alpha_0 x_n + \cdots + \alpha_{k-1} x_{n-k+1}}, \quad n = 0, 1, \ldots, \tag{E5}$$

where $p > -1$, $\alpha > 0$, $k \in \{1, 2, \ldots\}$, and $\alpha_i \geq 0$ for every $i \in \{0, \ldots, k-1\}$ with $\sum_{i=0}^{k-1} \alpha_i = 1$ and the initial conditions $x_{-k}, \ldots, x_0 \in (0, +\infty)$. He proved that (E3), (E4), and (E5) have positive solutions which decrease to the equilibrium.

The main theorem in this paper is motivated by the above studies and [17]. In this paper, we consider the following nonlinear difference equation:

$$x_{n+1} = f(x_{n-k}, x_{n-k+1}, \ldots, x_n), \quad n = 0, 1, \ldots, \tag{1.1}$$

where $k \in \{1, 2, \ldots\}$, the initial values $x_{-k}, x_{-k+1}, \ldots, x_0 \in (0, +\infty)$, and $f \in C(E^{k+1}, E)$, where $C(E^{k+1}, E)$ denotes the set of all continuous maps from E^{k+1} to E and $E = (0, +\infty)$ or $E = [0, +\infty)$. Using arguments similar to ones developed in the proof of main theorem in [18], we prove that under appropriate conditions (see (C_1)–(C_5) below) this difference equation has monotone solutions converging to the equilibrium $\overline{x}$.

2. Main result

In this section, we assume that f satisfies the following conditions.

(C_1) $f \in C(E^{k+1}, E)$ and $f(z_0, z_1, \ldots, z_k)$ is increasing in z_0 (i.e., $f(a, z_1, \ldots, z_k) > f(b, z_1, \ldots, z_k)$ if $a > b$), where $E = (0, +\infty)$ or $E = [0, +\infty)$ and $k \geq 1$ is an integer.

(C_2) Equation (1.1) has the unique nonnegative equilibrium, denoted by $\overline{x}$.

(C_3) $A = \{(z_0, z_1, \ldots, z_k) : z_0 \geq z_1 \geq \cdots \geq z_k \geq f(z_0, z_1, \ldots, z_k) \geq \overline{x}\}$ is an unbounded connected (closed) set.

Now we formulate and prove the main result of this paper.

Theorem 2.1. *Let f satisfy (C_1)–(C_3). If there exists $g \in C([\overline{x},+\infty)^{k+1},[\overline{x},+\infty))$ such that the following two conditions hold:*

(C_4) $A \subset B = \{(z_0,z_1,\dots,z_k) : g(z_0,z_1,\dots,z_k) \geq z_0 \geq z_1 \geq \cdots \geq z_k \geq \overline{x}\}$,

(C_5) $z_k = f(g(z_0,z_1,\dots,z_k),z_0,z_1,\dots,z_{k-1})$ *for any* $(z_0,z_1,\dots,z_k) \in [\overline{x},+\infty)^{k+1}$, *then* (1.1) *has a monotone positive solution which converges to the equilibrium* $\overline{x}$.

Proof. Define $F : A \to B$ by

$$F(z_0,z_1,\dots,z_k) = (u_0,u_1,\dots,u_k) \equiv (z_1,z_2,\dots,z_k,f(z_0,z_1,\dots,z_k)), \tag{2.1}$$

for all $(z_0,z_1,\dots,z_k) \in A$.

Claim 1. F is well defined.

Proof of Claim 1. From (2.1) and the definition of A, we have

$$\begin{gathered} u_i = z_{i+1}, \quad \text{for } i \in \{0,1,\dots,k-1\}, \\ u_k = f(z_0,z_1,\dots,z_k) \geq \overline{x}. \end{gathered} \tag{2.2}$$

It follows from (2.2) and (C_5) that

$$f(z_0,u_0,\dots,u_{k-1}) = u_k = f(g(u_0,u_1,\dots,u_k),u_0,u_1,\dots,u_{k-1}), \tag{2.3}$$

which with (C_1) implies

$$g(u_0,u_1,\dots,u_k) = z_0 \geq u_0 \geq \cdots \geq u_k \geq \overline{x}. \tag{2.4}$$

Thus, $(u_0,u_1,\dots,u_k) \in B$. Claim 1 is proved. □

Claim 2. F is a bijection from A to B.

Proof of Claim 2. Let $z = (z_0,z_1,\dots,z_k)$ and $y = (y_0,y_1,\dots,y_k) \in A$ with $z \neq y$. If $z_i \neq y_i$ for some $i \in \{1,\dots,k\}$, then $F(y) \neq F(z)$. If $z_0 \neq y_0$ and $z_i = y_i$ for every $i \in \{1,\dots,k\}$, then from (C_1) we have

$$f(z_0,z_1,\dots,z_k) \neq f(y_0,z_1,\dots,z_k), \tag{2.5}$$

which also implies $F(y) \neq F(z)$.

On the other hand, for any $u = (u_0,u_1,\dots,u_k) \in B$, we have

$$g(u_0,u_1,\dots,u_k) \geq u_0 \geq u_1 \geq \cdots \geq u_k \geq \overline{x}. \tag{2.6}$$

Choose

$$z = (z_0,z_1,\dots,z_k) \equiv (g(u_0,u_1,\dots,u_k),u_0,u_1,\dots,u_{k-1}). \tag{2.7}$$

It follows from (2.6), (2.7), and (C_5) that

$$z_k = u_{k-1} \geq u_k = f\left(g\left(u_0, u_1, \ldots, u_k\right), u_0, u_1, \ldots, u_{k-1}\right) = f\left(z_0, z_1, \ldots, z_k\right) \geq \overline{x}, \tag{2.8}$$

which implies $z \in A$. From (2.1) and (C_5), we obtain

$$\begin{aligned} F(z) &= \left(z_1, \ldots, z_k, f\left(z_0, z_1, \ldots, z_k\right)\right) \\ &= \left(u_0, \ldots, u_{k-1}, f\left(g\left(u_0, u_1, \ldots, u_k\right), u_0, u_1, \ldots, u_{k-1}\right)\right) \\ &= \left(u_0, u_1, \ldots, u_k\right) = u. \end{aligned} \tag{2.9}$$

Claim 2 is proved. □

Furthermore, since $F^{-1}(u_0, u_1, \ldots, u_k) = (g(u_0, u_1, \ldots, u_k), u_0, u_1, \ldots, u_{k-1})$ is continuous, F is a homeomorphism from A to B.

Since $A \subset B$ and F is a homeomorphism from A onto B, it follows that $F^{-1}(A) \subset F^{-1}(B) = A$. By induction, we have

$$\overline{\mathbf{x}} = \left(\overline{x}, \overline{x}, \ldots, \overline{x}\right) \in F^{-n}(A) \subset F^{-n+1}(A) \tag{2.10}$$

for every positive integer n. Because A is an unbounded connected closed set, we know that $F^{-n}(A)$ is an unbounded connected closed set for every positive integer n. Let

$$S = \bigcap_{i=0}^{\infty} F^{-i}(A). \tag{2.11}$$

Claim 3. S is an unbounded connected set.

Proof of Claim 3. Indeed, if S is a bounded connected closed set, then there exists $\beta > 0$ such that $S \subset B(\overline{\mathbf{x}}, \beta) \equiv \{\mathbf{x} \in E^{k+1} : d(\overline{\mathbf{x}}, \mathbf{x}) < \beta\}$. Since $F^{-n}(A)$ is an unbounded connected closed set for every positive integer n, it follows that $K_n = [\{\mathbf{x} : d(\overline{\mathbf{x}}, \mathbf{x}) \leq 2\beta\} - B(\overline{\mathbf{x}}, \beta)] \cap F^{-n}(A) \neq \varnothing$ and K_n is a bounded closed set. Let $\mathbf{x}_n \in K_n$, then there exist the positive integers $n_1 < n_2 < \cdots < n_k < \cdots$ and a point $\mathbf{v} \in \{\mathbf{x} : d(\overline{\mathbf{x}}, \mathbf{x}) \leq 2\beta\} - B(\overline{\mathbf{x}}, \beta)$ such that $\lim_{k \to \infty} \mathbf{x}_{n_k} = \mathbf{v}$. Notice that $\mathbf{v} \notin S$. On the other hand, for every positive integer n, there exists N such that $\mathbf{x}_{n_k} \in F^{-n}(A)$ if $n_k > N$, which implies $\mathbf{v} \in F^{-n}(A)$. Thus $\mathbf{v} \in S$, which is a contradiction. Claim 3 is proved. □

Now suppose that $\{x_n\}_{n=-k}^{\infty}$ is a positive solution of (1.1) with $(x_{-k}, \ldots, x_0) \in S - \overline{\mathbf{x}}$; we can show that for all positive integer n,

$$F^n\left(x_{-k}, \ldots, x_0\right) = \left(x_{n-k}, x_{n-k+1}, \ldots, x_n\right) \in A. \tag{2.12}$$

Thus, $\{x_n\}_{n=-k}^{\infty}$ is a monotone positive solution. Let

$$\lim_{n \to \infty} x_n = a, \tag{2.13}$$

then

$$a = f(a, a, \dots, a) \geq \overline{x}. \tag{2.14}$$

It follows from (C_2) that $a = \overline{x}$. Thus, $\{x_n\}_{n=-k}^{\infty}$ is a nontrivial monotone positive solution which converges to $\overline{x}$. Theorem 2.1 is proved. □

Remark 2.2. From the proof of Theorem 2.1, we can conclude that (1.1) has infinitely many monotone positive solutions which converge to the equilibrium $\overline{x}$.

Remark 2.3. In [21], Stević gave another proof of Claim 3 of Theorem 2.1 for the case of equation $x_n = x_{n-k}/(1 + x_{n-1} + \cdots + x_{n-(k-1)})$.

3. Example and some remarks

In this section, we will give an application of Theorem 2.1 and some remarks.

Example 3.1. Consider the equation

$$x_{n+1} = p + \frac{a + x_{n-k}}{b + \sum_{i=0}^{k-1} a_i x_{n-i}}, \quad n = 0, 1, \dots, \tag{3.1}$$

where $k \in \{1, 2, \dots\}$ and $a_i \geq 0$ for every $i \in \{0, \dots, k-1\}$ with $s = \sum_{i=0}^{k-1} a_i > 0$ and the initial conditions $x_{-k}, \dots, x_0 \in (0, +\infty)$. If $a, b, p \in [0, +\infty)$ satisfy one of the following conditions:

(i) $a = 0$,

(ii) $b/s \geq a > 0$,

then (3.1) has monotone positive solutions which converge to the unique nonnegative equilibrium.

Proof. Let $E = (0, +\infty)$ if $b = 0$ and let $E = [0, +\infty)$ if $b > 0$. Define $f \in C(E^{k+1}, E)$ by

$$f(z_0, z_1, \dots, z_k) = p + \frac{a + z_0}{b + \sum_{i=0}^{k-1} a_i z_{k-i}}, \tag{3.2}$$

for all $(z_0, z_1, \dots, z_k) \in E^{k+1}$. Then, (3.1) has the unique nonnegative equilibrium

$$\overline{x} = \frac{1 + ps - b + \sqrt{(1 + ps - b)^2 + 4s(pb + a)}}{2s} \geq p. \tag{3.3}$$

Let $A = \{(z_0, z_1, \dots, z_k) : z_0 \geq z_1 \geq \cdots \geq z_k \geq f(z_0, z_1, \dots, z_k) \geq \overline{x}\}$ and define

$$g(z_0, z_1, \dots, z_k) = (z_k - p)\left(b + \sum_{i=0}^{k-1} a_i z_{k-i-1}\right) - a, \tag{3.4}$$

for all $(z_0, z_1, \dots, z_k) \in [\overline{x}, +\infty)^{k+1}$; then

$$g(z_0, z_1, \dots, z_k) \geq (\overline{x} - p)(b + s\overline{x}) - a = \frac{a + \overline{x}}{b + s\overline{x}}(b + s\overline{x}) - a = \overline{x}. \tag{3.5}$$

Thus $g \in C([\overline{x}, +\infty)^{k+1}, [\overline{x}, +\infty))$.

It is easy to check that the conditions (C_1), (C_2), and (C_5) hold. Now we show that A is an unbounded connected set.

It follows from conditions (i) and (ii) that $b \geq as$; then

$$\begin{gathered} f(x, x, \dots, x) = p + \frac{a + x}{b + xs} = p + \frac{1}{s} - \frac{b - as}{s(b + sx)}, \\ F(x) = x - f(x, x, \dots, x) = \frac{sx^2 - (ps - b + 1)x - pb - a}{b + sx} \end{gathered} \tag{3.6}$$

are increasing in x in $[\overline{x}, +\infty)$. Thus $c \geq c \geq \cdots \geq c \geq f(c, c, \dots, c) \geq \overline{x}$ for any $c \geq \overline{x}$, which implies that $(c, \dots, c) \in A$ and A is unbounded.

Let $(z_0, z_1, \dots, z_k) \in A$ and $A_i = \{(z_0, \dots, z_0, tz_0 + (1 - t)z_i, z_{i+1}, \dots, z_k) : 0 \leq t \leq 1\}$ for $0 \leq i \leq k$; then A_i is a connected set. Since

$$\begin{gathered} z_0 \geq z_1 \geq \cdots \geq z_k \geq f(z_0, z_1, \dots, z_k) \geq \overline{x}, \\ f(x, x, \dots, x) = p + \frac{a + x}{b + xs} \end{gathered} \tag{3.7}$$

are increasing in x, we know that

$$\begin{aligned} z_0 &\geq \cdots \geq z_0 \geq tz_0 + (1 - t)z_i \geq z_{i+1} \geq \cdots \geq z_k \geq f(z_0, z_1, \dots, z_k) \\ &\geq f(z_0, \dots, z_0, tz_0 + (1 - t)z_i, z_{i+1}, \dots, z_k) \geq f(z_0, \dots, z_0) \geq \overline{x}, \end{aligned} \tag{3.8}$$

from which it follows that $A_i \subset A$. Again since $(z_0, \dots, z_0, z_{i+1}, \dots, z_k) \in A_i \cap A_{i+1}$ for any $0 \leq i \leq k - 1$, $\cup_{i=0}^{k} A_i \cup \{(c, c, \dots, c) : c \geq \overline{x}\}$ is a connected subset of A and $(z_0, z_1, \dots, z_k) \in A_0$, which implies that A is an unbounded connected set. Thus, the condition (C_3) holds.

On the other hand, let $z = (z_0, z_1, \dots, z_k) \in A$, then

$$z_0 \geq z_1 \geq \cdots \geq z_k \geq f(z_0, z_1, \dots, z_k) = p + \frac{a + z_0}{b + \sum_{i=0}^{k-1} a_i z_{k-i}} \geq \overline{x}. \tag{3.9}$$

It follows from (3.9) that

$$g(z_0, z_1, \dots, z_k) = (z_k - p)\left(b + \sum_{i=0}^{k-1} a_i z_{k-i-1}\right) - a \geq \frac{a + z_0}{b + \sum_{i=0}^{k-1} a_i z_{k-i}}\left(b + \sum_{i=0}^{k-1} a_i z_{k-i}\right) - a = z_0, \tag{3.10}$$

which implies $z \in B = \{(z_0, z_1, \dots, z_k) : g(z_0, z_1, \dots, z_k) \geq z_0 \geq z_1 \geq \cdots \geq z_k \geq \overline{x}\}$. Thus, condition (C_4) holds.

By Theorem 2.1, we know that (3.1) has monotone positive solutions which converge to the unique nonnegative equilibrium $\overline{x} = [1 + ps - b + \sqrt{(1 + ps - b)^2 + 4s(pb + a)}]/2s$. □

Remark 3.2. (i) Let $a = b = 0$ and $a_0 = 1 > a_1 = \cdots = a_{k-1} = 0$, then (3.1) reduces to (E2).

(ii) Take $a = p = 0$ and $b = 1$ in Example 3.1; then we have solved Open problem A.

Example 3.3. Consider the following equations:

$$x_{n+1} = f\left(x_{n-k}, x_{n-k+1}, \ldots, x_n\right) = \frac{x_{n-k}^3}{x_n}, \quad n = 0, 1, \ldots, \tag{3.11}$$

$$x_{n+1} = f\left(x_{n-k}, x_{n-k+1}, \ldots, x_n\right) = \frac{x_n^2}{1 + \sum_{i=0}^{k-1} a_i x_{n-i-1}}, \quad n = 0, 1, \ldots, \tag{3.12}$$

$$x_{n+1} = f\left(x_{n-k}, x_{n-k+1}, \ldots, x_n\right) = \frac{x_{n-k}}{1/2 + \sum_{i=0}^{k-1} a_i x_{n-i}}, \quad n = 0, 1, \ldots, \tag{3.13}$$

where $k \in \{1, 2, \ldots\}$ and $a_i \geq 0$ for every $i \in \{0, \ldots, k-1\}$ with $\sum_{i=0}^{k-1} a_i = 1$ and the initial conditions $x_{-k}, \ldots, x_0 \in (0, +\infty)$. Then

(i) equation (3.11) satisfies conditions (C_1) and (C_2), but $A = \{(z_0, z_1, \ldots, z_k) : z_0 \geq z_1 \geq \cdots \geq z_k \geq f(z_0, z_1, \ldots, z_k) \geq \overline{x}\} = \{(\overline{x}, \ldots, \overline{x})\}$ since $z_0 \geq z_1 \geq \cdots \geq z_k \geq z_0^3/z_k \geq \overline{x} = 1$ implies $z_0 = z_1 = \cdots = z_k = 1$; thus condition (C_3) does not hold;

(ii) using arguments similar to ones developed in the proof of Example 3.1, it is easy to check that conditions (C_2) and (C_3) hold, but $f(z_0, z_1, \ldots, z_k)$ is decreasing in z_0, which implies that condition (C_1) does not hold for (3.12);

(iii) equation (3.13) satisfies condition (C_1) and has two nonnegative equilibria: $\overline{x}_1 = 0$ and $\overline{x}_2 = 1/2$, which implies that condition (C_2) does not hold; using arguments similar to ones developed in the proof of Example 3.1, it is easy to check that $A = \{(z_0, z_1, \ldots, z_k) : z_0 \geq z_1 \geq \cdots \geq z_k \geq f(z_0, z_1, \ldots, z_k) \geq \overline{x}_2\}$ is an unbounded connected set.

Remark 3.4. From Example 3.3, we see that all the conditions (C_1), (C_2), and (C_3) are necessary, in the sense that no pair of such conditions implies the remaining condition.

Remark 3.5. If $k = 0$ and the conditions (C_1)–(C_3) are satisfied, then automatically the difference equation $x_{n+1} = f(x_n)$ has monotone positive solutions converging to $\overline{x}$.

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