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Research Article

Nonexistence and Radial Symmetry of Positive Solutions of Semilinear Elliptic Systems

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Nonexistence and radial symmetry of positive solutions for a class of semilinear elliptic systems are considered via the method of moving spheres.

1. Introduction

In this paper we consider the more general semilinear elliptic system

$$\begin{aligned} -\Delta u &= k_1 u^{p_1} + k_2 v^{p_2} + k_3 u^{p_3} v^{p_4}, \\ -\Delta v &= l_1 u^{q_1} + l_2 v^{q_2} + l_3 u^{q_3} v^{q_4}, \end{aligned} \quad \text{in } \mathbb{R}^N (N \geq 3), \tag{1.1}$$

where k_i and l_i $(i = 1, 2, 3)$ are nonnegative constants. The question is to determine for which values of the exponents p_i and q_i the only nonnegative solution (u, v) of (1.1) is $(u, v) = (0, 0)$. The solution here is taken in the classical sense, that is, $u, v \in C^2(\mathbb{R}^N)$. In the case of the Emden-Fowler equation

$$\Delta u + u^k = 0, \quad u \geq 0 \text{ in } \mathbb{R}^N. \tag{1.2}$$

When $1 \leq k < (N+2)/(N-2)$ $(N \geq 3)$, it has been proved in [1] that the only solution of (1.2) is $u = 0$. In dimension $N = 2$, a similar conclusion holds for $0 \leq k < \infty$. It is also well known that in the critical case, $k = (N+2)/(N-2)$, problem (1.2) has a two-parameter family of solutions given by

$$u(x) = \left(\frac{c}{d + |x - \overline{x}|^2}\right)^{(N-2)/2}, \tag{1.3}$$

where $c = [N(N-2)d]^{1/2}$ with $d > 0$ and $\overline{x} \in \mathbb{R}^N$. If $k_1 = k_2 = l_1 = l_2 = 0$, $k_3, l_3 > 0$, $p_3, q_4 > 1$, $p_4, q_3 \geq 0$ and $\min\{p_3 + 2p_4, q_4 + 2q_3\} \leq (N+2)/(N-2)$, using Pokhozhaev's second identity, Chen and Lu ([2, Theorem 2]) have proved that problem (1.1) has no positive radial solutions with $u(x) = u(|x|)$. Suppose that p_3, p_4, q_3, and q_4 satisfy $0 \leq p_3, q_4 \leq 1$, $p_4, q_3 > 1$ and other related conditions, using the method of integral relations, Mitidieri ([3, Theorem 1]) has proved that problem (1.1) has no positive solutions of $C^2(\mathbb{R}^N)$ with $k_3 = l_3 = 1$. In present paper, we study problem (1.1) by virtue of the method of moving spheres and obtain the following theorems of nonexistence and radial symmetry of positive solutions.

Theorem 1.1. *Suppose that $k_i, l_i \geq 0$ $(i = 1,2,3)$, but k_i and l_i are not equal to zero at the same time. Moreover, $\max\{p_1, p_2, p_3 + p_4\}$, $\max\{q_1, q_2, q_3 + q_4\} \leq (N+2)/(N-2)$ with $p_1, p_3, q_2, q_4 \geq 0$, $p_2, p_4, q_1, q_3 > 0$, but $p_1, p_2, p_3 + p_4$ and $q_1, q_2, q_3 + q_4$ are not both equal to $(N+2)/(N-2)$, then Problem (1.1) has no positive solution of $C^2(\mathbb{R}^N)$.*

Theorem 1.2. *Suppose that $k_i, l_i > 0$ $(i = 1,2,3)$, $p_j = q_j = (N+2)/(N-2)$ $(j = 1,2)$, and $p_3 + p_4 = q_3 + q_4 = (N-2)/(N+2)$, then the positive C^2 solution of (1.1) is of the form (1.3), that is, for some $d > 0$, $\overline{x} \in \mathbb{R}^N$,*

$$u(x) = \left(\frac{c_1}{d + |x - \overline{x}|^2}\right)^{(N-2)/2}, \qquad v(x) = \left(\frac{c_2}{d + |x - \overline{x}|^2}\right)^{(N-2)/2}, \tag{1.4}$$

where $c_1, c_2 > 0$ and satisfy the following equalities:

$$\begin{aligned} N(N-2)dc_1^{(N-2)/2} &= k_1 c_1^{(N+2)/2} + k_2 c_2^{(N+2)/2} + k_3 c_1^{((N-2)/2)p_3} c_2^{((N-2)/2)p_4}, \\ N(N-2)dc_1^{(N-2)/2} &= l_1 c_1^{(N+2)/2} + l_2 c_2^{(N+2)/2} + l_3 c_1^{((N-2)/2)q_3} c_2^{((N-2)/2)q_4}. \end{aligned} \tag{1.5}$$

Remark 1.3. Obviously Theorem 1.1 contains new region of k, t, p, and q which can not be covered by [2, Theorem 2] and [3, Theorem 1]. Moreover, Theorem 1.2 gives the exact forms of positive solutions of $C^2(\mathbb{R}^N)$.

There are some related works about problem (1.1). For $k_2 = l_1 = 1$ and $k_1 = k_3 = l_2 = l_3 = 0$, Figueiredo and Felmer (see [4]) proved Theorem 1.1 using the moving plane method and a special form of the maximum principle for elliptic systems. Busca and Manásevich obtained a new result (see [5, Theorem 2.1]) using the same method as in [4]. It allows p_2 and q_1 to reach regions where one of the two exponents is supercritical. In [6], Zhang et al. first introduced the Kelvin transforms and gave a different proof of Theorem 1.1 in [4] using the method of moving spheres. This approach was suggested in [7], while Li and Zhang who had made significant simplifications prove some Liouville theorems for a single equation in [8]. In this paper, we consider the general case of nonlinearities and do not need the maximum principle for elliptic systems. Moreover, the exact form of positive solution is proved in Theorem 1.2. If we can find a proper transforms instead of the Kelvin transforms, we suspect that [5, Theorem 2.1] can also be proved via the method of moving spheres. We leave this to the interested readers.

Let us emphasize that considerable attention has been drawn to Liouville-type results and existence of positive solutions for general nonlinear elliptic equations and systems, and that numerous related works are devoted to some of its variants, such as more general quasilinear operators and domains. We refer the interested reader to [9–15], and some of the references therein. We refer the interested reader to [16, 17].

2. Preliminaries and Moving Spheres

To prove Theorems 1.1 and 1.2, we will use the method of moving spheres. We first prove a number of lemmas as follows. For $x \in \mathbb{R}^N$ and $\lambda > 0$, let us introduce the Kelvin transforms

$$u_{x,\lambda}(y) = \frac{\lambda^{N-2}}{|y-x|^{N-2}} u\left(x + \frac{\lambda^2(y-x)}{|y-x|^2}\right), \qquad v_{x,\lambda}(y) = \frac{\lambda^{N-2}}{|y-x|^{N-2}} v\left(x + \frac{\lambda^2(y-x)}{|y-x|^2}\right), \tag{2.1}$$

which are defined for $y \in \mathbb{R}^N \setminus \{x\}$. For any $y \in \mathbb{R}^N \setminus \{x\}$, one verifies that $u_{x,\lambda}$ and $v_{x,\lambda}$ satisfy the system

$$\begin{aligned}
-\Delta u_{x,\lambda} &= k_1\left(\frac{\lambda}{|y-x|}\right)^{N+2-p_1(N-2)} u_{x,\lambda}^{p_1} + k_2\left(\frac{\lambda}{|y-x|}\right)^{N+2-p_2(N-2)} v_{x,\lambda}^{p_2} \\
&\quad + k_3\left(\frac{\lambda}{|y-x|}\right)^{N+2-(p_3+p_4)(N-2)} u_{x,\lambda}^{p_3} v_{x,\lambda}^{p_4}, \\
-\Delta v_{x,\lambda} &= l_1\left(\frac{\lambda}{|y-x|}\right)^{N+2-q_1(N-2)} u_{x,\lambda}^{q_1} + l_2\left(\frac{\lambda}{|y-x|}\right)^{N+2-q_2(N-2)} v_{x,\lambda}^{q_2} \\
&\quad + l_3\left(\frac{\lambda}{|y-x|}\right)^{N+2-(q_3+q_4)(N-2)} u_{x,\lambda}^{q_3} v_{x,\lambda}^{q_4}.
\end{aligned} \tag{2.2}$$

Our first lemma says that the method of moving spheres can get started.

Lemma 2.1. *For every $x \in \mathbb{R}^N$, there exists $\lambda_0(x) > 0$ such that $u_{x,\lambda}(y) \le u(y)$ and $v_{x,\lambda}(y) \le v(y)$, for all $0 < \lambda < \lambda_0(x)$ and $|y-x| \ge \lambda$.*

Proof. Without loss of generality we may take $x = 0$. We use u_λ and v_λ to denote $u_{0,\lambda}$, and $v_{0,\lambda}$, respectively. Clearly, there exists $r_0 > 0$ such that

$$\frac{d}{dr}\left(r^{(N-2)/2} u(r,\theta)\right) > 0, \quad \forall 0 < r < r_0, \ \theta \in \mathbb{S}^{N-1}. \tag{2.3}$$

Consequently,

$$u_\lambda(y) \leq u(y), \quad \forall 0 < \lambda \leq |y| < r_0. \tag{2.4}$$

By the superharmonicity of u and the maximum principle (see [4, Corollary 1.1]),

$$u(y) \geq \left(\min_{\partial B_{r_0}} u\right) r_0^{N-2} |y|^{2-N}, \quad \forall |y| \geq r_0. \tag{2.5}$$

Let

$$\widehat{\lambda}_0 = r_0 \left(\frac{\min_{\partial B_{r_0}} u}{\min_{\overline{B}_{r_0}} u}\right)^{1/(N-2)} \leq r_0. \tag{2.6}$$

Then for every $0 < \lambda < \widehat{\lambda}_0$, and $|y| \geq r_0$, we have

$$u_\lambda(y) \leq \frac{\widehat{\lambda}_0^{N-2}}{|y|^{N-2}} \max_{\overline{B}_{r_0}} u \leq \frac{r_0^{N-2} \min_{\partial B_{r_0}} u}{|y|^{N-2}}. \tag{2.7}$$

It follows from (2.4), (2.5), and (2.7) that for every $0 < \lambda < \widehat{\lambda}_0$,

$$u_\lambda(y) \leq u(y), \quad |y| \geq \lambda. \tag{2.8}$$

Similarly, there exists $\widetilde{\lambda}_0 > 0$ such that for every $0 < \lambda < \widetilde{\lambda}_0$, we obtain

$$v_\lambda(y) \leq v(y), \quad |y| \geq \lambda. \tag{2.9}$$

We can choose $\lambda_0 = \min\{\widehat{\lambda}_0, \widetilde{\lambda}_0\}$. □

Set, for $x \in \mathbb{R}^N$,

$$\begin{aligned} \overline{\lambda}_u(x) &= \sup\{\mu > 0 \mid u_{x,\lambda}(y) \leq u(y),\ \forall |y - x| \geq \lambda,\ 0 < \lambda \leq \mu\}, \\ \overline{\lambda}_v(x) &= \sup\{\mu > 0 \mid v_{x,\lambda}(y) \leq v(y),\ \forall |y - x| \geq \lambda,\ 0 < \lambda \leq \mu\}. \end{aligned} \tag{2.10}$$

By Lemma 2.1, $\overline{\lambda}_u(x)$ and $\overline{\lambda}_v(x)$ are well defined and $0 < \overline{\lambda}_u(x),\ \overline{\lambda}_v(x) \leq \infty$ for $x \in \mathbb{R}^N$. Let $\overline{\lambda} = \min\{\overline{\lambda}_u, \overline{\lambda}_v\}$, then we have the following

Lemma 2.2. *If $\overline{\lambda}(x) < \infty$ for some $x \in \mathbb{R}^N$, then $u_{x,\overline{\lambda}(x)} \equiv u$ and $v_{x,\overline{\lambda}(x)} \equiv v$ on $\mathbb{R}^N \setminus \{x\}$.*

Lemma 2.3. *If $\overline{\lambda}(\overline{x}) = \infty$ for some $\overline{x} \in \mathbb{R}^N$, then $\overline{\lambda}(x) = \infty$ for all $x \in \mathbb{R}^N$.*

Proof of Lemma 2.2. Without loss of generality, we assume that $\overline{\lambda} = \overline{\lambda}_u$ and take $x = 0$ and let $\overline{\lambda} = \overline{\lambda}(0)$, $u_\lambda = u_{0,\lambda}$ and $v_\lambda = v_{0,\lambda}$, and $\Sigma_\lambda = \{y; |y| > \lambda\}$. We wish to show $u_{\overline{\lambda}} \equiv u$ and $v_{\overline{\lambda}} \equiv v$ in $\mathbb{R}^N \setminus \{0\}$. Clearly, it suffices to show

$$u_{\overline{\lambda}} \equiv u, \qquad v_{\overline{\lambda}} \equiv v \quad \text{on } \Sigma_{\overline{\lambda}}. \tag{2.11}$$

We first prove $u_{\overline{\lambda}} \equiv u$. We know from the definition of $\overline{\lambda}$ that

$$u_{\overline{\lambda}} \le u, \qquad v_{\overline{\lambda}} \le v \quad \text{on } \Sigma_{\overline{\lambda}}. \tag{2.12}$$

In view of (1.1), a simple calculation yields

$$\begin{aligned} -\Delta u_\lambda = {}& k_1\left(\frac{\lambda}{|y|}\right)^{N+2-p_1(N-2)} u_\lambda^{p_1} + k_2\left(\frac{\lambda}{|y|}\right)^{N+2-p_2(N-2)} v_\lambda^{p_2} \\ & + k_3\left(\frac{\lambda}{|y|}\right)^{N+2-(p_3+p_4)(N-2)} u_\lambda^{p_3} v_\lambda^{p_4}, \quad \lambda > 0. \end{aligned} \tag{2.13}$$

Therefore,

$$\begin{aligned} -\Delta\left(u - u_{\overline{\lambda}}\right) = {}& k_1 u^{p_1} + k_2 v^{p_2} + k_3 u^{p_3} v^{p_4} - k_1\left(\frac{\overline{\lambda}}{|y|}\right)^{N+2-p_1(N-2)} u_{\overline{\lambda}}^{p_1} \\ & - k_2\left(\frac{\overline{\lambda}}{|y|}\right)^{N+2-p_2(N-2)} v_{\overline{\lambda}}^{p_2} - k_3\left(\frac{\overline{\lambda}}{|y|}\right)^{N+2-(p_3+p_4)(N-2)} u_{\overline{\lambda}}^{p_3} v_{\overline{\lambda}}^{p_4} \\ \ge {}& k_1\left(\frac{\overline{\lambda}}{|y|}\right)^{N+2-p_1(N-2)} \left(u^{p_1} - u_{\overline{\lambda}}^{p_1}\right) + k_2\left(\frac{\overline{\lambda}}{|y|}\right)^{N+2-p_2(N-2)} \left(v^{p_2} - v_{\overline{\lambda}}^{p_2}\right) \\ & + k_3\left(\frac{\overline{\lambda}}{|y|}\right)^{N+2-(p_3+p_4)(N-2)} \left(u^{p_3} v^{p_4} - u_{\overline{\lambda}}^{p_3} v_{\overline{\lambda}}^{p_4}\right) \\ \ge {}& 0 \quad \text{on } \Sigma_{\overline{\lambda}}. \end{aligned} \tag{2.14}$$

If $u - u_{\overline{\lambda}} \equiv 0$ on $\Sigma_{\overline{\lambda}}$, we stop. Otherwise, by the Hopf lemma and the compactness of $\partial B_{\overline{\lambda}}$, we have

$$\frac{d}{dr}\left(u - u_{\overline{\lambda}}\right)|_{\partial B_{\overline{\lambda}}} \ge C > 0. \tag{2.15}$$

By the continuity of ∇u, there exists $R > \overline{\lambda}$ such that

$$\frac{d}{dr}\left(u - u_\lambda\right) \ge \frac{C}{2} > 0, \quad \text{for } \overline{\lambda} \le \lambda \le R,\ \lambda \le r \le R. \tag{2.16}$$

Consequently, since $u - u_\lambda = 0$ on ∂B_λ, we have

$$u(y) - u_\lambda(y) > 0, \quad \text{for } \overline{\lambda} \le \lambda < R, \ \lambda < |y| \le R. \tag{2.17}$$

Set $c = \min_{\partial B_R}(u - u_{\overline{\lambda}}) > 0$. It follows from the superharmonicity of $u - u_{\overline{\lambda}}$ that

$$u - u_{\overline{\lambda}} \ge \frac{cR^{N-2}}{|y|^{N-2}}, \quad \forall |y| \ge R. \tag{2.18}$$

Therefore,

$$u(y) - u_\lambda(y) \ge \frac{cR^{N-2}}{|y|^{N-2}} - \left(u_\lambda(y) - u_{\overline{\lambda}}(y)\right), \quad \forall |y| \ge R. \tag{2.19}$$

By the uniform continuity of u on $\overline{B}_R$, there exists $0 < \epsilon < R - \overline{\lambda}$ such that for all $\overline{\lambda} \le \lambda \le \overline{\lambda} + \epsilon$,

$$\left|\lambda^{N-2} u\left(\frac{\lambda^2 y}{|y|^2}\right) - \overline{\lambda}^{N-2} u\left(\frac{\overline{\lambda}^2 y}{|y|^2}\right)\right| < \frac{cR^{N-2}}{2}, \quad \forall |y| \ge R. \tag{2.20}$$

It follows from (2.19) and the above inequality that

$$u(y) - u_\lambda(y) > 0, \quad \text{for } \overline{\lambda} \le \lambda \le \overline{\lambda} + \epsilon, \ |y| \ge R. \tag{2.21}$$

Estimates (2.17) and (2.21) violate the definition of $\overline{\lambda}$.

From $u_{\overline{\lambda}} \equiv u$ and (2.14), we easily know that $v_{\overline{\lambda}} \equiv v$ in $\Sigma_{\overline{\lambda}}$. Lemma 2.2 is proved. □

Proof of Lemma 2.3. Since $\overline{\lambda}(\overline{x}) = \infty$, we have

$$u_{\overline{x},\lambda}(y) \le u(y), \qquad v_{\overline{x},\lambda}(y) \le v(y), \quad \forall \lambda > 0, \ |y - \overline{x}| \ge \lambda. \tag{2.22}$$

It follows that

$$\lim_{|y| \to \infty} |y|^{N-2} u(y) = \infty. \tag{2.23}$$

On the other hand, if $\overline{\lambda}(x) < \infty$ for some $x \in \mathbb{R}^N$, then, by Lemma 2.2,

$$\lim_{|y| \to \infty} |y|^{N-2} u(y) = \lim_{|y| \to \infty} |y|^{N-2} u_{x,\overline{\lambda}(x)}(y) = \overline{\lambda}^{N-2}(x) u(x) < \infty, \tag{2.24}$$

which is a contradiction. Similarly, we also obtain a contradiction for v. □

3. Proofs of Theorems 1.1 and 1.2

In this section we first present two calculus lemmas taken from [8] (see also [7]).

Lemma 3.1 (See [8, Lemma 11.1]). *Let $f \in C^1(\mathbb{R}^N)$, $N \geq 1$, $\nu > 0$. Suppose that for every $x \in \mathbb{R}^N$, there exists $\lambda(x) > 0$ such that*

$$\left(\frac{\lambda(x)}{|y-x|}\right)^{\nu} f\left(x + \frac{\lambda^2(x)(y-x)}{|y-x|^2}\right) = f(y), \quad y \in \mathbb{R}^N \setminus \{x\}, \tag{3.1}$$

Then for some $c \geq 0$, $d > 0$, $\overline{x} \in \mathbb{R}^N$,

$$f(x) = \pm\left(\frac{c}{d + |x - \overline{x}|^2}\right)^{\nu/2}. \tag{3.2}$$

Lemma 3.2 (See [8, Lemma 11.2]). *Let $f \in C^1(\mathbb{R}^N)$, $N \geq 1$, $\nu > 0$. Assume that*

$$\left(\frac{\lambda}{|y-x|}\right)^{\nu} f\left(x + \frac{\lambda^2(y-x)}{|y-x|^2}\right) \leq f(y), \quad \forall \lambda > 0,\ x \in \mathbb{R}^N,\ |y-x| \geq \lambda, \tag{3.3}$$

Then $f \equiv constant$.

Proof of Theorem 1.1. We first claim that $\overline{\lambda}(x) = \infty$ for all $x \in \mathbb{R}^N$. We prove it by contradiction argument. If $\overline{\lambda}(\overline{x}) < \infty$ for some $\overline{x}$, then by Lemma 2.2, $u_{\overline{x},\overline{\lambda}(\overline{x})} \equiv u$ and $v_{\overline{x},\overline{\lambda}(\overline{x})} \equiv v$ on $\mathbb{R}^N \setminus \{\overline{x}\}$. But looking at equations in system (2.2) we realize that this is impossible. Therefore,

$$u_{x,\lambda}(y) \leq u(y), \qquad v_{x,\lambda}(y) \leq v(y), \quad \forall \lambda > 0,\ x \in \mathbb{R}^N,\ |y-x| \geq \lambda. \tag{3.4}$$

This, by Lemma 3.2, implies that $u, v \equiv constant$. From system (1.1) we know that it is also impossible. □

Proof of Theorem 1.2. We first claim that $\overline{\lambda}(x) < \infty$ for all $x \in \mathbb{R}^N$. We prove it by contradiction argument. If $\overline{\lambda}(\overline{x}) = \infty$ for some $\overline{x}$, then by Lemma 2.3, $\overline{\lambda}(x) = \infty$ for all x, that is,

$$u_{x,\lambda}(y) \leq u(y), \qquad v_{x,\lambda}(y) \leq v(y), \quad \forall \lambda > 0,\ x \in \mathbb{R}^N,\ |y-x| \geq \lambda. \tag{3.5}$$

This, by Lemma 3.2, implies that $u, v \equiv constant$, a contradiction to (1.1). Therefore, it follows from Lemma 2.2 that for every $x \in \mathbb{R}^N$, there exists $\overline{\lambda}(x) > 0$ such that $u_{x,\overline{\lambda}(x)} \equiv u$ and $v_{x,\overline{\lambda}(x)} \equiv v$. Then by Lemma 3.1, for some $c_i, d > 0$ $(i = 1, 2)$ and some $\overline{x} \in \mathbb{R}^N$,

$$u(x) \equiv \left(\frac{c_1}{d + |x - \overline{x}|^2}\right)^{(N-2)/2}, \qquad v(x) \equiv \left(\frac{c_2}{d + |x - \overline{x}|^2}\right)^{(N-2)/2}. \tag{3.6}$$

Theorem 1.2 follows from the above and the fact that (u, v) is a solution of (1.1). □

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