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Research Article

Stability Analysis of Discrete Hopfield Neural Networks with the Nonnegative Definite Monotone Increasing Weight Function Matrix

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The original Hopfield neural networks model is adapted so that the weights of the resulting network are time varying. In this paper, the Discrete Hopfield neural networks with weight function matrix (DHNNWFM) the weight changes with time, are considered, and the stability of DHNNWFM is analyzed. Combined with the Lyapunov function, we obtain some important results that if weight function matrix (WFM) is weakly (or strongly) nonnegative definite function matrix, the DHNNWFM will converge to a stable state in serial (or parallel) model, and if WFM consisted of strongly nonnegative definite function matrix and column (or row) diagonally dominant function matrix, DHNNWFM will converge to a stable state in parallel model.

1. Introduction

Discrete Hopfield neural network (DHNN) [1] is one of the famous neural networks with a wide range of applications. With the development of DHNN in theory and application, the model is more and more complex. It is well known that the nonautonomous phenomena often occur in many realistic systems. Particularly when we consider a long-term dynamical behavior of the system and consider seasonality of the changing environment, the parameters of the system usually will change with time [2, 3]. However, the original DHNN is difficult to adapt this change, because the matrixes of DHNN and DHNN with time or time-varying delay are constant matrixes [1, 4–22] and the parameters of DHNN, which change with time, are seldom considered. In order to implement a desired flow vector field distribution by using conventional matrix encoding scheme, a time-varying Hopfield model (TVHM) is proposed [23]. In many applications, the properties of periodic oscillatory solutions are of great interest.

For example, the human brain has been in periodic oscillatory or chaos state, hence it is of prime importance to study periodic oscillatory and chaos phenomenon of neural networks. So, the literature [2, 3] studies the global exponential stability and existence of periodic solutions of the high-order Hopfield-type neural networks. In [23, 24], we consider that the weight function matrix and the threshold function vector, respectively, converge to a constant matrix and a constant vector and the weight function matrix is a symmetric function matrix, and, we analyze the stability of the model. In this paper, with the stability of asymmetric Hopfield Neural Networks [4, 5], we work on the stability analysis of discrete Hopfield neural networks with the nonnegative definite monotone increasing weight function matrix.

This paper has the following organization. In Section 1, we provide the introduction. In Section 2, we introduce some basic concepts. In Section 3, we analyze the stability analysis of discrete Hopfield neural networks with the nonnegative definite monotone increasing weight function matrix. The last section offers the conclusions of this paper.

2. Basic Definitions

In this section, we will introduce basic concepts which will be used in the following to obtain some results.

DHNN with weight function matrix (DHNNWFM) varies with the discrete time factor t by step length h (in this paper, $h = 1$). Formally, let $N(t) = (W(t), \theta(t))$ be a DHNN with n neurons, which have the discrete time factor t with step length 1 and are denoted by $\{x_1, x_2, \ldots, x_n\}$. In the pair $N(t) = (W(t), \theta(t))$, $W(t) = (w_{ij}(t))$ is an $n \times n$ function matrix where $w_{ij}(t)$ changes with time t, representing the connection weight from x_j to x_i, and $\theta(t) = (\theta_i(t))$ is an n-dimensional function vector where $\theta_i(t)$ changes with time t, representing the threshold attached to the neuron x_i. The state of the i neuron at time t is denoted by $x_i(t)$. Each neuron is assumed to have two possible states: 1 and -1. The state of the network at time t is the vector $X(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in R^n$. In general, the state, $X(t+1)$, of the network at time $t+1$ is a function of $\{W(t), \theta(t)\}$ and the state, $X(t)$, of the network at time t. The network is, thus, completely determined by the parameters $\{W(t), \theta(t)\}$, the initial value, $X(0)$, of the states, and the manner in which the neurons are updated (evolved).

If at time step t, a neuron x_i is chosen to be updated, then at the next step

$$x_i(t+1) = \operatorname{sgn}(y_i(t)) = \begin{cases} 1, & y_i(t) \geq 0, \\ -1, & y_i(t) < 0, \end{cases} \tag{2.1}$$

where $y_i(t) = \sum_{j=1}^{n} w_{ij}(t) x_j(t) - \theta_i(t),\ i = 1, 2, \ldots, n$.

(1) The network is updated asynchronously, that is, only one neuron x_i is selected at time $t+1$. The updating rule is

$$x_j(t+1) = \begin{cases} \operatorname{sgn}(y_j(t)), & j = i, \\ x_j(t), & j \neq i. \end{cases} \tag{2.2}$$

(2) The network is updated synchronously, that is, every neuron x_j is selected at time $t+1$. The updating rule is

$$x_j(t+1) = \operatorname{sgn}(y_j(t)), \quad j = 1, 2, \dots, n. \tag{2.3}$$

Let $N(t) = (W(t), \theta(t))$ be a DHNNWFM. If the output of n neurons does not change any longer after a limited time interval t from its initial state $X(0)$, that is, $X(t+1) = X(t)$, then we can say that the network is a stable state, and we call $X(t+1) = X(t)$ a stable point of N. In addition, we say that the network converges about the initial state $X(t_0)$. It is easy to know if X is a stable point of DHNNWFM, then $X(t) = \operatorname{sgn}(W(t)X(t) - \theta(t))$. Sometimes we call X that satisfies the above formula a stable attraction factor of the network N. If X is an attraction factor DHNNWFM $N(t) = (W(t), \theta(t))$, we denote attraction domain of X as $\Gamma(X)$, which represents the set which consists of all of the initial state $X(0)$ attracted to X.

Let $N(t) = (W(t), \theta(t))$ be a DHNNWFM. $X(0), X(1), \dots, X(r-1), (r \geq 1)$ are n-dimensional vectors. If $X(t+1) = \operatorname{sgn}(W(t)X(t) - \theta(t))$, $t = 0, 1, 2, \dots, r-2$, and $X(0) = \operatorname{sgn}(W(r-1)X(r-1) - \theta(r-1))$, then we called a limit cycle attraction factor of $N(t)$, sometimes abbreviated to limit cycle; its length is r, and is denoted by $(X(0), X(1), \dots, X(r-1))$. Similar to the attraction domain of a stable attraction factor, the attraction domain of cycle attraction factor $(X(0), X(1), \dots, X(r-1))$, denoted by $\Gamma(X(0), X(1), \dots, X(r-1))$, represents the set that consists of all the possible initial states which are attracted to $(X(0), X(1), \dots, X(r-1))$.

Definition 2.1. $W(t)$ is a column or row-diagonally dominant function matrix, if it satisfies the following conditions:

$$w_{ii}(t) \geq \frac{1}{2} \sum_{1 \leq j \leq n, j \neq i} \left| w_{ji}(t) \right|, \quad i = 1, 2, \dots, n, \tag{2.4}$$

or

$$w_{ii}(t) \geq \frac{1}{2} \sum_{1 \leq j \leq n, j \neq i} \left| w_{ij}(t) \right|, \quad i = 1, 2, \dots, n. \tag{2.5}$$

Definition 2.2. let $W(t)$ be a column or row-diagonally dominant function matrix. $W(t)$ is called a column or row-diagonally dominant monotone increasing function matrix, if it satisfies the following conditions:

$$\Delta w_{ii}(t) \geq \frac{1}{2} \sum_{1 \leq j \leq n, j \neq i} \left| \Delta w_{ji}(t) \right|, \quad i = 1, 2, \dots, n, \tag{2.6}$$

or

$$\Delta w_{ii}(t) \geq \frac{1}{2} \sum_{1 \leq j \leq n, j \neq i} \left| \Delta w_{ij}(t) \right|, \quad i = 1, 2, \dots, n, \tag{2.7}$$

where $\Delta w_{ij}(t) = w_{ij}(t+1) - w_{ij}(t)$.

Definition 2.3. $W(t)$ is called a nonnegative definite matrix on the set $\{-2, 0, 2\}$, if it satisfies $P^T W(t) P \geq 0$ for each $P \in \{-2, 0, 2\}^n$.

Definition 2.4. let $W(t)$ be a nonnegative definite function matrix. $W(t)$ is called nonnegative definite monotone increasing function matrix, if it satisfies

$$P^T \Delta W(t) P \geq 0, \tag{2.8}$$

where $\Delta W(t) = W(t+1) - W(t)$, for each $P \in \{-2, 0, 2\}^n$.

Definition 2.5. $W(t)$ is called a weakly nonnegative definite function matrix, if it satisfies

$$w_{ii}(t) \geq \frac{1}{2} \sum_{1 \leq j \leq n} \left| w_{ji}(t) - w_{ij}(t) \right|, \quad i = 1, 2, \ldots, n. \tag{2.9}$$

Definition 2.6. let $W(t)$ be a weakly nonnegative definite function matrix. $W(t)$ is called weakly nonnegative definite monotone increasing function matrix, if it satisfies

$$\Delta w_{ii}(t) \geq \frac{1}{2} \sum_{1 \leq j \leq n} \left| \Delta w_{ji}(t) - \Delta w_{ij}(t) \right|, \quad i = 1, 2, \ldots, n, \tag{2.10}$$

where $\Delta w_{ij}(t) = w_{ij}(t+1) - w_{ij}(t)$.

For an $n \times n$ matrix $W(t)$, we denote in this paper the corresponding matrix $W^*(t) = (w^*_{ij}(t))$ with

$$w^*_{ij}(t) = \begin{cases} w_{ii}(t) - \dfrac{1}{2} \displaystyle\sum_{1 \leq k \leq n} \left| w_{ki}(t) - w_{ik}(t) \right|, & i = j, \\ w_{ij}(t), & i \neq j. \end{cases} \tag{2.11}$$

Definition 2.7. $W(t)$ is called a strongly nonnegative definite function matrix, if the corresponding matrix $W^*(t)$ is a nonnegative definite function matrix.

Definition 2.8. let $W(t)$ be a strongly nonnegative definite function matrix. $W(t)$ is called a strongly nonnegative definite monotone increasing function matrix, if the corresponding matrix $W^*(t)$ is nonnegative definite monotone increasing function matrix.

In this paper, the function matrix is the sum of the initial weight matrix and increment matrixes, that is,

$$W(t) = W(0) + \sum_{0 \leq s < t} \Delta W(s). \tag{2.12}$$

And the n-dimensional function vector is the sum of the initial vector and increment vectors, that is,

$$\theta(t) = \theta(0) + \sum_{0 \leq s < t} \Delta \theta(s). \tag{2.13}$$

In order to describe, let $L(t) = \{0, 1, \ldots, t-1\} \cup \{\overline{0}\}$, $I = \{1, 2, \ldots, n\}$, and $W(0) = \Delta W(\overline{0})$. $\overline{0}$ represents 0 of $W(0)$

$$
\begin{aligned}
L_1(t) &= \left\{ s \in L(t) \mid \Delta W(s) = \Delta W^T(s) \right\}, \\
L_2(t) &= \left\{ s \in L(t) \mid P^T \Delta W^*(s) P \geq 0 \right\}, \\
L_3(t) &= \left\{ \begin{matrix} s \in L(t) \mid \Delta W(s) \text{ is weakly} \\ \text{nonnegative definite function matrix} \end{matrix} \right\}.
\end{aligned}
\tag{2.14}
$$

3. Main Results

Theorem 3.1. *Let $N(t) = (W(t), \theta(t))$ be a DHNNWFM.*

(1) If $W(t)$ is a weakly nonnegative definite monotone increasing function matrix and $\lim_{t\to\infty} \theta(t) \to \theta$, then $N(t)$ will converge to a stable state in serial mode.

(2) If $W(t)$ is a strongly nonnegative definite monotone increasing function matrix and $\lim_{t\to\infty} \theta(t) \to \theta$, then $N(t)$ will converge to a stable state in parallel mode.

Proof. Based on (2.12) and (2.13), we have the following.

Let $\Delta\varepsilon_i(t) = \max\{\Delta\delta_i(t) + \Delta\theta_i(t) \mid \Delta\delta_i(t) + \Delta\theta_i(t) < 0\}$, where

$$
\Delta\delta_i(t) = \sum_{j=1}^{n} \Delta w_{ij}(t) x_j(t), \quad x_j(t) \in \{1, -1\},\ j \in I.
\tag{3.1}
$$

If $\forall x_j \in \{1, -1\}$, $j \in I$, $\Delta\delta_i(t) \geq 0$, then $\Delta\delta_i(t)$ is assigned an arbitrary negative number. Suppose $\Delta\theta(\overline{0}) = \theta(0)$, $\overline{\theta(t)} = \sum_{s \in L(t)} \overline{\Delta\theta(s)}$, where $\overline{\Delta\theta(s)} = (\overline{\Delta\theta_1(s)}, \ldots, \overline{\Delta\theta_n(s)})$ and $\overline{\Delta\theta_i(s)} = \Delta\theta_i(s) - (\Delta\varepsilon_i(s)/2)$, $i \in I$. Then we consider energy function (Lyapunov function) of the DHNNWFM as follows:

$$
E(t) = -\frac{1}{2} X^T(t) W(t) X(t) - X^T(t) \overline{\theta(t)}.
\tag{3.2}
$$

Combined with (2.12) and (2.13), we have

$$
\begin{aligned}
E(t) &= -\frac{1}{2} X^T(t) \left(\sum_{s \in L(t)} \Delta W(s) \right) X(t) - X^T(t) \left(\sum_{s \in L(t)} \overline{\Delta\theta(s)} \right), \\
E(t+1) &= -\frac{1}{2} X^T(t+1) W(t) X(t+1) - X^T(t+1) \overline{\theta(t)} + \chi(t),
\end{aligned}
\tag{3.3}
$$

where

$$
\chi(t) = -\frac{1}{2} X^T(t+1) \Delta W(t) X(t+1) - X^T(t+1) \overline{\Delta\theta(t)}.
\tag{3.4}
$$

$\chi(t)$that is the increasing energy for the connected weight matrix increases. So, the change of energy is

$$\Delta E(t) = E(t+1) - E(t) = \sum_{s \in L(t)} \beta_s(t) + \chi(t), \tag{3.5}$$

where

$$\begin{aligned}\beta_s(t) &= -\frac{1}{2}X^T(t+1)\Delta W(s)X(t+1) - X^T(t+1)\overline{\Delta\theta(s)} \\ &\quad + \frac{1}{2}X^T(t)\Delta W(s)X(t+1) + X^T(t)\overline{\Delta\theta(s)}.\end{aligned} \tag{3.6}$$

According to (3.4) and $\overline{\Delta\theta_i(t)} = \Delta\theta_i(t) - (\Delta\varepsilon_i(t)/2)$, we obtain

$$\begin{aligned}\chi(t) &= -\frac{1}{2}X^T(t+1)\Delta W(t)X(t+1) + X^T(t+1)\frac{\Delta\varepsilon(t)}{2} - X^T(t+1)\Delta\theta(t) \\ &= -\frac{1}{2}X^T(t+1)\Delta W(t)X(t+1) + \frac{1}{2}X^T(t+1)\Delta W(t)X(t) - X^T(t+1)\Delta\theta(t).\end{aligned} \tag{3.7}$$

By $\Delta X(t) = 2X(t+1) = -2X(t)$,

$$\chi(t) = -X^T(t+1)\Delta W(t)X(t+1) - X^T(t+1)\Delta\theta(t). \tag{3.8}$$

Because $\lim_{t\to\infty}\theta(t) \to \theta$ (i.e., $\lim_{t\to\infty}\Delta\theta(t) \to 0$) and $\Delta W(t)$ is nonnegative definite, we have

$$\chi(t) \le 0. \tag{3.9}$$

(1) Here, $W(t)$ is a weakly nonnegative definite monotone increasing function matrix, so $\Delta W(s),\ s \in L_3(t)$ is weakly nonnegative definite matrixes. Then, based on [4], when $N(t)$ is operating in serial mode, we obtain $\beta_s(t) \le 0,\ s \in L_3(t)$. Then $\Delta E(t) \le 0$. Therefore, $N(t)$ will converge to a stable state in serial mode.

(2) Here, $W(t)$ is a strongly nonnegative definite monotone increasing function matrix, so $\Delta W(s),\ s \in L_2(t)$ is strongly nonnegative definite matrixes. According to [4], we know that $\beta_s(t) \le 0$ in parallel mode. Then $\Delta E(t) \le 0$. Therefore, $N(t)$ will converge to a stable state in parallel mode. The proof is completed. □

Theorem 3.2. *Let $N(t) = (W(t), \theta(t))$ be a DHNNWFM.*

(1) If there exits an integer constant K such that $\Delta W(s),\ 0 \le s \le K$ is symmetric or weakly nonnegative definite matrix, $\Delta W(s),\ K < s$ is weakly nonnegative definite matrix and $\lim_{t\to\infty}\theta(t) \to \theta$, then $N(t)$ will converge to a stable state in serial mode.

(2) If there exits an integer constant K such that $\Delta W(s),\ 0 \le s \le K$ is symmetric or strongly nonnegative definite matrix $\Delta W(s),\ K < s$ is strongly nonnegative definite matrix and $\lim_{t\to\infty}\theta(t) \to \theta$, then $N(t)$ will converge to a limit cycle of length at most 2 or a stable state in parallel mode.

Proof. We consider energy function (Lyapunov function) of the DHNNWFM as follows:

$$E(t) = -\frac{1}{2}X^T(t)W(t)X(t) - X^T(t)\overline{\theta(t).} \tag{3.10}$$

According to the proof of Theorem 3.1, now we have $\Delta E(t) = \sum_{s\in L(t)} \beta_s(t) + \chi(t)$

(1) Here, we know $L(t) = L_1(t) \cup L_3(t) \wedge L_1(t) \cap L_3(t) = \emptyset$. Then

$$\Delta E(t) = \sum_{s\in L_1(t)} \beta_s(t) + \sum_{s\in L_3(t)} \beta_s(t) + \chi(t). \tag{3.11}$$

Based on [1, 4], when $N(t)$ is operating in serial mode, we obtain $\beta_s(t) \le 0,\ s \in L(t)$ for $\Delta W(s),\ s \in L(t)$ is symmetric or weakly nonnegative definite matrixes and according to (3.9) we know $\chi(t) \le 0$. Then $\Delta E(t) \le 0$. Therefore, $N(t)$ will converge to a stable state in serial mode.

(2) Here, we know $L(t) = L_1(t) \cup L_2(t) \wedge L_1(t) \cap L_2(t) = \emptyset$. Then

$$\Delta E(t) = \sum_{s\in L_1(t)} \beta_s(t) + \sum_{s\in L_2(t)} \beta_s(t) + \chi(t). \tag{3.12}$$

If $\Delta W(s)(s \in L_1(t))$, according to [4] we know that $\beta_s(t) \le 0$ if and only if $X(t) \ne X(t+1) = X(t-1)$ or $X(t) = X(t+1) = X(t-1)$ in parallel mode.

If $\Delta W(s)(s \in L_2(t))$, according to [4] we know that $\beta_s(t) \le 0$ in parallel mode.

According to (3.9), we know $\chi(t) \le 0$.

Based on the above, we obtain that if $X(t) \ne X(t+1) = X(t-1)$ or $X(t) = X(t+1) = X(t-1)$, then $\Delta E(t) \le 0$. So, $N(t)$ will converge to a limit cycle of length at most 2 or a stable state in parallel mode. The proof is completed. □

Combined with [25], we have the following.

Theorem 3.3. *Let $N(t) = (W(t) + U(t) + O(t), \theta(t))$ be a DHNNWFM. If $U(t)$ is column diagonally dominant monotone increasing function matrix, $O(t)$ is row diagonally dominant monotone increasing function matrix, and $W(t)$ is strongly nonnegative definite monotone increasing function matrix, then $N(t)$ will converge to a stable state in parallel mode.*

Proof. Let $\Delta\varepsilon_i(t) = \max\{\Delta\delta_i(t) + \Delta\theta_i(t) \mid \Delta\delta_i(t) + \Delta\theta_i(t) < 0\}$, where

$$\Delta\delta_i(t) = \sum_{j=1}^{n} \Delta w_{ij}(t)x_j(t), \quad x_j(t) \in \{1, -1\},\ j \in I. \tag{3.13}$$

If $\forall x_j \in \{1, -1\},\ j \in I,\ \Delta\delta_i(t) \ge 0$, then $\Delta\delta_i(t)$ is assigned an arbitrary negative number. Suppose $\Delta\theta(\overline{0}) = \theta(0)$, $\overline{\theta(t)} = \sum_{s\in L(t)} \overline{\Delta\theta(s)}$ where $\overline{\Delta\theta(s)} = (\overline{\Delta\theta_1(s)}, \ldots, \overline{\Delta\theta_n(s)})$ and $\overline{\Delta\theta_i(s)} = \Delta\theta_i(s) - (\Delta\varepsilon_i\ (s)/2)$, i ∈ I. Then we consider energy function (Lyapunov function) of the DHNNWFM as follows:

$$E(t) = -\frac{1}{2}X^T(t)W(t)X(t) - X^T(t)U(t)X(t) - X^T(t)O(t)X(t) - X^T(t)\overline{\theta(t)}, \tag{3.14}$$

where $W(t) = \sum_{s\in L(t)} \Delta W(s)$, $U(t) = \sum_{s\in L(t)} \Delta U(s)$, $O(t) = \sum_{s\in L(t)} \Delta O(s)$, and $\overline{\theta(t)} = \sum_{s\in L(t)} \overline{\Delta\theta(s)}$.

We have that the change of energy is

$$\Delta E(t) = E(t+1) - E(t) \tag{3.15}$$

$$\begin{aligned}
\Delta E(t) = {} & \frac{1}{2}\Delta X^T(t)W(t)X^T(t) - \frac{1}{2}X^T(t)W(t)\Delta X^T(t) \\
& - \frac{1}{2}\Delta X^T(t)W(t)\Delta X^T(t) - \Delta X^T(t)\Big((W(t)+U(t)+O(t))X^T(t)+\overline{\theta}\Big) \\
& - X^T(t+1)(U(t)+O(t))\Delta X(t) - \frac{1}{2}X^T(t+1)\Delta W(t)X(t+1) - X^T(t+1)\Delta U(t)X(t+1) \\
& - X^T(t+1)\Delta O(t)X(t+1) - X^T(t+1)\overline{\Delta\theta(t)} \\
= {} & -\varphi(t) - \eta(t) - \kappa(t) - \gamma(t),
\end{aligned} \tag{3.16}$$

where

$$\begin{aligned}
\varphi(t) &= -\frac{1}{2}\Delta X^T(t)W(t)X^T(t) + \frac{1}{2}X^T(t)W(t)\Delta X^T(t) + \frac{1}{2}\Delta X^T(t)W(t)\Delta X^T(t), \\
\eta(t) &= \Delta X^T(t)\Big((W(t)+U(t)+O(t))X^T(t)+\overline{\theta}\Big), \\
\kappa(t) &= X^T(t+1)(U(t)+O(t))\Delta X(t), \\
\gamma(t) &= \frac{1}{2}X^T(t+1)\Delta W(t)X(t+1) + X^T(t+1)\Delta U(t)X(t+1) \\
&\quad + X^T(t+1)\Delta O(t)X(t+1) + X^T(t+1)\overline{\Delta\theta(t)}.
\end{aligned} \tag{3.17}$$

Based on [4, Theorem 2], we obtain $\varphi(\mathrm{t}) \geq 0$.

Obviously, when $\Delta X(t) \neq 0$, $\Delta X(t) = 2X(t+1) = -2X(t)$, $\eta(t) \geq 0$.

When it is operating in parallel mode, let $I_1(t) = \{i \in I \mid \Delta x_i(t) \neq 0\}$, $I_2(t) = I \setminus I_1(t)$.

According to the property of column (or row) diagonally dominant matrix, we have

$$\begin{aligned}
\kappa(t) &= 2X^T(t+1)U(t)X(t+1) + 2X^T(t+1)O(t)X(t+1) \\
&\geq \sum_{i\in I_1(t)}\left(u_{ii}(t) - \sum_{j\neq i}\left|u_{ji}(t)\right|\right) + \sum_{i\in I_1(t)}\left(o_{ii}(t) - \sum_{j\neq i}\left|o_{ij}(t)\right|\right) \\
&\geq 0.
\end{aligned} \tag{3.18}$$

According to Definition 2.2 and (3.9), we know $\gamma(t) \geq 0$.

Based on the above, we have $\Delta E(t) \leq 0$. So, $N(t)$ will converge to a stable state in parallel mode. The proof is completed. □

4. Examples

Example 4.1. Let $N(t) = (W(t), \theta(t))$ be a DHNNWFM, where $W(t) = \begin{bmatrix} 2t & -t & t \\ t & 3t & -2t \\ t & 2t & 3t \end{bmatrix}$ and $\theta(t) = 0$. $N(t)$ will converge to a stable state in parallel mode.

Example 4.2. Let N(t) = (W(t), θ(t)) be a DHNNWFM, where $W(t) = \begin{bmatrix} 2t & -6t & t \\ t & t^2 & -2t \\ 8t^{1/2} & 2t & t^2 \end{bmatrix}$ and $\theta(t) = 0$. $N(t)$ will converge to a stable state in parallel mode.

Example 4.3. Let $N(t) = (W(t) + U(t) + O(t), \theta(t))$ be a DHNNWFM, where $W(t) = \begin{bmatrix} 2t & -t & t \\ t & 3t & -2t \\ t & 2t & 3t \end{bmatrix}$, $O(t) = 0$, $U(t) = \begin{bmatrix} 3t & 3t & t \\ 2t & 5t & -2t \\ -t & t & 3t \end{bmatrix}$ and $\theta(t) = 0$. $N(t)$ will converge to a stable state in parallel mode.

5. Conclusion

In this paper, we firstly introduce the DHNNWFM. Then we mainly discuss stability of DHNNWFM that WFM is a symmetric or nonnegative definite, or column (or row) diagonally dominant function matrix. This work widens the DHNN model. And we obtain some important results, which supply some theoretical principles to the application. DHNNWFM has many interesting phenomena. We will continue for the theoretic and the practical research about DHNNFWM.

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