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Research Article

Did Bank Capital Regulation Exacerbate the Subprime Mortgage Crisis?

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This contribution is the second in a series of papers on discrete-time modeling of bank capital regulation and its connection with the subprime mortgage crisis (SMC). The latter was caused by, amongst other things, the downturn in the U.S. housing market, risky lending and borrowing practices, inaccurate credit ratings, credit default swap contracts as well as excessive individual and corporate debt levels. The Basel II Capital Accord's primary tenet is that banks should be given more freedom to decide how much risk exposure to permit; a practice brought into question by the SMC. For instance, institutions worldwide have badly misjudged the risk related to investments ranging from subprime mortgage loans to mortgage-backed securities (MBSs). Also, analysts are now questioning whether Basel II has failed by allowing these institutions to provision less capital for subprime mortgage loan losses from highly rated debt, including MBSs. Other unintended consequences of Basel II include the procyclicality of credit ratings and changes in bank lending behavior. Our main objective is to model the dependence of bank credit and capital on the level of macroeconomic activity under Basel I and Basel II as well as its connection with banking behavior for the period before and during the SMC.

1. Introduction

This contribution is the second in a series of papers (of which [1] was the first) on modeling issues related to bank capital regulation and the subprime mortgage crisis (SMC) in a discrete-time framework. Some of the world's top banking experts spent nearly a decade designing regulation in the form of the Basel II Capital Accord in order to ensure the health of the global banking industry. However, the following question still remains: What if some of their suppositions were inaccurate?

The possibility that this may be answered in the affirmative was debated during the SMC. This crisis that started unraveling from 2007 onwards and was initiated by subprime mortgage loan losses in the U.S. brought into question the effectiveness of global macroeconomic policy, financial stability, and financial regulation such as the new Basel II capital adequacy framework for banks (see, e.g., [2]).

Basel II aims to address weaknesses in the Basel I capital adequacy framework for banks by incorporating more detailed calibration of credit risk and requiring the pricing of other forms of risk such as operational risk. However, the 2007–2008 implementation of Basel II corresponded to major losses suffered by some of the world's major banks due to financial crises. Furthermore, the risk models that underpin Basel II are similar to the ones many of those banks use. Under the Basel II framework, regulators allow large banks with sophisticated risk management systems to use risk assessment based on their own models in determining the minimum amount of capital they are required to hold by the regulators as a buffer against unexpected losses. Of concern is that by the end of 2008, in the U.S., non-risk-weighted capital adequacy ratios (CARs) were near historically low levels of about 7.0%. Naturally, these facts challenge the usefulness of important elements in the Basel II accord.

The SMC fallout did not originate from lightly-regulated hedge funds, but from banks regulated by governments. For instance, in the fourth quarter of 2007, Citigroup Inc. had its worst-ever quarterly loss of $ 9.83 billion and had to raise more than $ 20 billion in capital from outside investors, including foreign-government investment funds. This was done in order to augment the depleted capital on its balance sheet after bad investments in mortgage-backed securities (MBSs) and collateralized debt obligations (CDOs). According to the Federal Deposit Insurance Corporation (FDIC), at the time, Citigroup held $ 80 billion in core capital on its balance sheet to protect against its $ 1.1 trillion in assets. In the second half of 2007, Citigroup wrote down about $ 20 billion. Interestingly, at the end of 2007, major U.S. banks like J.P. Morgan Chase and Co., Wachovia Corp., Washington Mutual Inc., and Citigroup lobbied for leaner, European-style capital cushions. These banks urged the U.S. government

> "to help ensure U.S. banking institutions remain strong and competitive, the federal banking agencies should avoid imposing domestic capital regulation that provides an advantage to non-U.S. banks."

They argued that tighter rules would make it tougher for them to compete globally, since more of their money would be tied up in the capital cushion. Eventually, in July 2008, the U.S. Federal Reserve and regulators acceded to the banks' requests by allowing them to follow rules similar to those in Europe. That ruling potentially could enable American banks to hold looser, European-style capital. Ironically, by then, cracks in the global financial system were already spreading rapidly.

At the beginning of 2008, it appeared that the SMC had caused a higher degree of problems for non-U.S. financial institutions. The write-downs that British, European, and Asian institutions had to make on U.S. subprime mortgage debt were something that some analysts attributed to Basel II that gave institutions some carte blanche when it came to raising capital for securities with top credit ratings. For instance, on Friday, 14 September 2007, Northern Rock, the U.K.'s fifth-biggest mortgage lender, started experiencing a bank run after it was revealed that the bank was having trouble raising liquidity. Within one day, customers had withdrawn an estimated $ 1 billion resulting in the first bank run in the U.K. since 1866. Earlier, applying Basel II principles, Northern Rock announced it would boost its shareholder dividend by 30%—a step that depleted its capital even as regulators warned about the lender's condition. Adam Applegarth, Northern Rocks CEO at the time of the crisis, defended the lender's dividend boosting before a parliamentary inquiry on the bank run.

He argued that because of the high credit quality of its mortgage loans in Basel II terms, the lender could opt to hold less capital to cover potential losses. To finance its expansion, Northern Rock began to borrow heavily in global financial markets, rather than relying as much on traditional customer bank deposits. In fact, its deposits-to-total liabilities and equity ratio had decreased from 63% at the end of 1997 to 22% at the end of 2006, less than half the level of most fellow mortgage lenders. This meant that Northern Rock had access to insufficient cash when its own liquidity dried up after investors decided to stop financing its growth. Eventually, Northern Rock was nationalized by the British government. Even in Switzerland, where Basel II was devised, regulators have questioned capital regulation. For instance, UBS AG wrote down $ 18 billion in losses due to risk mismanagement and exposure to subprime mortgages and other risky assets. In December 2007, UBS disclosed plans to boost its capital with a $ 12.1 billion injection from the Government of Singapore Investment Corp. and an unidentified Middle Eastern investor. By Friday, 10 October 2008 (Black Friday), the Japanese company, Yamato Life, filed for bankruptcy becoming what is viewed as the first direct casualty in Japan from the fallout of the SMC. Yamato Life had $ 2.7 billion in liabilities at the time of the filing. Some analysts believed it to be a foreshadowing of things to come in Japan as the credit crisis began to affect the country. In short, the Basel rules are being questioned for containing inadequate prescriptions for monitoring a bank's liquidity; in other words, its ability to readily sell assets, or borrow affordably, to cover obligations. In principle, Basel II regulators worldwide are required to track a bank's risk-taking and to check how the bank monitors itself. Traditionally, it is not their task to prescribe to banks about the size and type of risks they can take. In the U.S., some regulators have recently shown a willingness to tighten Basel capital regulation. They are motivated by the failure of more than 1000 banks amid unforeseen risks related to interest rates and real estate during the 1980's S and L crisis. In addition, regulators both in the U.S. and abroad are gearing themselves to amend Basel II. In all likelihood, this will involve enforcing a higher level of capital against assets that may be construed to be risky in the wake of the SMC.

1.1. Relation to Previous Literature

In this subsection, we discuss terse literature reviews of subprime mortgage loans, Basel capital regulation as well as the SMC. More comprehensive reviews of some of these topics have already been done in [1] (see, also, [3]).

1.1.1. Brief Literature Review of Subprime Mortgage Loans

In [4], light is shed on the subprime mortgagor, the workings of a typical subprime mortgage loan, and the historical performance of subprime mortgage credit. In order to keep the discussion from becoming too abstract, they find it useful to frame many of these issues in the context of a real-life example. Using loan-level data, [5] analyzes the quality of subprime mortgage loans by adjusting their performance for differences in borrower characteristics, loan characteristics, and macroeconomic conditions. They find that the quality of loans deteriorated for six consecutive years before the SMC and that securitizers were, to some extent, aware of it.

The mortgage lenders that retained credit risk from subprime mortgages were the first to be affected by the SMC, as borrowers became unable or unwilling to make payments. Major banks and other financial institutions around the world have reported losses of approximately US $435 billion as of 17 July 2008 (see [6, 7]). Owing to a form of financial engineering called securitization, many mortgage lenders had passed the rights to the mortgage payments and related credit/default risk to third-party investors via mortgage-backed securities (MBSs) and collateralized debt obligations (CDOs). Corporate, individual and institutional investors holding MBSs or CDOs faced significant losses, as the value of the underlying mortgage assets declined. Stock markets in many countries declined significantly.

It is a widely accepted fact that subprime mortgages exhibit cyclical tendencies. The fact that credit ratings (profitability) behave procyclically by rising (falling) during economic booms and falling (rising) during recessions (see, e.g., [1, 8, 9]) is incorporated in our models. (Procyclicality is a normal feature of financial systems in which asset price increases and credit expansions support the business cycle and contribute to economic growth.) In particular, credit rating agencies (CRAs) are now under scrutiny for having given investment-grade ratings to CDOs and MBSs based on subprime mortgage loans during the boom. These high ratings were believed justified because of risk reducing practices, including overcollateralization (pledging collateral in excess of debt issued), credit default insurance, via, for instance, credit default swaps as well as the intervention of equity investors willing to bear the first losses such as for CDOs.

1.1.2. Brief Literature Review of Basel Capital Regulation

The most significant innovation of Basel II is the departure from a sole reliance on capital adequacy ratios. Basel II consists of three mutually reinforcing pillars, which together should contribute to safety and soundness in the financial system (see, e.g., [2]). To ensure that risks within an entire banking group are considered, Basel II is extended on a consolidated basis to holding companies of banking groups. The main objective of the Basel II Capital Accord is to promote standards for measurement and management of financial and operational risk in banking. Its approach to such risk issues has been severely criticized in the literature, inevitably leading to doubts about its practical implementation. In particular, many investigations have warned against the procyclicality induced by the IRB capital formula (see, e.g., [2]). Since the release of the Second Consultative Paper [10], many studies have assessed empirically the magnitude of procyclicality in the IRB capital formula (see, e.g., [11]). Also, there is overwhelming evidence to suggest that the movements of subprime mortgage loan extension, loan loss provisioning, capital and profitability are strongly correlated with the business cycle. While not providing an in-depth discussion of the first of the aforementioned problems, our contribution focusses strongly on issues related to subprime mortgage loan pricing and regulatory capital in both the Basel I and II paradigms. Also, we suggest that bank capital is less variable under Basel II than Basel I (see, e.g., [2, 10]).

1.1.3. Brief Literature Review of the Subprime Mortgage Crisis

It is generally accepted that the SMC resulted from the bursting of the U.S. housing bubble (see, e.g., [12]). Coupled to this was high default rates on “subprime” and adjustable rate mortgages (ARMs). Loan incentives, such as easy initial terms, in conjunction with an acceleration in rising housing prices encouraged borrowers to assume difficult mortgages in the belief they would be able to quickly refinance at more favorable terms. However, once housing prices started to drop moderately in 2006-2007 in many parts of the U.S., refinancing became more difficult. Defaults and foreclosure activity increased dramatically, as easy initial terms expired, home prices failed to go up as anticipated, and ARM interest rates reset higher. Foreclosures accelerated in the U.S. in late 2006 and triggered a global financial crisis through 2007 and 2008. During 2007, nearly $1.3 million U.S. housing properties were subject to foreclosure activity, up 79% from 2006 (see [13] for more details).

The working paper [5] makes several novel contributions to the literature on the SMC. Firstly, the authors, Demyanyk and Van Hemert, quantify how much different determinants have contributed to the observed high delinquency and foreclosure rates for vintage 2006 loans, which led up to the 2007 SMC. Their data analysis suggests that different loan-level characteristics as well as low house price appreciation were quantitatively too small to explain the bad performance of 2006 loans. Secondly, the authors uncover a downward trend in loan quality, determined as loan performance adjusted for differences in loan and borrower characteristics as well as subsequent house price appreciation. They further show that there was a deterioration of lending standards and a decrease in the subprime-prime mortgage rate spread during the 2001–2006 period. Together these results provide evidence that the rise and fall of the subprime mortgage market follows a classic lending boom-bust scenario, in which unsustainable growth leads to the collapse of the market. Thirdly, Demyanyk and Van Hemert show that continual deterioration of loan quality could have been detected long before the crisis by means of a simple statistical exercise. Fourth, securitizers were, to some extent, aware of this deterioration over time, as evidenced by changing determinants of mortgage rates. Furthermore, paper [5] documents that the poor performance of the vintage 2006 loans was not confined to a particular segment of the subprime mortgage market. In this regard, fixed-rate, adjustable-rate, purchase-money, cash-out refinancing, low-documentation, and full-documentation loans originated in 2006 all showed substantially higher delinquency and foreclosure rates than loans made in the preceding five years. As a consequence, [5] contradicts a widely held belief that the SMC was mostly confined to adjustable-rate or low-documentation mortgages.

Since mid 2007, role players in the financial industry have blamed the Basel II Capital Accord for certain aspects of the SMC. In this regard, the adequacy of capital levels in the banking industry, the role of credit rating agencies in financial regulation, the procyclicality of minimum capital requirements, and the fair-value assessment of banking assets have become the most studied topics. Paper [14] poses the following related questions. Is Basel II guilty of causing the subprime mortgage crisis? Is it appropriate to judge Basel II on the basis of features that are unlikely to have caused the subprime mortgage crisis? Should Basel II be completely abandoned or should an attempt rather be made to overcome its shortcomings? Paper [14] attempts to provide some answers to the questions raised above. After a short review of the main features of the financial crisis as well as of the rationale behind the Basel II rules, the authors try to describe the actual role played by the new prudential regulation in the crisis and discuss the main argument raised in the current debate. They conclude that, while aspects of Basel II need strengthening, there are not good enough reasons for abandoning the accord in its entirety.

1.2. Preliminaries

In this subsection, we provide preliminaries about balance sheets, subprime mortgage loans, bank profit, the Basel Capital Accords, and the SMC.

1.2.1. Preliminaries about Bank Balance Sheets

As is well known, the bank balance sheet consists of assets (uses of funds) and liabilities (sources of funds) that are balanced by bank capital (see, e.g., [15]) according to the well-known relation

$$\text{Total Assets } (A) = \text{Total Liabilities } (\Gamma) + \text{Total Bank Capital } (K). \tag{1.1}$$

In period t, the main on-balance sheet items in (1.1) can specifically be identified as

$$A_t = \Lambda_t^m + W_t + C_t + S_t + B_t, \qquad W_t = \mathtt{T}_t + R_t, \qquad \Gamma_t = D_t + B_t, \qquad K_t = n_t E_{t-1} + O_t + R_t^l, \tag{1.2}$$

where $\Lambda^m, C, S, B, \mathtt{T}, R, D, n, E, O,$ and R^l are the market value of short- and long-term loans, cash, short- and long-term securities, bonds, Treasuries, reserves, outstanding debt, number of shares, market price of the bank's common equity, subordinate debt, and loan loss reserves, respectively.

1.2.2. Preliminaries about Subprime Mortgage Loans

Subprime mortgage loans are loans whose interest rate repayment is below the prime rate. A study by the U.S. Federal Reserve found that the average difference between subprime and prime mortgage interest rates (the "subprime markup") declined from 280 basis points in 2001, to 130 basis points in 2007. In other words, the risk premium required by lenders to offer a subprime loan declined. This occurred even though the credit ratings of subprime borrowers, and the characteristics of subprime loans, both declined during the 2001–2006 period, which should have had the opposite effect. The combination is common to classic boom and recession credit cycles (see [5]).

The 2001 Interagency Expanded Guidance for Subprime Lending Programs defines the *subprime borrower* as one who generally displays a range of credit risk characteristics, including one or more of the following:

(i) two or more 30-day delinquencies in the last 12 months, or one or more 60-day delinquencies in the last 24 months;

(ii) judgment, foreclosure, repossession, or charge-off in the previous 24 months;

(iii) bankruptcy in the last 5 years;

(iv) relatively high default probability as evidenced by, for example, a credit bureau risk score (FICO) of 660 or below (depending on the product/collateral), or other bureau or proprietary scores with an equivalent default probability likelihood;

(v) debt service-to-income ratio of 50 percent or greater; or, otherwise limited ability to cover family living expenses after deducting total debt-service requirements from monthly income.

A diagrammatic overview of cash flows for subprime mortgage loans may be represented as shown in Figure 1.

From Figure 1, we note that 1A represents the cash flow from the mortgage lender for financing the portfolio of subprime mortgage loans. Also, 1b denotes the interest rate,

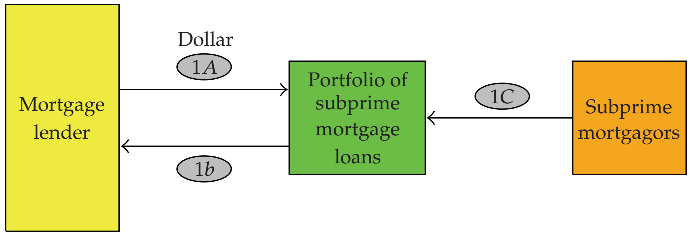

Figure 1: Diagrammatic overview of cash flows for subprime mortgage loans.

r^{Λ}, received from this subprime portfolio. The arrow numbered 1C is the mortgage cash payments by subprime mortgagors.

1.2.3. Preliminaries about Bank Profits

As far as profit, Π, is concerned, we closely follow paper [1] by using the basic fact that profits can be characterized as the difference between income and expenses that are reported in the bank's income statement. In our case, *income* is solely constituted by the return on intangible assets, $r_t^I I_t$, the return on subprime mortgage loans, $r_t^{\Lambda}\Lambda_t$, and the return on Treasuries, $r_t^{\mathtt{T}}\mathtt{T}_t$. In this regard, r^I, r^{Λ} and $r^{\mathtt{T}}$ denote the rates of return on intangible assets, subprime mortgage loans (that may include a component for provisions for expected loan losses), and Treasuries, respectively. Furthermore, we assume that the level of macro economic activity is denoted by $\mathtt{M_t}$. As *expenses*, in period t, we consider the cost of monitoring and screening of subprime mortgage loans and capital, $c^{\Lambda}\Lambda_t$, interest paid to depositors, $r_t^D D_t$, the cost of taking deposits, $c^D D_t$, the cost of deposit withdrawals, $c^w(W_t)$, the value of subprime mortgage loan losses, $L(\mathtt{M_t})$, and total loan loss provisions, $P(\mathtt{M_t})$. Here r^D and c^D are the deposit rate and cost of deposits, respectively. We assume all the aforementioned costs would sum to operating costs so that profit, Π, can be expressed as

$$\begin{aligned}\Pi_t = r_t^{\Lambda}\Lambda_t + r_t^{\mathtt{T}}\mathtt{T_t} + r_t^I I_t - c^{\Lambda}\Lambda_t - \left(r_t^D + c^D\right)D_t \\ - \left(r_t^B + c^B\right)B_t - c^w(W_t) - L(\mathtt{M_t}) - P(\mathtt{M_t}).\end{aligned} \tag{1.3}$$

1.2.4. Preliminaries about the Basel Capital Accords and the Subprime Mortgage Crisis

Many of the preliminaries presented in this subsection are directly related to the contents of [1, Sections 2 to 5]. Basel capital regulation has its roots in the 1980s, when bank regulations varied dramatically from country to country, making it tough for banks to compete across borders. Central bankers from around the globe congregated in Switzerland to agree upon basic standards, which were unveiled in 1988. Subsequently, Basel II focused on expanding those regulations in order to protect financial systems against complex new investment products being introduced by banks. Basel II went into effect in European countries in 2007 and 2008.

Banks rely on capital to cushion against loan losses and, ultimately, failure. Under pre-Basel II rules, setting the level of this capital was a simple process. Banks held a specific amount of capital, which was calculated based on the types of assets they hold. For example, MBSs did not require much capital because they were considered to be relatively safe. Basel II changes that by allowing banks to calculate their level of capital reserves based in part on their own assessments of risk and the opinion of CRAs. In support of Basel II, the U.S. Federal Reserve argued that its standards give banks incentives to bolster their risk management. In addition, Basel II requires institutions to maintain a safety net of capital to protect against trouble in "off balance sheet" assets they may have an issue that had largely escaped prior regulatory oversight. Some analysts claim that the SMC will strengthen Basel II by giving banks valuable new data to factor into their models.

1.3. Main Problems and Outline of the Paper

Undergirded by the analysis in [1], we extend aspects of the literature mentioned in Section 1.1 in several important directions. Firstly, in a Basel I framework, we investigate the dynamics of bank capital and credit and their sensitivity to changes in the level of macroeconomic activity. Here asset risk-weights are kept constant and we only consider credit and market risk. Next, we repeat the above in a Basel II paradigm where subprime mortgage loan risk-weights vary while risk-weights for intangible assets and shares remain constant. Also, á la Basel II, we consider credit and market risk as well as operational risk. Thirdly, we include a discussion of subprime mortgage loans and their reduced risk premiums. Finally, we consider the effect that Basel capital regulation has had on the SMC.

1.3.1. Main Problems

The main questions that we pose in this paper may be stated as follows.

Problem 1 (Bank Credit and Capital Under Basel I). What is the effect of changes in the level of macroeconomic activity on bank credit and capital when risk-weights are constant under Basel I? (see Theorem 3.1 as well as Propositions 3.2 and 3.3 of Section 3).

Problem 2 (Bank Credit and Capital Under Basel II). What is the effect of changes in the level of macroeconomic activity on bank credit and capital under Basel II? (see Theorem 4.1 as well as Propositions 4.2 and 4.3 of Section 4).

Problem 3 (The Basel Accords and the SMC). Did the Basel accords exacerbate the subprime mortgage crisis? (see Section 5).

1.3.2. Outline of the Paper

In the current subsection, an outline of our contribution is given. In Section 2, we present discrete-time stochastic models for subprime banking activities. Banks respond differently to shocks that affect subprime mortgage loan demand, Λ, when the minimum capital equirements are calculated by using risk-weighted assets. In the Hicksian case, these responses are usually sensitive to macroeconomic conditions that are related to the term $l_2\mathtt{M}_t$ in (2.3). Subprime mortgage loan defaults are independent of the capital adequacy paradigm that is chosen. In this regard, empirical evidence supports the opinion that enhanced levels of macroeconomic activity reduce the subprime mortgage loan default rate and thus the loan marginal cost.

Section 3 contains a discussion of the cyclicality of bank credit and capital where, as in Basel I, we assume that risk-weights are constant. In addition, borrowers are allowed to default on their subprime mortgage loan repayments. Furthermore, we present a result about the effects of changes in the level of macroeconomic activity on the quantity and price of subprime mortgage loans (see Theorem 3.1 of Section 3.1). Moreover, we establish the impact of changes in the business cycle on subprime mortgage loans and their loan rates when the capital constraint holds and when it does not (see, e.g., Proposition 3.2 of Section 3.2 and Proposition 3.3 of Section 3.3; also compare with (3.2)).

In Section 4, results analogous to those obtained in Section 3 are given for situations where subprime mortgage loan losses and loan risk-weights are a function of the level of macroeconomic activity (i.e., risk-weights vary with changes in business cycle phases). These situations are cast within the framework offered by the Basel II Capital Accord. Analogously, we study situations where the capital constraint holds and where it does not (see Theorem 4.1, Proposition 4.2, and Proposition 4.3 in Sections 4.1, 4.2, and 4.3, resp.). In Section 4.4, we discuss the cyclicality of subprime mortgage loans and their rates under Basel II in subsequent time periods.

In Section 5, we consider the relationship between the Basel Capital Accords and the SMC. In particular, we try to answer the main question posed by the paper, namely, whether the capital accords exacerbated the mortgage crisis.

Section 6 provides concluding remarks and a discussion about possible future research. Subsequent to the reference list we insert an appendix that contains a table summarizing the main results as well as two tables outlining the main differences between the Basel I and II Capital Accords.

2. Subprime Banking Models

This section is an adaptation of the models in [1] to the subprime case. Also, we present an optimization result achieved in preparation for Section 3.

2.1. Subprime Mortgage Loans

We suppose that, after providing liquidity, the bank lends in the form of tth period subprime mortgage loans, Λ_t, at the *bank's subprime loan rate*, r_t^Λ. This loan rate, for profit maximizing banks, is determined as follows:

$$\varrho_t = r_t^\Lambda - r_t \tag{2.1}$$

where r_t is the base rate and ϱ_t is the risk premium. As was mentioned before, the "subprime markup" declined considerably for the period 2001 to 2007. In other words, the risk premium required by lenders to offer a subprime loan declined. This occurred even though the credit ratings of subprime borrowers, and the characteristics of subprime loans, both declined during this period. Next, we introduce the generic variable, $\mathtt{M}_t$, that represents the level of macroeconomic activity in the bank's loan market. We suppose that $\mathtt{M} = \{\mathtt{M}_t\}_{t\geq 0}$ follows the first-order autoregressive stochastic process

$$\mathtt{M}_{t+1} = \mu^{\mathtt{M}}\mathtt{M}_t + \sigma^{\mathtt{M}_{t+1}}, \tag{2.2}$$

where $\sigma^{\mathtt{M}_{t+1}}$ denotes zero-mean stochastic shocks to macroeconomic activity.

In our case, the bank faces a *Hicksian demand for subprime mortgage loans* given by

$$\Lambda_t = l_0 - l_1 r_t^{\Lambda} + l_2\mathtt{M}_t + \sigma_t^{\Lambda}. \tag{2.3}$$

We note that the subprime mortgage loan demand in (2.3) is an increasing function of $\mathtt{M}$ and a decreasing function of r_t^{Λ}. Further, we suppose that σ_t^{Λ} is the *random shock to the subprime mortgage loan demand* with support $[\underline{\Lambda}, \overline{\Lambda}]$ that is independent of an exogenous stochastic variable, x_t, to be characterized below. Also, we assume that the *subprime mortgage loan supply process*, Λ, follows the first-order autoregressive stochastic process

$$\Lambda_{t+1} = \mu_t^{\Lambda}\Lambda_t + \sigma_{t+1}^{\Lambda}, \tag{2.4}$$

where $\mu_t^{\Lambda} = r_t^{\Lambda} - c^{\Lambda} - r^d(\mathtt{M}_t)$ and σ_{t+1}^{Λ} denotes zero-mean stochastic shocks to subprime mortgage loan supply.

An initial observation is that subprime mortgage loan losses are also dependent on macroeconomic activity. As a consequence, for the *value of subprime mortgage loan losses*, L, and the *default rate*, r^d, we set

$$L(\mathtt{M}_t) = r^d(\mathtt{M}_t)\Lambda_t, \tag{2.5}$$

where $r^d \in [0,1]$ increases when macroeconomic conditions deteriorate according to

$$0 \le r^d(\mathtt{M}_t) \le 1, \qquad \frac{\partial r^d(\mathtt{M}_t)}{\partial \mathtt{M}_t} < 0. \tag{2.6}$$

We note that the above description of the subprime mortgage loan loss rate is consistent with empirical evidence that suggests that bank losses on subprime mortgage loan portfolios are correlated with the business cycle under any capital adequacy regime (see, e.g., [1, 8]).

2.2. Reserves

Bank reserves are the deposits held in accounts with the central bank of a country plus money that is physically held by banks (vault cash). Such reserves constitute money that is not lent out but is earmarked to cater for withdrawals by depositors. Since it is uncommon for depositors to withdraw all of their funds simultaneously, only a portion of total deposits may be needed as reserves. As a result of this description, we may introduce a *reserve-deposit ratio*, γ, for which

$$R_t = \gamma D_t. \tag{2.7}$$

The bank uses the remaining deposits to earn profit, either by issuing subprime mortgage loans or by investing in assets such as Treasuries and stocks.

2.3. Risk-Weighted Assets

We consider risk-weighted assets (RWAs) that are defined by placing each on- and off-balance sheet item into a risk category. The more risky assets are assigned a larger weight in this study. As a result, RWAs are a weighted average of the various assets of the banks. In the sequel, we denote the risk-weights on intangible assets, subprime mortgage loans, Treasuries and reserves by $\omega^I, \omega^\Lambda, \omega^{\mathrm{T}}$ and ω^R, respectively. With regard to the latter, we can identify a special risk-weight on subprime mortgage loans, $\omega^\Lambda = \omega(\mathtt{M}_t)$, that is a decreasing function of current macroeconomic conditions so that

$$\frac{\partial \omega(\mathtt{M}_t)}{\partial \mathtt{M}_t} < 0. \tag{2.8}$$

This is in line with the procyclical notion that during booms, when macroeconomic activity increases, the risk-weights will decrease. On the other hand, during recessions, risk-weights may increase because of an elevated probability of default and/or loss given default on subprime mortgage loans.

2.4. Capital

For the purposes of our study, *regulatory capital*, K, is the book value of bank capital defined as the difference between the accounting value of the assets and liabilities. More specifically, Tier 1 capital is represented by period $t-1$'s market value of the bank equity, n_tE_{t-1}, where n_t is the number of shares and E_t is the period t market price of the bank's common equity. Tier 2 capital mainly consists of subordinate debt, O_t, that is subordinate to deposits and hence faces greater default risk and loan loss reserves, R_t^l. Subordinate debt issued in period $t-1$ is represented by a one-period bond that pays an interest rate, r^O. Also, we assume that loan loss reserves held in period $t-1$ changes at the rate, r^{R^l}. Tier 3 capital is not considered at all. In the sequel, we take the bank's total regulatory capital, K, in period t to be

$$K_t = n_tE_{t-1} + O_t + R_t^l. \tag{2.9}$$

For K_t given by (2.9), we obtain the balance sheet constraint

$$W_t = D_t + \mathrm{B}_t - \Lambda_t - C_t - B_t - S_t + K_t. \tag{2.10}$$

We define the *regulatory capital constraint* by the inequality

$$K_t \geq \rho(a_t + 12.5(\mathrm{mVaR} + \mathtt{O})), \tag{2.11}$$

where

$$a_t = \omega^I I_t + \omega^C C_t + \omega^B B_t + \omega^S S_t + \omega^\Lambda \Lambda_t + \omega^{\mathrm{T}} \mathrm{T}_t + \omega^R R_t, \tag{2.12}$$

and $\rho \approx 0.08$. If we assume that the risk-weights associated with intangible assets, shares, cash, bonds, Treasuries, reserves, and subprime mortgage loans may be taken to be $\omega^I \neq 0$, $\omega^S \neq 0$, $\omega^C = \omega^B = \omega^{\mathtt{T}} = \omega^R = 0$, and $\omega^\Lambda = \omega(\mathtt{M}_t)$, respectively, then (2.11) becomes the capital constraint

$$K_t \geq \rho\left[\omega(\mathtt{M}_t)\Lambda_t + \omega^I I_t + \omega^S S_t + 12.5(\text{mVaR} + \mathtt{O})\right]. \tag{2.13}$$

2.5. Profit

We assume that (2.5) holds. If we now add and subtract $r_t^{\mathtt{T}}\gamma D_t$ from (1.3) and use the fact that $W_t = \mathtt{T}_t + \gamma D_t$, we obtain

$$\begin{aligned}\Pi_t = {}& \left(r_t^\Lambda - c^\Lambda - r^d(\mathtt{M}_t)\right)\Lambda_t + r_t^{\mathtt{T}} W_t + r_t^I I_t - \left(r_t^D + c^D\right)D_t \\ & - \left(r_t^{\mathrm{B}} + c^{\mathrm{B}}\right)\mathrm{B}_t - c^w(W_t) - P(\mathtt{M}_t) - r_t^{\mathtt{T}}\gamma D_t.\end{aligned} \tag{2.14}$$

This is the cash flow constraint for a bank and will be used later. Furthermore, by considering $\partial P(\mathtt{M}_t)/\partial \mathtt{M}_t < 0$ and (2.14), we suspect that profit, Π, is an increasing function of current macroeconomic conditions, $\mathtt{M}$, so that

$$\frac{\partial \Pi_t}{\partial \mathtt{M}_t} > 0. \tag{2.15}$$

This is connected with procyclicality where we expect profitability to increase during booms, when macroeconomic activity increases. By contrast, profitability may decrease during recessions because of, among many other factors, an increase in provisioning (see (2.14)).

To establish the relationship between bank profitability and retained earnings, a model of bank financing is introduced that is based on [16]. We know that *bank profits*, Π_t, are used to meet the bank's commitments that include *dividend payments on equity*, $n_t d_t$ and *interest and principal payments on subordinate debt*, $(1 + r_t^O)O_t$. The *retained earnings*, E_t^r, subsequent to these payments may be computed by using

$$\Pi_t = E_t^r + n_t d_t + \left(1 + r_t^O\right)O_t. \tag{2.16}$$

In standard usage, retained earnings refer to earnings that are not paid out in dividends, interest, or taxes. They represent wealth accumulating in the bank and should be capitalized in the value of the bank's equity. Retained earnings are also defined to include bank charter value income. Normally, *charter value* refers to the present value of anticipated profits from future lending.

2.6. Valuation

In each period, banks invest in fixed assets (including buildings and equipment) which we denote by F_t. The bank is assumed to maintain these assets throughout its existence so that the bank must only cover the costs related to the *depreciation of fixed assets*, ΔF_t. These activities are financed through retaining earnings and the eliciting of additional debt and equity, so that

$$\Delta F_t = E_t^r + (n_{t+1} - n_t)E_t + O_{t+1} + R_{t+1}^l. \tag{2.17}$$

We can use (2.16) and (2.17) to obtain an expression for bank capital of the form

$$K_{t+1} = n_t(d_t + E_t) + \left(1 + r_t^O\right)O_t - \Pi_t + \Delta F_t, \tag{2.18}$$

where K_t is defined by (2.9).

If the expression for retained earnings given by (2.16) is substituted into (2.17), the *nett cash flow generated by the bank for a shareholder* is given by

$$N_t = \Pi_t - \Delta F_t = n_t d_t + \left(1 + r_t^O\right)O_t - K_{t+1} + n_t E_t. \tag{2.19}$$

In addition, we have the relationship

$$\text{Bank Value for a Shareholder} = \text{Nett Cash Flow} + \text{Ex-Dividend Bank Value}. \tag{2.20}$$

This translates to the expression

$$V_t = N_t + K_{t+1}, \tag{2.21}$$

where K_t is defined by (2.9). Furthermore, the stock analyst (acting in the interest of a shareholder) evaluates the expected future cash flows in j periods based on a *stochastic discount factor*$\delta_{t,j}$ such that the value of the bank is

$$V_t = N_t + \mathbf{E}_t\left[\sum_{j=1}^{\infty}\delta_{t,j}N_{t+j}\right]. \tag{2.22}$$

2.7. An Optimal Subprime Mortgage Loan Pricing Problem

In this subsection, we present the main features of the optimal subprime mortgage loan pricing problem solved in [1].

2.7.1. Statement of the Optimal Loan Pricing Problem

In the sequel, suppose that the bank's performance criterion, J, at t is given by

$$\begin{aligned} J_t = {} & \Pi_t + l_t\left[K_t - \rho\left(\omega(\mathtt{M}_t)\Lambda_t + \omega^I I_t + \omega^S S_t + 12.5(\mathrm{mVaR} + \mathtt{O})\right)\right] - c_t^{dw}[K_{t+1}] \\ & + \mathbf{E}_t[\delta_{t,1}V(K_{t+1}, x_{t+1})], \end{aligned} \tag{2.23}$$

where l_t is the Lagrangian multiplier for the total capital constraint (2.13), K_t is defined by (2.9), $\mathbf{E}_t[\cdot]$ is the expectation conditional on the bank's information at time t and x_t is the deposit withdrawals in period t with probability distribution $f(x_t)$. Also, c_t^{dw} is the deadweight cost of total capital consisting of debt and equity. We are now in a position to formally state the optimal valuation problem for banks that we solve in the sequel.

Problem 4 (Statement of the Optimal Loan Pricing Problem). Suppose that the total capital constraint and the performance criterion, J, are given by (2.13) and (2.23), respectively. The optimal subprime mortgage loan pricing problem is to maximize the value of the bank given by (2.22) from the point of view of a stock analyst, by choosing the subprime mortgage loan rate, deposits and regulatory capital for

$$V(K_t, x_t) = \max_{r_t^{\Lambda}, D_t, K_t} J_t, \tag{2.24}$$

subject to the subprime mortgage loan demand, balance sheet, cash flow, and financing constraints given by (2.3), (2.10), (2.14) and (2.18), respectively.

2.7.2. Solution of the Optimal Loan Pricing Problem

In this section, we find a solution to Problem 4 when the capital constraint is binding (see, e.g., [17]). In this regard, the main result can be stated and proved as follows.

Theorem 2.1 (Solution to the Optimal Loan Pricing Problem). *Suppose that J and V are given by* (2.23) *and* (2.24)*, respectively, and $P(\mathtt{M}_{\mathtt{t}}) > 0$. When the capital constraint given by* (2.13) *is binding (i.e., $l_t > 0$), a solution to the optimal subprime mortgage loan pricing problem stated in Problem 4 yields an optimal bank subprime mortgage loan supply and loan rate of the form*

$$\Lambda_t^* = \frac{K_t}{\rho\omega(\mathtt{M}_{\mathtt{t}})} - \frac{\omega^I I_t + \omega^S S_t + 12.5(mVaR + \mathtt{O})}{\omega(\mathtt{M}_{\mathtt{t}})}, \tag{2.25}$$

$$r_t^{\Lambda *} = \frac{1}{l_1}\left(l_0 + l_2\mathtt{M}_{\mathtt{t}} + \sigma_t^{\Lambda} - \frac{K_t}{\rho\omega(\mathtt{M}_{\mathtt{t}})} + \frac{\omega^I I_t + \omega^S S_t + 12.5(mVaR + \mathtt{O})}{\omega(\mathtt{M}_{\mathtt{t}})}\right), \tag{2.26}$$

respectively. In this case, the corresponding optimal deposits, provisions for deposit withdrawals, and profits are given by

$$\begin{aligned} D_t^* = {}& \overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left[r_t^{\mathtt{t}} - \frac{(r_t^D + c^D)}{1-\gamma}\right] + \frac{K_t}{\rho\omega(\mathtt{M}_t)} - \frac{\omega^I I_t + \omega^S S_t + 12.5(mVaR + \mathtt{O})}{\omega(\mathtt{M}_t)} \\ & + C_t + S_t + B_t - K_t - B_t, \\ W_t^* = {}& \overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left[r_t^{\mathtt{T}} - \frac{(r_t^D + c^D)}{1-\gamma}\right], \end{aligned}$$

$$\Pi_t^* = \left(\frac{K_t}{\rho\omega(\mathtt{M}_t)} - \frac{\omega^I I_t + \omega^S S_t + 12.5(mVaR + \mathtt{O})}{\omega(\mathtt{M}_t)}\right)$$
$$\times \left\{\frac{1}{l_1}\left(l_0 - \frac{K_t}{\rho\omega(\mathtt{M}_t)} + \frac{\omega^I I_t + \omega^S S_t + 12.5(mVaR + \mathtt{O})}{\omega(\mathtt{M}_t)} + l_2\mathtt{M}_t + \sigma_t^{\Lambda}\right)\right.$$
$$\left. -\left(c^{\Lambda} + \left(r_t^D + c^D + r_t^{\mathrm{T}}\gamma\right) + r^d(\mathtt{M}_t)\right)\right\} - \left(r_t^D + c^D + r_t{}^{\mathrm{t}}\gamma\right)(C_t + B_t + S_t - K_t - B_t)$$
$$+ \left(\overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left[r_t^{\mathrm{T}} - \frac{(r_t^D + c^D)}{1-\gamma}\right]\right)\left((1-\gamma)r_t^{\mathrm{T}} - \left(r_t^D + c^D\right)\right)$$
$$- \left(r_t^{\mathrm{B}} + c^{\mathrm{B}}\right)\mathrm{B}_t - c^w(W_t) - P(\mathtt{M}_t) + r_t^I I_t + r_t^C C_t + r_t^B B_t + r_t^S S_t, \tag{2.27}$$

respectively.

In the proof of Theorem 2.1, we note that the first-order conditions are given by

$$\frac{\partial \Pi_t}{\partial r_t^{\Lambda}}\left[1 + c_t^{dw} - \mathbf{E}_t\left\{\int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1}\frac{\partial V}{\partial K_{t+1}} dF\left(\sigma_{t+1}^{\Lambda}\right)\right\}\right] + l_t\rho l_1\omega(\mathtt{M}_t) = 0, \tag{2.28}$$

$$\frac{\partial \Pi_t}{\partial D_t}\left[1 + c_t^{dw} - \mathbf{E}_t\left\{\int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1}\frac{\partial V}{\partial K_{t+1}} dF\left(\sigma_{t+1}^{\Lambda}\right)\right\}\right] = 0, \tag{2.29}$$

$$\rho\left[\omega(\mathtt{M}_t)\Lambda_t + \omega^I I_t + \omega^S S_t + 12.5(m\mathrm{VaR} + \mathtt{O})\right] \le K_t, \tag{2.30}$$

$$-c_t^{dw} + \mathbf{E}_t\left\{\int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1}\frac{\partial V}{\partial K_{t+1}} dF\left(\sigma_{t+1}^{\Lambda}\right)\right\} = 0. \tag{2.31}$$

Here $F(\cdot)$ is the cumulative distribution of the shock to the subprime mortgage loans.

In the case where the constraint (2.13) does not hold, the following corollary follows directly.

Corollary 2.2 (solution to the optimal loan pricing problem (slack)). *Suppose that J and V are given by* (2.23) *and* (2.24)*, respectively, and* $P(\mathtt{M}_t) > 0$. *When the capital constraint* (2.13) *does not hold (i.e., $l_t = 0$), a solution to the optimal loan pricing problem stated in Problem 2.1 yields the optimal bank subprime mortgage loan supply and its rate*

$$\Lambda_t^{n*} = \frac{1}{2}\left(l_0 + l_2\mathtt{M}_t + \sigma_t^{\Lambda}\right) - \frac{l_1}{2}\left(c^{\Lambda} + r^d(\mathtt{M}_t) + \left(r_t^D + c^D\right) + r_t{}^{\mathrm{t}}\gamma\right),$$
$$r_t^{\Lambda^{n*}} = \frac{1}{2l_1}\left(l_0 + l_2\mathtt{M}_t + \sigma_t^{\Lambda}\right) + \frac{1}{2}\left(c^{\Lambda} + r^d(\mathtt{M}_t) + \left(r_t^D + c^D\right) + r_t{}^{\mathrm{t}}\gamma\right), \tag{2.32}$$

respectively. In this case, the corresponding W_t, deposits and profits are given by

$$
\begin{aligned}
W_t^{n*} &= \overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left(r^{\mathtt{T}_\mathtt{t}} - \frac{(r_t^D + c^D)}{1-\gamma}\right), \\
D_t^{n*} &= \overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left(r_t^{\mathtt{T}} - \frac{(r_t^D + c^D)}{1-\gamma}\right) + \Lambda_t^{n*} + C_t + B_t + S_t - K_t - B_t, \\
\Pi_t^{n*} &= \frac{1}{2}\left(l_0 + l_2\mathtt{M}_t + \sigma_t^{\Lambda}\right) - \frac{l_1}{2}\left(c^{\Lambda} + \left(r_t^D + c^D\right) + r^d(\mathtt{M}_t) + r_t^{\mathtt{T}}(\gamma)\right) \\
&\quad \times \left\{\frac{1}{2l_1}\left(l_0 + l_2\mathtt{M}_t + \sigma_t^{\Lambda}\right) - \frac{1}{2}\left(c^{\Lambda} + \left(r_t^D + c^D\right) + r^d(\mathtt{M}_t) + r_t{}^{\mathtt{t}}\gamma\right)\right\} \\
&\quad - \left(r_t^D + c^D + r_t^{\mathtt{T}}\gamma\right)(C_t + B_t + S_t - K_t - B_t) \\
&\quad + \left(\overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left[r_t{}^{\mathtt{t}} - \frac{(r_t^D + c^D)}{1-\gamma}\right]\right)\left((1-\gamma)r_t{}^{\mathtt{t}} - \left(r_t^D + c^D\right)\right) \\
&\quad - (r_t^{\mathtt{B}} + c^{\mathtt{B}})\mathtt{B}_t - c^w(W_t) - P(\mathtt{M}_t) + r_t^I I_t + r_t^C C_t + r_t^B B_t + r_t^S S_t,
\end{aligned}
\tag{2.33}
$$

respectively.

3. Bank Credit and Capital under Basel I (Constant Risk-Weights)

In this section, we use the models developed in Section 1 to discuss the cyclicality of bank credit and capital in a Basel I paradigm. Two standing assumptions are that borrowers may default on subprime mortgage loans and that their behavior may depend on the phase of the business cycle. We note, for instance, that in line with the prescripts of Basel I, risk-weights are kept constant and operational risk is not considered as in Basel II. In this situation, the capital constraint (2.13) may be adapted to become

$$K_t \geq \rho\left[\omega^{\Lambda}\Lambda_t + \omega^I I_t + \omega^{\mathrm{S}}\mathrm{S}_t + 12.5\,\mathrm{mVaR}\right]. \tag{3.1}$$

Furthermore, since risk-weights on subprime mortgage loans, intangible assets, and shares are kept constant, that is, $\omega^{\Lambda} = \omega^I = \omega^{\mathrm{S}} = 1$, (3.1) has the simpler form

$$K_t \geq \rho[\Lambda_t + I_t + \mathrm{S}_t + 12.5\,\mathrm{mVaR}]. \tag{3.2}$$

Where applicable, in the remainder of this section, we take the binding capital constraint to be (3.2).

3.1. Quantity and Price of Loans and Bank Capital under Basel I

In this subsection, under Basel I, we examine how subprime mortgage loan quantity and pricing as well as bank capital are affected by changes in the level of macroeconomic activity, $\mathtt{M}$.

Theorem 3.1 (cyclicality of bank capital under Basel I). *Suppose that* $S(\mathtt{M}_t) > 0$ *and* $\omega(\mathtt{M}_t) = \omega^I = \omega^{\mathrm{S}} = 1$ *(i.e, risk-weights are constant). It follows that*

(1) *if* $\partial\sigma_{t+1}^{\Lambda*}/\partial\mathtt{M}_t < 0$ *then* $\partial K_{t+1}/\partial\mathtt{M}_t > 0$;

(2) *if* $\partial\sigma_{t+1}^{\Lambda*}/\partial\mathtt{M}_t > 0$ *then* $\partial K_{t+1}/\partial\mathtt{M}_t < 0$.

Proof. In order to prove Theorem 3.1, we have to appeal to the implicit function theorem. Before using this theorem, we explicitly determine the critical shock to the demand for subprime mortgage loans, σ_t^{Λ}, such that the total capital constraint (3.2) will just hold, that is, $\Lambda_t^{n*} = \Lambda_t^*$. By equating the optimal subprime mortgage loans from the two problems (with $l_t = 0$ and $l_t > 0$), we obtain

$$\frac{1}{2}\left(l_0 + l_2\mathtt{M}_t + \sigma_t^{\Lambda}\right) - \frac{l_1}{2}\left(c^{\Lambda} + r^d(\mathtt{M}_t) + \left(r_t^D + c^D\right) + r_t^{\mathrm{T}}\gamma\right) = \frac{K_t}{\rho} - (I_t + \mathrm{S}_t + 12.5\,\mathrm{mVaR}). \quad (3.3)$$

Solving for σ_t^{Λ}, we get

$$\sigma_t^{\Lambda*} = 2\left(\frac{K_t}{\rho} - (I_t + \mathrm{S}_t + 12.5\,\mathrm{mVaR})\right) - (l_0 + l_2\mathtt{M}_t) + l_1\left(c^{\Lambda} + r^d(\mathtt{M}_t) + \left(r_t^D + c^D\right) + r_t^{\mathrm{T}}\gamma\right). \quad (3.4)$$

Using the Euler condition for the bank's subprime loan rate, r^{Λ}, we have that

$$\Lambda_t - l_1\left(r_t^{\Lambda} - c^{\Lambda} - r^d(\mathtt{M}_t)\right) + l_1\left(r_t^{\mathrm{T}} + r_t^C + r_t^B + r_t^{\mathrm{S}}\right) + l_1\frac{r_t^p}{\overline{D}}\left(\overline{D} - W_t\right) + l_1\rho l_t = 0. \quad (3.5)$$

Furthermore, substituting provisions for deposit withdrawals we obtain

$$l_1\rho l_t = l_1\left(r_t^{\Lambda} - c^{\Lambda} - r^d(\mathtt{M}_t) - \left(r_t^D + c^D + \gamma r_t^{\mathrm{T}}\right)\right) - \Lambda_t. \quad (3.6)$$

By substituting $r_t^{\Lambda^*}$ and Λ_t^* into the expression above, it follows that

$$l_t^* = \frac{\sigma_t^{\Lambda} - \sigma_t^{\Lambda*}}{\rho l_1}, \quad \sigma_t^{\Lambda*} \le \sigma_t^{\Lambda} \le \overline{\Lambda}. \quad (3.7)$$

Using (2.24) to find the partial derivative of the value function with respect to bank capital yields

$$\frac{\partial V}{\partial K_t} = \begin{cases} l_t + \left(r_t^D + c^D + \gamma r_t^{\mathrm{T}}\right), & \\ \left(r_t^D + c^D + \gamma r_t^{\mathrm{T}}\right), & \text{for } \underline{\Lambda} \leq \sigma_t^{\Lambda} \leq \sigma_t^{\Lambda *}, \\ \left(r_t^D + c^D + \gamma r_t^{\mathrm{T}}\right) + \dfrac{\sigma_t^{\Lambda} - \sigma_t^{\Lambda *}}{\rho l_1}, & \text{for } \sigma_t^{\Lambda *} \leq \sigma_t^{\Lambda} \leq \overline{\Lambda}. \end{cases} \tag{3.8}$$

By substituting the above expression into the optimal condition for total capital given by

$$-c_t^{dw} + \mathbf{E}_t \left\{ \int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1} \frac{\partial V}{\partial K_{t+1}} dF\left(\sigma_{t+1}^{\Lambda}\right) \right\} = 0, \tag{3.9}$$

we obtain

$$c_t^{dw} - \mathbf{E}_t \left[\delta_{t,1} \left(r_t^D + c^D + \gamma r_t^{\mathrm{T}} \right) \right] - \frac{1}{\rho l_1} \mathbf{E}_t \left[\int_{\sigma_{t+1}^{\Lambda *}}^{\overline{\Lambda}} \delta_{t,1} \left(\sigma_{t+1}^{\Lambda} - \sigma_{t+1}^{\Lambda *} \right) dF\left(\sigma_{t+1}^{\Lambda}\right) \right] = 0. \tag{3.10}$$

If we denote the left-hand side of the above expression by X, then it follows that

$$X = \frac{1}{\rho l_1} \mathbf{E}_t \left[\int_{\sigma_{t+1}^{\Lambda *}}^{\overline{\Lambda}} \delta_{t,1} \left(\sigma_{t+1}^{\Lambda} - \sigma_{t+1}^{\Lambda *} \right) dF\left(\sigma_{t+1}^{\Lambda}\right) \right] = 0. \tag{3.11}$$

By the implicit function theorem we have

$$\frac{\partial K_{t+1}}{\partial \mathtt{M}_t} = -\frac{\partial X / \partial \mathtt{M}_t}{\partial X / \partial K_{t+1}}. \tag{3.12}$$

In order to calculate $\partial X / \partial \mathtt{M}_t$, we utilize (3.11) to obtain

$$\frac{\partial X}{\partial \mathtt{M}_t} = \frac{1}{\rho l_1} \left\{ \frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial \mathtt{M}_t} \mathbf{E}_t \left[\int_{\sigma_{t+1}^{\Lambda *}}^{\Lambda} \delta_{t,1} dF\left(\sigma_{t+1}^{\Lambda}\right) \right] \right\}, \tag{3.13}$$

where

$$\begin{aligned} \frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial \mathtt{M}_t} &= -l_2 \mu^{\mathtt{M}_t} + l_1 \mu^{\mathtt{M}_t} \frac{\partial r^d}{\partial \mathtt{M}_{t+1}}, \\ \frac{\partial X}{\partial K_{t+1}} &= \frac{2}{l_1 \rho^2} \mathbf{E}_t \left[\int_{\sigma_{t+1}^{\Lambda *}}^{\Lambda} \delta_{t,1} dF\left(\sigma_{t+1}^{\Lambda}\right) \right]. \end{aligned} \tag{3.14}$$

Therefore

$$\frac{\partial K_{t+1}}{\partial \mathtt{M}_t} = \frac{1}{2} l_1 \rho \mu^{\mathtt{M}_t} \left(-\frac{l_2}{l_1} + \frac{\partial r^d}{\partial \mathtt{M}_{t+1}} \right). \tag{3.15}$$

From the above, we may conclude that in this case $\partial K_{t+1}/\partial \mathtt{M}_t > 0$, where $\partial \sigma_{t+1}^{\Lambda *}/\partial \mathtt{M}_t < 0$. This concludes the proof of Theorem 3.1. □

3.2. *Subprime Mortgage Loans and Loan Rates under Basel I (Constraint Slack)*

In this subsection, we establish the impact of changes in the level of macroeconomic activity, $\mathtt{M}_t$, on subprime mortgage loans, Λ, and the bank's subprime loan rate, r^{Λ}. We first present a result in the case where the capital constraint (3.2) is slack.

Proposition 3.2 (subprime mortgage loans and loan rates under Basel I (constraint slack)). *If* $l_t = 0$, *then*

$$\begin{aligned} \frac{\partial \Lambda_{t+j}^{n*}}{\partial \mathtt{M}_t} &= \frac{1}{2} \mu_j^{\mathtt{M}} \left(l_2 - l_1 \frac{\partial r^d (\mathtt{M}_{t+j})}{\partial \mathtt{M}_{t+j}} \right), \\ \frac{\partial r_{t+j}^{\Lambda^{n*}}}{\partial \mathtt{M}_t} &= \frac{1}{2} \mu_j^{\mathtt{M}} \left(\frac{l_2}{l_1} + \frac{\partial r^d (\mathtt{M}_{t+j})}{\partial \mathtt{M}_{t+j}} \right). \end{aligned} \tag{3.16}$$

Proof. In order to prove Proposition 3.2 under a slack constraint, we have to determine the partial derivatives of the optimal subprime mortgage loan supply, Λ^*, and loan rate, $r^{\Lambda *}$, with respect to $\mathtt{M}_t$ of (2.32), respectively. Of course, we do not have to include operational risk as in Basel II. We also consider the condition $\partial r^d(\mathtt{M}_{t+j})/\partial \mathtt{M}_{t+j} < 0$. Next, we can calculate

$$\begin{aligned} \frac{\partial \Lambda_{t+j}^{n*}}{\partial \mathtt{M}_t} \left(\frac{1}{2} \left(l_0 + l_2 \mathtt{M}_t + \sigma_t^{\Lambda} \right) - \frac{l_1}{2} \left(c^{\Lambda} + r^d (\mathtt{M}_t) + \left(r^D + c^D \right) + r_t^{\mathrm{T}} \gamma \right) \right) &= \frac{1}{2} \mu_j^{\mathtt{M}} \left(l_2 - l_1 \frac{\partial r^d (\mathtt{M}_{t+j})}{\partial \mathtt{M}_{t+j}} \right), \\ \frac{\partial r_{t+j}^{\Lambda^{n*}}}{\partial \mathtt{M}_t} \left(\frac{1}{2 l_1} \left(l_0 + l_2 \mathtt{M}_t + \sigma_t^{\Lambda} \right) + \frac{1}{2} \left(c^{\Lambda} + r^d (\mathtt{M}_t) + \left(r^D + c^D \right) + r_t^{\mathrm{T}} \gamma \right) \right) &= \frac{1}{2} \mu_j^{\mathtt{M}} \left(\frac{l_2}{l_1} + \frac{\partial r^d (\mathtt{M}_{t+j})}{\partial \mathtt{M}_{t+j}} \right), \end{aligned} \tag{3.17}$$

as required. □

3.3. *Subprime Mortgage Loans and Loan Rates under Basel I (Constraint Holding)*

In this subsection, we repeat the analysis in Section 3.2 in the analogous case where the capital constraint (3.2) holds.

Proposition 3.3 (subprime mortgage loans and loan rates under Basel I (constraint holding)). *When $l_t > 0$ the subprime mortgage loan supply is determined by the total capital constraint* (3.2) *and*

$$\Lambda_t^* = \frac{K_t}{\rho} - (I_t + \mathtt{S}_t + 12.5\,mVaR), \tag{3.18}$$

while the subprime mortgage loan rate response to changes in the level of macroeconomic activity is

$$\frac{\partial r_t^{\Lambda^*}}{\partial \mathtt{M}_t} = \frac{l_2}{l_1}. \tag{3.19}$$

Proof. In order to prove Proposition 3.3 under a holding constraint, we have to find the partial derivatives of the optimal subprime mortgage loan supply, Λ^*, and loan rate, $r^{\Lambda *}$, with respect to $\mathtt{M}_t$. This involves considering (2.25) and (2.26) in order to find $\partial\Lambda_{t+j}^*/\partial\mathtt{M}_t$ and $\partial r_{t+j}^{\Lambda^*}/\partial\mathtt{M}_t$, respectively. In this case, we obtain

$$\begin{gathered}\frac{\partial\Lambda_{t+j}^*}{\partial\mathtt{M}_t}\left(\frac{K_t}{\rho\omega(\mathtt{M}_t)} - \left[\frac{\omega^I I_t + 12.5\,\mathrm{mVaR}}{\omega(\mathtt{M}_t)}\right]\right) = 0,\\ \frac{l_2}{l_1} = \frac{\partial r_{t+j}^{\Lambda^*}}{\partial\mathtt{M}_t}\left(\frac{1}{l_1}\left(l_0 + l_2\mathtt{M}_t + \sigma_t^{\Lambda} - \frac{K_t}{\rho\omega(\mathtt{M}_t)} + \left[\frac{\omega^I I_t + 12.5\,\mathrm{mVaR}}{\omega(\mathtt{M}_t)}\right]\right)\right).\end{gathered} \tag{3.20}$$

□

4. Bank Credit and Capital under Basel II (Varying Loan Risk-Weights)

In this section, similar results to the ones in Section 3 will be derived for a model where both subprime mortgage loan losses and loan risk-weights are a function of the current level of macroeconomic activity, $\mathtt{M}_t$. The capital constraint is described by the expression in (2.13), where the risk-weights for intangible assets, $\omega^I \neq 0$, and risk-weights on short- and long-term shares, $\omega^{\mathtt{S}} \neq 0$, are considered. Also, in this situation, the risk-weight on subprime mortgage loans, $\omega(\mathtt{M}_t)$, is a decreasing function of the current level of macroeconomic activity, that is, $\partial\omega(\mathtt{M}_t)/\partial\mathtt{M}_t < 0$. In particular, we keep the risk-weights for intangible assets and short-and long-term shares constant, that is, $\omega^I = \omega^{\mathtt{S}} = 1$. In this case, the capital constraint (2.13) becomes

$$K_t \geq \rho[\omega(\mathtt{M}_t)\Lambda_t + I_t + \mathtt{S}_t + 12.5(\mathrm{mVaR} + \mathtt{O})]. \tag{4.1}$$

4.1. Quantity and Price of Bank Loans and Capital under Basel II

In this subsection, we examine how bank capital, K, and the quantity and price of loans, Λ, are affected by changes in the level of macroeconomic activity, $\mathtt{M}$, when subprime mortgage loan risk-weights, $\omega(\mathtt{M}_t)$, are allowed to vary.

Theorem 4.1 (subprime mortgage loans and capital under Basel II). *Suppose that $S(\mathtt{M}_t) > 0$ and the subprime mortgage loan risk-weights, $\omega(\mathtt{M}_t)$, are allowed to vary. In this case, one has that*

(1) *if* $\partial\sigma^{\Lambda*}_{t+1}/\partial\mathtt{M}_t < 0$ *then* $\partial K_{t+1}/\partial\mathtt{M}_t > 0$;

(2) *if* $\partial\sigma^{\Lambda*}_{t+1}/\partial\mathtt{M}_t > 0$ *then* $\partial K_{t+1}/\partial\mathtt{M}_t < 0$.

Proof. As in the proof of Theorem 3.1, we equate the optimal subprime mortgage loans for the problems with $l_t = 0$ and $l_t > 0$ in order to obtain

$$\frac{1}{2}\left(l_0 + l_2\mathtt{M}_t + \sigma^{\Lambda}_t\right) - \frac{l_1}{2}\left(c^{\Lambda} + r^d(\mathtt{M}_t) + \left(r^D_t + c^D\right) + r^{\mathrm{T}}_t\gamma\right) = \frac{K_t}{\omega(\mathtt{M}_t)\rho} - \frac{I_t + \mathtt{S}_t + 12.5(\mathrm{mVaR} + \mathtt{O})}{\omega(\mathtt{M}_t)}. \tag{4.2}$$

Solving for σ^{Λ}_t, we get

$$\sigma^{\Lambda*}_t = 2\left(\frac{K_t}{\omega(\mathtt{M}_t)\rho} - \frac{I_t + \mathtt{S}_t + 12.5(\mathrm{mVaR} + \mathtt{O})}{\omega(\mathtt{M}_t)}\right) - (l_0 + l_2\mathtt{M}_t) + l_1\left(c^{\Lambda} + r^d(\mathtt{M}_t) + \left(r^D_t + c^D\right) + r^{\mathrm{T}}_t\gamma\right). \tag{4.3}$$

Using the Euler condition for the subprime mortgage loan rate and substituting provisions for the deposit withdrawals, we obtain

$$l_1\rho\omega(\mathtt{M}_t)l_t = l_1\left(r^{\Lambda}_t - c^{\Lambda} - r^d(\mathtt{M}_t) - \left(r^D_t + c^D + \gamma r^{\mathrm{T}}_t\right)\right) - \Lambda_t. \tag{4.4}$$

Substitute $r^{\Lambda^*}_t$ and Λ^*_t into the expression above to obtain

$$l^*_t = \frac{\sigma^{\Lambda}_t - \sigma^{\Lambda*}_t}{\omega(\mathtt{M}_t)\rho l_1}. \tag{4.5}$$

Using (2.24) to find the partial derivative of the value function with respect to bank capital, we obtain

$$\frac{\partial V}{\partial K_t} = \begin{cases} l_t + (r^D_t + c^D + \gamma r^{\mathrm{T}}_t), & \\ (r^D_t + c^D + \gamma r^{\mathrm{T}}_t), & \text{for } \underline{\Lambda} \le \sigma^{\Lambda}_t \le \sigma^{\Lambda*}_t, \\ (r^D_t + c^D + \gamma r^{\mathrm{T}}_t) + \dfrac{\sigma^{\Lambda}_t - \sigma^{\Lambda*}_t}{\omega(\mathtt{M}_t)\rho l_1}, & \text{for } \sigma^{\Lambda*}_t \le \sigma^{\Lambda}_t \le \overline{\Lambda}. \end{cases} \tag{4.6}$$

By substituting the above expression into the optimal condition for total capital (2.31), we obtain

$$c^{dw}_t - \mathbf{E}_t\left[\delta_{t,1}\left(r^D_t + c^D + \gamma r^{\mathrm{T}}_t\right)\right] - \frac{1}{\omega(\mathtt{M}_{t+1})\rho l_1}\mathbf{E}_t\left[\int_{\sigma^{\Lambda*}_{t+1}}^{\overline{\Lambda}} \delta_{t,1}\left(\sigma^{\Lambda}_{t+1} - \sigma^{\Lambda*}_{t+1}\right)dF\left(\sigma^{\Lambda}_{t+1}\right)\right] = 0. \tag{4.7}$$

We denote the left-hand side of the above expression by Υ, so that

$$\Upsilon = \frac{1}{\omega(\mathtt{M}_{t+1})\rho l_1}\mathbf{E}_t\left[\int_{\sigma^{\Lambda*}_{t+1}}^{\overline{\Lambda}}\delta_{t,1}\left(\sigma^{\Lambda}_{t+1}-\sigma^{\Lambda*}_{t+1}\right)dF\left(\sigma^{\Lambda}_{t+1}\right)\right]. \tag{4.8}$$

From (3.12), we can calculate $\partial\Upsilon/\partial\mathtt{M}_t$ by using (4.8) in order to obtain

$$\begin{aligned}\frac{\partial\Upsilon}{\partial\mathtt{M}_t} &= -\frac{1}{\rho l_1}\frac{-\mu^{\mathtt{M}_t}(\partial\omega/\partial\mathtt{M}_{t+1})}{[\omega(\mathtt{M}_{t+1})]^2}\mathbf{E}_t\left[\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda}\delta_{t,1}\left(\sigma^{\Lambda}_{t+1}-\sigma^{\Lambda*}_{t+1}\right)dF\left(\sigma^{\Lambda}_{t+1}\right)\right]\\ &\quad-\frac{1}{\rho l_1\omega(\mathtt{M}_{t+1})}\frac{\partial\sigma^{\Lambda*}_{t+1}}{\partial\mathtt{M}_t}\mathbf{E}_t\left[\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda}\delta_{t,1}dF\left(\sigma^{\Lambda}_{t+1}\right)\right],\end{aligned} \tag{4.9}$$

where

$$\begin{aligned}\frac{\partial\sigma^{\Lambda*}_{t+1}}{\partial\mathtt{M}_t} &= -\frac{2}{\rho}\left(\frac{K_t-\rho(I_t+\mathtt{S}_t+12.5(\mathrm{mVaR}+\mathtt{O}))}{[\omega(\mathtt{M}_{t+1})]^2}\right)\mu^{\mathtt{M}_t}\frac{\partial\omega}{\partial\mathtt{M}_{t+1}}-l_2\mu^{\mathtt{M}_t}+l_1\mu^{\mathtt{M}_t}\frac{\partial r^d}{\partial\mathtt{M}_{t+1}},\\ \frac{\partial\Upsilon}{\partial K_{t+1}} &= \frac{2}{l_1[\omega(\mathtt{M}_{t+1})\rho]^2}\mathbf{E}_t\left[\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda}\delta_{t,1}dF\left(\sigma^{\Lambda}_{t+1}\right)\right].\end{aligned} \tag{4.10}$$

As a consequence, we have that $\partial K_{t+1}/\partial\mathtt{M}_t>0$ only if $\partial\sigma^{\Lambda*}_{t+1}/\partial\mathtt{M}_t<0$. □

4.2. Subprime Mortgage Loans and Loan Rates under Basel II (Constraint Slack)

Next, we consider the effect of a shock to the current level of macroeconomic activity $\mathtt{M}_t$ on subprime mortgage loans, Λ, and the bank's subprime loan rate, r^{Λ}. In particular, we analyze the case where the capital constraint (4.1) is slack.

Proposition 4.2 (subprime mortgage Loans under Basel II (constraint slack)). *Under the same hypothesis as Theorem 4.1 when $l_t=0$ one has that*

$$\begin{aligned}\frac{\partial\Lambda^{n*}_{t+j}}{\partial\mathtt{M}_t} &= \frac{1}{2}\mu^{\mathtt{M}}_j\left(l_2-l_1\frac{\partial r^d(\mathtt{M}_{t+j})}{\partial\mathtt{M}_{t+j}}\right),\\ \frac{\partial r^{\Lambda^{n*}}_{t+j}}{\partial\mathtt{M}_t} &= \frac{1}{2}\mu^{\mathtt{M}}_j\left(\frac{l_2}{l_1}+\frac{\partial r^d(\mathtt{M}_{t+j})}{\partial\mathtt{M}_{t+j}}\right).\end{aligned} \tag{4.11}$$

Proof. In order to prove Proposition 4.2, we find the partial derivatives of the optimal subprime mortgage loan supply, Λ^*, and the bank's subprime loan rate, $r^{\Lambda*}$, with respect to the current level of macroeconomic activity, $\mathtt{M}_t$. Here, we consider (2.32), and the condition $\partial r^d(\mathtt{M}_{t+j})/\partial \mathtt{M}_{t+j} < 0$. We are now able to calculate

$$\begin{aligned}
&\frac{\partial \Lambda_{t+j}^{n*}}{\partial \mathtt{M}_t}\left(\frac{1}{2}\left(l_0 + l_2\mathtt{M}_t + \sigma_t^{\Lambda}\right) - \frac{l_1}{2}\left(c^{\Lambda} + r^d(\mathtt{M}_t) + \left(r_t^D + c^D\right) + r_t^{\mathrm{T}}\gamma\right)\right) \\
&\quad = \frac{1}{2}\mu_j^{\mathtt{M}}\left(l_2 - l_1\frac{\partial r^d(\mathtt{M}_{t+j})}{\partial \mathtt{M}_{t+j}}\right), \\
&\frac{\partial r_{t+j}^{\Lambda^{n*}}}{\partial \mathtt{M}_t}\left(\frac{1}{2l_1}\left(l_0 + l_2\mathtt{M}_t + \sigma_t^{\Lambda}\right) + \frac{1}{2}\left(c^{\Lambda} + r^d(\mathtt{M}_t) + \left(r_t^D + c^D\right) + r_t^{\mathrm{T}}\gamma\right)\right) \\
&\quad = \frac{1}{2}\mu_j^{\mathtt{M}}\left(\frac{l_2}{l_1} + \frac{\partial r^d(\mathtt{M}_{t+j})}{\partial \mathtt{M}_{t+j}}\right).
\end{aligned} \tag{4.12}$$

□

4.3. *Subprime Mortgage Loans and Loan Rates under Basel II (Constraint Holding)*

Next, we present results about the effect of changes in the level of macroeconomic activity, $\mathtt{M}$, on subprime mortgage loans when the capital constraint (4.1) holds.

Proposition 4.3 (subprime mortgage loans under Basel II (constraint holding)). *Assume that the same hypothesis as in Theorem 4.1 holds. If $l_t > 0$ then by taking the first derivatives of* (2.25) *with respect to $\mathtt{M}_t$ and using the fact that the risk-weights for intangible assets, ω^I, and short- and long-term shares, ω^{S}, are constant one obtains*

$$\frac{\partial \Lambda_t^*}{\partial \mathtt{M}_t} = -\frac{K_t - \rho(I_t + \mathtt{S}_t + 12.5(mVaR + \mathtt{O}))}{[\omega(\mathtt{M}_t)]^2\rho}\frac{\partial \omega(\mathtt{M}_t)}{\partial \mathtt{M}_t}. \tag{4.13}$$

In this situation, the subprime mortgage loan rate response to changes in the level of macroeconomic activity is given by

$$\frac{\partial r_t^{\Lambda^*}}{\partial \mathtt{M}_t} = \frac{l_2}{l_1} + \frac{K_t - \rho(I_t + \mathtt{S}_t + 12.5(mVaR + \mathtt{O}))}{[\omega(\mathtt{M}_t)]^2\rho l_1}\frac{\partial \omega(\mathtt{M}_t)}{\partial \mathtt{M}_t}. \tag{4.14}$$

Proof. In order to prove Proposition 4.3, we find the partial derivatives of the optimal subprime mortgage loan supply, Λ^*, and bank's subprime loan rate, r^{Λ}, with respect to $\mathtt{M}_t$.

This involves using (2.25) and (2.26) and the condition $\partial\omega(\mathtt{M}_{t+j})/\partial\mathtt{M}_{t+j} < 0$ in order to find $\partial\Lambda^*{}_{t+j}/\partial\mathtt{M}_t$ and $\partial r^{\Lambda^*}_{t+j}/\partial\mathtt{M}_t$, respectively. We are now able to determine that

$$\begin{aligned}
&\frac{\partial\Lambda^*_{t+j}}{\partial\mathtt{M}_t}\left(\frac{K_t}{\rho\omega(\mathtt{M}_t)}-\left[\frac{\omega^I I_t+12.5(\mathrm{mVaR}+\mathtt{O})}{\omega(\mathtt{M}_t)}\right]\right)\\
&\quad=-\frac{K_t-\rho\left(12.5(\mathrm{mVaR}+\mathtt{O})+\omega^I I_t\right)}{[\omega(\mathtt{M}_t)]^2\rho}\frac{\partial\omega(\mathtt{M}_t)}{\partial\mathtt{M}_t},\\
&\frac{\partial r^{\Lambda^*}_{t+j}}{\partial\mathtt{M}_t}\left(\frac{1}{l_1}\left(l_0+l_2\mathtt{M}_t+\sigma^{\Lambda}_t-\frac{K_t}{\rho\omega(\mathtt{M}_t)}+\frac{\omega^I I_t+12.5(\mathrm{mVaR}+\mathtt{O})}{\omega(\mathtt{M}_t)}\right)\right)\\
&\quad=\frac{l_2}{l_1}+\frac{K_t-\rho\left(12.5(\mathrm{mVaR}+\mathtt{O})+\omega^I I_t\right)}{[\omega(\mathtt{M}_t)]^2\rho l_1}\frac{\partial\omega(\mathtt{M}_t)}{\partial\mathtt{M}_t}.
\end{aligned} \tag{4.15}$$

as required to complete the proof of Proposition 4.3. □

4.4. *Subprime Mortgage Loans and Loan Rates under Basel II (Future Time Periods)*

In the sequel, we examine the effect of a current macroeconomic shock in future periods on subprime mortgage loans, Λ, and loan rates, r^{Λ}.

4.4.1. *Capital Constraint Slack*

If the capital constraint is slack, the response of subprime mortgage loans and loan rates in period $j \geq 1$ to current fluctuations in the level of macroeconomic activity is described by Theorem 4.1. Nevertheless, as time goes by, the impact of the macroeconomic shock is minimized since $\mu^{\mathtt{M}}_j < 1$.

4.4.2. *Capital Constraint Holding*

In future, if the capital constraint holds, the response of subprime mortgage loans and loan rates to a change in the level of macroeconomic activity, $\mathtt{M}_t$, is described by

$$\begin{aligned}
\frac{\partial\Lambda^*_{t+j}}{\partial\mathtt{M}_t}&=\frac{\mu^{\mathtt{M}}_{j-1}}{\omega(\mathtt{M}_{t+j})\rho}\left[\frac{\partial\left(K_{t+j}-\rho\left(I_{t+j}+S_{t+j}+12.5(\mathrm{mVaR}+\mathtt{O})\right)\right)}{\partial\mathtt{M}_{t-1+j}}\right]\\
&\quad-\frac{\mu^{\mathtt{M}}_{j-1}}{\omega(\mathtt{M}_{t+j})\rho}\left[\frac{\mu^{\mathtt{M}}}{\omega(\mathtt{M}_{t+j})}\left(K_{t+j}-\rho\left(I_{t+j}+S_{t+j}+12.5(\mathrm{mVaR}+\mathtt{O})\right)\right)\frac{\partial\omega(\mathtt{M}_{t+j})}{\partial\mathtt{M}_{t+j}}\right],
\end{aligned} \tag{4.16}$$

$$\frac{\partial r^{\Lambda^*}_{t+j}}{\partial \mathtt{M}_t} = \frac{l_2}{l_1}\mu^{\mathtt{M}}_j - \frac{\mu^{\mathtt{M}}_{j-1}}{\omega(\mathtt{M}_{t+j})\rho l_1}\frac{\partial\left(K_{t+j} - \rho\left(I_{t+j} + S_{t+j} + 12.5(\mathrm{mVaR} + \mathtt{O})\right)\right)}{\partial \mathtt{M}_{t-1+j}}$$
$$+ \frac{\mu^{\mathtt{M}}_j}{\left[\omega(\mathtt{M}_{t+j})\right]^2\rho}\left(K_{t+j} - \rho\left(I_{t+j} + S_{t+j} + 12.5(\mathrm{mVaR} + \mathtt{O})\right)\right)\frac{\partial\omega(\mathtt{M}_{t+j})}{\partial \mathtt{M}_{t+j}}. \tag{4.17}$$

From (4.16), it can be seen that future subprime mortgage loans can either rise or fall in response to positive macroeconomic shocks. This process depends on the relative magnitudes of the terms in (4.16). If capital rises in response to positive macroeconomic shocks, subprime mortgage loans can fall provided that the effect of the shock on capital is greater than the effect of the shock on subprime mortgage loan risk-weights.

5. Connections between the Basel Accords and the Subprime Mortgage Crisis

The turmoil in financial markets, that resulted from the ongoing SMC in the U.S., necessitated the transformation of regulation and supervision of financial institutions. The question is whether these institutions would have been more stable if Basel II had already been fully implemented by 2007. Basel II represents a dramatic change in capital regulation of large banks in the countries represented on the BCBS. Its IRB approaches to capital regulation will allow large banks to use their own credit risk models to set minimum capital requirements. The BCBS itself implicitly acknowledged in spring 2008 that the revised framework would not have been adequate to contain the risks exposed by the SMC and needed strengthening. This crisis has highlighted two more basic questions about Basel II: one, is the method of capital regulation incorporated in the revised framework fundamentally misguided? Two, even if the basic Basel II approach has promise as a paradigm for domestic regulation, is the effort at extensive international harmonization of capital rules and supervisory practice useful and appropriate? In this section, we discuss the relationships between the Basel I and II Capital Accords and the SMC.

There is substantial evidence to suggest that credit rating changes exhibit procyclical behavior or systematic variation (see, e.g., [18]). Thus in the arguments below, we are justified in using the credit rating, $\mathtt{C}$, as a proxy for the level of macroeconomic activity, $\mathtt{M}$. Concerns about credit ratings have resurfaced under Basel II, where banks have been allowed to use ratings to determine the risk attached to their subprime mortgage loans and hence to calculate minimum capital requirements. An illustration of how problems can arise from this is outline below.

(1) Banks sell subprime mortgage loans of their balance sheet to a special purpose vehicle (SPV) that is highly rated (maybe AAA). In this case, minimal capital reserves against potential loan losses are required to be held by the bank.

(2) If the SPV borrows funds from the aforementioned bank to buy securities, the bank needs to make minimal, if any, capital reserve provisions.

(3) In the event that the SPV's rating is downgraded, the bank is then required to make substantial capital reserve provisions of up to 8% of the loans extended. If these loans to the SPV are considered to be doubtful, then the bank must make explicit provisions by increasing the capital it holds.

(4) Also, under Basel II, the banks own risk assessment may be comprised in order to show that they face a lower risk so that the capital requirements are lower. Banks prefer to hold less capital because control is not diluted. Instead it increases leverage (able to use more borrowed funds) and hence profits during times of elevated levels of macroeconomic activity. On the other hand, when credit ratings decrease, as they have been in 2008 and 2009, banks have to meet a higher than normal minimum capital requirement. In times of low macroeconomic activity when share values are depressed, banks are unable to raise additional capital on the market. In this situation, they have to reduce new loan extensions and attempt to increase their profit margins by increasing interest rates beyond the prescribed level

(5) Inevitably, the process above leads to contagion. As a consequence, banks do not lend to each other because they suspect each other of underhand dealings. Eventually balance sheets lose transparency and the interbank lending market grinds to a halt. Banks hoard cash and consequently liquidity dries up.

In the light of the above, stakeholders have made extensive recommendations for the reform of the credit rating industry, the use of statistical risk models and have called for parts of the Basel II Capital Accord to be referred back to the Bank for International Settlements (BIS) for re-evaluation.

5.1. Basel I and the Subprime Mortgage Crisis

In this subsection, we discuss the connections between Basel I and the SMC.

5.1.1. Basel I: Bank Capital and the SMC

From the statement of Theorem 3.1 in Section 3.1 and the equivalence of $\mathtt{C}$ and $\mathtt{M}$, we can deduce that

$$\text{if } \frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial \mathtt{C}_t} < 0 \quad \text{then } \frac{\partial K_{t+1}}{\partial \mathtt{C}_t} > 0, \qquad \text{if } \frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial \mathtt{C}_t} > 0 \quad \text{then } \frac{\partial K_{t+1}}{\partial \mathtt{C}_t} < 0. \tag{5.1}$$

The amended form of Theorem 3.1 given above suggests that an interesting relationship exists between subprime mortgage loan quantity and price and bank capital. Under the assumptions that subprime mortgage loan losses may occur and asset risk-weights are constant, we can study the relationship between Basel I regulation and the boom period before the SMC as well as the connection between Basel I and the recessionary period during the SMC.

Before the SMC, in a boom period of increasing credit ratings, empirical data supports the fact that a decrease (increase) in the volatility of mortgage loan supply leads to less (more) capital being held by banks in order to cover unexpected losses. On the other hand, in Theorem 3.1, when credit ratings increase, the decreasing (increasing) volatility of mortgage loan supply implied that an increasing (decreasing) amount of capital is held. This means that the reality experienced in the boom period before the SMC is contrary to the outcome of Theorem 3.1. The main reason for this is that the assumptions made about bank capital in our Basel I models do not accurately reflect the reality in the U.S. economy before the SMC. For instance, these assumptions do not take into account that banking behavior before the

SMC differed quite dramatically in terms of compliance with accepted banking practices as encapsulated by Basel I. Also, our subprime banking models are not based on any assumptions about the cyclicality of bank capital. This is a very controversial issue with some experts insisting that bank capital is acyclical while others claim that capital is procyclical (see, e.g., [18]). A consideration of bank capital cyclicality is way beyond the scope of this paper and will be deferred to later studies.

During the SMC, in a recessionary environment of decreasing credit ratings, empirical data shows that a increase (decrease) in the volatility of mortgage loan supply implied that more (less) capital was held by banks in order to cover unexpected losses. By contrast, in Theorem 3.1, when credit ratings decrease, the increasing (decreasing) volatility of mortgage loan supply implies that a decreasing (increasing) amount of capital is held. As before, the reasons for this are related to banking behavior and the cyclicality of bank capital during the SMC. During the SMC, credit ratings decreased while the banks attempted to hoard cash in order to boost their capital. In fact, in the U.S., CRAs lowered the credit ratings on $ 1.9 trillion in MBSs from the third quarter of 2007 to the second quarter of 2008. Financial institutions felt they had to lower the value of their MBSs and acquire additional capital so as to maintain capital ratios. The latter subject falls beyond the scope of this paper and will be considered in future research.

5.1.2. Basel I: Subprime Mortgage Loans and Their Rates and the SMC under a Slack Capital Constraint

From Proposition 3.2 in Section 3.2 and the equivalence of $\mathtt{C}$ and $\mathtt{M}$, we can deduce that

$$\text{if } l_t = 0, \quad \text{then } \frac{\partial \Lambda^{n*}_{t+j}}{\partial \mathtt{C}_t} = \frac{1}{2}\mu^{\mathtt{C}}_j \left(l_2 - l_1 \frac{\partial r^d(\mathtt{C}_{t+j})}{\partial \mathtt{C}_{t+j}} \right), \qquad \frac{\partial r^{\Lambda^{n*}}_{t+j}}{\partial \mathtt{C}_t} = \frac{1}{2}\mu^{\mathtt{C}}_j \left(\frac{l_2}{l_1} + \frac{\partial r^d(\mathtt{C}_{t+j})}{\partial \mathtt{C}_{t+j}} \right). \tag{5.2}$$

We can draw the following conclusions about the dependence of subprime mortgage loans and their rates on credit ratings from the form of Proposition 3.2 given in (5.2). In particular, when the capital constraint (3.2) is slack, subprime mortgage loan extension increases in sympathy with an increase in credit ratings. Here, the subprime mortgage loan rate, r^{Λ}, can either rise or fall depending on the characteristics of the mortgage loan demand, Λ, and loan default rate, r^d. However, Proposition 3.2 conjectures that the loan rate generally increases in response to an increase in credit ratings.

Before the SMC, high credit ratings encouraged investors to buy MBSs, thereby helping to finance the housing boom. The reliance on agency ratings and the way ratings was used to justify investments led many investors to treat securitized products—some based on subprime mortgage loans—as equivalent to higher quality securities. This was exacerbated by the removal of regulatory barriers by the U.S. Securities and Exchange Commission (SEC) and its reduction of disclosure requirements. The above discussion is generally consistent with the hypothesis of Proposition 3.2 involving subprime mortgage loans and their rates.

During the SMC, lower credit ratings discouraged investors from buying MBSs, thereby helping to slow and even reverse the housing boom. This is consistent with the hypothesis of Proposition 3.2.

5.1.3. Basel I: Subprime Mortgage Loans and Their Rates and the SMC under a Binding Capital Constraint

From the statement of Proposition 3.3 in Section 3.3 and the equivalence of $\mathtt{C}$ and $\mathtt{M}$, we can deduce that when $l_t > 0$ the subprime mortgage loan supply is determined by the total capital constraint (3.2) and

$$\Lambda_t^* = \frac{K_t}{\rho} - (I_t + \mathtt{S}_t + 12.5\,\mathrm{mVaR}), \tag{5.3}$$

while the subprime mortgage loan rate response to changes in the credit rating is

$$\frac{\partial r_t^{\Lambda^*}}{\partial \mathtt{C}_t} = \frac{l_2}{l_1}. \tag{5.4}$$

We make conclusions about bank capital as well as subprime mortgage loans and their rates from the alternative formulation of Proposition 3.3 given above. Subsequent to an increase in the credit rating the likelihood that the capital constraint (3.2) will hold is greater with banks increasing the amount of capital they hold. Also, when the capital constraint (3.2) holds, bank lending behavior does not change in response to an increase in the credit rating. The subprime mortgage loan rate, however, rises as a consequence of a higher loan demand. When the capital constraint is slack, subprime mortgage loans and its rate rise as a result of an increased credit rating. On the other hand, if the capital constraint (3.2) holds, a rise in the subprime mortgage loan demand will result in a rise in the mortgage loan rate, which will leave the loan supply unchanged.

Before the SMC, highly rated subprime mortgages and their securities resulted in less capital being held by banks in order to cover unexpected losses. In this boom period, there is conclusive empirical evidence to suggest that when the capital constraint (3.2) holds, bank lending behavior changed in response to an increase in the credit rating. This is contrary to what is suggested by Proposition 3.3 where mortgage extension does not change in response to an increase in the credit rating. This may be due to the fact that such behavior in our Basel I models does not take excessive procyclicality in mortgage extension into account. However, by way of complying with Proposition 3.3, before the SMC the rate of return of adjustable rate mortgages (ARMs) rose in sympathy with higher loan demand. In this regard, when the capital constraint was slack, subprime mortgage loan extension and its rate rose as a result of increased credit ratings.

During the SMC, CRAs aggressively downgraded large amounts of mortgage-backed debt. As credit ratings decreased banks attempted to hoard cash in order to boost their capital. This is generally consistent with the outcomes of Proposition 3.3.

5.2. Basel II and the Subprime Mortgage Crisis

In this subsection, we discuss the connections between Basel II and the SMC.

5.2.1. Basel II: Bank Capital and the SMC

The formulation of Theorem 4.1 in Section 4.1 and the equivalence of $\mathtt{C}$ and $\mathtt{M}$ imply that

$$\text{if } \frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial \mathtt{C}_t} < 0 \quad \text{then } \frac{\partial K_{t+1}}{\partial \mathtt{C}_t} > 0, \qquad \text{if } \frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial \mathtt{C}_t} > 0 \quad \text{then } \frac{\partial K_{t+1}}{\partial \mathtt{C}_t} < 0. \tag{5.5}$$

This alternative form of Theorem 4.1 enables us to forge a connection between subprime mortgage loan quantity and price and bank capital. Under the assumptions of positive loan losses and a risk-sensitive capital constraint, banks can either raise or lower their capital holdings in response to an increase in the credit rating. Their choice depends on the effect that the changes in $\mathtt{C}_t$ has on the likelihood of the capital constraint holding in the next time period. In this case, we can study the relationship between Basel II and the boom period before the SMC as well as the connection between Basel II and the recessionary period during the SMC.

Before the SMC, several so-called "quantitative impact studies" (QISs) were conducted under the auspices of the Basel Committee on Banking Supervision (BCBS) to explore the consequences of shifting from Basel I to Basel II for large banks. These studies show that bank capital requirements will fall further for many banks when Basel II is fully implemented. For instance, in the U.S., the QIS results indicate potential reductions in required capital of more than 50% for some major banks. The need to recapitalize banks reveals that the internal risk models of many banks performed poorly and greatly underestimated risk exposure, forcing banks to reassess and reprice credit risk. To some extent, this reflects the difficulties of accounting for low probability but large events (see, e.g., [3]). Before the SMC, banks both raised and lowered their capital holdings in response to an increase in the credit rating which is consistent with the conjecture in Theorem 4.1.

During the SMC, credit ratings decreased while the banks attempted to hoard cash in order to boost their capital. In this regard, it is clear that the relationship between the banks and the CRAs during the real estate bubble has had and will have a long-lasting impact on banks' ability to recover from the current crisis. More specifically, the CRAs, who are remunerated by the loan issuers, gave high ratings to securities backed by subprime mortgage loans. In order to compensate for this situation, the pace of downgrades by credit agencies on MBSs has accelerated considerably during the SMC. This created additional problems since every time their portfolios are hit by significant credit downgrades, banks are compelled to raise their capital adequacy ratios. Often this results in the issuance of new equity which leads to dilution as shareholders at Citigroup, Merrill Lynch and Washington Mutual have experienced. During the SMC, a decrease in the credit rating led to banks both raising and lowering their capital holdings which is consistent with the conjecture in Theorem 4.1.

5.2.2. Basel II: Subprime Mortgage Loans and Their Rates and the SMC under a Slack Capital Constraint

Proposition 4.2 in Section 4.2 and the equivalence of $\mathtt{C}$ and $\mathtt{M}$ imply that

$$\text{if } l_t = 0, \quad \text{then } \frac{\partial \Lambda_{t+j}^{n*}}{\partial \mathtt{C}_t} = \frac{1}{2}\mu_j^{\mathtt{C}}\left(l_2 - l_1 \frac{\partial r^d(\mathtt{C}_{t+j})}{\partial \mathtt{C}_{t+j}} \right), \qquad \frac{\partial r_{t+j}^{\Lambda^{n*}}}{\partial \mathtt{C}_t} = \frac{1}{2}\mu_j^{\mathtt{C}}\left(\frac{l_2}{l_1} + \frac{\partial r^d(\mathtt{C}_{t+j})}{\partial \mathtt{C}_{t+j}} \right). \tag{5.6}$$

Firstly, we deduce that when the capital constraint (4.1) is slack, the models with constant and varying subprime mortgage loan risk-weights (while keeping the risk-weights for intangible assets and shares constant) yield the same results. Under the same hypothesis as Theorem 4.1, when the capital constraint (4.1) is slack, subprime mortgage loans, Λ, increase as a result of an increase in the credit rating, $\mathtt{C}$. Furthermore, the bank's subprime loan rate, r^{Λ}, can either increase or decrease depending on the parameters characterizing the subprime mortgage loan default rate and the loan demand function. However, Proposition 4.2 conjectures that the loan rate generally increases in response to an increase in credit ratings.

The SMC-related dynamics of the subprime mortgage loans and their rates under a slack capital constraint are consistent with the conjectures of Proposition 4.2. In this regard, the arguments are analogous to those of Section 5.1.2.

5.2.3. Basel II: Subprime Mortgage Loans and the SMC under a Binding Capital Constraint

From Proposition 4.3 in Section 4.3 and the equivalence of $\mathtt{C}$ and $\mathtt{M}$, we can deduce that if $l_t > 0$ then by taking the first derivatives of (2.25) with respect to $\mathtt{M}_t$ and using the fact that the risk-weights for intangible assets, ω^I, and short- and long-term shares, ω^S, are constant we obtain

$$\frac{\partial \Lambda_t^*}{\partial \mathtt{C}_t} = -\frac{K_t - \rho\ (I_t + \mathtt{S}_t + 12.5(\mathrm{mVaR} + \mathtt{O}))}{[\omega(\mathtt{C}_t)]^2 \rho} \frac{\partial \omega(\mathtt{C}_t)}{\partial \mathtt{C}_t}. \tag{5.7}$$

In this situation, the subprime mortgage loan rate response to changes in the credit rating is given by

$$\frac{\partial r_t^{\Lambda^*}}{\partial \mathtt{C}_t} = \frac{l_2}{l_1} + \frac{K_t - \rho(I_t + \mathtt{S}_t + 12.5(\mathrm{mVaR} + \mathtt{O}))}{[\omega(\mathtt{C}_t)]^2 \rho l_1} \frac{\partial \omega(\mathtt{C}_t)}{\partial \mathtt{C}_t}. \tag{5.8}$$

Under the same hypothesis as Theorem 4.1, when the capital constraint (4.1) holds, bank lending rises in response to an increase in the credit rating. The subprime mortgage loan rate, r^{Λ}, can either rise or fall depending on the parameters characterizing the subprime mortgage loan demand, Λ, and mortgage loan risk-weights, $\omega(\mathtt{C}_t)$. In the Basel II Capital Accord, a change in the credit rating does not only affect the subprime mortgage loan demand but also the risk-weights in the bank's capital adequacy ratios (CARs). If the capital constraint (4.1) is slack, subprime mortgage loans rise as in Basel I. On the other hand, if the capital constraint holds, banks can still expand their credit supply, but to a lesser degree compared to the case where the capital constraint is slack. Banks are able to do so because an increase in the credit rating results in the lowering of risk-weights and ultimately leads to a more relaxed capital requirement. The lower rate can either rise or fall, depending on the relative size of the change in subprime mortgage loan demand and the capital adequacy ratio. Similarly, a decrease in the credit rating results in a possibly greater reduction of credit extension than in the Basel I model because of both a decrease in subprime mortgage loan demand as well as a tightening of the capital constraint. Furthermore, under Basel II, an indication of the change in bank capital held is undetermined because credit rating increases have two counteracting effects on the equilibrium values of bank capital. On the one hand, increased credit ratings have a continual positive effect on subprime mortgage loan demand and so raise the probability of the capital constraint (4.1) holding in future. At the same time, the CAR increases so that the chance of the capital constraint being lower exists.

Before the SMC, high credit ratings for subprime mortgages and their securities resulted in less capital being held by banks in order to cover unexpected losses. In this boom period, there is strong empirical evidence to suggest that when the capital constraint (4.1) holds, bank lending increased in response to an increase in the credit rating. This coincides with the conjecture in Proposition 4.3 where mortgage extension increases in response to an increase in the credit rating. However, before the SMC the rate of return of adjustable rate mortgages (ARMs) rose in sympathy with higher loan demand. In this regard, when the capital constraint was slack, subprime mortgage loan extension and its rate rose as a result of increased credit ratings. This reality is not reflected in Proposition 4.3.

During the SMC, CRAs aggressively downgraded large amounts of mortgage-backed debt. As credit ratings decreased banks attempted to hoard cash in order to boost their capital. This is not consistent with the outcomes of Proposition 4.3.

6. Concluding Remarks and Future Directions

In many respects, our modeling choices have been validated by events happening before and during the SMC. In other respects, there have been practical contradictions of our rigorous conjectures. The simple truth is that many SMC-related events have consistently violated conventional economic wisdoms with their complete description requiring more sophisticated modeling assumptions. This is beyond the scope of the current paper and is deferred to a future investigation.

In general, Basel capital regulation seems to have exacerbated the subprime mortgage crisis. In this regard, the main questions to have been answered by this paper may be posed as follows.

> Under a Basel capital accord dispensation, did banks understand how much risk they were exposing themselves to?
>
> Did they know how much capital they needed to cushion the shock from defaulting subprime mortgage loans?
>
> Did they prepare themselves adequately for the reduction in their ability to easily sell their subprime securities or mortgage loans (i.e., reduction in “iquidity”)?

The answer to all three questions appears to be “no.” As a result, it is reasonable to conclude that banks have misjudged and mispriced risks and have been unable to assess the creditworthiness of borrowers. It appears that this situation has been exacerbated by changes in international banking regulation brought about under the auspices of Basel I and II. Such regulation has allowed banks to systematically underestimate risk and to make inadequate capital reserve provision for unexpected loan losses. Consequently, Basel capital regulation has implicitly encouraged banks to prioritize growth and profit above prudent behavior.

6.1. Concluding Remarks

In this paper, in a Basel I context, we firstly established the impact of changes in levels of macroeconomic activity (and credit ratings) on bank capital as well as subprime mortgage

Table 1: Summary of results for cyclicality of bank credit and capital under Basel I and II.

	Bank capital	Loans	Loan rate
Basel I (slack)	Increase (Section 3.1)	Increase (Section 3.2)	Increase (Section 3.2)
Basel I (holding)	Increase (Section 3.3)	Unchanged (Section 3.3)	Increase (Section 3.3)
Basel II (slack)	Undetermined (Section 4.1)	Increase (Section 4.2)	Increase (Section 4.2)
Basel II (holding)	Undetermined (Section 4.3)	Increase (Section 4.3)	Undetermined (Section 4.3)

Table 2: Main differences between Basel I and II.

Focus	Basel I	Basel II
Risk measure	Single risk measure	Counterparty and transaction specific risk measures
Risk sensitivity	Broad brush approach	Granularity and risk sensitivity
Credit risk mitigation	Limited recognition	Comprehensive recognition
Operational risk	Excluded	Included
Flexibility	One size fits all	Menu of approaches
Supervisory review	Implicit	Explicit
Market discipline	Not addressed	Supervisory role conferred on market
Incentives	Not addressed	Explicit and well defined
Economic capital	Divergence	Convergence

loans and their rates when the capital constraint holds and when it does not (see, e.g., Sections 3.2 and 3.3, resp.; also compare with (3.2)). In Section 4 results analogous to those obtained in Section 3 are given for the situation where subprime mortgage loan losses and their risk-weights were a function of macroeconomic conditions (i.e., risk-weights vary with changes in the phases of the business cycle). This situation mimics the paradigm suggested by Basel II. Furthermore, our analysis also involved the situations where the capital constraint holds and where it does not (see Sections 4.2 and 4.3, resp.). In Section 4.4, we presented cyclicality of subprime mortgage loans and their rates under Basel II in future periods. The relationship between this regulatory issues and the SMC is explored in Section 5, where credit ratings are considered to be a proxy for macroeconomic activity.

6.2. Future Directions

Some issues that require further investigation are briefly discussed below. The SMC, which has caused large banks to take substantial losses and search for significant new capital, indicates that Basel II should not be implemented until a number of significant changes are considered. We propose the following improvements in order to rectify some deficiencies in banking regulation. Firstly, we suggest that the BCBS should conduct further quantitative impact studies that use observations from the ongoing SMC before permitting banks to use internal models for calculating capital adequacy. Secondly, as is the case in the U.S., we encourage the adoption of an additional non-risk-weighted leverage ratio requirement to supplement Basel II risk-weighted capital requirements. This will compel banks to hold a minimal capital buffer, even when risk-based Basel II capital requirements suggest lower risk. Adequate capital, for instance, allows banks to provision for loan losses and mitigate the effects of financial shocks during crises and recessions. Thirdly, we recommend that the Basel II approach using idiosyncratic banking risk models should be complemented by market discipline that is both credible and effective. In this regard, although Basel II insists

Table 3: Continued differences between Basel I and II.

Basel I	Basel II
Not risk sensitive; no reliance on risk ratings or maturity.	Risk sensitive
Based on simplistic or crude categories of obligors and assets with fixed risk-weighted asset percentages; no minimal recognition of risk mitigation	Based on risk ratings, maturity substantial of recognition risk mitigation
Calculated in aggregate	Calculated at facility/transaction level
No acknowledgment of operational risk as a separate risk discipline/category	Operational risk as a separate discipline/category
No relationship with risk capital	Alignment with risk capital, advanced risk management concepts
Largely finance-driven off of finance systems	Operationally intense–places major emphasis on rigorous risk management processes, models systems/technology and integration of risk/finance systems and processes

Table 4: Differences in economic conditions before and during the SMC.

Before SMC (Year < 2007)	During SMC (Year ≥ 2007)
High level of macroeconomic activity	Lower level of macroeconomic activity
Boom conditions	Recessionary conditions
Low perceived credit risk	Higher perceived credit risk
High credit ratings	Lower credit ratings
Low delinquency rate	Higher delinquency rate
Low foreclosure rate	Higher foreclosure rate
Regret-averse lenders	Risk-averse lenders
House prices increase	House prices decline
Low counterparty risk	Higher counterparty risk
High rate of securitization of Subprime mortgage loans	Lower rate of securitization of Subprime mortgage loans
Low investment in safe assets such as treasury securities	Higher investment in safe assets such as treasury securities
High spreads	Lower spreads
No credit crunches	Credit crunches
Highly leveraged financial institutions	Less highly leveraged financial institutions

on information disclosure, it fails to incentivize the use of this information by investors in an optimal way. Currently, investors holding bank liabilities believe that major banks are too big to fail and that their deposits are fully insured so that they will not loose their own money. This will lessen their incentives to use the disclosed information. In this regard, major banks' incentives to reduce capital may be mitigated by a compulsory requirement to issue credibly uninsured subordinated debt as part of the regulatory capital requirement. This steps the potential to enhance market discipline.

Appendix

This appendix contains Table 1 that shows the effect of booms on banks' lending behavior under Basel I and II and summarizes the main results for the cyclicality of bank credit and capital under the Basel accords as well as tables outlining the main differences between Basel I and Basel II (see Tables 2 and 3). Furthermore, in Table 4, we expose some of the differences between economic conditions before and during the SMC.

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