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Research Article

Public Spending in a Model of Endogenous Growth with Habit Formation

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This paper introduces habit-forming preferences in a Barro-type endogenous growth model with productive public services. Government expenditure, which may be subject to congestion, is financed by distortionary income taxation. Different from the standard time-separable model, the presence of habits makes the economy feature transitional dynamics, which are solved in closed form. Setting the income tax so as to equate the elasticity of public services in production is shown to maximize both long-run growth and welfare as in the standard model. This second-best solution coincides with the first-best outcome only in the presence of proportional congestion.

1. Introduction

Following the seminal work of Barro [1], the effect of productive government expenditure has been the subject of active research in the theoretical literature on endogenous growth (see, e.g., Irmen and Kuehnel [2], for a recent survey). This interest has been stimulated by empirical work, starting from Aschauer [3], providing compelling evidence that productive public spending has a positive impact on growth (see, e.g., the review by Romp and de Haan [4]).

Barro [1] introduces the flow of government expenditure, which is fully financed by a distortionary income tax, as a productive input to private production. (Another strand of the literature, starting from Futagami et al. [5], considers that it is the accumulated stock of public capital, rather than the flow of current expenditure, that enters in the production function (see, e.g., Irmen and Kuehnel [2]).) Thus, public policy has two opposite effects on growth, which offset each other if the tax rate on income is equal to the elasticity of public spending in production. Under Cobb-Douglas technology, this growth-maximizing tax rate also maximizes intertemporal utility in a second-best sense. The ensuing literature has enriched the original Barro's setup to incorporate other relevant features such as, among others, adjustment costs [6], congestion [7, 8], public consumption services [9, 10], elastic labor supply [11], alternative government financing [9, 12], progressive taxation [13], rent-seeking [14, 15], finite lifetimes [16, 17], two countries [18], or stochastic growth [19, 20].

This paper adds to this literature by extending the Barro [1] model to incorporate habit-persistent preferences. In models with habit formation, individual's utility depends not only on her level of current consumption but also on how it compares to a reference level—the habits stock. Recent empirical works by Chen and Ludvigson [21], Korniotis [22], and Grishchenko [23] have reported supporting evidence for the presence of habits that enter utility in a subtractive manner. Furthermore, subtractive habits have been widely incorporated into dynamic equilibrium models in order to address several empirical facts that are difficult to explain under standard time-separable preferences. (A partial list includes the equity premium puzzle, [24, 25] , time-varying expected returns [26] , the term structure of interest rates [27] , the foreign exchange risk premium [28] , some stylized facts of business cycles [29] , current account dynamics [30] , and the effects of monetary policy [31]. Subtractive habits have also become an important feature of many dynamic stochastic general equilibrium models [32, 33].) Based on these works, we assume that habits enter subtractively in utility. The literature distinguishes between internal habits (IH), when individual's habits depend on her own past consumption, and external habits (EH), when habits are formed from average past consumption in the economy. However, whether habits are formed in an internal, external, or hybrid internal-external form appears to be an open empirical question yet. Therefore, we consider a fairly general specification of the habit formation process that nests the cases of internal, external, and hybrid internal-external habits. Besides its own interest, the introduction of habit-persistent preferences is also relevant because it allows to overcome one important limitation of the flow models based on the Barro [1] setup, namely, that they lack transitional dynamics, so that the economy is always on its balanced growth path and all variables grow at a constant rate. (Actually, this also occurs in stock models in which public capital fully depreciates in one period (e.g., [34–36]).)

This paper develops an endogenous growth model with habit persistence in utility and productive government expenditure in production, which is financed by distortionary income taxation. As Barro and Sala-i-Martin [7] argue, almost all public services are subject to some degree of congestion. Therefore, public spending is allowed to be subject to congestion, so that the service of public expenditure to the agent depends on the usage of her capital stock relative to the aggregate capital stock (e.g., [6, 7]). First, we analyze the equilibrium dynamics of the decentralized economy, and characterize the tax rate on income that maximizes the representative agent's utility; that is, the second-best solution. Next, we study the first-best outcome that a benevolent central planner would implement, and compare it with the second-best solution. Finally, some numerical simulations are presented.

The main results of this paper are twofold. First, we show that, unlike the standard time-separable Barro [1] model, the introduction of habit formation makes that, more realistically, the model features transitional dynamics. Furthermore, we give closed-form solutions for the dynamics of both the market and the socially-planned economies. Thus, this paper also adds to a recent literature that investigates the existence of closed-form solutions to growth models as, e.g., Mehlum [37], Smith [38, 39], and Guerrini [40] for the Ramsey model, and Bethmann [41], Ruiz-Tamarit [42], Boucekkine and Ruiz-Tamarit [43], Chilarescu [44, 45], and Hiraguchi [46], among others, for the two-sector Lucas [47] model. This is interesting because such explicit solutions are analytically tractable, and numerical simulations can be performed without error. Second, we find that the results obtained by Barro [1] are robust to the inclusion of habit formation and, consequently, to the existence of transition dynamics. On one hand, within a decentralized economy set-up, the growth-maximizing and welfare-maximizing (in a second-best sense) income tax rates coincide, and are equal to the elasticity of public spending in production. On the other hand, in a first-best setting, the optimal ratio of government expenditure to output is equal to the elasticity of public services in production. Furthermore, the first-best solution can be decentralized as a market equilibrium only in the presence of proportional congestion, when the second-best outcome coincides with the first-best solution. In any other case, the first-best solution cannot be decentralized. The numerical results illustrate the implications for the dynamics of the economy of the introduction of habit formation.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the equilibrium dynamics of a decentralized economy. Section 4 studies the first-best solution attainable by a social planner. Some numerical results are presented in Section 5. Section 6 concludes.

2. The Model

We study a closed economy populated by a large but fixed number N of identical infinitely-lived individuals, each of whom has an infinite planning horizon and possesses perfect foresight. Each individual is the owner of a unique firm that produces the single output in the economy, which can be devoted to consumption or investment. The government provides productive services to firms, which are financed by an income tax. With all agents being identical, aggregate private quantities are simply multiples of individual quantities. We will denote individual quantities by lower case letters and aggregate quantities by corresponding upper case letters, so that $X = Nx$.

2.1. Preferences

The intertemporal utility derived by the agent depends both on its current consumption, c_t, and a reference consumption level or habits stock, h_t, according to

$$U = \sum_{t=0}^{\infty} u(c_t - \gamma h_t)\beta^t, \quad 0 \le \gamma < 1, \ 0 < \beta < 1, \tag{2.1}$$

where u is the instantaneous utility function, and β is the time discount rate. (Another commonly-used specification is the "multiplicative" one (e.g., [48–51]), in which habits-adjusted consumption is defined as the geometric mean of absolute consumption and relative consumption as a ratio, $z_t = c_t^{1-\gamma}(c_t/h_t)^{\gamma}$.) The habits-adjusted consumption, $z_t \equiv c_t - \gamma h_t$, can be rewritten as $z_t = (1-\gamma)c_t + \gamma(c_t - h_t)$. Hence, the parameter γ reflects the importance of absolute versus relative-to-habits consumption in utility: the higher γ, the greater the importance of relative consumption. The case $\gamma = 0$ corresponds to the standard time-separable model in which only the absolute level of consumption matters. We will assume that the instantaneous utility function u is twice differentiable, with $u'(z) > 0$ and $u''(z) < 0$ for all $z > 0$, and that u' is homogeneous of degree $-\varepsilon < 0$. Therefore, the utility function is isoelastic. This last assumption is imposed for guaranteeing the existence of a balanced growth path (see, e.g., Acemoglu [52] and Alonso-Carrera et al. [53]). We also assume that u satisfies standard Inada conditions, $\lim_{z\to 0}u'(z) = \infty$ and $\lim_{z\to\infty}u'(z) = 0$.

Following Gómez [54], the habits stock evolves according to

$$h_{t+1} = h_t + \rho[m(c_t, \overline{c}_t) - h_t], \quad 0 < \rho \leq 1, \tag{2.2}$$

where $\overline{c}_t$ denotes the economy-wide average level of consumption. The parameter ρ, which governs the speed with which the habits stock adjusts to previous-period consumption, determines the relative weight of consumption at different dates: the larger ρ, the more important is agent's consumption in the recent past and the smaller is the importance of the old values of the habits stock. If $\rho = 1$, then $h_{t+1} = m(c_t, \overline{c}_t)$, so that the habits stock depends only on previous-period consumption. Note that if $\rho = 0$, then $h_t = h_0$ for all t, so that the stock of habits would remain constant at its initial value and would not depend on past consumption levels.

In the internal-habits model, the reference consumption stock depends only on individual's own past consumption, so that $m(c_t, \overline{c}_t) = c_t$. In the external-habits model, habits are formed only from average past consumption in the economy, so that $m(c_t, \overline{c}_t) = \overline{c}_t$. Thus, in this case average past consumption, which is taken as given by the individual agent, exerts an external effect in utility. In the hybrid internal-external case, the habits stock is determined by both its own consumption and the economy-wide average level of consumption, which are combined by means of a continuously differentiable homogeneous mean. Thus, in this case m is a continuously differentiable function such that $m(c, c) = c$ for all $c > 0$, strictly increasing in its components, and (positively) homogeneous of degree one. Differentiating $m(c, c) = c$, and using its linear homogeneity, we get that $m_1(c, c) = \phi$ and $m_2(c, c) = 1 - \phi$ for all $c > 0$, with $0 \leq \phi \leq 1$, where m_i denotes the partial derivative of m with respect to its ith argument. Hence, the case $\phi = 1$ corresponds to the internal-habits model; the case $\phi = 0$ to the external-habits model, and the case $0 < \phi < 1$ to the model with hybrid internal-external habits. This specification comprises as particular cases the weighted geometric mean, $m(c_t, \overline{c}_t) = c_t^{\phi}\overline{c}_t^{1-\phi}$, proposed by Abel [48], and the weighted arithmetic mean, $m(c_t, \overline{c}_t) = \phi c_t + (1-\phi)\overline{c}_t$, considered, for example, by Marrero and Novales [36]. Most of the literature has considered that habits are formed in an external or internal way, and the papers that introduce hybrid internal-external habits use a weighted geometric mean or (to a much lesser extent) a weighted arithmetic mean. All of them are encompassed by the specification assumed which, therefore, is fairly general.

2.2. Production

At each moment of time, the representative agent is endowed with a fixed and constant stock of labor, $l_t = l$. Each individual firm produces output, y_t, using the inelastically supplied labor input $l_t = l$, the capital stock k_t, and public services provided by the government, P_t, in accordance with the Cobb-Douglas production function:

$$y_t = \widetilde{A}k_t^{\alpha}l^{1-\alpha}P_t^{1-\alpha} = Ak_t^{\alpha}P_t^{1-\alpha}, \tag{2.3}$$

where $A = \widetilde{A}l^{1-\alpha} > 0$.

Following Turnovsky [6, 55], the services derived by the agent from public investment are represented by

$$P_t = G_t \left(\frac{k_t}{K_t} \right)^{1-\sigma}, \quad 0 \leq \sigma \leq 1, \tag{2.4}$$

where K_t denotes the aggregate capital stock and G_t denotes the aggregate public expenditure. The former formulation incorporates the potential for public investment to be associated with relative congestion. The case $\sigma = 1$, so that $P_t = G_t$, corresponds to a nonrival nonexcludable public good that is available equally to each individual firm, independent of the usage of others. Thus, there is no congestion. However, as Barro and Sala-i-Martin [7] argue, almost all public services are subject to some degree of congestion, so that the pure public good should be viewed as only a benchmark. As σ deviates from 1, the nonexcludable public service losses its nonrival nature, and as this occurs the level of public services enjoyed by the individual firm from a given level of public expenditure is enhanced as his individual capital stock increases relative to the aggregate. In the case $\sigma = 0$, the public good is like a private good in that, since $k_t/K_t = 1/N$, the individual receives his proportional share $P_t = G_t/N$. This case is referred to as proportional relative congestion.

Combining (2.3) and (2.4), firm's output can be expressed as

$$y_t = Ak_t^{1-\sigma(1-\alpha)} G_t^{1-\alpha} K_t^{(\sigma-1)(1-\alpha)}. \tag{2.5}$$

Thus, the productivity of individual capital depends upon the usual elasticity of private capital, α, and a component, $(1-\sigma)(1-\alpha)$, which reflects the fact that, from the perspective of the individual agent, increasing her stock of capital will increase the level of government services she derives in the presence of relative congestion. Note that the presence of congestion introduces a distortion because the individual firm takes aggregate capital K_t as given in her production function (2.5). However, a central planner would take into account that $K_t = Nk_t$.

2.3. Government

The government levies a tax on income at a constant rate τ. Tax revenues are used to finance productive public expenditure,

$$G_t = \tau Y_t, \tag{2.6}$$

so that the government runs a balanced budget. In particular, this means that lump-sum taxes (or transfers) are not available.

2.4. Individual's Optimization

Output can be used for consumption or investment. Thus, the agent's budget constraint is

$$\mathrm{k}_{t+1} = k_t + (1-\tau) y_t - c_t. \tag{2.7}$$

At time $t = 0$, the agent chooses $\{c_t, k_{t+1}, h_{t+1}\}_{t=0}^{\infty}$ to maximize the lifetime utility (2.1) subject to the budget constraint (2.7)—where y_t is given by (2.5)—and the constraint on the habits stock accumulation (2.2), taking as given the paths of economy-wide average consumption, $\{\bar{c}_t\}_{t=0}^{\infty}$, public expenditure, $\{G_t\}_{t=0}^{\infty}$, and aggregate capital, $\{K_t\}_{t=0}^{\infty}$, the income tax rate, τ, and the initial conditions on capital $k_0 > 0$ and habits stock $h_0 > 0$.

To derive the optimization conditions, let us set up the Lagrangian of the agent's maximization problem,

$$L = \sum_{t=0}^{\infty} \beta^t \{u(c_t - \gamma h_t) + \lambda_t [k_t + (1-\tau) y_t - c_t - k_{t+1}] + \mu_t [(1-\rho) h_t + \rho m(c_t, \bar{c}_t) - h_{t+1}]\}, \tag{2.8}$$

where λ_t and μ_t are the shadow values of capital and habits stock, respectively. The first-order conditions with respect to c_t, k_{t+1}, and h_{t+1} are given by

$$c_t: \ u'(c_t - \gamma h_t) + \rho \mu_t m_1(c_t, \bar{c}_t) = \lambda_t, \tag{2.9}$$

$$k_{t+1}: \ \beta[1 + (1-\tau)(1-\sigma(1-\alpha)) y_t / k_t] \lambda_{t+1} = \lambda_t, \tag{2.10}$$

$$h_{t+1}: \ \beta(1-\rho)\mu_{t+1} - \beta\gamma u'(c_{t+1} - \gamma h_{t+1}) = \mu_t, \tag{2.11}$$

together with the transversality condition

$$\lim_{t \to \infty} \beta^t \lambda_t k_{t+1} = \lim_{t \to \infty} \beta^t \mu_t h_{t+1} = 0. \tag{2.12}$$

Equation (2.9) equates the marginal utility of consumption, adjusted by its effect on the future stock of habits, to the shadow price of capital. Equation (2.10) equates the rate of return on capital to the rate of return on consumption. From (2.11) and (2.12), we get

$$\mu_t = -\beta\gamma u'(c_{t+1} - \gamma h_{t+1}) - \beta \sum_{j=t+1}^{\infty} [\beta(1-\rho)]^{j-t} \gamma u'(c_{j+1} - \gamma h_{j+1}). \tag{2.13}$$

This condition states that the shadow cost of the habits stock is determined as the present discounted value of the stream of extra utils that would be lost by a marginal unit of habits, which depreciates at the rate ρ. Note that (2.13) entails that the shadow value of the habits stock is negative, $\mu_t < 0$.

3. Equilibrium

An equilibrium for this economy is as a set of paths $\{c_t, k_{t+1}, h_{t+1}\}_{t=0}^{\infty}$ that solves the agent's utility maximization problem when $\bar{c}_t = c_t$ and $K_t = N k_t$ for all t, and such that the government obeys its budget constraint.

3.1. Dynamics in the Aggregate Economy

We will focus on the equilibrium dynamics of aggregate variables, $C_t = Nc_t$, $K_t = Nk_t$, and $H_t = NH_t$. Furthermore, let $Z_t \equiv C_t - \gamma H_t = Nz_t$ denote the aggregate habit-adjusted consumption, and $\widehat{q}_t \equiv \mu_t/\lambda_t$ the relative shadow cost of habits. Note first that, using the government budget constraint (2.6) to substitute for G_t in the production function (2.5), individual output, y_t, and aggregate output, $Y_t = Ny_t$, can be expressed as

$$y_t = \tau^{(1-\alpha)/\alpha} A^{1/\alpha} N^{\sigma(1-\alpha)/\alpha} k_t = Bk_t, \tag{3.1}$$

$$Y_t = \tau^{(1-\alpha)/\alpha} A^{1/\alpha} N^{\sigma(1-\alpha)/\alpha} K_t = BK_t, \tag{3.2}$$

where $B = \tau^{(1-\alpha)/\alpha} A^{1/\alpha} N^{\sigma(1-\alpha)/\alpha}$.

Along a balanced growth path (BGP), C_t, H_t, K_t, and therefore Z_t, all grow at the same constant rate $\widehat{g}$. Appendix A derives the following system, which drives the dynamics of the economy in terms of the variables $\widehat{Z}_t \equiv Z_t(1+\widehat{g})^{-t}$, $\widehat{K}_t \equiv K_t(1+\widehat{g})^{-t}$, $\widehat{H}_t \equiv H_t(1+\widehat{g})^{-t}$, and $\widehat{q}_t \equiv \mu_t/\lambda_t$, which are constant along a BGP:

$$(1+\widehat{g})\widehat{Z}_{t+1} = \left\{ \frac{1-\rho\phi\widehat{q}_t}{1-\rho\phi\widehat{q}_{t+1}} \beta[1+(1-\tau)(1-\sigma(1-\alpha))B] \right\}^{1/\varepsilon} \widehat{Z}_t, \tag{3.3}$$

$$(1+\widehat{g})\widehat{K}_{t+1} = (1+(1-\tau)B)\widehat{K}_t - \widehat{Z}_t - \gamma\widehat{H}_t, \tag{3.4}$$

$$(1+\widehat{g})\widehat{H}_{t+1} = \widehat{H}_t + \rho\left(\widehat{Z}_t - (1-\gamma)\widehat{H}_t\right), \tag{3.5}$$

$$\left[1-\rho(1-\gamma\phi)\right]\widehat{q}_{t+1} = [1+(1-\tau)(1-\sigma(1-\alpha))B]\widehat{q}_t + \gamma. \tag{3.6}$$

The following proposition, which is proved in Appendix B, yields a closed-form expression for the equilibrium dynamics of the decentralized economy.

Proposition 3.1. *The decentralized economy has a unique feasible equilibrium with positive long-run growth which is described by*

$$\widehat{H}_t - \widehat{H}_\infty = \left(\frac{1-\rho(1-\gamma)}{1+\widehat{g}}\right)^t \left(\widehat{H}_0 - \widehat{H}_\infty\right), \tag{3.7}$$

$$\widehat{C}_t - \widehat{C}_\infty = \gamma\left(\widehat{H}_t - \widehat{H}_\infty\right), \tag{3.8}$$

$$\widehat{K}_t - \widehat{K}_\infty = \frac{\gamma}{(1-\tau)B + \rho(1-\gamma)}\left(\widehat{H}_t - \widehat{H}_\infty\right), \tag{3.9}$$

which converges to a BGP given by

$$\widehat{C}_\infty = \frac{(\widehat{g}+\rho)\widehat{Z}_\infty}{\widehat{g}+\rho(1-\gamma)}, \tag{3.10}$$

$$\widehat{K}_\infty = \frac{(\widehat{g}+\rho)\widehat{Z}_\infty}{((1-\tau)B-\widehat{g})(\widehat{g}+\rho(1-\gamma))}, \tag{3.11}$$

$$\widehat{H}_\infty = \frac{\rho\widehat{Z}_\infty}{\widehat{g}+\rho(1-\gamma)}, \tag{3.12}$$

where

$$\widehat{Z}_\infty = ((1-\tau)B-\widehat{g})\left[K_0 - \frac{\gamma(\rho K_0 + H_0)}{(1-\tau)B+\rho}\right], \tag{3.13}$$

and the long-run growth rate is

$$\widehat{g} = \{\beta[1+(1-\tau)(1-\sigma(1-\alpha))B]\}^{1/\varepsilon} - 1, \tag{3.14}$$

if and only if parameter values and initial conditions are that

$$\widehat{g} > 0 > \beta(1+\widehat{g})^{1-\varepsilon} - 1, \tag{3.15}$$

$$\frac{\gamma H_0}{K_0} < (1-\tau)B + \rho(1-\gamma). \tag{3.16}$$

The equilibrium paths of aggregate consumption, capital, and habits stock are then given by $C_t = \widehat{C}_t(1+\widehat{g})^t$, $K_t = \widehat{K}_t(1+\widehat{g})^t$, and $H_t = \widehat{H}_t(1+\widehat{g})^t$. Note, in particular, that the initial value of consumption is given by $C_0 = \widehat{C}_0 = \widehat{Z}_\infty + \gamma H_0$.

One important consequence of Proposition 3.1 is that the equilibrium dynamics of the economy is invariant to the specific homogeneous mean m chosen; that is, to the implied value of ϕ. Hence, the dynamics are invariant to whether habits are internally or externally formed and, therefore, the presence of externalities associated to past consumption in the external-habits model does not provoke equilibrium inefficiency. This theoretical finding seems to accord with empirical evidence supporting the habit-formation hypothesis but inconclusive about whether habits are formed in an internal or external way.

If $H_0 = \widehat{H}_0 > \widehat{H}_\infty$, from (3.7)–(3.12) in Proposition 3.1, we can find that

$$\frac{\widehat{H}_t}{\widehat{H}_\infty} > \frac{\widehat{C}_t}{\widehat{C}_\infty} > \frac{\widehat{K}_t}{\widehat{K}_\infty} > 1. \tag{3.17}$$

Furthermore, as the economy evolves, $\widehat{H}_t$, $\widehat{C}_t$, and $\widehat{K}_t$ decrease monotonically toward their respective steady-state values. Intuitively, to maintain a flat pattern of detrended adjusted-consumption, given the relatively high level of the habits stock (with respect to its stationary value), the agent starts with a relatively high level of consumption, $\widehat{C}_t$, and a low level of investment. Therefore, the capital stock, $\widehat{K}_t$, decreases. As the economy evolves, in order to drive the de-trended habits stock to its long-run level, consumption decreases steadily towards its stationary value. These results should be reversed in the case that $H_0 = \widehat{H}_0 < \widehat{H}_\infty$.

The explicit expressions derived in Proposition 3.1 allow us, for example, to compute analytically the speed of convergence towards the balanced growth path. Adapting the measure proposed by Eicher and Turnovsky [56] (see also Papageorgiou and Perez-sebastian [57]), the speed of convergence of a variable x_t is defined as

$$\Phi_t = -\frac{(x_{t+1} - x_t) - (x_{\infty,t+1} - x_{\infty,t})}{x_t - x_{\infty,t}}, \tag{3.18}$$

where $x_{\infty,t}$ is the equilibrium balanced growth path, which may or may not be stationary. In particular, if x_t converges to a (constant) stationary value, then $x_{\infty,t+1} = x_{\infty,t} = x_\infty$. If $\gamma \neq 0$, the convergence speed of the aggregate variables, H, K and C, is given by $\widehat{\Phi} = \rho(1-\gamma)$. Hence, the convergence speed is increasing in the speed of adjustment of habits to consumption, ρ, and decreasing in the strength of habits in utility, γ. If $\gamma = 0$, we recover the standard time-separable Barro [1] model. As it is wellknown, in this case the adjustment is instantaneous: at the initial time, consumption jumps to its BGP value, $C_0 = \widehat{C}_\infty = ((1-\tau)B - \widehat{g})K_0$, and thereafter consumption and capital both increase at the constant rate $\widehat{g}$.

3.2. Second-Best Solution

Among the different competitive equilibria indexed by the value of the income tax rate, τ, this section determines the one that maximizes individual welfare; that is, the second-best outcome.

Using that $z_t = c_t - \gamma h_t = \widehat{Z}_\infty (1+\widehat{g})^t / N$, the lifetime utility obtained is

$$\widehat{U} = \sum_{t=0}^{\infty} u\left[\frac{\widehat{Z}_\infty (1+\widehat{g})^t}{N}\right]\beta^t. \tag{3.19}$$

Therefore, we are looking for the value of the tax rate on income, τ, that maximizes (3.19). Using (3.14), we can obtain that

$$(1-\tau)B = \frac{(1+\widehat{g})^{\varepsilon} - \beta}{\beta(1-\sigma(1-\alpha))}, \tag{3.20}$$

which can be used to substitute $(1-\tau)B$ as a function of $\widehat{g}$ in (3.19) by means of (3.13), and allows us to express the lifetime utility $\widehat{U}$ as a function of $\widehat{g}$. Now, we have that $d\widehat{U}/d\tau = (d\widehat{U}/d\widehat{g})(d\widehat{g}/d\tau)$. Appendix D shows that $d\widehat{U}/d\widehat{g} > 0$. Hence, the first-order condition to maximize the lifetime utility $\widehat{U}$ with respect to τ becomes

$$\frac{d\widehat{g}}{d\tau} = \frac{(1+\widehat{g})^{\varepsilon-1} B\beta(1-\sigma(1-\alpha))(1-\alpha-\tau)}{\alpha\varepsilon\tau} = 0, \tag{3.21}$$

which yields the familiar result that the optimal income tax is equal to the elasticity of public services in production,

$$\hat{\tau} = 1 - \alpha. \tag{3.22}$$

Given that

$$\frac{d^2\hat{g}}{d\tau^2}(\hat{\tau}) = -\frac{(1+\hat{g})^{\varepsilon-1}B\beta(1-\sigma(1-\alpha))(1-\alpha)}{\alpha\varepsilon\hat{\tau}^2} < 0, \tag{3.23}$$

lifetime utility $\widehat{U}$ is maximized at the income tax rate $\hat{\tau} = 1-\alpha$, which also maximizes long-run growth.

This result is similar to that obtained by Barro [1] in the time-separable model. There is a hump-shaped relationship between growth (and welfare) and the income tax rate or, equivalently, the ratio of public spending to output: long-run growth first increases, reaches its maximum value, and then decreases as the tax rate increases. This shape results from the combination of two opposing forces on the marginal product of capital, which determines long-run growth: the positive effect caused by the increased provision of productive public services, and the negative effect of a higher tax rate needed to finance them. The growth- and welfare-maximizing tax rate on income is equal to the elasticity of public services in goods production.

4. The Socially-Planned Economy

We now turn to the first-best solution that a benevolent social planner would implement. The planner takes into account that private and aggregate outputs are related by $K_t = Nk_t$ in the production function, and so,

$$Y_t = AN^{\sigma(1-\alpha)}K_t^{\alpha}G_t^{1-\alpha}. \tag{4.1}$$

The planner also takes into account that $\overline{c}_t = c_t$ or, in aggregate terms, $N\overline{c}_t = C_t$, and, therefore, the constraint on the habits stock accumulation becomes

$$H_{t+1} = H_t + \rho(C_t - H_t). \tag{4.2}$$

Thus, given the initial conditions on capital, $K_0 > 0$, and habits stock, $H_0 > 0$, the planner chooses $\{C_t, G_t, K_{t+1}, H_{t+1}\}_{t=0}^{\infty}$ to maximize the lifetime utility

$$\sum_{t=0}^{\infty} u\left[\frac{(C_t - \gamma H_t)}{N}\right]\beta^t dt, \tag{4.3}$$

subject to the resource's constraint

$$K_{t+1} = K_t + Y_t - C_t - G_t, \tag{4.4}$$

where Y_t is given by (4.1), and the constraint on the habits stock accumulation (4.2).

4.1. First-Best Solution

The Lagrangian of the planner's problem is

$$\overline{L} = \sum_{t=0}^{\infty} \beta^t \left\{ u\left[\frac{(C_t - \gamma H_t)}{N}\right] + \overline{\lambda}_t [K_t + Y_t - C_t - G_t - K_{t+1}] + \overline{\mu}_t [(1-\rho)H_t + \rho C_t - H_{t+1}] \right\}, \tag{4.5}$$

where $\overline{\lambda}_t$ and $\overline{\mu}_t$ are the shadow values of capital and habits stocks, respectively. The first-order conditions with respect to C_t, G_t, K_{t+1}, and H_{t+1} are given by

$$C_t: \; u'(C_t - \gamma H_t)N^{\varepsilon-1} + \rho\overline{\mu}_t = \overline{\lambda}_t, \tag{4.6}$$

$$G_t: \; \overline{\lambda}_t \left(\frac{(1-\alpha)Y_t}{G_t} - 1\right) = 0, \tag{4.7}$$

$$K_{t+1}: \; \beta\left(1 + \frac{\alpha Y_t}{K_t}\right)\overline{\lambda}_{t+1} = \overline{\lambda}_t, \tag{4.8}$$

$$H_{t+1}: \; \beta(1-\rho)\overline{\mu}_{t+1} - \beta\gamma u'(C_{t+1} - \gamma H_{t+1})N^{\varepsilon-1} = \overline{\mu}_t, \tag{4.9}$$

where it has been used that u' is homogeneous of degree $-\varepsilon$ in (4.6) and (4.9), together with the transversality condition

$$\lim_{t\to\infty} \beta^t \overline{\lambda}_t K_{t+1} = \lim_{t\to\infty} \beta^t \overline{\mu}_t H_{t+1} = 0. \tag{4.10}$$

Let $\pi_t \equiv G_t/Y_t$ denote the ratio of public expenditure to output. Equation (4.7) yields the standard rule that the optimal ratio of public expenditure to output is the (constant) elasticity of public services in production,

$$\pi_t = \overline{\pi} = \frac{G_t}{Y_t} = 1 - \alpha. \tag{4.11}$$

Substituting (4.11) into (4.1), we obtain the following expression for aggregate output:

$$Y_t = (1-\alpha)^{(1-\alpha)/\alpha} A^{1/\alpha} N^{\sigma(1-\alpha)/\alpha} K_t = \overline{B} K_t, \tag{4.12}$$

where $\overline{B} \equiv (1-\alpha)^{(1-\alpha)/\alpha} A^{1/\alpha} N^{\sigma(1-\alpha)/\alpha}$.

Let $\overline{g}$ denote the long-run growth rate, and let $\overline{q}_t \equiv \overline{\mu}_t/\overline{\lambda}_t$ denote the relative shadow cost of habits. Proceeding in a similar manner as in the market economy, taking into account (4.11), we can derive the system that drives the dynamics of the centralized economy in terms of the variables $\overline{Z}_t \equiv Z_t(1+\overline{g})^{-t}$, $\overline{K}_t \equiv K_t(1+\overline{g})^{-t}$, $\overline{H}_t \equiv H_t(1+\overline{g})^{-t}$, and $\overline{q}_t$, which are constant along a BGP

$$(1+\overline{g})\overline{Z}_{t+1} = \left[\frac{1-\rho\overline{q}_t}{1-\rho\overline{q}_{t+1}}\beta\left(1+\alpha\overline{B}\right)\right]^{1/\varepsilon}\overline{Z}_t, \tag{4.13}$$

$$(1+\overline{g})\overline{K}_{t+1} = \left(1+\alpha\overline{B}\right)\overline{K}_t - \overline{Z}_t - \gamma\overline{H}_t, \tag{4.14}$$

$$(1+\overline{g})\overline{H}_{t+1} = \overline{H}_t + \rho\left(\overline{Z}_t - (1-\gamma)\overline{H}_t\right), \tag{4.15}$$

$$[1-\rho(1-\gamma)]\overline{q}_{t+1} = \left(1+\alpha\overline{B}\right)\overline{q}_t + \gamma. \tag{4.16}$$

The following proposition, which is proved in Appendix C, yields a closed-form solution for the dynamics of the efficient (first-best) solution.

Proposition 4.1. *The socially-planned economy has a unique feasible optimal solution with positive long-run growth which is described by*

$$\overline{H}_t - \overline{H}_\infty = \left(\frac{1-\rho(1-\gamma)}{1+\overline{g}}\right)^t\left(\overline{H}_0 - \overline{H}_\infty\right), \tag{4.17}$$

$$\overline{C}_t - \overline{C}_\infty = \gamma\left(\overline{H}_t - \overline{H}_\infty\right), \tag{4.18}$$

$$\overline{K}_t - \overline{K}_\infty = \frac{\gamma}{\alpha\overline{B}+\rho(1-\gamma)}\left(\overline{H}_t - \overline{H}_\infty\right), \tag{4.19}$$

where the (constant) ratio of public expenditure to output is

$$\pi_t = \frac{G_t}{Y_t} = 1-\alpha, \tag{4.20}$$

which converges to a BGP given by

$$\overline{C}_\infty = \frac{(\overline{g}+\rho)\overline{Z}_\infty}{\overline{g}+\rho(1-\gamma)}, \tag{4.21}$$

$$\overline{K}_\infty = \frac{(\overline{g}+\rho)\overline{Z}_\infty}{\left(\alpha\overline{B}-\overline{g}\right)(\overline{g}+\rho(1-\gamma))}, \tag{4.22}$$

$$\overline{H}_\infty = \frac{\rho\overline{Z}_\infty}{\overline{g}+\rho(1-\gamma)}, \tag{4.23}$$

where

$$\overline{Z}_\infty = \left(\alpha\overline{B} - \overline{g}\right)\left[K_0 - \frac{\gamma(\rho K_0 + H_0)}{\alpha\overline{B} + \rho}\right], \tag{4.24}$$

and the long-run growth rate is

$$\overline{g} = \left[\beta\left(1 + \alpha\overline{B}\right)\right]^{1/\varepsilon} - 1 \tag{4.25}$$

if and only if parameter values and initial conditions are that

$$\overline{g} > 0 > \beta(1 + \overline{g})^{1-\varepsilon} - 1, \tag{4.26}$$

$$\frac{\gamma H_0}{K_0} < \alpha\overline{B} + \rho(1 - \gamma). \tag{4.27}$$

The efficient paths of aggregate consumption, capital, and habits stock can be easily computed as $C_t = \overline{C}_t(1 + \overline{g})^t$, $K_t = \overline{K}_t(1 + \overline{g})^t$, and $H_t = \overline{H}_t(1 + \overline{g})^t$. In particular, the initial value of consumption is given by $C_0 = \overline{C}_0 = \overline{Z}_\infty + \gamma H_0$.

As in the Barro [1] model, the share of output claimed to finance public services that maximizes welfare is equal to the elasticity of public services in production. The intuition is simple. On one hand, an increase in public expenditure raises the marginal product of capital and this has positive welfare effects. On the other hand, an increase in public expenditure also decreases the amount of output available for consumption. The optimal size of government balances off these two effects.

If $\gamma \neq 0$, the convergence speed of K, C, and H towards their respective BGPs in the socially-planned economy can be easily shown to be constant and equal to the one in the decentralized economy, $\overline{\Phi} = \widehat{\Phi} = \rho(1 - \gamma)$. The case $\gamma = 0$ corresponds to the standard time-separable Barro [1] model, in which the economy displays no transitional dynamics: at the initial time, consumption jumps to its balanced growth path, $C_0 = \overline{C}_\infty = \overline{Z}_\infty = (\alpha\overline{B} - \overline{g})K_0$, and thereafter consumption and capital both increase at a constant rate $\overline{g}$.

4.2. First-Best versus Second-Best Solution

In the second-best solution, the income tax rate is set according to (3.22) at $\widehat{\tau} = 1 - \alpha$, and so, $B = \overline{B}$. Comparing the transition paths of $\widehat{H}_t$, $\widehat{C}_t$, and $\widehat{K}_t$ in the market economy given by (3.7)–(3.9) when $\tau = \widehat{\tau} = 1 - \alpha$ with their counterparts $\overline{H}_t$, $\overline{C}_t$, and $\overline{K}_t$ in the centrally-planned economy given by (4.17)–(4.19), it is immediate to see that they coincide if and only if the long-run growth rates in the market and the centralized economies coincide, $\widehat{g} = \overline{g}$. Comparing (3.14) with (4.11), we observe that the optimal and equilibrium long-run growth rates are related through $\widehat{g} \leq \overline{g}$, with equality if and only if $\sigma = 0$. Thus, the first-best and the second-best solutions coincide if and only if there is proportional congestion associated to public expenditure. With proportional congestion, $\sigma = 0$, setting the income tax rate according to (3.22) ensures that the social marginal return to capital, as viewed by the central planner, $\alpha\overline{B}$, coincides with the private after-tax return, as viewed by the individual agent, $(1-\tau)B$. Intuitively, the agent ignores the negative externality caused by congestion and overaccumulates capital relative to the optimum. Thus, an income tax can restore efficiency by increasing the effective marginal product of capital, which discourages capital accumulation.

The dynamics of consumption in the first-best and second-best solutions can be easily compared by using the closed-form expressions derived in Propositions 3.1 and 4.1. Comparison of (4.24) and (3.13) when $\tau = \hat{\tau} = 1-\alpha$, and so, $B = \overline{B}$, reveals that

$$\frac{\overline{Z}_{\infty}}{\widehat{Z}_{\infty}} = \frac{\left(\alpha\overline{B} - \overline{g}\right)}{\left(\alpha\overline{B} - \widehat{g}\right)} \leq 1, \tag{4.28}$$

with equality if and only if $\overline{g} = \widehat{g}$. Hence, the initial value of consumption in the second-best solution, $\widehat{C}_0$, is greater than that in the centrally-planned economy, $\overline{C}_0$. However, as the optimal long-run growth rate of consumption is higher than the equilibrium one, consumption in the first-best solution eventually catches up the corresponding one in the second-best solution.

5. Some Numerical Results

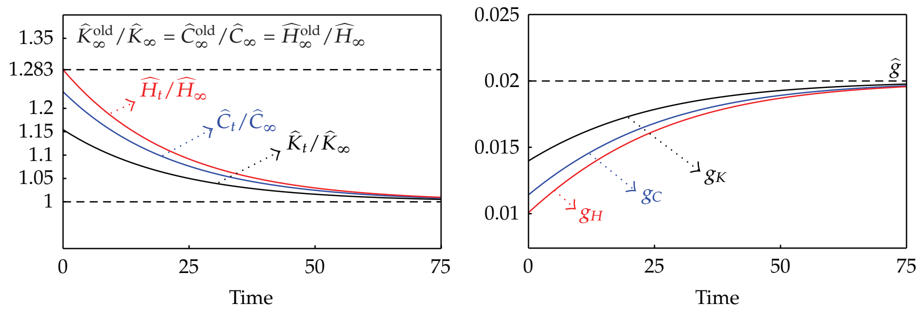
This section illustrates the implications for the dynamics of the economy of introducing habits in the Barro [1] model. To this end, we assume that the instantaneous utility function is given by

$$u(c_t, h_t) = \begin{cases} \dfrac{\left(c_t - \gamma h_t\right)^{1-\varepsilon} - 1}{1-\varepsilon} & \text{if } \varepsilon \neq 1, \\ \ln\left(c_t - \gamma h_t\right) & \text{if } \varepsilon = 1, \end{cases} \tag{5.1}$$

where ε is the relative risk aversion.

The baseline parameterization is shown in Table 1. The values of the relative risk aversion, ε, and the time discount rate, β, are standard. As usual in AK-type models, the stock of capital, K_t, comprises both physical and human capital, which explains the value of $\alpha = 0.9$. The tax rate on income is set at its (second-best) optimal value. As a benchmark, we assume that there is no congestion, $\sigma = 1$. Given these parameter values, the value of B is chosen so that the long-run growth rate in the market economy, $\widehat{g}$, is 2 percent. For example, normalizing $N = 1$, this entails a value of $A = 0.1832$. This calibration yields a plausible share of consumption in output of 72.8 percent. In the baseline, we set the strength of habits in utility to $\gamma = 0.9$, following empirical work suggesting a value of γ near unity. The value $\rho = 0.25$ is then chosen so as to obtain a realistic rate of convergence, $\widehat{\Phi}$, of 2.5 percent. Notwithstanding, we will illustrate the effect of assuming different values of ρ and γ on the equilibrium dynamics.

We perform an exercise similar to that in Carroll et al. [49], and consider the effect of a 10 percent unanticipated reduction in the capital stock of an economy that was initially on its balanced growth path. It should be noted that this shock does not affect the long-run growth rate. Figure 1(a) illustrates the time paths of consumption, capital and habits relative to their



(a) Time paths of $\widehat{K}_t$, $\widehat{C}_t$, and $\widehat{H}_t$.

(b) Growth rates of C_t, K_t, and H_t.

Figure 1: Transition dynamics after a 10% reduction in capital.

Table 1: Parameter and steady-state values.

β	ε	α	B	σ	ρ	γ	τ	$\widehat{g}$	$\widehat{C/Y}$	$\widehat{\Phi}$
0.95	2	0.90	0.1175	1	0.25	0.90	0.10	0.02	0.728	0.025

respective balanced growth path values. At $t = 0$, the capital stock experiments a reduction of 10 percent. Hence, income also falls a 10 percent at the outset, which causes a reduction in consumption. The percentage reduction in consumption is lower than the one in capital in order to keep a flat shape of adjusted-consumption. Hereafter, $\widehat{K}_t$, $\widehat{C}_t$, and, consequently, $\widehat{H}_t$ decrease steadily towards their respective steady-state values, which are a 28.3 percent lower than the initial ones. Figure 1(b) depicts the evolution of the growth rates of aggregate consumption, capital, and habits. At the initial time, the growth rates fall below their common preshock stationary value. Subsequently, they increase monotonically towards their common steady sate. Note that in the standard Barro [1] model, the economy would jump immediately to its new steady state, while the long-run growth rate would keep constant at 2 percent.

Figure 2 depicts the growth rate of consumption for different values of ρ and γ. As (3.14) shows, variations of these parameters do not affect the long-run growth rate of the economy. The behaviour of the growth rate of consumption reflects the fact that the convergence speed, $\widehat{\Phi} = \rho(1-\gamma)$, decreases as ρ decreases and/or γ increases.

Figure 3 compares the transition dynamics of the growth rates of consumption and capital in the first-best and second-best outcomes. We assume that the economy was initially at the BGP of the decentralized economy, and at time $t = 0$, it experiments a shock that reduces its capital stock a 10 percent. Figure 3 depicts the time paths of the growth rates in the socially-planned and the decentralized economies. Note that in the standard time-separable Barro [1] model the adjustment would be instantaneous; that is, the equilibrium growth rates of the market economy would keep constant at their common value $\widehat{g}$, and the optimal growth rates of the socially-planned economy would jump at the initial time to their new common value, $\overline{g}$.

6. Conclusion

This paper develops an endogenous growth model with habit-persistent preferences and productive public services. Government expenditure, which may be subject to congestion,

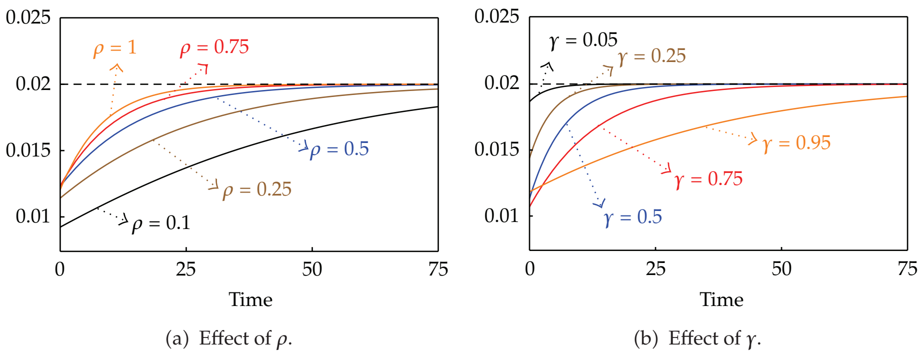


Figure 2: Growth rate of consumption after a 10% reduction in capital.

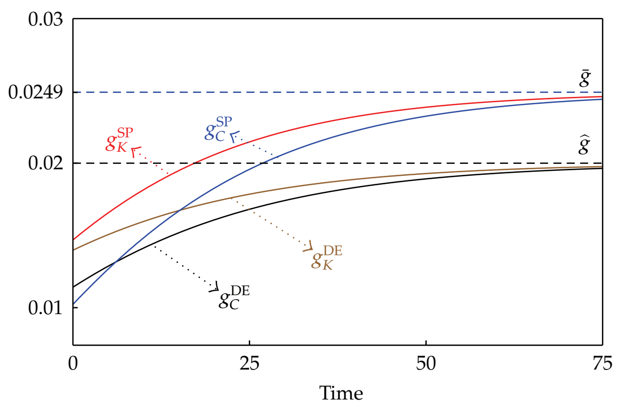


Figure 3: Growth rates of capital and consumption after a 10% reduction in capital: First-best versus second-best solution.

is financed by an income tax. Unlike the standard time-separable Barro [1] model, the introduction of habit formation makes the model to feature transitional dynamics, which are solved in closed form. As in the Barro [1] model, within a decentralized economy setup, the growth- and welfare-maximizing (in a second-best sense), income tax rates coincide and are equal to the elasticity of public services in production. This is also the optimal share of government expenditure on income in a first-best setting. The first-best solution can be decentralized as a market equilibrium only in the presence of proportional congestion.

In this paper, we have considered that it is the current flow of public investment which is productive. Another strand of the literature assumes instead that the accumulated stock, rather than the current flow, is the source of contribution to productive capacity. One interesting extension would be to analyze a model with public capital or with a mix of productive capital and public expenditure. Another valuable extension would be to consider a more general parameterization of technology, such as CES production. These topics will be the subject of future research.

Appendices

A. Derivation of the Dynamic System (3.3)–(3.6)

Since u' is homogeneous of degree $-\varepsilon$, and using the fact that $\bar{c}_t = c_t$, (2.9) can be rewritten as

$$u'(c_t - \gamma h_t) = u'\left(\frac{Z_t}{N}\right) = N^{\varepsilon} Z_t^{-\varepsilon} u'(1) = \lambda_t(1 - \rho\phi\widehat{q}_t). \tag{A.1}$$

Given that (3.1) entails that $y_t/k_t = B$, (2.10) becomes

$$\lambda_{t+1} = \frac{\lambda_t}{\beta[1 + (1-\tau)(1-\sigma(1-\alpha))B]}. \tag{A.2}$$

Now, evaluating (A.1) at $t = t+1$, dividing the result by (A.1), and using (A.2) to eliminate λ_{t+1}/λ_t from the ensuing expression, we get

$$\left(\frac{Z_{t+1}}{Z_t}\right)^{-\varepsilon} = \frac{\lambda_{t+1}(1-\rho\phi\widehat{q}_{t+1})}{\lambda_t(1-\rho\phi\widehat{q}_t)} = \frac{(1-\rho\phi\widehat{q}_{t+1})/(1-\rho\phi\widehat{q}_t)}{\beta[1 + (1-\tau)(1-\sigma(1-\alpha))B]}. \tag{A.3}$$

Using (A.1) evaluated at $t = t+1$ to eliminate $u'(c_{t+1} - \gamma h_{t+1})$ from (2.11), we get

$$\beta(1-\rho)\mu_{t+1} - \beta\gamma\lambda_{t+1}(1-\rho\phi\widehat{q}_{t+1}) = \mu_t. \tag{A.4}$$

Now, (3.3) follows immediately from (A.3). Equations (3.4) and (3.5) result from aggregating the agent's budget constraint (2.7) and the habits stock accumulation law (2.2), respectively, over the N identical individuals and multiplying both sides of the resulting expressions by $(1+\widehat{g})^{-t}$. Equation (3.6) can be obtained after dividing (A.4) by (A.2) and solving out for $\widehat{q}_{t+1}$ in the ensuing expression.

B. Proof of Proposition 3.1

Firstly, the long-run growth rate (3.14) can be immediately derived from (3.3), using that $\widehat{q}_t$, and $\widehat{Z}_t$ are constant at the BGP. Taking into account the homogeneity of degree $-\varepsilon$ of u', the transversality condition (2.12) is equivalent to $\beta(1+\widehat{g})^{1-\varepsilon} < 1$, which combined with the requirement of positive growth yields (3.15). Note that, if $\widehat{g} > 0$, the transversality condition is automatically satisfied if $\varepsilon \geq 1$.

Equation (3.6) is a linear difference equation that depends solely on $\widehat{q}_t$, which is a jump variable. Its steady state,

$$\widehat{q}_{\infty} = -\frac{\gamma}{\rho(1-\gamma\phi) + (1-\tau)(1-\sigma(1-\alpha))B}, \tag{B.1}$$

is unstable because

$$\frac{d\widehat{q}_{t+1}}{d\widehat{q}_t} = \frac{1 + (1-\tau)(1-\sigma(1-\alpha))B}{1-\rho(1-\gamma\phi)} > 1. \tag{B.2}$$

Hence, $\widehat{q}_t$ displays no transition dynamics: it jumps at the initial time to its stationary value $\widehat{q}_\infty$. Substituting $\widehat{q}_{t+1} = \widehat{q}_t$ into (3.3), and using (3.14), we get that $\widehat{Z}_{t+1} = \widehat{Z}_t = \widehat{Z}_\infty$ at any time, where $\widehat{Z}_\infty = Z_0$ denotes the stationary (and the initial) value of $\widehat{Z}_t$. Now, the solution to the linear difference equation (3.5) with initial value $\widehat{H}_0 = H_0$ is

$$\widehat{H}_t = \frac{\rho\widehat{Z}_\infty}{\widehat{g}+\rho(1-\gamma)} + \left[H_0 - \frac{\rho\widehat{Z}_\infty}{\widehat{g}+\rho(1-\gamma)}\right]\left(\frac{1-\rho(1-\gamma)}{1+\widehat{g}}\right)^t, \tag{B.3}$$

and so,

$$\widehat{C}_t = \widehat{Z}_t + \gamma\widehat{H}_t = \frac{(\widehat{g}+\rho)\widehat{Z}_\infty}{\widehat{g}+\rho(1-\gamma)} + \gamma\left[H_0 - \frac{\rho\widehat{Z}_\infty}{\widehat{g}+\rho(1-\gamma)}\right]\left(\frac{1-\rho(1-\gamma)}{1+\widehat{g}}\right)^t. \tag{B.4}$$

Substituting $\widehat{H}_t$ from (B.3) and $\widehat{Z}_t = \widehat{Z}_\infty$ into (3.4), the solution to the resulting difference equation is

$$\begin{aligned}\widehat{K}_t = {} & \frac{(\widehat{g}+\rho)\widehat{Z}_\infty}{((1-\tau)B-\widehat{g})(\widehat{g}+\rho(1-\gamma))} + \Omega\left(\frac{1+(1-\tau)B}{1+\widehat{g}}\right)^t \\ & + \frac{\gamma}{(1-\tau)B+\rho(1-\gamma)}\left[H_0 - \frac{\rho\widehat{Z}_\infty}{\widehat{g}+\rho(1-\gamma)}\right]\left(\frac{1-\rho(1-\gamma)}{1+\widehat{g}}\right)^t,\end{aligned} \tag{B.5}$$

where Ω is a constant to be determined. As condition (3.15) implies that $(1-\tau)B - \widehat{g} > 0$, convergence of $\widehat{K}_t$ to its stationary value $\widehat{K}_\infty$ as t tends to infinity requires that $\Omega = 0$ and, therefore,

$$\begin{aligned}\widehat{K}_t = {} & \frac{(\widehat{g}+\rho)\widehat{Z}_\infty}{((1-\tau)B-\widehat{g})(\widehat{g}+\rho(1-\gamma))} \\ & + \frac{\gamma}{(1-\tau)B+\rho(1-\gamma)}\left[H_0 - \frac{\rho\widehat{Z}_\infty}{\widehat{g}+\rho(1-\gamma)}\right]\left(\frac{1-\rho(1-\gamma)}{1+\widehat{g}}\right)^t.\end{aligned} \tag{B.6}$$

The stationary value $\widehat{Z}_\infty$ in (3.13) can now be derived by evaluating (B.6) at $t = 0$, taking into account that $\widehat{K}_0 = K_0$. From (3.13), the feasibility condition $Z_0 = \widehat{Z}_\infty > 0$ is satisfied if and only if condition (3.16) holds, which also ensures that the equilibrium paths are feasible. The BGP of the market economy given by (3.10), (3.11), and (3.12) can be easily computed by computing the limit as t goes to infinity of expressions (B.4), (B.6), and (B.3), respectively. Substituting the BGP values into (B.4), (B.6), and (B.3), we can readily obtain (3.8), (3.9), and (3.7).

C. Proof of Proposition 4.1

The proof is similar to that in Proposition 3.1. Firstly, the long-run growth rate (4.25) can be immediately obtained from (4.13), using that $\overline{q}_t$ and $\overline{Z}_t$ are constant at the BGP. Taking into account the homogeneity of degree $-\varepsilon$ of u', the transversality condition (4.10) is equivalent to $\beta(1+\overline{g})^{1-\varepsilon} < 1$, which combined with the requirement of positive growth yields (4.26).

Equation (4.16) is a linear difference equation that depends solely on $\overline{q}_t$, a jump variable. Its steady state,

$$\overline{q}_\infty = -\frac{\gamma}{\rho(1-\gamma)+\alpha\overline{B}}, \tag{C.1}$$

is unstable because

$$\frac{d\overline{q}_{t+1}}{d\overline{q}_t} = \frac{1+\alpha\overline{B}}{1-\rho(1-\gamma)} > 1. \tag{C.2}$$

Hence, $\overline{q}_t$ displays no transition dynamics: it jumps at the initial time to its stationary value $\overline{q}_\infty$. Substituting $\overline{q}_{t+1} = \overline{q}_t$ into (4.13), we get $\overline{Z}_{t+1} = \overline{Z}_t = \overline{Z}_\infty$ at any time, where $\overline{Z}_\infty = Z_0$ denotes the stationary (and the initial) value of $\overline{Z}_t$. Now, the solution to the linear difference equation (4.15) with initial value $\overline{H}_0 = H_0$ is

$$\overline{H}_t = \frac{\rho\overline{Z}_\infty}{\overline{g}+\rho(1-\gamma)} + \left[H_0 - \frac{\rho\overline{Z}_\infty}{\overline{g}+\rho(1-\gamma)}\right]\left(\frac{1-\rho(1-\gamma)}{1+\overline{g}}\right)^t, \tag{C.3}$$

and so,

$$\overline{C}_t = \overline{Z}_t + \gamma\overline{H}_t = \frac{(\overline{g}+\rho)\overline{Z}_\infty}{\overline{g}+\rho(1-\gamma)} + \gamma\left[H_0 - \frac{\rho\overline{Z}_\infty}{\overline{g}+\rho(1-\gamma)}\right]\left(\frac{1-\rho(1-\gamma)}{1+\overline{g}}\right)^t. \tag{C.4}$$

As in Proposition 3.1, since condition (4.26) entails that $\alpha\overline{B} - \overline{g} > 0$, the solution to (4.14) with terminal condition $\lim_{t\to\infty}\overline{K}_t = \overline{K}_\infty$ is

$$\begin{aligned}\overline{K}_t = {} & \frac{(\overline{g}+\rho)\overline{Z}_\infty}{\left(\alpha\overline{B}-\overline{g}\right)(\overline{g}+\rho(1-\gamma))} \\ & + \frac{\gamma}{\alpha\overline{B}+\rho(1-\gamma)}\left[H_0 - \frac{\rho\overline{Z}_\infty}{\overline{g}+\rho(1-\gamma)}\right]\left(\frac{1-\rho(1-\gamma)}{1+\overline{g}}\right)^t.\end{aligned} \tag{C.5}$$

The stationary value $\overline{Z}_\infty$ in (4.24) can now be derived from making $\overline{K}_0 = K_0$ in (C.5). The feasibility condition $Z_0 = \overline{Z}_\infty > 0$ is satisfied if and only if the initial values of capital and habits stock are such that condition (4.27) holds, which also ensures that the optimal growth paths are feasible. The BGP of the socially-planned economy given by (4.21), (4.22), and (4.23) can be easily computed by computing the limit as t goes to infinity of expressions (C.4), (C.5), and (C.3), respectively. Substituting these BGP values into (C.4), (C.5), and (C.3), we can readily obtain (4.18), (4.19), and (4.17).

D. Second-Best Solution

This appendix shows that $d\widehat{U}/d\widehat{g} > 0$. Using that u' is homogeneous of degree $-\varepsilon$, and taking into account the condition (3.15), we get that

$$\begin{aligned}
\frac{d\widehat{U}}{d\widehat{g}} &= \frac{1}{N} \sum_{t=0}^{\infty} \left(\frac{d\widehat{Z}_\infty}{d\widehat{g}} (1+\widehat{g})^t + \widehat{Z}_\infty (1+\widehat{g})^{t-1} t \right) u' \left[\frac{\widehat{Z}_\infty (1+\widehat{g})^t}{N} \right] \beta^t \\
&= \frac{N^{\varepsilon-1} u'(1)}{\widehat{Z}_\infty^{\varepsilon}} \sum_{t=0}^{\infty} \left(\frac{d\widehat{Z}_\infty}{d\widehat{g}} (1+\widehat{g})^{-(\varepsilon-1)t} + \widehat{Z}_\infty (1+\widehat{g})^{-(\varepsilon-1)t-1} t \right) \beta^t \\
&= \frac{N^{\varepsilon-1} u'(1)}{\widehat{Z}_\infty^{\varepsilon} \left(1-\beta(1+\widehat{g})^{1-\varepsilon}\right)} \left[\frac{d\widehat{Z}_\infty}{d\widehat{g}} + \frac{\beta(1+\widehat{g})^{-\varepsilon} \widehat{Z}_\infty}{1-\beta(1+\widehat{g})^{1-\varepsilon}} \right].
\end{aligned} \tag{D.1}$$

Using (3.20) to substitute $(1-\tau)B$ as a function of $\widehat{g}$ into (3.13), we can get that

$$\frac{d\widehat{Z}_\infty}{d\widehat{g}} = \frac{\varepsilon(1+\widehat{g})^{\varepsilon-1} \gamma(\rho K_0 + H_0)((1-\tau)B - \widehat{g})}{\beta(1-\sigma(1-\alpha))((1-\tau)B+\rho)^2} + \frac{\varepsilon(1+\widehat{g})^{\varepsilon-1} - \beta(1-\sigma(1-\alpha))}{\beta(1-\sigma(1-\alpha))((1-\tau)B-\widehat{g})} \widehat{Z}_\infty, \tag{D.2}$$

which substituted into (D.1) yields, after simplification,

$$\begin{aligned}
\frac{d\widehat{U}}{d\widehat{g}} = \frac{N^{\varepsilon-1} u'(1)}{\widehat{Z}_\infty^{\varepsilon} \left(1-\beta(1+\widehat{g})^{1-\varepsilon}\right)} \Bigg\{ & \frac{((1-\tau)B - \widehat{g}) \varepsilon (1+\widehat{g})^{\varepsilon-1} \gamma(\rho K_0 + H_0)}{\beta(1-\sigma(1-\alpha))((1-\tau)B+\rho)^2} \\
& + \frac{\left[\left((1+\widehat{g})^{\varepsilon-1} - \beta\right)\varepsilon + (1-\alpha)\beta\left(1-\beta(1+\widehat{g})^{-\varepsilon}\right)\sigma\right]}{((1-\tau)B-\widehat{g})\left(1-\beta(1+\widehat{g})^{1-\varepsilon}\right)\beta(1-\sigma(1-\alpha))} \widehat{Z}_\infty \Bigg\} > 0,
\end{aligned} \tag{D.3}$$

where we have used condition (3.15) to determine its sign.

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