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Research Article

Dynamic Output Feedback Stabilization of Controlled Positive Discrete-Time Systems with Delays

Zhenbo Li[1] and Shuqian Zhu[2]

[1] *School of Statistics and Mathematics, Shandong Economic University, Jinan 250014, China*
[2] *School of Mathematics, Shandong University, Jinan 250100, China*

Correspondence should be addressed to Zhenbo Li, lizhenbo@126.com

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The problem of stabilization by means of dynamic output feedback is studied for discrete-time delayed systems with possible interval uncertainties. The control is under positivity constraint, which means that the resultant closed-loop system must be stable and positive. The robust resilient controller is respect to additive controller gain variation which also belongs to an interval. Necessary and sufficient/sufficient conditions are established for the existence of the dynamic output feedback controller. The desired controller gain matrices can be determined effectively via the cone complementarity linearization techniques.

1. Introduction

A dynamical system is called positive if any trajectory of the system starting from nonnegative initial states remains forever nonnegative. Such systems abound in almost all fields, for instance, engineering, ecology, economics, biomedicine, and social science [1–3]. Since the states of positive systems are confined within a "cone" located in the positive quadrant rather than in linear spaces, many well-established results for general linear systems cannot be readily applied to positive systems. This feature makes the analysis and synthesis of positive systems a challenging and interesting job, and many results have been obtained, see [4–12]. It should be pointed out that in [9–12], the governed system is not necessarily positive, while a control strategy can be designed such that the closed-loop system is positive. We call systems in this class controlled positive.

The reaction of real world systems to exogenous signals is never instantaneous and always infected by certain time delays. The delay presence may induce complex behaviors, such as oscillations, instability, and poor performance [13]. Recently, the study on delayed positive systems has drawn increasing attention and many important results have been obtained, see [14–17] for stability and [18–20] for control. It has been shown that the stability of delayed positive systems has nothing to do with the amplitude of delays.

It should be noted that in most literature aforementioned for delayed systems, it is assumed that the parameters of systems are exactly known, and the controller takes the form of state feedback. However, in practical applications, it is inevitable that uncertainties enter the system parameters and it is often impossible to obtain the full information on the state variables. Hence, it is necessary to investigate the output feedback stabilization problem of uncertain positive system with delays. On the other hand, in practice, instead of being precise or exactly implemented, many controllers do have a certain degree of errors and may be sensitive to these errors. Such controllers are often termed "fragile". Therefore, it is considered beneficial to design a "resilient" controller being capable of tolerating some level of controller gain variations [21, 22]. All of the above motivate our research.

This paper deals with the dynamic output feedback stabilization problem for discrete-time delayed systems (not necessarily positive) under the positivity constraint, which means that the closed-loop systems are not only stable, but also positive. First, a new necessary and sufficient condition is given for the stability of discrete-time positive systems with delays, which is more useful for designing output feedback controllers. Then for systems with/without uncertainties, necessary and sufficient/sufficient conditions for the existence of the dynamic output feedback controllers are established in terms of linear matrix inequalities (LMIs) together with a matrix equality constraint. The controller gain matrices can be determined via the cone complementarity linearization techniques.

Notations

$\mathbb{R}$, $\mathbb{R}^n$, and $\mathbb{R}^n_{0,+}$ denote the reals, the n-dimensional linear vector space over the reals the nonnegative quadrant of $\mathbb{R}^n$, respectively. $\mathbb{R}^{n\times m}$ denotes the set of all $n \times m$ real matrices. $A \succeq 0 (\preceq 0)$ means that the elements of A are nonnegative (nonpositive). For matrices $A, B \in \mathbb{R}^{n\times m}$, the notation $A \succeq B$ or $B \preceq A$ means that $A - B \succeq 0$. $A > 0\ (< 0)$ stands for a symmetric positive (negative) definite matrix A. The symbol $\rho(A)$ denotes the spectral radius of matrix A, that is, $\rho(A) = \max\{|\lambda| : \lambda \in \sigma(A)\}$ with $\sigma(A)$ being the spectrum of A. The superscript T represents the transpose. The symbol $*$ will be used in some matrix expressions to induce a symmetric structure.

2. Mathematical Preliminaries

In this section, we will give some definitions and lemmas about positive discrete-time delayed systems.

Consider the discrete-time system with delay

$$\begin{aligned} x(t+1) &= Ax(t) + A_\tau x(t-\tau), \\ y(t) &= Cx(t), \\ x(t) &= \phi(t), \quad t = -\tau, -(\tau-1), \ldots, 0, \end{aligned} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $y(t) \in \mathbb{R}^m$ is the measurable output, A, A_τ, and C are known real constant matrices, $\tau \in \mathbb{N}$ is a constant delay and $\phi(t) : [-\tau, -(\tau-1), \ldots, 0] \to \mathbb{R}^n_{0,+}$ is the vector valued initial function.

First, some definitions and lemmas are given.

Definition 2.1 (see [17]). System (2.1) is said to be positive if for any $\phi(t) : [-\tau, -(\tau - 1), \ldots, 0] \to \mathbb{R}^n_{0,+}$, one has $x(t) \succeq 0$ and $y(t) \succeq 0$ for all $t \in \mathbb{N}$.

Lemma 2.2 (see [17]). *System* (2.1) *is positive if and only if $A \succeq 0$, $A_\tau \succeq 0$ and $C \succeq 0$.*

Definition 2.3 (see [15]). A square matrix A is called a Schur matrix if $\rho(A) < 1$.

Lemma 2.4 (see [15]). *Positive system* (2.1) *is asymptotically stable if and only if $(A + A_\tau)$ is a Schur matrix.*

Lemma 2.5 (see [3]). *A matrix $A \succeq 0$ is a Schur matrix if and only if there exists a diagonal matrix $P > 0$ such that $A^T PA - P < 0$.*

Combining the above lemmas, we have

Lemma 2.6. *Positive system* (2.1) *is asymptotically stable if and only if there exists a diagonal matrix $P > 0$ such that*

$$(A + A_\tau)^T P(A + A_\tau) - P < 0. \tag{2.2}$$

Lemma 2.7 (see [2]). *For two matrices $A, B \in \mathbb{R}^{n\times n}$, if $A \succeq B \succeq 0$, then $\rho(A) \geq \rho(B)$.*

3. Dynamic Output Feedback Stabilization

Now consider the discrete-time system with delay and control

$$\begin{aligned} x(t+1) &= Ax(t) + A_\tau x(t-\tau) + Bu(t), \\ y(t) &= Cx(t), \\ x(t) &= \phi(t), \quad t = -\tau, -(\tau-1), \ldots, 0, \end{aligned} \tag{3.1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^p$, $y(t) \in \mathbb{R}^m$ are, respectively, the state, the control input and the measurable output. A, A_τ, B and C are known constant matrices, $\tau \in \mathbb{N}$ is a constant delay and $\phi(t) : [-\tau, -(\tau-1), \ldots, 0] \to \mathbb{R}^n_{0,+}$ is the vector valued initial function.

The purpose of this section is to design a dynamic output feedback controller

$$\begin{aligned} \delta(t+1) &= A_k\delta(t) + B_k y(t), \\ u(t) &= C_k\delta(t) + D_k y(t), \\ \delta(0) &= \delta_0 \end{aligned} \tag{3.2}$$

such that the resultant closed-loop system

$$\begin{bmatrix} x(t+1) \\ \delta(t+1) \end{bmatrix} = \begin{bmatrix} A + BD_kC & BC_k \\ B_kC & A_k \end{bmatrix} \begin{bmatrix} x(t) \\ \delta(t) \end{bmatrix} + \begin{bmatrix} A_\tau & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(t-\tau) \\ \delta(t-\tau) \end{bmatrix},$$
$$y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \delta(t) \end{bmatrix}, \tag{3.3}$$
$$x(t) = \phi(t), \quad t = -\tau, -(\tau-1), \dots, 0,$$
$$\delta(0) = \delta_0$$

is positive and asymptotically stable. Where $\delta(t) \in \mathbb{R}^r$ is the state of the controller, $\delta_0 \in \mathbb{R}^r_{0,+}$, A_k, B_k, C_k and D_k are the controller gain matrices to be determined. The above stabilization problem will be called Problem DOFS (Dynamic Output Feedback Stabilization).

Remark 3.1. r may be either equal to or less than n. In the case of $r = n$ or $r < n$, controller (3.2) is called the full-order or reduced-order dynamic output feedback controller for system (3.1), respectively.

First, similar to [8], in order to design dynamic output feedback controller for system (3.1), we give an equivalent form of Lemma 2.6.

Theorem 3.2. *Positive system* (2.1) *is asymptotically stable if and only if there exist diagonal matrices* $P > 0$ *and* $Q > 0$ *satisfying the LMI*

$$\begin{bmatrix} -P & (A + A_\tau)^T \\ * & -Q \end{bmatrix} < 0, \tag{3.4}$$

and the matrix equality constraint

$$PQ = I. \tag{3.5}$$

Proof. By Schur complement, it is easy to see that (2.2) holds if and only if the following inequality holds:

$$\begin{bmatrix} -P & (A + A_\tau)^T \\ * & -P^{-1} \end{bmatrix} < 0. \tag{3.6}$$

Taking $P^{-1} = Q$, we have that there exist a diagonal matrix $P > 0$ satisfying (2.2) if and only if there exist diagonal matrices $P > 0$ and $Q > 0$ satisfying (3.4) and (3.5). □

Remark 3.3. Comparing with Lemma 2.6, the conditions in Theorem 3.2 are a little more complicated since a matrix equality constraint is introduced. However, it can be seen that in Theorem 3.2, the Lyapunov matrix P and the system parametric matrices have been decoupled. Hence, Theorem 3.2 is more useful when designing output feedback controllers for system (3.1).

Based on Theorem 3.2, we will establish the necessary and sufficient conditions for the solvability of Problem DOFS.

Theorem 3.4. *For discrete-time delayed system* (3.1) *with* $A_\tau \geq 0$, $C \geq 0$, *there exists a solution to Problem DOFS if and only if there exist matrices* L_i, $i = 1,2,3,4$, *and diagonal matrices* $P_j > 0$, $Q_j > 0$, $j = 1,2$, *satisfying the LMIs*

$$\begin{gathered} L_1 \geq 0, \\ L_2 C \geq 0, \\ B L_3 \geq 0, \\ A + B L_4 C \geq 0, \\ \begin{bmatrix} -P_1 & 0 & A^T + C^T L_4^T B^T + A_\tau^T & C^T L_2^T \\ * & -P_2 & L_3^T B^T & L_1^T \\ * & * & -Q_1 & 0 \\ * & * & * & -Q_2 \end{bmatrix} < 0 \end{gathered} \tag{3.7}$$

and the matrix equality constraints

$$\begin{gathered} P_1 Q_1 = I, \\ P_2 Q_2 = I. \end{gathered} \tag{3.8}$$

In this case, the controller gain matrices in (3.2) *are designed as*

$$A_k = L_1, \qquad B_k = L_2, \qquad C_k = L_3, \qquad D_k = L_4. \tag{3.9}$$

4. Robust Resilient Stabilization of Interval Systems

In this section, we consider the discrete-time interval uncertain system (3.1), where the system parametric matrices are all uncertain with

$$\begin{gathered} A \in [A_m, A_M], \qquad A_\tau \in [A_{\tau m}, A_{\tau M}], \qquad B \in [B_m, B_M], \qquad C \in [C_m, C_M], \\ A_{\tau m} \geq 0, \qquad B_m \geq 0, \qquad C_m \geq 0 \end{gathered} \tag{4.1}$$

and $A_m, A_M, A_{\tau m}, A_{\tau M}, B_m, B_M, C_m, C_M$ are known constant matrices.

For uncertain system (3.1), we will design a resilient dynamic output feedback controller

$$\begin{aligned} \delta(t+1) &= (A_k + \Delta A_k)\delta(t) + (B_k + \Delta B_k) y(t), \\ u(t) &= (C_k + \Delta C_k)\delta(t) + (D_k + \Delta D_k) y(t), \\ \delta(0) &= \delta_0, \end{aligned} \tag{4.2}$$

such that the resultant closed-loop system

$$\begin{gathered}
\eta(t+1) = A_c\eta(t) + A_{c\tau}\eta(t-\tau), \\
y(t) = C_c\eta(t), \\
x(t) = \phi(t), \quad t = -\tau, -(\tau-1), \dots, 0, \\
\delta(0) = \delta_0,
\end{gathered} \tag{4.3}$$

with

$$\begin{gathered}
\eta(t) = \begin{bmatrix} x(t) \\ \delta(t) \end{bmatrix}, \qquad A_c = \begin{bmatrix} A + BD_kC + B\Delta D_kC & BC_k + B\Delta C_k \\ B_kC + \Delta B_kC & A_k + \Delta A_k \end{bmatrix}, \\
A_{c\tau} = \begin{bmatrix} A_\tau & 0 \\ 0 & 0 \end{bmatrix}, \qquad C_c = \begin{bmatrix} C & 0 \end{bmatrix},
\end{gathered} \tag{4.4}$$

is positive and robustly stable. In (4.2), $\delta(t) \in \mathbb{R}^r$ is the state of the controller, A_k, B_k, C_k, and D_k are the controller gain matrices to be determined and ΔA_k, ΔB_k, ΔC_k, and ΔD_k are the controller gain variations which are assumed to satisfy

$$\begin{gathered}
\Delta A_k \in [A_{km}, A_{kM}], \qquad \Delta B_k \in [B_{km}, B_{kM}], \\
\Delta C_k \in [C_{km}, C_{kM}], \qquad \Delta D_k \in [D_{km}, D_{kM}]
\end{gathered} \tag{4.5}$$

with A_{km}, A_{kM}, B_{km}, B_{kM}, C_{km}, C_{kM}, D_{km}, D_{kM} being known constant matrices.

In the sequel, the above stabilization problem will be stated as Problem RRDOFS (Robust Resilient Dynamic Output Feedback Stabilization).

Assumption 4.1. Assume that

$$\begin{gathered}
A_{kM} \succeq 0, \qquad B_{kM} \succeq 0, \qquad C_{kM} \succeq 0, \qquad D_{kM} \succeq 0, \qquad A_{km} = -A_{kM}, \\
B_{km} = -B_{kM}, \qquad C_{km} = -C_{kM}, \qquad D_{km} = -D_{kM}.
\end{gathered} \tag{4.6}$$

Remark 4.2. In fact, Assumption 4.1 is without loss of generality. For example, for any matrices A_{km} and A_{kM} with $A_{km} \preceq A_{kM}$, let $\overline{A_k} = (A_{km} + A_{kM})/2$, $\underline{A_k} = (A_{kM} - A_{km})/2 \succeq 0$, then $A_k + \Delta A_k$ can be rewritten as

$$A_k + \Delta A_k = A_k + \overline{A_k} + \Delta A_k - \overline{A_k} = \widehat{A}_k + \Delta\widehat{A}_k, \tag{4.7}$$

where $\widehat{A}_k = A_k + \overline{A_k}$ is the new controller gain matrix to be determined and $\Delta\widehat{A}_k = \Delta A_k - \overline{A_k}$ is the new controller gain variation which satisfies $\Delta\widehat{A}_k =\in [-\underline{A_k}, \underline{A_k}]$. Similarly, we can prove the generality of the assumption on ΔB_k, ΔC_k, and ΔD_k.

Next, we will establish the sufficient conditions for the solvability of Problem RRDOFS.

Theorem 4.3. *For the interval uncertain delayed system* (3.1) *with the parametric matrices satisfying* (4.1)*, there exists a solution to Problem RRDOFS if there exist matrices* L_1*,* $L_i \succeq 0$*,* $H_i \preceq 0$*,* $i = 2,3,4$ *and diagonal matrices* $P_j > 0$*,* $Q_j > 0$*,* $j = 1,2$*, satisfying the following LMIs:*

$$L_1 + A_{km} \succeq 0, \tag{4.8a}$$

$$L_2 C_m + H_2 C_M + B_{km} C_M \succeq 0, \tag{4.8b}$$

$$B_m L_3 + B_M H_3 + B_M C_{km} \succeq 0, \tag{4.8c}$$

$$A_m + B_m L_4 C_m + B_M H_4 C_M + B_M D_{km} C_M \succeq 0, \tag{4.8d}$$

$$\begin{bmatrix} -P_1 & 0 & A_M^T + C_M^T L_4^T B_M^T + C_m^T H_4^T B_m^T + C_M^T D_{kM}^T B_M^T + A_{\tau M}^T & C_M^T L_2^T + C_m^T H_2^T + C_M^T B_{kM}^T \\ * & -P_2 & L_3^T B_M^T + H_3^T B_m^T + C_{kM}^T B_M^T & L_1^T + A_{kM}^T \\ * & * & -Q_1 & 0 \\ * & * & * & -Q_2 \end{bmatrix} < 0 \tag{4.8e}$$

and the matrix equality constraints (3.8)*. In this case, the controller gain matrices in* (4.2) *are designed as*

$$A_k = L_1, \qquad B_k = L_2 + H_2, \qquad C_k = L_3 + H_3, \qquad D_k = L_4 + H_4. \tag{4.9}$$

Proof. Letting

$$B_k = B_{k1} + B_{k2}, \qquad C_k = C_{k1} + C_{k2}, \qquad D_k = D_{k1} + D_{k2} \tag{4.10}$$

with $B_{k1} \succeq 0$, $B_{k2} \preceq 0$, $C_{k1} \succeq 0$, $C_{k2} \preceq 0$, $D_{k1} \succeq 0$, $D_{k2} \preceq 0$, and noting that (4.1) and (4.5)-(4.6), we get

$$\begin{aligned} &A_m + B_m D_{k1} C_m + B_M D_{k2} C_M + B_M D_{km} C_M \\ &\quad \preccurlyeq A + B D_k C + B \Delta D_k C \\ &\quad = A + B D_{k1} C + B D_{k2} C + B \Delta D_k C \\ &\quad \preccurlyeq A_M + B_M D_{k1} C_M + B_m D_{k2} C_m + B_M D_{kM} C_M, \end{aligned} \tag{4.11a}$$

$$\begin{aligned} &B_m C_{k1} + B_M C_{k2} + B_M C_{km} \preceq B C_k + B \Delta C_k \\ &\quad = B C_{k1} + B C_{k2} + B \Delta C_k \preceq B_M C_{k1} + B_m C_{k2} + B_M C_{kM}, \end{aligned} \tag{4.11b}$$

$$\begin{aligned} &B_{k1} C_m + B_{k2} C_M + B_{km} C_M \preceq B_k C + \Delta B_k C \\ &\quad = B_{k1} C + B_{k2} C + \Delta B_k C \preceq B_{k1} C_M + B_{k2} C_m + B_{kM} C_M, \end{aligned} \tag{4.11c}$$

$$A_k + A_{km} \preceq A_k + \Delta A_k \preccurlyeq A_k + A_{kM}, \tag{4.11d}$$

$$A_c+A_{c\tau} \preceq M := \begin{bmatrix} A_M+B_MD_{k1}C_M+B_mD_{k2}C_m+B_MD_{kM}C_M+A_{\tau M} & B_MC_{k1}+B_mC_{k2}+B_MC_{kM} \\ B_{k1}C_M + B_{k2}C_m + B_{kM}C_M & A_k + A_{kM} \end{bmatrix}. \tag{4.11e}$$

From (4.8a)–(4.8d), (4.9)–(4.11a), (4.11b), (4.11c), (4.11d), and (4.11e) and $B_m \succeq 0$, $C_m \succeq 0$, $L_i \succeq 0$, $H_i \preceq 0$, $i = 2,3,4$, we obtain that $A_c \succeq 0$, $A_{c\tau} \succeq 0$, $C_c \succeq 0$ for all uncertainties and $M \succeq 0$. By using Lemma 2.2, we conclude that the closed-loop system (4.3) is positive.

Noting (4.8e) and by using Lemma 2.4 and Theorem 3.2, we have that $M \succeq 0$ is a Schur matrix considering (4.9). From (4.11e) and Lemma 2.7, we get $A_c + A_{c\tau} \succeq 0$ is also a Schur matrix for all uncertainties. Hence, the positive system (4.3) is robustly stable. □

Remark 4.4. From the proof of Theorem 4.3, we can see that the condition in (4.1) that $B_m \succeq 0$, $C_m \succeq 0$ is given for the purpose to find the upper bound and the lower bound about the parametric matrices of the closed-loop system (4.3).

If in system (3.2), there are no uncertainties in the parametric matrices $B \succeq 0$ and $C \succeq 0$, that is, $B \succeq 0$ and $C \succeq 0$ are known constant matrices, we will obtain the the necessary and sufficient conditions for the solvability of Problem RRDOFS, which will be given as follows.

Theorem 4.5. *For the interval uncertain delayed system* (3.1) *with* (4.1) *and* $B \succeq 0$ *and* $C \succeq 0$ *being known constant matrices, there exists a solution to Problem RRDOFS if there exist matrices* L_i, $i = 1,2,3,4$ *and diagonal matrices* $P_j > 0$, $Q_j > 0$, $j = 1,2$, *satisfying the following LMIs:*

$$L_1 + A_{km} \succeq 0, \tag{4.12a}$$

$$L_2C + B_{km}C \succeq 0, \tag{4.12b}$$

$$BL_3 + BC_{km} \succeq 0, \tag{4.12c}$$

$$A_m + BL_4C + BD_{km}C \succeq 0, \tag{4.12d}$$

$$\begin{bmatrix} -P_1 & 0 & A_M^T + C^TL_4^TB^T + C^TD_{kM}^TB^T + A_{\tau M}^T & C^TL_2^T + C^TB_{kM}^T \\ * & -P_2 & L_3^TB^T + C_{kM}^TB^T & L_1^T + A_{kM}^T \\ * & * & -Q_1 & 0 \\ * & * & * & -Q_2 \end{bmatrix} < 0 \tag{4.12e}$$

and the matrix equality constraints (3.8). *In this case, the controller gain matrices in* (4.2) *are designed as*

$$A_k = L_1, \qquad B_k = L_2, \qquad C_k = L_3, \qquad D_k = L_4. \tag{4.13}$$

Proof. The sufficiency can be easily obtained from Theorem 4.3 by letting $H_i = 0$, $i = 2,3,4$ and $B_m = B_M = B$, $C_m = C_M = C$.

Now we will prove the necessity. Suppose that for the interval uncertain delayed system (3.1) with (4.1) and $B \succeq 0$ and $C \succeq 0$ being known constant matrices, Problem

RRDOFS is solvable, that is, there exists matrices A_k, B_k, C_k and D_k such that the closed-loop (4.3) is positive and asymptotically stable for any $A \in [A_m, A_M]$, $A_\tau \in [A_{\tau m}, A_{\tau M}]$ and $\Delta A_k \in [A_{km}, A_{kM}]$, $\Delta B_k \in [B_{km}, B_{kM}]$, $\Delta C_k \in [C_{km}, C_{kM}]$, $\Delta D_k \in [D_{km}, D_{kM}]$, then we have that both the systems

$$\begin{bmatrix} x(t+1) \\ \delta(t+1) \end{bmatrix} = \begin{bmatrix} A_m + BD_kC + BD_{km}C & BC_k + BC_{km} \\ B_kC + B_{km}C & A_k + A_{km} \end{bmatrix} \begin{bmatrix} x(t) \\ \delta(t) \end{bmatrix} + \begin{bmatrix} A_{\tau m} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(t-\tau) \\ \delta(t-\tau) \end{bmatrix}, \tag{4.14a}$$

$$\begin{bmatrix} x(t+1) \\ \delta(t+1) \end{bmatrix} = \begin{bmatrix} A_M + BD_kC + BD_{kM}C & BC_k + BC_{kM} \\ B_kC + B_{kM}C & A_k + A_{kM} \end{bmatrix} \begin{bmatrix} x(t) \\ \delta(t) \end{bmatrix} + \begin{bmatrix} A_{\tau M} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(t-\tau) \\ \delta(t-\tau) \end{bmatrix} \tag{4.14b}$$

are positive and asymptotically stable.

From $A_{\tau m} \succeq 0$, we obtain that system (4.14a) is positive if and only if

$$\begin{bmatrix} A_m + BD_kC + BD_{km}C & BC_k + BC_{km} \\ B_kC + B_{km}C & A_k + A_{km} \end{bmatrix} \succeq 0. \tag{4.15}$$

Thus (4.12a)–(4.12d) hold considering (4.13).

From the positivity and stability of system (4.14b) and using Theorem 3.2 again, we conclude that there exist matrices L_i, $i = 1, 2, 3, 4$ and diagonal matrices $P_j > 0$, $Q_j > 0$, $j = 1, 2$, satisfying (3.8) and (4.12e). The necessity is proved. □

Remark 4.6. We stress out that the conditions in above theorems do not impose the restriction on the governed system that the system matrix $A \succeq 0$. That is, the free system is not necessarily positive. Therefore, the governed system considered in this paper is called controlled positive system.

Remark 4.7. The matrix equality constraint in the above theorems can be solved via the cone complementarity linearization techniques [8].

5. Numerical Examples

Example 5.1. Consider the discrete-time delayed system (3.1) with

$$A = \begin{bmatrix} 0.2 & -0.1 \\ 0.4 & 0.6 \end{bmatrix}, \qquad A_\tau = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 \end{bmatrix}. \tag{5.1}$$

It is easy to see that A is not nonnegative, which implies that the unforced system (3.1) is not positive. By solving the conditions in Theorem 3.4, after 1 iteration, we obtain the full-order DOFS controller gain matrices

$$A_k = \begin{bmatrix} 0.2638 & 0.2638 \\ 0.2638 & 0.2638 \end{bmatrix}, \quad B_k = \begin{bmatrix} 0.1092 \\ 0.1092 \end{bmatrix},$$
$$C_k = \begin{bmatrix} -0.3517 & -0.3517 \\ 0.3763 & 0.3763 \end{bmatrix}, \quad D_k = \begin{bmatrix} -0.6241 \\ -2.8714 \end{bmatrix} \tag{5.2}$$

and the reduced-order DOFS controller gain matrices

$$A_k = 0.2460, \qquad B_k = 0.2079, \qquad C_k = \begin{bmatrix} -0.5911 \\ 0.6413 \end{bmatrix}, \qquad D_k = \begin{bmatrix} -0.6268 \\ -2.8685 \end{bmatrix}. \tag{5.3}$$

Example 5.2. Consider the uncertain discrete-time delayed system (3.1) with

$$A_m = \begin{bmatrix} 0.1 & -0.08 \\ 0.3 & 0.5 \end{bmatrix}, \qquad A_M = \begin{bmatrix} 0.2 & 0.33 \\ 0.4 & 0.6 \end{bmatrix}, \qquad A_{\tau m} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad A_{\tau M} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$B_m = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.15 \end{bmatrix}, \qquad B_M = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad C_m = \begin{bmatrix} 0 & 1 \end{bmatrix}, \qquad C_M = \begin{bmatrix} 0 & 1.2 \end{bmatrix},$$
$$A_{kM} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, \qquad B_{kM} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \qquad C_{kM} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, \qquad D_{kM} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}. \tag{5.4}$$

It is easy to see that A_M is nonnegative while A_m is not, which implies that the unforced system (3.1) is not always positive within the set of uncertain system matrices. And computation shows that the eigenvalues of $A_M + A_{\tau M}$ are 0.1853, 1.0147. From Lemma 2.4, we know that the unforced system (3.1) is not always asymptotically stable.

Solving the conditions in Theorem 4.3 gives the RRDOFS controller gain matrices

$$A_k = \begin{bmatrix} 0.1019 & 0.1019 \\ 0.1019 & 0.1019 \end{bmatrix}, \qquad B_k = \begin{bmatrix} 0.1206 \\ 0.1206 \end{bmatrix}, \qquad C_k = \begin{bmatrix} 0.1390 & 0.1390 \\ 0.1389 & 0.1389 \end{bmatrix}, \qquad D_k = \begin{bmatrix} 0.6945 \\ -1.9818 \end{bmatrix} \tag{5.5}$$

after 1 iteration.

6. Conclusions and Future Works

In this paper, we have studied the dynamic output feedback stabilization problem for delayed systems with/wihtout interval uncertainties. The controller/resilient controller which has additive controller gain variation belonging to an interval, is designed to guarantee that the resulting closed-loop systems are not only stable, but also positive. Necessary and sufficient/sufficient conditions for the existence of such controllers are established in terms of linear matrix inequalities together with a matrix equality constraint. And the controller gain matrices can be determined via the cone complementarity linearization techniques. The approach presented in this paper can also solve the corresponding problems for continuous-time delayed systems.

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