Research Article

Modelling and Analysis of a Pest-Control Pollution Model with Integrated Control Tactics

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A hybrid impulsive pest control model with stage structure for pest and Holling II functional response is proposed and investigated, in which the effects of impulsive pesticide input in the environment and in the organism are considered. Sufficient conditions for global attractiveness of the pest-extinction periodic solution and permanence of the system are obtained, which show that there exists a globally asymptotically stable pest-extinction periodic solution when the number of natural enemies released is more than some critical value, whereas the system can be permanent when the number of natural enemies released is less than another critical value. Furthermore, numerical simulations are carried out to illustrate our theoretical results and facilitate their interpretation.

1. Introduction

Since the beginning of recorded history, outbreaks of pests have plagued humanity, coming in direct competition with people for life-sustaining food. Reportedly, an estimated 67,000 different pest species attack agricultural crops, and about 35% of the yearly agricultural crop production is lost to pests worldwide [1, 2]. That problem is one of how to control or suppress damaging populations of pests over widespread areas. As we know, the most effective strategy for controlling pests may be to combine methods in an approach known as integrated pest management (IPM) that emphasizes preventing pest damage. In IPM, information about pests and available pest-control methods (including biological, cultural, and chemical) is used to manage pest damage by the most economical means and with the least possible hazard to people, property, and environment [3–5].

Biological control of pests in agriculture is a method of controlling pests (including insects, mites, weeds, and plant diseases) that relies on predation, parasitism, herbivory, or other natural mechanisms. It can be an important component of integrated pest management

(IPM) programs. It is defined as the reduction of pest populations by natural enemies and typically involves an active human role such as augmentation which involves the supplemental release of natural enemies. Biological control is not a "quick fix" for most pest problems. Natural enemies usually take longer to suppress a pest population than other forms of pest-control, and farmers often regard this as a disadvantage. Cultural controls are manipulations of the agroecosystem that make the cropping system less friendly to the establishment and proliferation of pest populations. Although they are designed to have positive effects on farm ecology and pest management, negative impacts may also result, due to variations in weather or changes in crop management [6]. Another important method for pest-control is chemical control. Chemical control is the approach of controlling pests through the spraying pesticide which is liable to reduce the pest populations considerably and which is indispensable when there are not enough natural enemies to decrease pest populations. In most cropping systems, insecticides are still the principal means of controlling pests once the economic threshold has been reached. They can be relatively cheap and are easy to apply, fast acting, and in most instances reliable in controling the pests [7]. Despite the advantages of conventional insecticides, the problems associated with their use have been well documented. These include the resurgence of pest populations after decimation of the natural enemies, development of insecticide-resistant populations, and negative impacts on nontarget organisms within and outside the crop system [8]. When considering these actions, in the process of effective control of pest, excessive use of a single control strategy is undesirable. Wherever possible, different pest-control techniques should work together rather than against each other. Even so, in many cases, the most effective release rate or spraying rate has not been identified as it will vary depending on crop type and target host density. Therefore, human beings have been forced to face the new challenge in the integrated pest management (IPM) program. One of the most important questions in IPM is how many natural enemies should be released and what fraction of the pest population should be killed to avoid economic damage and reduce the pesticide applications when the pest population reaches or exceeds the economic threshold level.

According to the idea of IPM, many mathematical models have been constructed and studied for understanding the range of possible ecological interactions between pest, natural enemy, and pesticides in the last decades. In order to consider the consequences of especially spraying pesticide and introducing additional predators into a natural pest-predator system, impulsive differential equations have been employed to describe such a system by many researchers [9–13], and the references cited therein. Impulsive differential equations are found in almost every domain of applied sciences [14, 15] and have been studied in many investigations [16–18]. They generally describe phenomena which are subject to steep or instantaneous changes. In IPM, impulsive reduction of the pest population is possible by trapping the pests and/or by poisoning them with chemicals. An impulsive increase of the natural enemy density can be achieved by releasing the natural enemy based on laboratory breeding into the field [5, 11]. Unfortunately, most of the pest-control models in the literature, which were modeled by impulsive differential equations, assume that at every impulsive spraying period, the pest population (including the natural enemies) may be killed immediately, and the instant killing rate of pesticide is a proportional constant. However, the actual situation is not always the same. Generally, pesticide appears in environment first, then it is absorbed by organism, and the individuals are affected, that is, the toxicity of pesticide does not act on the organism at once; in other words, it will last for some time before toxins are capable of decreasing the average growth rate of the species [19]. This fact urges us to consider the effect of pollution time delay on the extinction and permanence of population in

a polluted environment. With this in mind, it is necessary to introduce the pollution model to model the process of pest-control problems and study its dynamics, and this is different from the previous pest-control model which assumed that pests were reduced proportionally by spraying pesticides.

As we know, since Hallam and his coworkers proposed a toxicant-population model in the early 1980s [20–23], mathematical models of single or multiple populations with toxicant effect have been constructed and studied extensively [19, 24–29]. However, the majority of these studies have been focused on the effects of toxicant emitted into the environment from industrial and household resources on biological species, and only a few attempts have been made to combine pollution model to study pest-control problems with pesticide (toxin) input. Recently, by using pollution model and impulsive delay differential equation, Liu et al. [19] constructed, and investigated, a pest-control model with age structure for pest by introducing a constant periodic pesticide input and releasing natural enemies at different fixed moment. It is assumed in their model that each individual has the same dose response parameter to the organismal toxicant concentration regardless of the difference in many aspects between the immature and mature pest populations. However, in the natural world, there are many species whose individual members have a life story that takes them through two stages, immature and mature. Those species hatch from egg. Moreover, the immature and mature species express great differences in many aspects. One of the facts is that only the mature individuals are affected by the toxin (pesticide) and the immature individuals are not. For example, locust and salt-cedar leaf beetle, and so forth, are such species whose immature individuals (eggs) are protected by their eggshell and hardly injured by pesticides.

Based on all the above points, in this paper, we propose and investigate a pest-control model with a constant periodic pesticide input and natural enemies release at different fixed moment, in which the effects of impulsive pesticide input in the environment and in the organism are considered. Moreover, we assume that the pest individuals have two life stages: immature (egg) and mature with a constant maturation time delay, pesticide (toxin) has no effect on the immature individuals, and the capacity of the environment is so large that the change of toxin in the environment that comes from uptake and egestion by the organisms can be ignored. On the other hand, it is well known that functional response is a basic modeling unit in community ecology [30]. So, we further assume that natural enemy (predator) only feeds on mature pest (prey), and the functional response of natural enemy (predator) to mature pest (prey) species takes the Holling type II form. Meanwhile, because we may artificially pick on the appropriate releasing time when there is the lowest chance of adversely affecting natural enemies; thus, we further assume that pesticide input has little influence on the natural enemies, that is, the effect of pesticide input on natural enemies can be ignored. We are interested in a theoretical study about the effects of our control tactics on dynamical behavior of populations and attempt to obtain a theoretical threshold value which determines extinction of pest species and permanence of the system.

The organization of this paper is as follows. In Section 2, we set up our model and introduce some notations, definitions, and lemmas. In Section 3, sufficient conditions for extinction of the pest species and permanence of the system are given, respectively. The numerical simulations are carried out to study the effects of the impulsive varying parameters on the system as well as to illustrate our theoretical results in Section 4. Finally, a brief discussion is given to conclude this work.

2. Model and Preliminaries

According to the above analysis and assumption, we construct a pest-control pollution model with stage structure for the pest and Holling II functional response concerning integrated control tactics. The model takes the following form:

$$
\begin{aligned}
&\frac{dx_j(t)}{dt} = ax(t) - b_1 x_j(t) - ae^{-b_1\tau}x(t-\tau), \\
&\frac{dx(t)}{dt} = ae^{-b_1\tau}x(t-\tau) - fx^2(t) - \frac{\beta x(t)y(t)}{\eta + x(t)} - rc_o(t)x(t), \\
&\frac{dy(t)}{dt} = \lambda\frac{\beta x(t)y(t)}{\eta + x(t)} - b_2 y(t), \qquad\qquad\qquad t \neq (n+l-1)T, \quad t \neq nT, \\
&\frac{dc_o(t)}{dt} = kc_e(t) - gc_o(t) - mc_o(t), \\
&\frac{dc_e(t)}{dt} = -hc_e(t), \\
&\Delta x_j(t) = 0, \quad \Delta x(t) = 0, \quad \Delta y(t) = \mu_1, \quad \Delta c_o(t) = 0, \quad \Delta c_e(t) = 0, \quad t = (n+l-1)T, \\
&\Delta x_j(t) = 0, \quad \Delta x(t) = 0, \quad \Delta y(t) = 0, \quad \Delta c_o(t) = 0, \quad \Delta c_e(t) = \mu_2, \quad t = nT.
\end{aligned}
\tag{2.1}
$$

The initial conditions are

$$
\begin{aligned}
&\left(x_j(t), x(t), y(t), c_o(t), c_e(t)\right) = \left(\phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t), \phi_5(t)\right) \in C_5^+, \\
&C_5^+ = C\left([-\tau, 0], R_+^5\right), \quad t \in [-\tau, 0], \ \phi_i(0) > 0, \ i = 1, 2, 3, 4, 5,
\end{aligned}
\tag{2.2}
$$

where $x_j(t)$, $x(t)$, and $y(t)$ represent the density of the immature pest (egg), mature pest and natural enemy at time t, respectively, $c_e(t)$ represents the concentration of pesticide in the environment at time t; $c_0(t)$ represents the concentration of pesticide in the organism for the mature pest at time t, a is the growth rate of the immature pest; b_1 and b_2 show the death rate of the immature pest and natural enemy, respectively, τ represents a constant time to maturity, f represents the intraspecific competition coefficient of mature species, expression $\beta x(t)/(\eta + x(t))$ is Holling II functional response function, $\beta > 0, \eta > 0, \lambda$ represents the rate of conversion of consumed mature pest to natural enemy, r represents the decreasing rate of the intrinsic growth rate associated with the uptake of pesticide in the organism for the mature pest, $kc_e(t)$ represents the organism's net uptake pesticide from the environment, $gc_0(t)$ and $mc_0(t)$ represent the egestion and depuration rates of pesticide in the organism for the mature pest, respectively, $-hc_e(t)$ represents the loss of pesticide in the environment due to natural degradation, $\Delta x_j(t) = x_j(t^+) - x_j(t)$, $\Delta c_0(t) = c_0(t^+) - c_0(t)$, $\Delta c_e(t) = c_e(t^+) - c_e(t); 0 \leq l \leq 1, T$ is the period of impulsive effect, $n \in Z_+ = \{1, 2, \ldots\}$, μ_1 is the releasing amount of the natural enemy at time $t = (n+l-1)T$, and μ_2 is the amount of pesticide input at time $t = nT$.

Obviously, the first equation of system (2.1) can be written as

$$x_j(t) = \int_{t-\tau}^{t} ae^{-b_1(t-s)}x(s)ds, \tag{2.3}$$

$$x_j(0) = \int_{-\tau}^{0} ae^{b_1 s}x(s)ds \tag{2.4}$$

which means that the property of $x_j(t)$ can be investigated by $x(t)$. Moreover, the condition (2.4) presents the total surviving immature population from the observed birth on $-\tau \leq t \leq 0$. On the other hand, because the immature pest (egg) does little harm to the crops and it cannot breed, we just need to consider the control of the mature pest. Meanwhile, note that the variable $x_j(t)$ does not appear in the second, third, fourth, and fifth equations of system (2.1); hence, we only need to consider the subsystem of (2.1) as follows:

$$
\begin{aligned}
\frac{dx(t)}{dt} &= ae^{-b_1\tau}x(t-\tau) - fx^2(t) - \frac{\beta x(t)y(t)}{\eta + x(t)} - rc_o(t)x(t), \\
\frac{dy(t)}{dt} &= \lambda \frac{\beta x(t)y(t)}{\eta + x(t)} - b_2 y(t), \\
\frac{dc_o(t)}{dt} &= kc_e(t) - gc_o(t) - mc_o(t), \\
\frac{dc_e(t)}{dt} &= -hc_e(t),
\end{aligned}
\qquad t \neq (n+l-1)T, \ t \neq nT,
\tag{2.5}
$$

$$\Delta x(t) = 0, \quad \Delta y(t) = \mu_1, \quad \Delta c_o(t) = 0, \quad \Delta c_e(t) = 0, \quad t = (n+l-1)T,$$

$$\Delta x(t) = 0, \quad \Delta y(t) = 0, \quad \Delta c_o(t) = 0, \quad \Delta c_e(t) = \mu_2, \quad t = nT.$$

The initial conditions for system (2.5) are

$$
\begin{aligned}
&\left(x(t), y(t), c_o(t), c_e(t)\right) = \left(\phi_2(t), \phi_3(t), \phi_4(t), \phi_5(t)\right) \in C_4^+, \\
&C_4^+ = C\left([-\tau, 0], R_+^4\right), \quad t \in [-\tau, 0], \ \phi_i(0) > 0, \ i = 2, 3, 4, 5.
\end{aligned}
\tag{2.6}
$$

Furthermore, since $c_0(t)$ and $c_e(t)$ are the concentration of toxicant, to ensure $0 \leq c_0(t) \leq 1$ and $0 \leq c_e(t) \leq 1$, we assume that condition $g \leq k \leq g + m$, $\mu_2 \leq 1 - e^{-hT}$ holds in this paper. Meanwhile, considering the biological meaning, we assume that $k < h$.

In Sections 3 and 4, we mainly consider the global stability of pest-extinction solution and the uniform permanence of system (2.1); before introducing our main results, we give some preliminaries needed in next sections.

Let $R_+ = [0, \infty)$ and $R_+^5 = \{X \in R^5 : X > 0\}$. Denote $F = (f_1, f_2, f_3, f_4, f_5)$ as the map defined by the right hand of system (2.1). The solution of (2.1), denoted by $X(t) = (x_j(t), x(t), y(t), c_o(t), c_e(t)) : R_+ \rightarrow R_+^5$, is continuous on $((n-1)T, (n+l-1)T]$ and $((n+l-1)T, nT]$. $X((n+l-1)T^+) = \lim_{t \rightarrow (n+l-1)T^+} X(t)$ and $X(nT^+) = \lim_{t \rightarrow nT^+} X(t)$ exist. Obviously, the global existence and uniqueness of solutions of (2.1) is guaranteed by the smoothness properties of F (see [15]). Furthermore, the following lemma is easily obtained.

Lemma 2.1. *If $X(t)$ is a solution of system* (2.1) *with* $(\phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t), \phi_5(t)) > 0$ $(-\tau \leq t \leq 0)$, *then* $X(t) > 0$ *for all* $t \geq 0$.

Consider the following system

$$
\frac{dy(t)}{dt} = -b_2 y(t), \quad t \neq (n + l - 1)T,
$$
$$
\Delta y(t) = \mu_1, \quad t = (n + l - 1)T.
$$
(2.7)

Lemma 2.2 (see [19]). *System* (2.7) *has a unique positive periodic solution given by*

$$
y^*(t) = \frac{\mu_1 e^{-b_2 [t - (n+l-1)T]}}{1 - e^{-b_2 T}}, \quad \text{for } (n + l - 1)T < t \leq (n + l)T
$$
(2.8)

which is globally asymptotically stable.

Consider the following system

$$
\frac{dv(t)}{dt} = p - q v(t), \quad t \neq (n + l - 1)T,
$$
$$
\Delta v(t) = \mu, \quad t = (n + l - 1)T.
$$
(2.9)

Lemma 2.3 (see [19]). *System* (2.9) *has a unique positive periodic solution given by*

$$
v^*(t) = \frac{p}{q} + \frac{\mu e^{-q[t - (n+l-1)T]}}{1 - e^{-qT}}, \quad \text{for } (n + l - 1)T < t \leq (n + l)T
$$
(2.10)

which is globally asymptotically stable.

Now we consider some basic properties of the following subsystem of system (2.5)

$$
\frac{dc_o(t)}{dt} = k c_e(t) - g c_o(t) - m c_o(t),
$$
$$
\frac{dc_e(t)}{dt} = -h c_e(t), \qquad\qquad t \neq nT
$$
$$
\Delta c_o(t) = 0, \quad \Delta c_e(t) = \mu_2, \quad t = nT,
$$
$$
0 \leq c_o(0) \leq 1, \quad 0 \leq c_e(0) \leq 1.
$$
(2.11)

Lemma 2.4 (see [19]). *System* (2.11) *has a unique positive T-periodic solution given by*

$$c_0^*(t) = c_0^*(0)e^{-(g+m)(t-nT)} + \frac{kc_e^*(0)\left[e^{-(g+m)(t-nT)} - e^{-h(t-nT)}\right]}{h - g - m},$$

$$c_e^*(t) = c_e^*(0)e^{-h(t-nT)},$$

$$c_0^*(0) = \frac{kc_e^*(0)\left[e^{-(g+m)T} - e^{-hT}\right]}{(h - g - m)\left[1 - e^{-(g+m)T}\right]},$$

$$c_e^*(0) = \frac{\mu_2}{1 - e^{-hT}},$$

(2.12)

for $nT < t \leq (n+1)T$, *which is globally asymptotically stable.*

Lemma 2.5 (see [19]). *Considering the following equation*

$$\frac{dx(t)}{dt} = ax(t - \tau) - bx(t) - cx^2(t),$$

(2.13)

where a, b, c, *and* τ *are all positive constants,* $x(t) > 0$ *for* $-\tau \leq t \leq 0$, *one has*

(1) *if* $a < b$, *then* $\lim_{t \to +\infty} x(t) = 0$;

(2) *if* $a > b$, *then* $\lim_{t \to +\infty} x(t) = (a - b)/c$.

Definition 2.6. System (2.5) is said to be permanent if there are constants $M_1, M_2 > 0$ (independent of initial value) and a finite time T_0 such that for every positive solution $(x(t), y(t), c_o(t), c_e(t)) \in R_4^+$ with initial conditions, (2.6) satisfies $M_1 \leq x(t) \leq M_2$, $M_1 \leq y(t) \leq M_2$, $M_1 \leq c_o(t) \leq M_2$, $M_1 \leq c_e(t) \leq M_2$ for all $t \geq T_0$. Hence, T_0 may depend on the initial conditions (2.6).

3. Extinction and Permanence

Firstly, we show that all solutions of system (2.1) are uniformly ultimately bounded.

Theorem 3.1. *There exists a constant* $L > 0$ *such that* $x_j(t) \leq L/\lambda$, $x(t) \leq L/\lambda$, $y(t) \leq L$, $c_o(t) \leq L$, $c_e(t) \leq L$ *for each solution* $(x_j(t), x(t), y(t), c_o(t), c_e(t))$ *of system* (2.1) *with large enough t.*

Obviously, when the pest individuals are entirely absent from the model, that is, $x(t) = 0$ for $t \geq 0$, $y(t)$ satisfies the system (2.7). Accordingly, by Lemmas 2.2 and 2.4, we can get that system (2.1) exists for an immature and mature pest-extinction periodic solution $(0, 0, y^*(t), c_o^*(t), c_e^*(t))$, whose global attractiveness is equivalent to global attractiveness of the mature pest-extinction periodic solution $(0, y^*(t), c_o^*(t), c_e^*(t))$ of system (2.5). In the following, we give the sufficient conditions for global attractiveness of solution $(0, y^*(t), c_o^*(t), c_e^*(t))$.

If we denote $A = e^{-b_2 T}/(1 - e^{-b_2 T})$, $B = k[e^{-(g+m)T} - e^{-hT}]e^{-(g+m)T}/(h - g - m)(1 - e^{-hT})[1 - e^{-(g+m)T}]$, $b = \min\{b_1, b_2, h - k, g + m\}$, $\sigma_1 = e^{blT}/(e^{bT} - 1)$, $\sigma_2 = e^{bT}/(e^{bT} - 1)$, $M_0 = \lambda(a + b)^2/4f$, then we have the following.

Theorem 3.2. *The mature pest-extinction periodic solution* $(0, y^*(t), c_o^*(t), c_e^*(t))$ *of system* (2.5) *is globally attractive provided that*

$$\mu_1 > \frac{ae^{-b_1\tau}(b\lambda\eta + M_0) - (b\lambda\eta rB - \sigma_2 bae^{-b_1\tau} + M_0 rB)\mu_2 - \sigma_2 rbB\mu_2^2}{b(\beta\lambda A - \sigma_1 ae^{-b_1\tau} + \sigma_1 rB\mu_2)} \tag{3.1}$$

holds true.

Remark 3.3. The pest-extinction periodic solution $(0, 0, y^*(t), c_o^*(t), c_e^*(t))$ of system (2.1) is also globally attractive if the condition (3.1) holds.

Now, we give the sufficient conditions for permanence of system (2.5). If we denote $\sigma = 1/(1 - e^{-b_2 T})$, $\delta = k[e^{-(g+m)T} - e^{-hT}]/(h - g - m)(1 - e^{-hT})[1 - e^{-(g+m)T}] + k/|h - g - m|(1 - e^{-hT})$, and then we have the following.

Theorem 3.4. *System* (2.5) *is permanent provided that*

$$\mu_1 < \frac{\eta(ae^{-b_1\tau} - r\delta\mu_2)}{\beta\sigma} \tag{3.2}$$

holds true.

Remark 3.5. System (2.1) is also permanent if the condition (3.2) holds.

For convenience, the proofs of Theorems 3.1, 3.2, and 3.4 are given in Appendices A–C, respectively.

The above results show that many factors including maturation time delay, functional response of the predator, the organism's net uptake pesticide from the environment, the egestion and depuration rates of pesticide in an organism, the loss of pesticide in the environment due to natural degradation, the natural enemy releasing amount, the pesticide spraying amount, and the releasing and spraying period, can induce variation in the characteristics of populations. Meanwhile, the results imply that the modelling methods described can help in the design of appropriate control strategies and assist management decision-making. In fact, the conditions (3.1) and (3.2) imply that there exist two theoretical criteria values in system (2.1), which can be, respectively, denoted as follows:

$$\mu_1^* = \frac{ae^{-b_1\tau}(b\lambda\eta + M_0) - (b\lambda\eta rB - \sigma_2 bae^{-b_1\tau} + M_0 rB)\mu_2 - \sigma_2 rbB\mu_2^2}{b(\beta\lambda A - \sigma_1 ae^{-b_1\tau} + \sigma_1 rB\mu_2)},$$

$$\mu_1^{**} = \frac{\eta(ae^{-b_1\tau} - r\delta\mu_2)}{\beta\sigma}. \tag{3.3}$$

Moreover, if $\mu_1 > \mu_1^*$, the pest-extinction periodic solution is globally asymptotically stable; if $\mu_1 < \mu_1^{**}$, the insect pests and the natural enemies can coexist, that is, system (2.1) that

we consider permanent. It is well known that, in a definitive ecological environment, the appropriate artificial release of natural enemies and spraying of pesticides play an important role in the success of pest-control. Due to the antagonism between chemical and biological methods, we should reduce the pesticide application to avoid antagonism and especially negative impacts on nontarget organisms. Theorems 3.2 and 3.4 indicate that we can choose the appropriate impulsive parameters to reduce pests to tolerable levels with little economical cost and minimal effect on the environment. Therefore, our impulsive strategy is more effective than the classical one if the chemical control is adopted rationally. To confirm our mathematical findings and facilitate their interpretation, we proceed to investigate further by using numerical simulations in the following section.

4. Numerical Simulations

In this section, numerical simulations are carried out to investigate effects of impulsive varying parameters on dynamical behaviors of system (2.1) as well as to illustrate our theoretical results. Owing to the lack of biologically realistic parametric values, the solution of the system with initial conditions in the first octant is obtained numerically for biologically feasible ranges of parametric values dominated by Theorems 3.2 and 3.4. For convenience, we assume that some parametric values of system (2.1) are kept as

$$a = 0.8, \quad b_1 = 0.9, \quad \tau = 0.7, \quad f = 0.2, \quad \beta = 0.9, \quad \eta = 1, \quad r = 0.7, \quad b_2 = 0.3,$$
$$\lambda = 0.9, \quad k = 1, \quad g = 0.5, \quad m = 0.7, \quad h = 2, \quad l = 0.1, \quad T = 2, \quad \mu_2 = 0.1.$$

(4.1)

Firstly, we give numerical results of the system, in which there are no impulsive perturbations (including natural enemy releasing and pesticide spraying), in other words, that is the unforced system of (2.1). The model takes the following form:

$$\frac{dx_j(t)}{dt} = ax(t) - b_1 x_j(t) - ae^{-b_1\tau}x(t - \tau),$$

$$\frac{dx(t)}{dt} = ae^{-b_1\tau}x(t - \tau) - fx^2(t) - \frac{\beta x(t)y(t)}{\eta + x(t)},$$

$$\frac{dy(t)}{dt} = \lambda\frac{\beta x(t)y(t)}{\eta + x(t)} - b_2 y(t),$$

(4.2)

where the value of parameters for model (4.2) can be seen in (4.1). We can easily plot the time series of every population and phase portrait of the system and find that the solution of (4.2) with initial values $x_j(0) = 0.2$, $x(0) = 0.1$, and $y(0) = 0.1$ would tend to a positive equilibrium solution (see Figures 1(a) and 1(b) in details). From the following discussion, we can observe that the solution of the unforced system would become unstable via impulsive perturbation. Further, it indicates that the system is impulsively controllable.

From theoretical criteria values formula (3.3) and the above parameter hypothesis (4.1), by a straightforward calculation, we can obtain that two theoretical criteria values of system (2.1) are $\mu_1^* = 5.5247$ and $\mu_1^{**} = 0.1654$, respectively.

Let $\mu_1 > \mu_1^* = 5.5247$, that is, the condition (3.1) holds true; we know that the pest-extinction periodic solution is globally asymptotically stable from Theorem 3.2. that is, if we
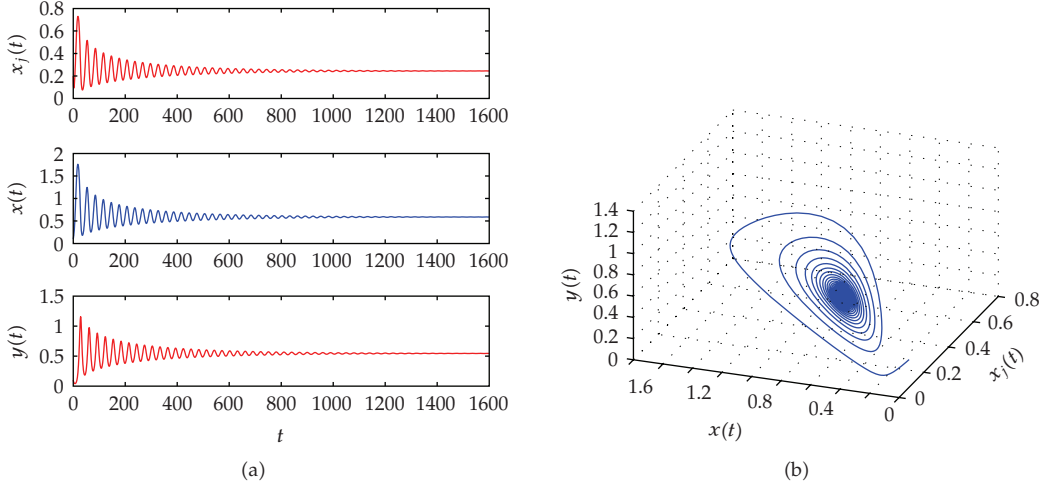
Figure 1: Dynamical behavior of system (4.2) with $a = 0.8$, $b_1 = 0.9$, $\tau = 0.7$, $f = 0.2$, $\beta = 0.9$, $\eta = 1$, $b_2 = 0.3$, $\lambda = 0.9$, initial values $x_j(0) = 0.2$, $x(0) = 0.1$, $y(0) = 0.1$. (a) Time series of system (4.2). (b) Phase portrait of system (4.2).

let $\mu_1 = 5.6 > \mu_1^*$, a typical pest-eradication periodic solution of system (2.1) with initial values $x_j(0) = 0.2$, $x(0) = 0.1$, $y(0) = 0.1$, $c_o(t) = 0.01$, and $c_e(t) = 0.01$ is shown in **Figure 2(a)**, where we observe how the predator (natural enemy) $y(t)$ and the concentration and $c_e(t)$ of pesticide in the environment and the concentration $c_0(t)$ of pesticide in the organism periodically oscillate; in contrast, both the immature pest $x_j(t)$ and mature pest $x(t)$ rapidly decrease to zero. If we continue to increase μ_1 and let $\mu_1 = 6.5 > \mu_1^*$ and $\mu_1 = 7.5 > \mu_1^*$, from Figures 2(b) and 2(c), the same phenomenon as above can be observed, respectively. This illustrates that the pest-extinction periodic solution of system (2.1) is globally asymptotically stable.

Let $\mu_1 < \mu_1^{**} = 0.1654$, that is, the condition (3.2) holds true; we know that system (2.1) that we consider is permanent from **Theorem 3.4**. That is, if we let $\mu_1 = 0.16 < \mu_1^{**}$, a positive periodic solution of system (2.1) with initial values $x_j(0) = 0.2$, $x(0) = 0.1$, $y(0) = 0.1$, $c_o(t) = 0.0$, $c_e(t) = 0.01$ is shown in **Figure 3**, where we observe that each population of system (2.1) can coexist on a stable limit cycle. If we continue to decrease μ_1, and let $\mu_1 = 0.12 < \mu_1^{**}$ and $\mu_1 = 0.08 < \mu_1^{**}$, from Figures 4 and 5, the same phenomenon as above can be observed, respectively. This illustrates that system (2.1) is permanent.

We must emphasize here that condition (3.1) and condition (3.2) are the only sufficient conditions which, respectively, assure global attractiveness of the pest-extinction periodic solution of system (2.1) and permanence of the populations. Accordingly, $\mu_1^* = 5.5247$ and $\mu_1^{**} = 0.1654$ are only two theoretical criteria values, not the threshold. Concerning the mathematical formula of theoretical threshold, we leave this for future work. We only give here an approximate threshold which can be obtained by numerical simulations. Indeed, by plotting the bifurcation diagram, we may observe that the theoretical threshold of parameter μ_1 is approximately equal to 0.29 (see **Figure 6** in details). That is to say, when $\mu_1 > 0.29$, the pest-extinction periodic solution of system (2.1) is globally asymptotically stable; reversely, when $\mu_1 < 0.29$, system (2.1) that we consider is permanent.
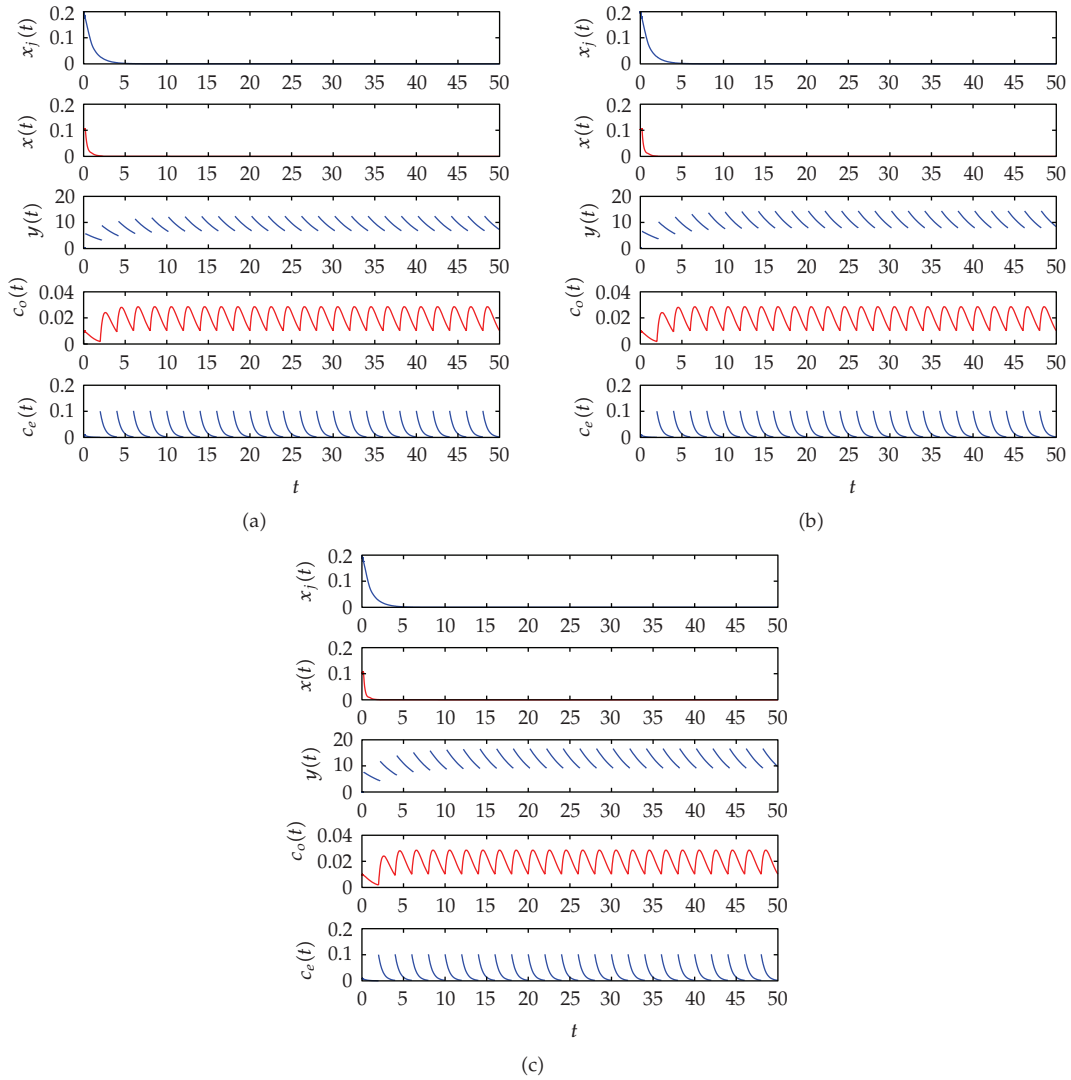
(a)

(b)

(c)

Figure 2: Dynamical behavior (extinction) of system (2.1) with $a = 0.8$, $b_1 = 0.9$, $\tau = 0.7$, $f = 0.2$, $\beta = 0.9$, $\eta = 1$, $r = 0.7$, $b_2 = 0.3$, $\lambda = 0.9$, $k = 1$, $g = 0.5$, $m = 0.7$, $h = 2$, $l = 0.1$, $T = 2$, $\mu_2 = 0.1$, initial values $x_j(0) = 0.2$, $x(0) = 0.1$, $y(0) = 0.1$, $c_o(t) = 0.01$, $c_e(t) = 0.01$ (*corresponding theoretical criteria value: $\mu_1^* = 5.5247$. (a) Time-series of system (2.1) with $\mu_1 = 5.6$. (b) Time-series of system (2.1) with $\mu_1 = 6.5$. (c) Time-series of system (2.1) with $\mu_1 = 7.5$.

According to the bifurcation theory, the properties of a dynamic system depend on certain parameter, and dynamic system with different parameters may have different dynamic behaviors. The above numerical results that we have investigated depend on parameter μ_1, that is, μ_1 is control parameter. In fact, from condition (3.1) and condition (3.2), the control parameter may also choose the other parameter as T, μ_2, l, or τ, and then the same argument as above can be continued. We only give here two numerical examples. Figure 7(a) is plotted by changing the parameter $\tau = 0.7$ of Figure 3 to $\tau = 1.5$. Figure 7(b) is plotted by changing the parameter $T = 2$ of Figure 3 to $T = 1$. As against Figure 3, Figure 7
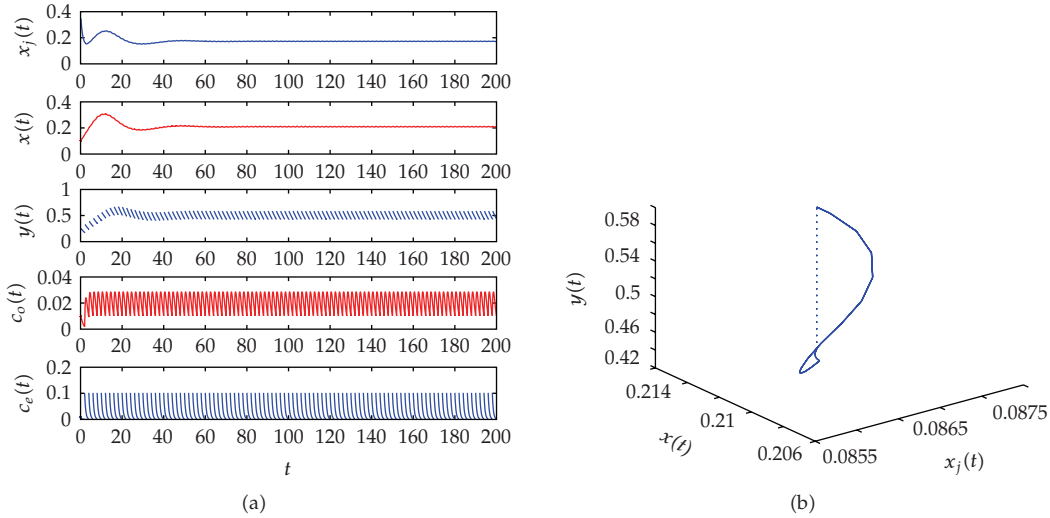
(a)   (b)

Figure 3: Dynamical behavior (permanence) of system (2.1) with $a = 0.8$, $b_1 = 0.9$, $\tau = 0.7$, $f = 0.2$, $\beta = 0.9$, $\eta = 1$, $r = 0.7$, $b_2 = 0.3$, $\lambda = 0.9$, $k = 1$, $g = 0.5$, $m = 0.7$, $h = 2$, $l = 0.1$, and $T = 2$, and $\mu_2 = 0.1$; initial value $x_j(0) = 0.2$, $x(0) = 0.1$, $y(0) = 0.1$, $c_o(t) = 0.01$, and $c_e(t) = 0.01$ (*corresponding theoretical criteria value: $\mu_1^{**} = 0.1654$). (a) Time-series of system (2.1) with $\mu_1 = 0.16$. (b) Solution $x_j(t)$, $x(t)$, $y(t)$ with $\mu_1 = 0.16$ will finally tend to a T-periodic solution.
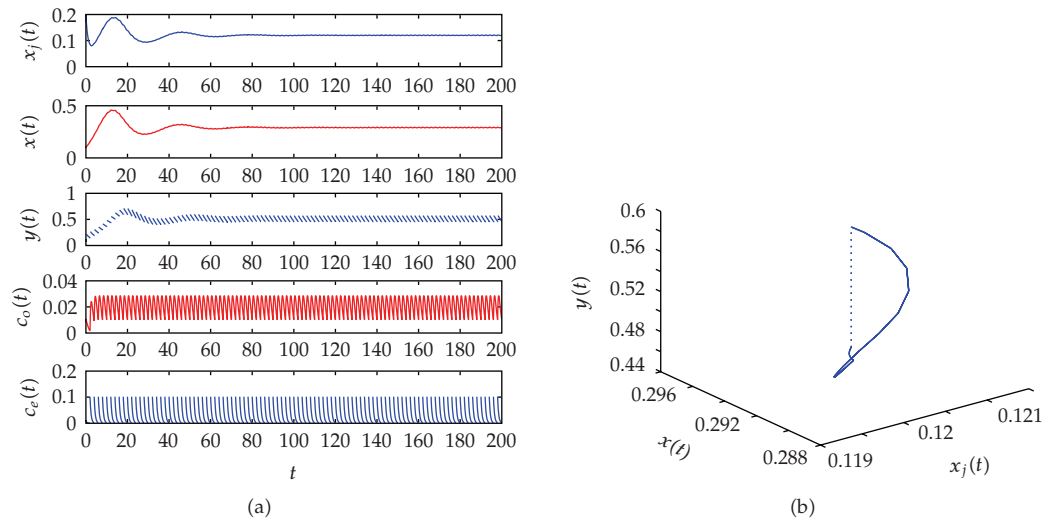

(a)   (b)

Figure 4: Dynamical behavior (permanence) of system (2.1) with $a = 0.8$, $b_1 = 0.9$, $\tau = 0.7$, $f = 0.2$, $\beta = 0.9$, $\eta = 1$, $r = 0.7$, $b_2 = 0.3$, $\lambda = 0.9$, $k = 1$, $g = 0.5$, $m = 0.7$, $h = 2$, $l = 0.1$, $T = 2$, and $\mu_2 = 0.1$, initial values $x_j(0) = 0.2$, $x(0) = 0.1$, $y(0) = 0.1$, $c_o(t) = 0.01$, and $c_e(t) = 0.01$ (*corresponding theoretical criteria value: $\mu_1^{**} = 0.1654$. (a) Time-series of system (2.1) with $\mu_1 = 0.12$. (b) Solution $x_j(t)$, $x(t)$, $y(t)$ with $\mu_1 = 0.12$ will finally tend to a T-periodic solution.

implies that long maturation time delay and short impulsive period may induce variation in the characteristics of populations and cause pests eradication.
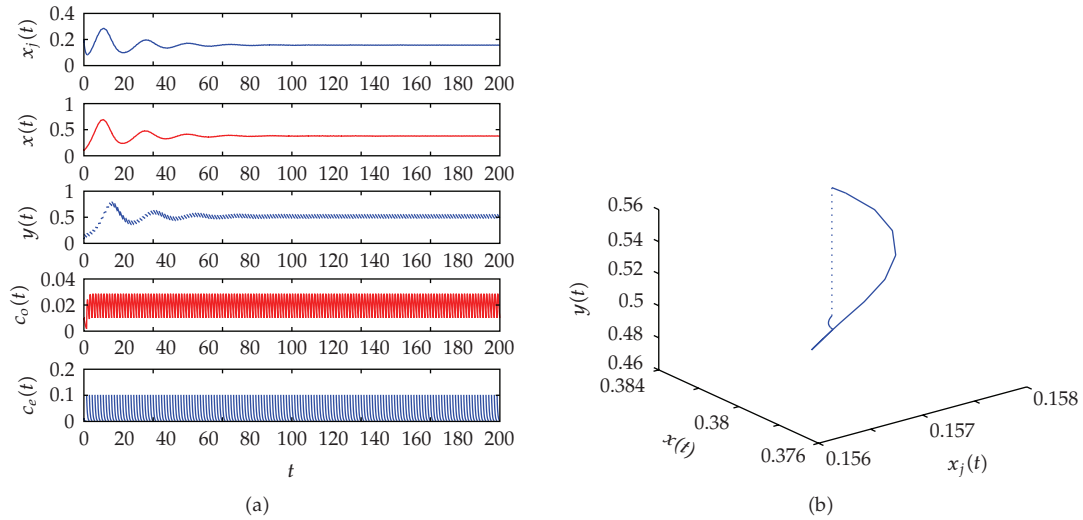
(a)

(b)

Figure 5: Dynamical behavior (permanence) of system (2.1) with $a = 0.8$, $b_1 = 0.9$, $\tau = 0.7$, $f = 0.2$, $\beta = 0.9$, $\eta = 1$, $r = 0.7$, $b_2 = 0.3$, $\lambda = 0.9$, $k = 1$, $g = 0.5$, $m = 0.7$, $h = 2$, $l = 0.1$, $T = 2$, $\mu_2 = 0.1$, initial values $x_j(0) = 0.2$, $x(0) = 0.1$, $y(0) = 0.1$, $c_o(t) = 0.01$, $c_e(t) = 0.01$ (*corresponding theoretical criteria value: $\mu_1^{**} = 0.1654$. (a) Time series of system (2.1) with $\mu_1 = 0.08$. (b) Solution $x_j(t)$, $x(t)$, $y(t)$ with $\mu_1 = 0.08$ will finally tend to a T-periodic solution.
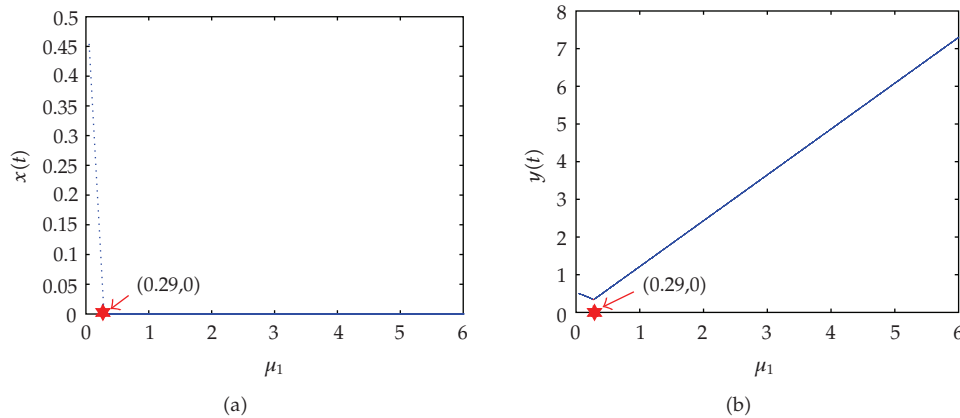

(a)

(b)

Figure 6: Bifurcation diagrams of system (2.1), showing the effect of μ_1 with $a = 0.8$, $b_1 = 0.9$, $\tau = 0.7$, $f = 0.2$, $\beta = 0.9$, $\eta = 1$, $r = 0.7$, $b_2 = 0.3$, $\lambda = 0.9$, $k = 1$, $g = 0.5$, $m = 0.7$, $h = 2$, $l = 0.1$, $T = 2$, and $\mu_2 = 0.1$, initial values $x_j(0) = 0.2$, $x(0) = 0.1$, $y(0) = 0.1$, $c_o(t) = 0.01$, and $c_e(t) = 0.01$. (a) $x(t)$ is plotted for μ_1 over $[0.05, 6]$. (b) $y(t)$ is plotted for μ_1 over $[0.05, 6]$.

5. Conclusion

In this paper, in order to investigate the consequences of periodically spraying pesticides and releasing natural enemies at different fixed moment in pest-natural enemy system, a hybrid impulsive pest-control model with stage structure for pest and Holling II
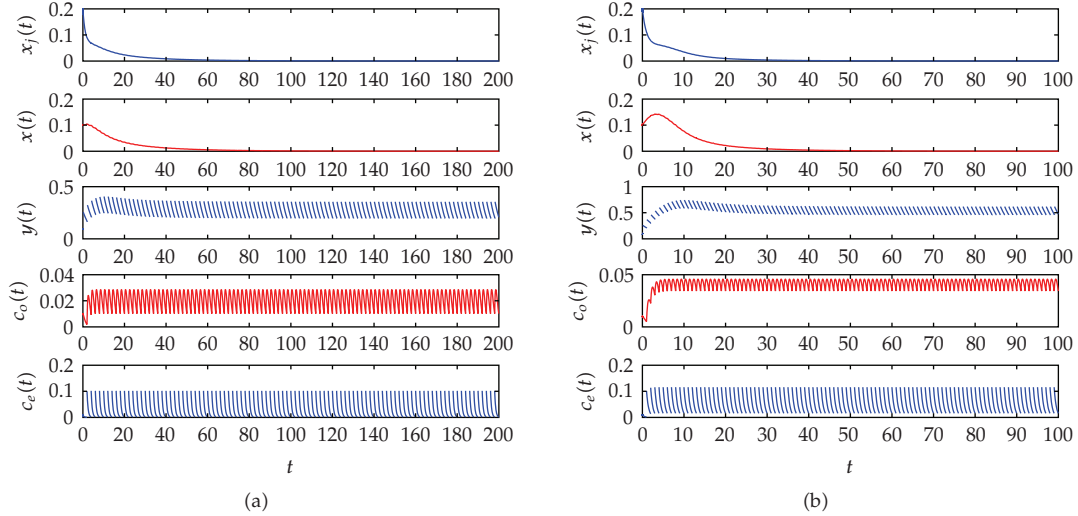
Figure 7: Dynamical behavior (extinction) of system (2.1) with $a = 0.8$, $b_1 = 0.9$, $f = 0.2$, $\beta = 0.9$, $\eta = 1$, $r = 0.7$, $b_2 = 0.3$, $\lambda = 0.9$, $k = 1$, $g = 0.5$, $m = 0.7$, $h = 2$, $l = 0.1$, and $\mu_2 = 0.1$, initial values $x_j(0) = 0.2$, $x(0) = 0.1$, $y(0) = 0.1$, $c_o(t) = 0.01$, and $c_e(t) = 0.01$. (a) Time series of system (2.1) with $\tau = 1.5, T = 2$, $\mu_1 = 0.16$. (b) Time series of system (2.1) with $\tau = 0.7$, $T = 1$, and $\mu_1 = 0.16$.

functional response is proposed, in which the effects of impulsive pesticide input in the environment and in the organism are considered. Sufficient conditions for global attractiveness of the pest-extinction periodic solution and permanence of the system have been obtained, which shows that there exists a globally asymptotically stable pest-eradication periodic solution when the number μ_1 of natural enemies released is more than some critical value μ_1^* (see Figure 2), whereas the system can be permanent when the number μ_1 of natural enemies released is less than another critical value μ_1^{**} (see Figures 3, 4, and 5). Meanwhile, numerical simulation results for biologically feasible ranges of parametric values can confirm our mathematical findings and facilitate their interpretation. We also note that the conditions for the extinction or permanence in system (2.1) are quite different from the corresponding system (4.2) without impulse. For example, the system (4.2) has a positive equilibrium which is orbitally asymptotically stable (see Figure 1); however, this properties are changed via additional impulsive perturbation (see Figures 2–7). Furthermore, by plotting the bifurcation diagram (see Figure 6), we obtained the theoretical threshold of control parameter μ_1, which is crucial for extinction or permanence of the population if the other parameters of system (2.1) are fixed. Finally, the numerical results, which show that long maturation time delay and short impulsive period may cause pests eradicat, have been given (see Figure 7). Obviously, these results indicate that the models proposed in this paper can help us understand pest-natural enemy interactions, to design appropriate control strategies and to make management decisions in insect pest-control. We would like to mention here that an interesting but challenging problem associated with the studies of system (2.1) should be how to optimize the number of periodically releasing natural enemy and the dosage of spraying pesticides to reduce pests to tolerable levels with little economical cost and minimal effect on the environment. We leave this for future work.

Appendices

A. Proof of Theorem 3.1

Define $V(t) = \lambda x_j(t) + \lambda x(t) + y(t) + c_o(t) + c_e(t)$, $b = \min\{b_1, b_2, h-k, g+m\}$. When $t \neq (n+l-1)T$ and $t \neq nT$, we have

$$\frac{dV(t)}{dt} + bV(t) \leq \lambda(a+b)x(t) - f\lambda x^2(t) \leq M_0, \tag{A.1}$$

where $M_0 = \lambda(a+b)^2/(4f)$. In addition, $V((n+l-1)T^+) = V((n+l-1)T) + \mu_1$, $V(nT^+) = V(nT) + \mu_2$.

By a straightforward calculation, when $0 < (n+l-1)T < nT < t < (n+1+l-1)T$, we have

$$V(t) \leq V(0)e^{-bt} + \int_0^t M_0 e^{-b(t-s)}ds + \sum_{i=1}^n \left(\mu_1 e^{-b[t-(i+l-1)T]} + \mu_2 e^{-b(t-iT)} \right), \tag{A.2}$$

and when $0 < nT < (n+1+l-1)T < t < (n+1)T$, we have

$$V(t) \leq V(0)e^{-bt} + \int_0^t M_0 e^{-b(t-s)}ds + \sum_{i=1}^{n+1} \left(\mu_1 e^{-b[t-(i+l-1)T]} \right) + \sum_{i=1}^n \left(\mu_2 e^{-b(t-iT)} \right). \tag{A.3}$$

Accordingly, we have

$$V(t) \leq \frac{M_0}{b} + \frac{\mu_1 e^{blT} + \mu_2 e^{bT}}{e^{bT} - 1} \stackrel{\triangle}{=} L \quad \text{as } t \to \infty. \tag{A.4}$$

So $V(t)$ is uniformly ultimately bounded. By the definition of $V(t)$, we have $x_j(t) \leq L/\lambda$, $x(t) \leq L/\lambda$, $y(t) \leq L$, $c_o(t) \leq L$, $c_e(t) \leq L$ for large enough t. The proof is completed.

B. Proof of Theorem 3.2

Suppose that $(x(t), y(t), c_o(t), c_e(t))$ is any solution of system (2.5) with initial conditions (2.6). From system (2.5), we have

$$\frac{dy(t)}{dt} \geq -b_2 y(t), \quad t \neq (n+l-1)T,$$

$$\Delta y(t) = \mu_1, \quad t = (n+l-1)T. \tag{B.1}$$

By Lemma (2.2), we know that $u^*(t) = \mu_1 e^{-b_2[t-(n+l-1)T]}/(1-e^{-b_2T})$ is the unique positive periodic solution of impulsive differential equation as follows:

$$\frac{du(t)}{dt} = -b_2 u(t), \quad t \neq (n+l-1)T,$$

$$\Delta u(t) = \mu_1, \quad t = (n+l-1)T.$$

(B.2)

By comparison theorem of impulsive equation [15], for any small enough $\varepsilon_1 > 0$, there exists an integer N_1 such that

$$y(t) > u^*(t) - \varepsilon_1, \quad \text{for } (N_1+l-1)T < t \leq (N_1+l)T.$$

(B.3)

Accordingly, we obtain

$$y(t) > \mu_1 A - \varepsilon_1, \quad \text{for } (N_1+l-1)T < t \leq (N_1+l)T.$$

(B.4)

On the other hand, from **Lemma 2.4**, we can easily obtain that for any small enough $\varepsilon_2 > 0$, there exists an integer N_2 such that

$$c_o(t) > \mu_2 B - \varepsilon_2, \quad \text{for } N_2 T < t \leq (N_2+1)T.$$

(B.5)

Leting $\tilde{T} = \max\{(N_1+l-1)T, N_2T\}$, from the first equation of system (2.5), (B.4), (B.5), and Theorem 3.1, we have

$$\frac{dx(t)}{dt} < ae^{-b_1\tau}x(t-\tau) - \left[\frac{\beta(\mu_1 A - \varepsilon_1)}{\eta + L/\lambda} + r(\mu_2 B - \varepsilon_2)\right]x(t) - fx^2(t), \quad \text{for } t > \tilde{T} + \tau,$$

(B.6)

where $L = M_0/b + (\mu_1 e^{blT} + \mu_2 e^{bT})/(e^{bT}-1) = M_0/b + \mu_1\sigma_1 + \mu_2\sigma_2$ is obtained from Proof of Theorem 3.1. In the following we consider the comparison equation

$$\frac{dz(t)}{dt} = ae^{-b_1\tau}z(t-\tau) - \left[\frac{\beta(\mu_1 A - \varepsilon_1)}{\eta + L/\lambda} + r(\mu_2 B - \varepsilon_2)\right]z(t) - fz^2(t).$$

(B.7)

Because the condition (3.1) is equivalent to the condition $ae^{-b_1\tau} < \beta A\mu_1/(\eta + L/\lambda) + rB\mu_2$, therefore, we can choose $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ small enough such that

$$ae^{-b_1\tau} < \frac{\beta(\mu_1 A - \varepsilon_1)}{\eta + L/\lambda} + r(\mu_2 B - \varepsilon_2).$$

(B.8)

For any solution $z(t)$ of (B.7), by **Lemma 2.5** and (B.8), we can get $\lim_{t\to+\infty}z(t) = 0$. Thus by the comparison theorem in delay differential equation and **Lemma 2.1**, we obtain that $\lim_{t\to+\infty}x(t) \leq \lim_{t\to+\infty}z(t) = 0$, and $\lim_{t\to+\infty}x(t) = 0$.

Further, for any small enough $\varepsilon_3 > 0$ and large enough t, we have $0 < x(t) < \varepsilon_3$. Without loss of generality, we may assume $0 < x(t) < \varepsilon_3$ for $t \geq 0$. And then from system (2.5), we obtain

$$\frac{dy(t)}{dt} \leq \frac{\lambda\beta L\varepsilon_3}{\eta + \varepsilon_3} - b_2 y(t), \quad t \neq (n+l-1)T,$$

$$\Delta y(t) = \mu_1, \quad t = (n+l-1)T. \tag{B.9}$$

By Lemma 2.3 and the comparison theorem in impulsive differential equation [15], for any $\varepsilon_4 > 0$ is small enough, when large enough t, we have

$$y(t) < Y^*(t) + \varepsilon_4, \tag{B.10}$$

where $Y^*(t) = \lambda\beta L\varepsilon_3/b_2(\eta+\varepsilon_3) + \mu_1 e^{-b_2[t-(n+l-1)T]}/(1-e^{-b_2 T})$ for $(n+l-1)T < t \leq (n+l)T$ is the unique positive periodic solution of impulsive differential equation as follows

$$\frac{dY(t)}{dt} = \frac{\lambda\beta L\varepsilon_3}{\eta + \varepsilon_3} - b_2 Y(t), \quad t \neq (n+l-1)T,$$

$$\Delta Y(t) = \mu_1, \quad t = (n+l-1)T. \tag{B.11}$$

Combining (B.3) with (B.10), when t is large enough, we obtain

$$u^*(t) - \varepsilon_1 < y(t) < Y^*(t) + \varepsilon_4 \tag{B.12}$$

which implies $\lim_{t \to +\infty} y(t) = y^*(t)$ since $\varepsilon_1, \varepsilon_3, \varepsilon_4$ are all sufficiently small positive constants. Moreover, by Lemma 2.4, when $t \to +\infty$, we have $c_o(t) \to c_o^*(t)$, $c_e(t) \to c_e^*(t)$. Thus the proof is completed.

C. Proof of Theorem 3.4

Suppose that $(x(t), y(t), c_o(t), c_e(t))$ is any solution of system (2.5) with initial conditions (2.6). By Theorem 3.1, we have proved that there exists a constant $L = M_0/b + (\mu_1 e^{blT} + \mu_2 e^{bT})/(e^{bT} - 1) > 0$ such that $x(t) \leq L/\lambda$, $y(t) \leq L$, $c_o(t) \leq L$, $c_e(t) \leq L$ for large enough t. From Proof of Theorem 3.2, we know that $y(t) > \mu_1 A - \varepsilon_1$, $c_o(t) > \mu_2 B - \varepsilon_2$ for large enough t (see (B.4), and (B.5)). By Lemma 2.4, we easily obtain that $c_e(t) > \mu_2 e^{-hT}/(1 - e^{-hT}) - \varepsilon_2$ for large enough t. Thus, from Definition 2.6, we only need to find a constant $M_1 > 0$ such that $x(t) \geq M_1$ for t large enough. We will do it in the following two steps.

(1) we prove that there exists a constant $m_1 > 0$ such that $x(t) < m_1$ cannot hold for all $t \geq t_0$. Otherwise, there is a constant $t_0 > 0$ such that $x(t) < m_1$ for all $t \geq t_0$. Thus, from system (2.5), when $t \geq t_0$, we have

$$\frac{dy(t)}{dt} < \frac{\lambda\beta m_1 L}{\eta + m_1} - b_2 y(t), \quad t \neq (n+l-1)T,$$

$$\Delta y(t) = \mu_1, \quad t = (n+l-1)T. \tag{C.1}$$

By Lemma (2.4) and comparison theorem of impulsive equation [15], for any $\varepsilon > 0$ small enough, there exists a $T_1 \geq t_0$ such that

$$y(t) < \frac{\lambda\beta m_1 L}{b_2(\eta + m_1)} + \frac{\mu_1}{1 - e^{-b_2 T}} + \varepsilon = \frac{\lambda\beta m_1 L}{b_2(\eta + m_1)} + \mu_1 \sigma + \varepsilon. \tag{C.2}$$

For the above ε, by Lemma 2.4, there exists a $T_2 \geq t_0$ such that

$$c_o(t) < \mu_2 \delta + \varepsilon, \quad \text{for } t > T_2. \tag{C.3}$$

Because the first equation of (2.5) can be rewritten as

$$\frac{dx(t)}{dt} = \left[ae^{-b_1 \tau} - fx(t) - \frac{\beta y(t)}{\eta + x(t)} - rc_o(t) \right] x(t) - ae^{-b_1 \tau} \frac{d}{dt} \int_{t-\tau}^{t} x(s) ds. \tag{C.4}$$

Now, we define

$$V(t) = x(t) + ae^{-b_1 \tau} \int_{t-\tau}^{t} x(s) ds. \tag{C.5}$$

By calculating the derivative of $V(t)$ along system (2.5), we have

$$\frac{dV(t)}{dt} = \left[ae^{-b_1 \tau} - fx(t) - \frac{\beta y(t)}{\eta + x(t)} - rc_o(t) \right] x(t). \tag{C.6}$$

Let $\widehat{T} = \max\{T_1, T_2\}$, then for $t > \widehat{T}$, combining (C.2), (C.3), and (C.6), we have

$$\frac{dV(t)}{dt} > \left[ae^{-b_1 \tau} - fm_1 - \frac{\beta}{\eta} \left(\frac{\lambda\beta m_1 L}{b_2(\eta + m_1)} + \mu_1 \sigma + \varepsilon \right) - r(\mu_2 \delta + \varepsilon) \right] x(t). \tag{C.7}$$

Since condition (3.2) holds, we can choose m_1 and ε to be small enough such that

$$ae^{-b_1 \tau} > fm_1 + \frac{\beta}{\eta} \left(\frac{\lambda\beta m_1 L}{b_2(\eta + m_1)} + \mu_1 \sigma + \varepsilon \right) + r(\mu_2 \delta + \varepsilon). \tag{C.8}$$

Leting $m_2 = \min_{t \in [\widehat{T}, \widehat{T} + \tau]} x(t)$, we show that $x(t) \geq m_2$ for $t > \widehat{T}$. Otherwise, there is a nonnegative constant T_3 such that $x(t) \geq m_2$ for $t \in [\widehat{T}, \widehat{T} + \tau + T_3]$, $x(\widehat{T} + \tau + T_3) = m_2$, and $x'(\widehat{T} + \tau + T_3) < 0$. Further, from the first equation of (2.5), we obtain that

$$
\frac{dx(\widehat{T} + \tau + T_3)}{dt} = ae^{-b_1\tau} x(\widehat{T} + T_3) - fx^2(\widehat{T} + \tau + T_3)
$$

$$
- \frac{\beta x(\widehat{T} + \tau + T_3) y(\widehat{T} + \tau + T_3)}{\eta + x(\widehat{T} + \tau + T_3)} - rc_0(\widehat{T} + \tau + T_3) x(\widehat{T} + \tau + T_3) \quad \text{(C.9)}
$$

$$
\geq m_2 \left[ae^{-b_1\tau} - fm_1 - \frac{\beta}{\eta} \left(\frac{\lambda\beta m_1 L}{b_2(\eta + m_1)} + \mu_1\sigma + \varepsilon \right) - r(\mu_2\delta + \varepsilon) \right] > 0.
$$

This is a contradiction. So, we obtain that $x(t) \geq m_2$ for $t > \widehat{T}$. Combining (C.7) and (C.8), we have

$$
\frac{dV(t)}{dt} > \left[ae^{-b_1\tau} - fm_1 - \frac{\beta}{\eta} \left(\frac{\lambda\beta m_1 L}{b_2(\eta + m_1)} + \mu_1\sigma + \varepsilon \right) - r(\mu_2\delta + \varepsilon) \right] m_2 > 0, \quad \text{(C.10)}
$$

for $t > \widehat{T}$. It implies that as $t \to +\infty$, $V(t) \to +\infty$. Meanwhile, by the definition of $V(t)$, we easily obtained that $V(t) \leq L(1 + \tau ae^{-b_1\tau})/\lambda$. This is contradiction. Hence, for any constant $t_0 > 0$, $x(t) < m_1$ cannot hold for all $t \geq t_0$.

(2) If $x(t) \geq m_1$ holds true for all large enough t, then our aim is obtained. Otherwise, $x(t)$ is oscillatory about m_1. Thus there exist two positive constant $\bar{t}, \theta$ such that $x(\bar{t}) = x(\bar{t} + \theta) = m_1$ and $x(t) \leq m_1$ for $\bar{t} < t < \bar{t} + \theta$. Let $m_3 = \min\{m_1/2, m_1 e^{-(f/\lambda + \beta/\eta + r)\tau L}\}$. In the following, we firstly show that $x(t) \geq m_3$ for $\bar{t} \leq t \leq \bar{t} + \theta$ and then address that $x(t) \geq m_3$ for t large enough.

From system (2.5), we know that $x(t)$ is continuous and bounded. So, there exists a constant T_4 ($0 < T_4 < \tau$ and independent of the choice of $\bar{t}$) such that $x(t) \geq m_1/2$ for all $\bar{t} \leq t \leq \bar{t} + T_4$. Moreover, when $\bar{t}$ is large enough, by Theorem 3.1 and the first equation of (2.5), we have

$$
\frac{dx(t)}{dt} \geq -\left(\frac{f}{\lambda} + \frac{\beta}{\eta} + r \right) Lx(t), \quad \text{for } \bar{t} \leq t \leq \bar{t} + \theta. \quad \text{(C.11)}
$$

Accordingly, if $\theta \leq T_4$, our aim is obtained; if $T_4 < \theta \leq \tau$, from (C.11), we have

$$
x(t) \geq m_1 e^{-(f/\lambda + \beta/\eta + r)\tau L} \quad \text{for } \bar{t} < t \leq \bar{t} + \theta \leq \bar{t} + \tau. \quad \text{(C.12)}
$$

It is obvious that $x(t) \geq m_3$ for $\bar{t} \leq t \leq \bar{t} + \theta$; if $\theta > \tau$, from (C.11), we can obtain that $x(t) \geq m_3$ for $\bar{t} < t \leq \bar{t} + \tau$. The same argument can be continued, so we can obtain that $x(t) \geq m_3$ for $\bar{t} + \tau < t \leq \bar{t} + \theta$. Since two positive constants, $\bar{t}, \theta$, are arbitrarily chosen, we only assure $\bar{t}$ to be large enough, and then we get that $x(t) \geq m_3$ for t large enough.

According to the above analysis, we can find a constant $M_1 > 0$ such that $x(t) \geq M_1$ for large enough t. Thus the proof is completed.

Acknowledgments

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