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*Research Article*

# Stability and Robust Stability of 2D Discrete Stochastic Systems

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New stability and robust stability results are given based on weaker conservative assumptions. First, new boundary condition is designed. It is less conservative and has broader application range than that has been given. Then, we derive the results which have the same form, but under a weaker conservative assumption. Meanwhile, the process of the proofs has been simplified. Finally, an example is given to illustrate our results. Our results can be extended to the fields of stabilization, filtering and state estimation, and so forth.

## 1. Introduction

Over the past decades, Fornasini-Marchesini (FM) model has been applied in many practical problems, for example, the control of sheet-forming processes [1], circuits, signal processing, and discrimination of some partial differential equations with initial-boundary conditions [2–6]. Asymptotic stability for 2D deterministic systems based on FM models also has been developed quite successfully. Several methods have been proposed, for example, using Lyapunov function [7, 8], using LMI technique [9], and using the nonnegative matrices theory [10–12]. For linear 2D model in general case, stability has been discussed in [13]. However, very few effort has been made toward the analysis and synthesis of 2D stochastic systems (2DSSs) with stochastic system matrices. The mean-square stability of 2DSS has been discussed in [14, 15]. The state estimation problem has been discussed in [16]. The $H_\infty$ filtering problem has been discussed in [17]. But all of them are based on the conservative assumption which is listed as follows.

*Assumption 1.1.* The boundary condition of (2.1) is independent of $v(i,j)$ and is assumed to satisfy

$$\lim_{N\to\infty}\mathbb{E}\left\{\sum_{k=1}^{N}\left(\|x(0,k)\|^2+\|x(k,0)\|^2\right)\right\}<\infty. \tag{1.1}$$

The main goal of the present paper is to find stability and robust stability criteria for two dimensional stochastic systems based on weaker conservative assumptions. First, new boundary condition is designed. It is less conservative and has broader application range than Assumption 1.1. Then, we derive the results which have the same form, but under a weaker conservative assumption. Meanwhile, the process of the proofs has been simplified. At last, an example is given to illustrate our results.

The following notation is used in this paper. For an $n$-dimensional vector of real elements $x\in\mathbb{R}^n$, $\|x\|=(x^Tx)^{1/2}$ denotes the 2-norm, where the superscript $T$ stands for matrix transposition. $\mathbb{E}\{x\}$ denotes the expected value of $x$. In symmetric block matrices or long matrix expressions, we use an asterisk (*) to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

## 2. 2D Stochastic System Model

First, we rewrite the 2D stochastic system model as follows:

$$x(i+1,j+1)=\left[A_1+M_1v(i,j)\right]x(i,j+1)+\left[A_2+M_2v(i,j)\right]x(i+1,j), \tag{2.1}$$

where $A_1$, $A_2$ are system matrices with compatible dimensions, $M_1$, $M_2$ are appropriately dimensioned matrices, and $v(i,j)$ is a standard random scalar signal satisfying $\mathbb{E}\{v(i,j)\}=0$ and

$$\mathbb{E}\{v(i,j)v(m,n)\}=\begin{cases}1 & (i,j)=(m,n),\\ 0 & (i,j)\neq(m,n).\end{cases} \tag{2.2}$$

We make the following assumption on the boundary condition which is less conservative.

*Assumption 2.1.* The boundary condition is independent of $v(i,j)$ and is assumed to satisfy

$$\begin{gathered}\lim_{k\to\infty}\mathbb{E}\left\{\|x(0,k)\|^2\right\}=0,\qquad \lim_{k\to\infty}\mathbb{E}\left\{\|x(k,0)\|^2\right\}=0,\\ \mathbb{E}\left\{\|x(k,0)\|^2\right\}<\infty,\quad E\left\{\|x(k,0)\|^2\right\}<\infty\quad\text{for any } k\geq 1.\end{gathered} \tag{2.3}$$

*Remark 2.2.* Assumption 2.1 does not yield any loss of generality since the 2DDS (2.1) is not asymptotically stable if the initial states do not satisfy Assumption 2.1. We can see that Assumption 2.1 is necessary if the 2DDS (2.1) is asymptotically stable.

*Remark 2.3.* Assumption 2.1 is less conservative and has broader application range than Assumption 1.1.

For example, we assume the boundary state of system (2.1) satisfies $\mathbb{E}\{\|x(0,k)\|^2\} = \mathbb{E}\{\|x(k,0)\|^2\} = 1/k$, where $k = 1, 2, \ldots, \infty$. So, we have

$$\lim_{k\to\infty} \mathbb{E}\left\{ \|x(0,k)\|^2 \right\} = 0,$$
$$\lim_{k\to\infty} \mathbb{E}\left\{ \|x(k,0)\|^2 \right\} = 0. \tag{2.4}$$

Clearly, the above boundary state does not meet Assumption 1.1, so the results in [14–17] can not be used on system (2.1) with Assumption 1.1. Although the above-boundary state meets Assumption 2.1, we can use our conclusions on system (2.1) with Assumption 2.1.

Similar to [14], we give the following definition which will be used throughout the paper.

*Definition 2.4.* The two-dimensional discrete stochastic system (2.1) with Assumption 2.1 is said to be mean-square asymptotically stable if under the zero input and for every initial condition $\mathbb{E}\{\|x(0,0)\|^2\} < \infty$,

$$\lim_{i+j\to\infty} \mathbb{E}\left\{ \|x(i,j)\|^2 \right\} = 0. \tag{2.5}$$

*Remark 2.5.* Definition 2.4 is more general and has broader application range than Definition 2 given in [14].

## 3. Asymptotic Stability and Robust Stability

In this section, we discuss mean-square asymptotic stability for 2D discrete stochastic systems (2.1). Then, we extend the result into the fields of robust stability.

**Lemma 3.1** (Schur's Complement [18]). *Given constant matrices $C$, $L$, and $D$ of appropriate dimensions where $C$ and $D$ are symmetric and $D > 0$, then the inequality $C + L^T D L < 0$ holds if and only if*

$$\begin{bmatrix} C & L^T \\ L & -D^{-1} \end{bmatrix} < 0 \quad or \quad \begin{bmatrix} -D^{-1} & L \\ L^T & C \end{bmatrix} < 0. \tag{3.1}$$

### 3.1. Asymptotic Stability

**Theorem 3.2.** *The 2D discrete stochastic system* (2.1) *is mean-square asymptotically stable if there exist two positive-definite matrices $P_1$ and $P_2$ satisfying*

$$\left[\begin{array}{cc|cc} -P_1 & 0 & A_1^T P & M_1^T P \\ * & -P_2 & A_2^T P & M_2^T P \\ \hline * & * & -P & 0 \\ * & * & * & -P \end{array}\right] < 0, \tag{3.2}$$

*where $P = P_1 + P_2$.*

*Proof.* Let

$$C = \begin{bmatrix} -P_1 & 0 \\ 0 & -P_2 \end{bmatrix}, \quad L^T = \begin{bmatrix} A_1^T P & M_1^T P \\ M_2^T P & M_2^T P \end{bmatrix}, \quad D = \begin{bmatrix} P^{-1} & 0 \\ 0 & P^{-1} \end{bmatrix}, \tag{3.3}$$

by Lemma 3.1, and LMI (3.2) is equivalent to

$$\Psi = \begin{bmatrix} A_1^T P A_1 + M_1^T P M_1 - P_1 & A_1^T P A_2 + M_1^T P M_2 \\ * & A_2^T P A_2 + M_2^T P M_2 - P_2 \end{bmatrix} < 0. \tag{3.4}$$

Let

$$\Delta V(i+1, j+1) = V(i+1, j+1) - V_1(i, j+1) - V_2(i+1, j), \tag{3.5}$$

where

$$\begin{aligned} V_1(i,j) &= \mathbb{E}\left\{ x^T(i,j) P_1 x(i,j) \right\}, \\ V_2(i,j) &= \mathbb{E}\left\{ x^T(i,j) P_2 x(i,j) \right\}. \end{aligned} \tag{3.6}$$

Substitute (2.1) into (3.5) and let $\tilde{x} := \begin{bmatrix} x(i,j+1) \\ x(i+1,j) \end{bmatrix}$, then we have

$$\begin{aligned} \Delta V(i+1, j+1) &= \mathbb{E}\left\{ x^T(i+1, j+1) P_1 x(i+1, j+1) \right\} + \mathbb{E}\left\{ x^T(i+1, j+1) P_2 x(i+1, j+1) \right\} \\ &\quad - \mathbb{E}\left\{ x^T(i, j+1) P_1 x(i, j+1) \right\} - \mathbb{E}\left\{ x^T(i+1, j) P_2 x(i+1, j) \right\} \\ &= \mathbb{E}\left\{ x^T(i+1, j+1) [P_1 + P_2] x(i+1, j+1) \right\} \\ &\quad - \mathbb{E}\left\{ x^T(i, j+1) P_1 x(i, j+1) \right\} - \mathbb{E}\left\{ x^T(i+1, j) P_2 x(i+1, j) \right\} \\ &= \mathbb{E}\left\{ \begin{array}{c} ([A_1 + M_1 v(i,j)] x(i, j+1) + [A_2 + M_2 v(i,j)] x(i+1, j))^T P \\ ([A_1 + M_1 v(i,j)] x(i, j+1) + [A_2 + M_2 v(i,j)] x(i+1, j)) \end{array} \right\} \\ &\quad - \mathbb{E}\left\{ x^T(i, j+1) P_1 x(i, j+1) \right\} + \mathbb{E}\left\{ x^T(i+1, j) P_2 x(i+1, j) \right\} \\ &= (A_1 x(i, j+1) + A_2 x(i+1, j))^T P (A_1 x(i, j+1) + A_2 x(i+1, j)) \\ &\quad + (M_1 x(i, j+1) + M_2 x(i+1, j))^T P (M_1 x(i, j+1) + M_2 x(i+1, j)) \\ &\quad - \mathbb{E}\left\{ x^T(i, j+1) P_1 x(i, j+1) \right\} + \mathbb{E}\left\{ x^T(i+1, j) P_2 x(i+1, j) \right\} \end{aligned}$$

$$= \tilde{x}^T \begin{bmatrix} A_1^T \\ A_2^T \end{bmatrix} P[A_1 \quad A_2]\tilde{x} + \tilde{x}^T \begin{bmatrix} M_1^T \\ M_2^T \end{bmatrix} P[M_1 \quad M_2]\tilde{x} - \tilde{x}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \tilde{x}$$
$$= \mathbb{E}\left\{ \tilde{x}^T \Psi \tilde{x} \right\}. \tag{3.7}$$

Hence, for any $\tilde{x} \neq 0$, we have

$$\Delta V(i+1, j+1) = \mathbb{E}\left\{ \tilde{x}^T \Psi \tilde{x} \right\} < 0. \tag{3.8}$$

From Assumption 2.1, we have $\mathbb{E}\{\|x(i,0)\|^2\} \to 0$ and $\mathbb{E}\{\|x(0,i)\|^2\} \to 0$ as $i \to \infty$.

From Definition 2.4, we can see that, to show mean-square asymptotically stable, we only need to prove that $\lim_{i+j\to\infty}\mathbb{E}\{\|x(i,j)\|^2\} = 0$, that is, to prove that $\mathbb{E}\{\|x(i,0)\|^2\} \to 0$, $\mathbb{E}\{\|x(i,1)\|^2\} \to 0, \ldots, \mathbb{E}\{\|x(i,\infty)\|^2\} \to 0$, and $\mathbb{E}\{\|x(0,i)\|^2\} \to 0$, $\mathbb{E}\{\|x(1,i)\|^2\} \to 0, \ldots, \mathbb{E}\{\|x(\infty,i)\|^2\} \to 0$ as $i \to \infty$. So, we only need to prove that for any natural number $k \geq 1$, $\mathbb{E}\{\|x(i,k)\|^2\} \to 0$ and $\mathbb{E}\{\|x(k,i)\|^2\} \to 0$ as $i \to \infty$. To prove this, we need the following two steps.

*Step 1.* Let $k = 1$, to prove $\mathbb{E}\{\|x(i,1)\|\} \to 0$ and $\mathbb{E}\{\|x(1,i)\|\} \to 0$ as $i \to \infty$.

*Step 2.* Let $k = 2, \ldots, \infty$, to prove $\mathbb{E}\{\|x(i,k)\|\} \to 0$ and $\mathbb{E}\{\|x(k,i)\|\} \to 0$ as $i \to \infty$.

Now, we prove that $\mathbb{E}\{\|x(i,1)\|\} \to 0$ as $i \to \infty$. From (3.5) and (3.8), we have

$$V_1(i+1, j+1) + V_2(i+1, j+1) - V_1(i, j+1) - V_2(i+1, j) < 0, \tag{3.9}$$

which implies

$$V_1(i+1, j+1) < V_1(i, j+1) \quad \text{or} \quad V_2(i+1, j+1) < V_2(i+1, j) \quad \text{if } \mathbb{E}\{\|x(i,1)\|\} > 0. \tag{3.10}$$

Clearly, let $i \to \infty$ and $j = 0$, and substitute them into (3.10), then we get

$$V_1(\infty, 1) < V_1(\infty, 1) \quad \text{or} \quad V_2(\infty, 1) < V_2(\infty, 0) = 0 \quad \text{if } \mathbb{E}\{\|x(\infty,1)\|\} > 0. \tag{3.11}$$

Since both $V_1(\infty,1) < V_1(\infty,1)$ and $V_2(\infty,1) < 0$ are false, (3.11) is false. Hence, the condition $\mathbb{E}\{\|x(\infty,1)\|\} > 0$ in (3.11) is false. Thus, it yields $\mathbb{E}\{\|x(\infty,1)\|\} = 0$. Similarly, we can get that $\mathbb{E}\{\|x(1,\infty)\|\} = 0$.

Continue this procedure, and we can obtain that $\mathbb{E}\{\|x(i,k)\|^2\} \to 0$ and $\mathbb{E}\{\|x(k,i)\|^2\} \to 0$ as $i \to \infty$ for any natural number $k \geq 2$. It implies that $\lim_{i+j\to\infty}\mathbb{E}\{\|x(i,j)\|^2\} = 0$. Therefore, from Definition 2.4, the system (2.1) is mean-square asymptotically stable. The proof is completed. □

*Remark 3.3.* Theorem 3.2 gives a sufficient condition for the mean-square asymptotical stability of system (2.1). It is equivalent to [14, Theorem 2] in the form. However, it has broader application range because the assumption is weaker.

*Remark 3.4.* Theorem 3.2 is also equivalent to [14, Theorem 1] from [14, Theorem 3] in the form. However, it has broader application range.

Before proceeding further, we give the following lemma which will be used in the following proofs frequently.

**Lemma 3.5** (see [15]). *Given appropriately dimensioned matrices $R_1$, $R_2$, $R_3$, with $R_1^T = R_1$, then*

$$R_1 + R_3 W_{i,j} R_2 + R_2^T W_{i,j}^T R_3^T < 0 \tag{3.12}$$

*holds for all $W_{i,j}$ satisfying $W_{i,j}^T W_{i,j} \leq I$ if and only if, for some $\delta > 0$,*

$$R_1 + \delta^{-1} R_3 R_3^T + \delta R_2^T R_2 < 0. \tag{3.13}$$

Next, we present the robust stability result for system (2.1) with norm-bounded uncertain matrices.

## 3.2. Robust Stability

The main task of this subsection is to establish the robust mean-square asymptotic stability for two-dimensional stochastic system (2.1) with uncertain matrix data.

First, we give the following assumptions.

*Assumption 3.6.* Assume that the matrices $A_1$, $A_2$, $M_1$, $M_2$ of system (2.1) have the following form:

$$\begin{aligned} A_1 &= A_{10} + \Delta A_1, \qquad M_1 = M_{10} + \Delta M_1, \\ A_2 &= A_{20} + \Delta A_2, \qquad M_2 = M_{20} + \Delta M_2, \end{aligned} \tag{3.14}$$

where $A_{10}$, $A_{20}$, $M_{10}$, $M_{20}$ are known constant matrices with appropriate dimensions. $\Delta A_1$, $\Delta A_2$, $\Delta M_1$, $\Delta M_2$ are real-valued time-varying matrix functions representing norm-bounded parameter uncertainties satisfying

$$\begin{pmatrix} \Delta A_1 & \Delta A_2 & \Delta M_1 & \Delta M_2 \end{pmatrix} = G \Delta_{i,j} \begin{pmatrix} H_1 & H_2 & H_3 & H_4 \end{pmatrix}, \tag{3.15}$$

where $\Delta_{i,j}$ is a real uncertain matrix function with Lebesgue measurable elements satisfying

$$\Delta_{i,j}^T \Delta_{i,j} \leq I, \tag{3.16}$$

and $G$, $H_1$, $H_2$, $H_3$, $H_4$ are known real constant matrices with appropriate dimensions. These matrices specify how the uncertain parameters in $\Delta_{i,j}$ enter the nominal matrices $A_{10}$, $A_{20}$, $M_{10}$, $M_{20}$.

Now, we have the robust stability result for system (2.1) with norm-bounded uncertain matrices.

**Theorem 3.7.** *The 2D discrete stochastic system* (2.1) *with Assumption 3.6 is robustly mean-square asymptotically stable if there exist two positive-definite matrices $P_1$, $P_2$ and a scalar $\delta > 0$ satisfying*

$$\left[\begin{array}{cccc|cc} -P_1+\delta\left(H_1^TH_1+H_3^TH_3\right) & \delta\left(H_1^TH_2+H_3^TH_4\right) & A_{10}^TP & M_{10}^TP & 0 & 0 \\ * & -P_2\delta\left(H_2^TH_2+H_4^TH_4\right) & A_{20}^TP & M_{20}^TP & 0 & 0 \\ * & * & -P & 0 & PG & 0 \\ * & * & * & -P & 0 & PG \\ \hline * & * & * & * & -\delta I & 0 \\ * & * & * & * & * & -\delta I \end{array}\right] < 0, \tag{3.17}$$

*where $P = P_1 + P_2$.*

*Proof.* With the result of Theorem 3.2, substituting the norm-bounded uncertain matrices $A_1$, $A_2$, $M_1$, $M_2$ defined in (3.14) into (3.2), we have

$$\left[\begin{array}{cc|cc} -P_1 & 0 & (A_{10}+\Delta A_1)^TP & (M_{10}+\Delta M_1)^TP \\ * & -P_2 & (A_{20}+\Delta A_2)^TP & (M_{20}+\Delta M_2)^TP \\ \hline * & * & -P & 0 \\ * & * & * & -P \end{array}\right] < 0, \tag{3.18}$$

where $P = P_1 + P_2$.

It can be written as (3.12) with

$$R_1 = \begin{bmatrix} -P_1 & 0 & A_{10}^TP & M_{10}^TP \\ * & -P_2 & A_{20}^TP & M_{20}^TP \\ * & * & -P & 0 \\ * & * & * & -P \end{bmatrix}, \quad R_2 = \begin{bmatrix} H_1 & H_2 & 0 & 0 \\ H_3 & H_4 & 0 & 0 \end{bmatrix}, \quad R_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ PG & 0 \\ 0 & PG \end{bmatrix}, \tag{3.19}$$

$$W_{i,j} = \begin{bmatrix} \Delta_{i,j} & 0 \\ 0 & \Delta_{i,j} \end{bmatrix}.$$

By Lemma 3.5, we get

$$\begin{bmatrix} -P_1+\delta\left(H_1^TH_1+H_3^TH_3\right) & \delta\left(H_1^TH_2+H_3^TH_4\right) & A_{10}^TP & M_{10}^TP \\ * & -P_2+\delta\left(H_2^TH_2+H_4^TH_4\right) & A_{20}^TP & M_{20}^TP \\ * & * & -P+\delta^{-1}PG(PG)^T & 0 \\ * & * & * & -P+\delta^{-1}PG(PG)^T \end{bmatrix} < 0. \tag{3.20}$$

It can be rewritten as

$$C + L^T D L < 0, \tag{3.21}$$

with

$$C = \begin{bmatrix} -P_1 + \delta\left(H_1^T H_1 + H_3^T H_3\right) & \delta\left(H_1^T H_2 + H_3^T H_4\right) & A_{10}^T P & M_{10}^T P \\ * & -P_2 + \delta\left(H_2^T H_2 + H_4^T H_4\right) & A_{20}^T P & M_{20}^T P \\ * & * & -P & 0 \\ * & * & * & -P \end{bmatrix},$$

$$L^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ PG & 0 \\ 0 & PG \end{bmatrix}, \qquad D = \begin{bmatrix} \delta^{-1} I & 0 \\ 0 & \delta^{-1} I \end{bmatrix}. \tag{3.22}$$

Using Lemma 3.1 (Schur's Complement), we get

$$\begin{bmatrix} C & L^T \\ L & -D^{-1} \end{bmatrix} < 0, \tag{3.23}$$

which is (3.17). The proof is completed. □

*Remark 3.8.* Theorem 3.7 is equivalent to [14, Theorem 4] in the form. However, it has broader application range.

*Remark 3.9.* We can get similar results correspond to [14, Theorems 5–8].

*Remark 3.10.* We can get similar results about robust $H_\infty$ filtering correspond to [14, Theorems 1–5].

## 4. Example

In this section, we illustrate our results for 2D discrete stochastic system (2.1) through an example. All computations in this section are carried out by Matlab 7.8.0.347.

Consider two-dimensional stochastic system (2.1) with two state variables $x_1$, $x_2$, and the following system matrices:

$$A_1 = \begin{bmatrix} 0.1 & 0 \\ 0.5 & 0.15 + 0.2\rho \end{bmatrix}, \qquad M_1 = \begin{bmatrix} 0.06 & 0 \\ 0 + 0.1\rho & 0.05 + 0.2\rho \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.2 & 0+0.1\rho \\ 0.3 & 0.2 \end{bmatrix}, \qquad M_2 = \begin{bmatrix} 0 & 0.05 \\ 0.05+0.15\rho & 0 \end{bmatrix},$$

$$\mathbb{E}\{v(i,j)\} = 0,$$

$$\mathbb{E}\{v(i,j)v(m,n)\} = \begin{cases} 1 & \text{for } (i,j) = (m,n), \\ 0 & \text{for } (i,j) \neq (m,n). \end{cases} \tag{4.1}$$

The boundary condition is assumed to satisfy Assumption 3.6 but does not satisfy Assumption 2.1. For example,

$$\|x(0,k)\|^2 = \|x(k,0)\|^2 = \frac{1}{k}. \tag{4.2}$$

First, we assume that the system matrices are perfectly known, that is, $\rho = 0$. We can not determine the stability of the system using the conclusions of [14] because the boundary condition does not satisfy the assumption given in [14]. However, we can determine the stability of the system by Theorem 3.2 because the boundary condition satisfies our assumption. Using Matlab to solve the inequality (3.2), we get that there exist two positive matrices $P_1 = P_2 = \left[\begin{smallmatrix} 0.8612 & -0.0497 \\ -0.0497 & 0.7026 \end{smallmatrix}\right]$, such that inequality (3.2) is true. So the system is stable.

Figures 1 and 2 show the two state variables of the above system. It can be seen that the system is asymptotically stable too.

Now, we assume that the uncertain parameter $\rho$, satisfying $|\rho| \leq 1$. we have the matrices in Assumption 3.6 as follows:

$$A_{10} = \begin{bmatrix} 0.1 & 0 \\ 0.5 & 0.15 \end{bmatrix}, \quad M_{10} = \begin{bmatrix} 0.06 & 0 \\ 0 & 0.05 \end{bmatrix}, \quad A_{20} = \begin{bmatrix} 0.2 & 0 \\ 0.3 & 0.2 \end{bmatrix}, \quad M_{20} = \begin{bmatrix} 0 & 0.05 \\ 0.05 & 0 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0 & 0.1 \\ 0 & 0 \end{bmatrix}, \quad H_3 = \begin{bmatrix} 0 & 0 \\ 0.1 & 0.2 \end{bmatrix}, \quad H_4 = \begin{bmatrix} 0 & 0 \\ 0.15 & 0 \end{bmatrix},$$

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \Delta_{ij} = \begin{bmatrix} \rho & 0 \\ 0 & \rho \end{bmatrix},$$

$$\mathbb{E}\{v(i,j)\} = 0,$$

$$\mathbb{E}\{v(i,j)v(m,n)\} = \begin{cases} 1 & \text{for } (i,j) = (m,n), \\ 0 & \text{for } (i,j) \neq (m,n). \end{cases} \tag{4.3}$$

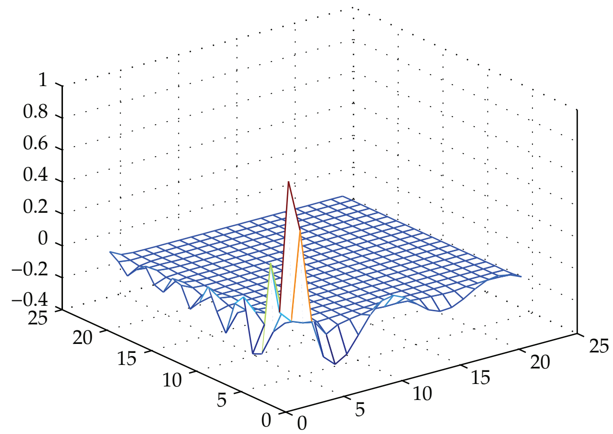


**Figure 1:** State variable $x_1$ of system (2.1).

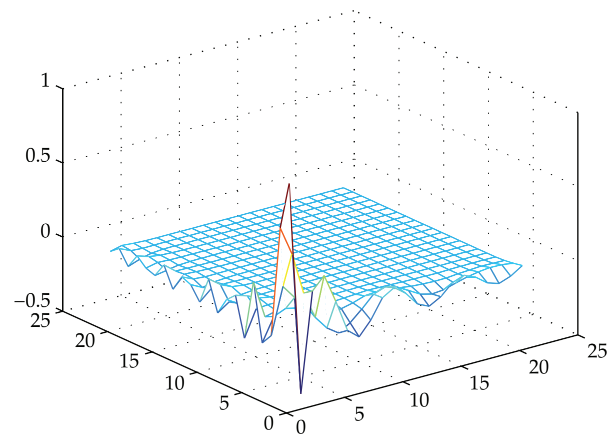


**Figure 2:** State variable $x_2$ of system (2.1).

Similar to the case of stability, we can not determine the robust stability of the system using the conclusions of [14]. However, we can determine the stability of the system by Theorem 3.7 because the boundary condition satisfies our assumption. Using Matlab to solve the inequality (3.17), we get that there exist two positive matrices

$$P_1 = \begin{bmatrix} 2.0592 & 0.0717 \\ 0.0717 & 0.9343 \end{bmatrix}, \qquad P_2 = \begin{bmatrix} 2.2844 & -0.1540 \\ -0.1540 & 0.5537 \end{bmatrix}, \tag{4.4}$$

and a scalar $\delta = 6.5538 > 0$, such that inequality (3.17) is true. So the system is robustly stable.

## 5. Conclusions

In this paper, new stability and robust stability results are given based on weaker conservative assumptions. A new boundary condition is designed. It is less conservative and has broader application range than that has been given. Then, we derive the results which have the same form, but under a weaker conservative assumption. Meanwhile, the process of the proofs has been simplified. Our results can be extended to the fields of stabilization, filtering and state estimation, and so forth.

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