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Research Article

Permanence in Multispecies Nonautonomous Lotka-Volterra Competitive Systems with Delays and Impulses

Xiaomei Feng,[1,2] Fengqin Zhang,[2] Kai Wang,[3] and Xiaoxia Li[2]

[1] *College of Mathematics and System Sciences, Xinjiang University, Urumqi 830046, China*
[2] *Department of Mathematics, Yuncheng University, Yuncheng 044000, China*
[3] *Department of Medical Engineering and Technology, Xinjiang Medical University, Urumqi 830011, China*

Correspondence should be addressed to Kai Wang, wangkaimath@sina.com

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This paper studies multispecies nonautonomous Lotka-Volterra competitive systems with delays and fixed-time impulsive effects. The sufficient conditions of integrable form on the permanence of species are established.

1. Introduction

In this paper, we consider the nonautonomous n-species Lotka-Volterra type competitive systems with delays and impulses

$$\begin{gathered} x_i'(t) = x_i(t)\left[a_i(t) - b_i(t)x_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j\left(t-\tau_{ij}(t)\right)\right], \quad t \neq t_k, \\ x_i\left(t_k^+\right) = h_{ik}x_i(t_k), \quad i = 1,2,\ldots,n, \ k = 1,2,\ldots, \end{gathered} \tag{1.1}$$

where $x_i(t)$ represents the population density of the ith species at time t, the functions $a_i(t)$, $b_i(t)$, $a_{ij}(t)$, and $\tau_{ij}(t)$ $(i,j = 1,2,\ldots,n)$ are bounded and continuous functions defined on $R_+ = [0,+\infty)$, $a_{ij}(t) \geq 0$, $b_i(t) \geq 0$, $\tau_{ij}(t) \geq 0$ for all $t \in R_+$, and impulsive coefficients h_{ik} for any $i = 1,2,\ldots,n$ and $k = 1,2,\ldots$ are positive constants.

In particular, when the delays $\tau_{ij}(t) \equiv 0$ for all $t \in R_+$ and $i, j = 1, 2, \ldots, n$, then the system (1.1) degenerate into the following nondelayed non-autonomous n-species Lotka-volterra system

$$x_i'(t) = x_i(t)\left[a_i(t) - \sum_{j=1}^{n} b_{ij}(t)x_j(t)\right], \quad t \neq t_k,$$
$$x_i\left(t_k^+\right) = h_{ik}x_i(t_k), \quad i = 1, 2, \ldots, n, \ k = 1, 2, \ldots, \tag{1.2}$$

where $b_{ii}(t) = b_i(t) + a_{ii}(t)$ and $b_{ij}(t) = a_{ij}(t)$ for $i, j = 1, 2, \ldots, n$ and $i \neq j$. For system (1.2), the author establish some new sufficient condition on the permanence of species and global attractivity in [1].

As we well know, systems like (1.1) and (1.2) without impulses are very important in the models of multispecies populations dynamics. Many important results on the permanence, extinction, global asymptotical stability for the two species or multi-species non-autonomous Lotka-Volterra systems and their special cases of periodic and almost periodic systems can be found in [2–14] and the references therein.

However, owing to many natural and man-made factors (e.g., fire, flooding, crop-dusting, deforestation, hunting, harvesting, etc.), the intrinsic discipline of biological species or ecological environment usually undergoes some discrete changes of relatively short duration at some fixed times. Such sudden changes can often be characterized mathematically in the form of impulses. In the last decade, much work has been done on the ecosystem with impulsive(see [1, 15–21] and the reference therein). Specially, the following system is considered in [22]:

$$x_i'(t) = x_i(t)\left[a_i(t) - b_{ii}(t)x_i(t) - \sum_{j=1, j\neq i}^{n}\int_{-\infty}^{0} k_j(s)x_j(t+s)\mathrm{d}s\right], \quad t \neq t_k,$$
$$x_i\left(t_k^+\right) = h_{ik}x_i(t_k), \quad i = 1, 2, \ldots, n, \ k = 1, 2, \ldots. \tag{1.3}$$

The author establish some new sufficient conditions on the permanence of species and global attractivity for system (1.3). However, the effect of discrete delays on the possibility of species survival has been an important subject in population biology. We find that infinite delays are considered in the system (1.3). In this paper, it is very meaningful that discrete delays are proposed in the impulsive system (1.1).

2. Preliminaries

Let $\tau = \sup\{\tau_{ij}(t),\ t \geq 0,\ i, j = 1, 2, \ldots, n\}$. We define $C^n[-\tau, 0]$ the Banach space of bounded continuous function $\phi : [-\tau, 0] \to R^n$ with the supremum norm defined by:

$$\|\phi\|_c = \sup_{-\tau \leq s \leq 0}\left|\phi(s)\right|, \tag{2.1}$$

where $\phi = (\phi_1, \phi_2, \ldots, \phi_n)$, and $|\phi(s)| = \sum_{i=1}^{n}|\phi_i(s)|$. Define $C_+^n[-\tau, 0] = \{\phi = (\phi_1, \phi_2, \ldots, \phi_n) \in C^n[-\tau, 0] : \phi_i(s) \geq 0$, and $\phi_i(0) \geq 0$ for all $s \in [-\tau, 0]$ and $i = 1, 2, \ldots, n\}$.

Motivated by the biological background of system (1.1), we always assume that all solutions $(x_1(t), x_2(t), \ldots, x_n(t))$ of system (1.1) satisfy the following initial condition:

$$x_i(s) = \phi_i(s) \quad \forall s \in [-\tau, 0],\ i = 1, 2, \ldots, n, \tag{2.2}$$

where $\phi = (\phi_1, \phi_2, \ldots, \phi_n) \in C_+^n[-\tau, 0]$.

It is obvious that the solution $(x_1(t), x_2(t), \ldots, x_n(t))$ of system (1.1) with initial condition (2.2) is positive, that is, $x_i(t) > 0$ $(i = 1, 2, \ldots, n)$ on the interval of the existence and piecewise continuous with points of discontinuity of the first kind t_k $(k \in N)$ at which it is left continuous, that is, the following relations are satisfied:

$$x_i(t_k^-) = x_i(t_k), \quad x_i(t_k^+) = h_{ik} x_i(t_k), \quad i = 1, 2, \ldots, n,\ k \in N. \tag{2.3}$$

For system (1.1), we introduce the following assumptions:

(H$_1$) functions $a_i(t), b_i(t), a_{ij}(t)$ and $\tau_{ij}(t)$ are bounded continuous on $[0, +\infty]$, and $b_i(t)$, $a_{ij}(t)$ and $\tau_{ij}(t)$ $(i, j = 1, 2, \ldots, n)$ are nonnegative for all $t \geq 0$.

(H$_2$) for each $1 \leq i \leq n$, there are positive constants $\omega_i > 0$ such that

$$\liminf_{t \to \infty} \left(\int_t^{t+\omega_i} b_i(s) \mathrm{d}s \right) > 0, \tag{2.4}$$

and the functions

$$h_i(t, \mu) = \sum_{t \leq t_k < t+\mu} \ln h_{ik} \tag{2.5}$$

are bounded for all $t \in R_+$ and $\mu \in [0, \omega_i]$.

First, we consider the following impulsive logistic system

$$\begin{aligned} x'(t) &= x(t)\left[\alpha(t) - \beta(t) x(t)\right], \quad t \neq t_k, \\ x(t_k^+) &= h_k x(t_k), \quad k = 1, 2, \ldots, \end{aligned} \tag{2.6}$$

where $\alpha(t)$ and $\beta(t)$ are bounded and continuous functions defined on R_+, $\beta(t) \geq 0$ for all $t \in R_+$, and impulsive coefficients h_k for any $k = 1, 2, \ldots$ are positive constants. We have the following results.

Lemma 2.1. *Suppose that there is a positive constant ω such that*

$$\begin{gathered} \liminf_{t \to \infty} \left(\int_t^{t+\omega} \beta(s) \mathrm{d}s \right) > 0, \\ \liminf_{t \to \infty} \left(\int_t^{t+\omega} \alpha(s) \mathrm{d}s + \sum_{t \leq t_k < t+\omega} \ln h_k \right) > 0, \end{gathered} \tag{2.7}$$

and function

$$h(t,\mu) = \sum_{t\le t_k<t+\omega} \ln h_k \tag{2.8}$$

is bounded on $t \in R_+$ *and* $\mu \in [0,\omega]$. *Then we have*

(a) *there exist positive constants* m *and* M *such that*

$$m \le \liminf_{t\to\infty} x(t) \le \limsup_{t\to\infty} x(t) \le M, \tag{2.9}$$

for any positive solution $x(t)$ *of system* (2.6);

(b) $\lim_{t\to\infty}(x^{(1)}(t) - x^{(2)}(t)) = 0$ *for any two positive solutions* $x^{(1)}(t)$ *and* $x^{(2)}(t)$ *of system* (2.6).

The proof of Lemma 2.1 can be found as Lemma 2.1 in [1] by Hou et al.

On the assumption (H_2), we firstly have the following result.

Lemma 2.2. *If assumption* (H_2) *holds, then there exist constants* $d > 0$ *and* $D > 0$ *such that for any* $t_2 \ge t_1 \ge 0$

$$\left|\sum_{t_1\le t_k<t_2} \ln h_{ik}\right| \le d(t_2 - t_1) + D, \quad i = 1,2,\ldots,n. \tag{2.10}$$

The proof of Lemma 2.2 is simple, we hence omit it here.

3. Main Results

Let $x_{i0}(t)$ be some fixed positive solution of the following impulsive logistic systems as the subsystems of system (1.1):

$$\begin{aligned} x_i'(t) &= x_i(t)[a_i(t) - b_i(t)x_i(t)], \quad t \ne t_k, \\ x_i(t_k^+) &= h_{ik}x_i(t_k), \quad k = 1,2,\ldots. \end{aligned} \tag{3.1}$$

On the permanence of all species x_i $(i = 1,2,\ldots,n)$ for system (1.1), we have the following result.

Theorem 3.1. *Suppose that assumptions* (H_1)-(H_2) *hold. If there exist positive constants* ω_i *such that for each* $1 \le i \le n$:

$$\liminf_{t\to\infty}\left(\int_t^{t+\omega_i}\left(a_i(s) - \sum_{j\ne i}^{n} a_{ij}(s)x_{j0}(s-\tau_{ij}(s))\right)ds + \sum_{t\le t_k<t+\omega_i} \ln h_{ik}\right) > 0, \tag{3.2}$$

and the functions

$$h_i(t,\mu) = \sum_{t \le t_k < t+\mu} \ln h_{ik} \tag{3.3}$$

are bounded for all $t \in R_+$ *and* $\mu \in [0, \omega_i]$. *Then the system* (1.1) *is permanent, that is, there are positive constants* $\gamma > 0$ *and* $M > 0$ *such that*

$$\gamma \le \liminf_{t \to \infty} x_i(t) \le \limsup_{t \to \infty} x_i(t) \le M, \quad i = 1, 2, \dots, n, \tag{3.4}$$

for any positive solution $x(t) = (x_1(t), x_2(t), \dots, x_n(t))$ *of system* (1.1).

Proof. Let $x(t) = (x_1(t), x_2(t), \dots, x_n(t))$ be any positive solution of system (1.1). We first prove that the components x_i $(i = 1, 2, \dots, n)$ of system (1.1) are bounded. From assumption (H_1) and the ith equation of system (1.1), we have

$$\begin{aligned} x_i'(t) &\le x_i(t)[a_i(t) - b_i(t)x_i(t)], \quad t \ne t_k, \\ x_i(t_k^+) &= h_{ik} x_i(t_k), \quad k \in N. \end{aligned} \tag{3.5}$$

by the comparison theorem of impulsive differential equation, we have

$$x_i(t) \le y_i(t), \quad \forall t \ge 0, \tag{3.6}$$

where $y_i(t)$ is the solution of (3.1) with initial value $y_i(0) = x_i(0)$. From the condition (3.2), we directly have

$$\liminf_{t \to \infty} \left(\int_t^{t+\omega_i} a_i(s) \mathrm{d}s + \sum_{t \le t_k < t+\omega_i} \ln h_{ik} \right) > 0, \quad i = 1, 2, \dots, n. \tag{3.7}$$

Hence, from conclusion (a) of Lemma 2.1, we can obtain a constant $M_{i1} > 0$, and there is a $T_{i1} > 0$ such that $y_i(t) < M_{i1}$ for all $t \ge T_{i1}$. Let $M = \max_{1 \le i \le n}\{M_{i1}\}$ and $T_1 = \max_{1 \le i \le n}\{T_{i1}\}$, we have

$$x_i(t) \le M, \quad \forall t \ge T_1, \ i = 1, 2, \dots, n. \tag{3.8}$$

Hence, we finally have

$$\limsup_{t \to \infty} x(t) \le M. \tag{3.9}$$

□

Next, we prove that there is a constant $\gamma > 0$ such that

$$\liminf_{t \to \infty} x(t) \ge \gamma, \quad i = 1, 2, \dots, n. \tag{3.10}$$

For any t_1 and t_2 directly from system (1.1), we have

$$x_i(t_1) = x_i(t_2)\exp\left(\int_{t_2}^{t_1}\left[a_i(t) - b_i(t)x_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t-\tau_{ij}(t))\right]\mathrm{d}t + \sum_{t_2\le t_k\le t_1}\ln h_{ik}\right). \quad (3.11)$$

From condition (3.2), we can choose constants $0 < \varepsilon < 1$ small enough and $T_2 > 0$ large enough such that

$$\int_{t}^{t+\omega_i}\left(a_i(s) - [b_i(s) + a_{ii}(s)]\varepsilon - \sum_{j\ne i}^{n} a_{ij}(s)\left[x_{j0}(s-\tau_{ij}(s)) + \varepsilon\right]\right)\mathrm{d}s + \sum_{t\le t_k<t+\omega_i}\ln h_{ik} > \varepsilon, \quad (3.12)$$

for all $t \ge T_2$ and $i = 1, 2, \ldots, n$. Considering (3.5), by the comparison theorem of impulsive differential equation and the conclusion (b) of Lemma 2.1., we obtain for the above $\varepsilon \ge 0$ that there is a $T_3 > T_2$ such that

$$x_i(t) \le x_{i0}(t) + \varepsilon \quad \forall t \ge T_3,\ i = 1, 2, \ldots, n, \quad (3.13)$$

where $x_{i0}(t)$ is a globally uniformly attractive positive solution of system (3.1).

Claim 1. There is a constant $\eta > 0$ such that $\limsup_{t\to\infty} x_i(t) > \eta$ $(i = 1, 2, \ldots, n)$ for any positive solution $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))$ of system (1.1). In fact, if Claim 1 is not true, then there is an integer $k \in \{1, 2, \ldots, n\}$ and a positive solution $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))$ of system (1.1) such that

$$\limsup_{t\to\infty} x_k(t) < \varepsilon. \quad (3.14)$$

Hence, there is a constant $T_4 > T_3$ such that

$$x_k(t) < \varepsilon \quad \forall t \ge T_4. \quad (3.15)$$

On the other hand, by (3.13) there is a $T_5 \ge T_4$ such that

$$x_i(t) \le x_{i0}(t) + \varepsilon \quad \forall t \ge T_5, \quad (3.16)$$

where $i = 1, 2, \ldots, n$ and $i \neq k$. By (3.11) and (3.16), we obtain

$$
\begin{aligned}
x_k(t) &= x_k(T_5+\tau)\exp\Bigg(\int_{T_5+\tau}^{t}\Bigg[a_k(s) - b_k(s)x_k(t) - \sum_{j=1}^{n} a_{kj}(s)x_j\big(s-\tau_{ij}(s)\big)\Bigg]\mathrm{d}s \\
&\qquad + \sum_{T_5+\tau\le t_k\le t} \ln h_{kk}\Bigg) \\
&\ge x_k(T_5+\tau)\exp\Bigg(\int_{T_5+\tau}^{t}\Bigg[a_k(s) - \big(b_k(s)+a_{kk}(s)\big)\varepsilon - \sum_{j=1, j\neq k}^{n} a_{ij}(s)\big(x_{j0}\big(s-\tau_{ij}(s)\big)+\varepsilon\big)\Bigg]\mathrm{d}s \\
&\qquad + \sum_{T_5+\tau\le t_k\le t} \ln h_{kk}\Bigg),
\end{aligned}
\tag{3.17}
$$

for all $t \ge T_5 + \tau$. Thus, from (3.12) we finally obtain $\lim_{t\to\infty} x_k(t) = \infty$, which lead to a contradiction.

Claim 2. There is a constant $\gamma > 0$ such that $\liminf_{t\to\infty} x_i(t) > \gamma$ $(i = 1, 2, \ldots, n)$ for any positive solution of system (1.1).

If Claim 2 is not true, then there is an integer $k \in \{1, 2, \ldots, n\}$ and a sequence of initial function $\{\phi_m\} \subset C_+[-\tau, 0]$ such that

$$
\liminf_{t\to\infty} x_k(t, \phi_m) < \frac{\eta}{m^2} \quad \forall m = 1, 2, \ldots, \tag{3.18}
$$

where constant η is given in Claim 1. By Claim 1, for every m there are two time sequences $s_q^{(m)}$ and $t_q^{(m)}$, satisfying:

$$
0 < s_1^{(m)} < t_1^{(m)} < s_2^{(m)} < t_2^{(m)} < \cdots < s_q^{(m)} < t_q^{(m)} < \cdots, \qquad \lim_{q\to\infty} s_q^{(m)} = \infty, \tag{3.19}
$$

such that

$$
x_k\Big(s_q^{(m)}, \phi_m\Big) \ge \frac{\eta}{m}, \qquad x_k\Big(t_q^{(m)}, \phi_m\Big) \le \frac{\eta}{m^2}, \tag{3.20}
$$

$$
\frac{\eta}{m^2} \le x_k(t, \phi_m) \le \frac{\eta}{m} \quad \forall t \in \Big(s_q^{(m)}, t_q^{(m)}\Big). \tag{3.21}
$$

From the above proof, there is a constant $T^{(m)} \geq T_2$ such that $x_i(t, \phi_m) < M$ $(i = 1, 2, \ldots, n)$ for all $t \geq T^{(m)}$. Further, there is an integer $K_1^{(m)} > 0$ such that $s_q^{(m)} > T^{(m)}$ for all $q > K_1^{(m)}$. From (3.11) and lemma 2.2., we can obtain

$$\begin{aligned} x_k\left(t_q^{(m)}, \phi_m\right) &\geq x_k\left(s_q^{(m)}, \phi_m\right) \exp\left(\int_{s_q^{(m)}}^{t_q^{(m)}} \left[a_k(s) - b_k(s)M - \sum_{j=1}^{n} a_{kj}(s)M\right] \mathrm{d}s + \sum_{s_q^{(m)} \leq t_k \leq t_q^{(m)}} \ln h_{kk}\right) \\ &\geq x_k\left(s_q^{(m)}, \phi_m\right) \exp\left(-(r_1 + d)\left(t_q^{(m)} - s_q^{(m)}\right) - D\right), \end{aligned} \tag{3.22}$$

where $r_1 = \sup_{t \geq 0}\{|a_i(t)| + b_i(t)M + \sum_{j=1}^{n} a_{ij}(t)M\}$. Consequently, from (3.20) we have

$$t_q^{(m)} - s_q^{(m)} \geq \frac{\ln m - D}{r_1 + d} \quad \forall q > K_1^{(m)}. \tag{3.23}$$

By (3.12), there is a large enough $P > 0$ such that for all $t \geq T_2$, $a \geq P$ and $a \in [l w_i, (l+1) w_i)$ and $i = 1, 2, \ldots, n$, then, we obtain

$$\begin{aligned} &\int_{t}^{t+a} \left(a_i(s) - [b_i(s) + a_{ii}(s)]\varepsilon - \sum_{j \neq i}^{n} a_{ij}(s)\left[x_{j0}(s - \tau_{ij}(s)) + \varepsilon\right]\right) \mathrm{d}s + \sum_{t \leq t_k < t+a} \ln h_{ik} \\ &\quad = \int_{t}^{t+l w_i} \left(a_i(s) - [b_i(s) + a_{ii}(s)]\varepsilon - \sum_{j \neq i}^{n} a_{ij}(s)\left[x_{j0}(s - \tau_{ij}(s)) + \varepsilon\right]\right) \mathrm{d}s + \sum_{t \leq t_k < t + l w_i} \ln h_{ik} \\ &\quad\quad + \int_{t+l w_i}^{t+a} \left(a_i(s) - [b_i(s) + a_{ii}(s)]\varepsilon - \sum_{j \neq i}^{n} a_{ij}(s)\left[x_{j0}(s - \tau_{ij}(s)) + \varepsilon\right]\right) \mathrm{d}s + \sum_{t + l w_i \leq t_k < t+a} \ln h_{ik} \\ &\quad > l\varepsilon - r_2 w_i, \end{aligned} \tag{3.24}$$

where $r_2 = \sup_{t \geq 0}\{|a_i(t)| + [b_i(t) + a_{ii}(t)]\varepsilon + \sum_{j \neq i}^{n} a_{ij}(s)[x_{j0}(s - \tau_{ij}(s)) + \varepsilon]\}$. So, we choose $L = 2 + (r_2 w_i / \varepsilon)$ such that for all $l > L$, we have

$$\int_{t}^{t+a} \left(a_i(s) - [b_i(s) + a_{ii}(s)]\varepsilon - \sum_{j \neq i}^{n} a_{ij}(s)\left[x_{j0}(s - \tau_{ij}(s)) + \varepsilon\right]\right) \mathrm{d}s + \sum_{t \leq t_k < t+a} \ln h_{ik} > \varepsilon. \tag{3.25}$$

From (3.23), there is an integer N_0 such that for any $m > N_0$ and $q > K_1^{(m)}$, we have

$$\frac{\eta}{m} < \varepsilon, \qquad t_q^{(m)} - s_q^{(m)} > 2Q, \tag{3.26}$$

where constant $Q > P + \tau$.

So, when $m > N_0$ and $q > K_1^{(m)}$, for any $t \in [s_q^{(m)} + Q + \tau, t_q^{(m)}]$, from (3.11), (3.21), (3.25), and (3.26) we can obtain

$$
\begin{aligned}
x_k\left(t_q^{(m)}, \phi_m\right) = x_k\left(s_q^{(m)} + Q + \tau, \phi_m\right) \\
\times \exp\Bigg(\int_{s_q^{(m)}+Q+\tau}^{t_q^{(m)}} \left[a_k(t) - b_k(t)x_k(t, \phi_m) - \sum_{j=1}^{n} a_{kj}(t)x_{j0}(t - \tau_{kj}(t)), \phi_m \right] \mathrm{d}t \\
+ \sum_{s_q^{(m)}+Q+\tau \le t_k \le t_q^{(m)}} \ln h_{kk} \Bigg)
\end{aligned}
\tag{3.27}
$$

Consequently, from (3.20) and (3.25) it follows

$$
\begin{aligned}
\frac{\eta}{m^2} &\ge \frac{\eta}{m^2} \\
&\quad \times \exp\Bigg(\int_{s_q^{(m)}+Q+\tau}^{t_q^{(m)}} \left[a_k(t) - (b_k(t) + a_{kk}(t))\varepsilon - \sum_{j=1, j \ne k}^{n} a_{kj}(t)x_{j0}(t - \tau_{kj}(t)) + \varepsilon \right] \mathrm{d}t \\
&\quad + \sum_{s_q^{(m)}+Q+\tau \le t_k \le t_q^{(m)}} \ln h_{kk} \Bigg) \\
&> \frac{\eta}{m^2}.
\end{aligned}
\tag{3.28}
$$

This leads to a contradiction. Therefore, Claim 2 is true. This completes the proof.

When system (1.1) degenerates into the periodic case, then we can assume that there is a constant $\omega > 0$ and an integer $q > 0$ such that $a_i(t + \omega) = a_i(t)$, $b_i(t + \omega) = b_i(t)$, $a_{ij}(t + \omega) = a_{ij}(t)$, $t_{k+q} = t_k + \omega$ and $h_{ik+q} = h_{ik}$ for all $t \in R_+$, $k = 1, 2, \ldots$ and $i, j = 1, 2, \ldots, n$. From Remarks 2.3 and 2.4 in [1], we can see the fixed positive solution x_{j0} of system (3.1) can be chosen to be the ω-periodic solution of system (3.1). Therefore, as a consequence of Theorem 3.1. we have the following result.

Corollary 3.2. *Suppose that system* (1.1) *is ω-periodic and for each $i = 1, 2, \ldots, n$,*

$$
\begin{gathered}
\int_0^{\omega} b_i(s)\mathrm{d}s > 0, \\
\int_0^{\omega} \left(a_i(s) - \sum_{j \ne i}^{n} a_{ij}(s)x_{j0}(s - \tau_{ij}(s)) \right) \mathrm{d}s + \sum_{k=1}^{q} \ln h_{ik} > 0.
\end{gathered}
\tag{3.29}
$$

Then, system (1.1) *is permanent.*

4. Numerical Example

In this section, we will give an example to demonstrate the effectiveness of our main results. We consider the following two species competitive system with delays and impulses:

$$\begin{aligned} &\left.\begin{aligned} x_1'(t) &= x_1(t)\left[a_1(t) - b_1(t)x_1(t) - a_{11}(t)x_1(t-\tau_{11}(t)) - a_{12}(t)x_2(t-\tau_{12}(t))\right],\\ x_2'(t) &= x_2(t)\left[a_2(t) - b_2(t)x_2(t) - a_{21}(t)x_1(t-\tau_{21}(t)) - a_{22}(t)x_2(t-\tau_{22}(t))\right], \end{aligned}\right\} \quad t \neq t_k\\ &\left.\begin{aligned} x_1(t_k^+) &= h_{1k}x_1(t_k),\\ x_2(t_k^+) &= h_{2k}x_2(t_k), \end{aligned}\right\} \quad k = 1, 2, \ldots. \end{aligned} \tag{4.1}$$

We take $a_1(t) = 2$, $a_2(t) = b_1(t) = b_2(t) = a_{11}(t) = a_{12}(t) = a_{22}(t) = 1$, $a_{21} = 1 - |\sin(\pi/2)t|$, $\tau_{ij}(t) = 2$, $h_{1k} = e^{-1}$, $h_{2k} = e$, $t_k = k$. Obviously, system (4.1) is periodic with period $\omega = 2$. For $q = 2$, we have $t_{k+q} = t_k + \omega$, $h_{1k+q} = h_{1k}$ and $h_{2k+q} = h_{2k}$ for all $k = 1, 2, \ldots$. Consider the following impulsive logistic systems as the subsystems of system (4.1):

$$\begin{aligned} &\left.\begin{aligned} x_1'(t) &= x_1(t)(2 - x_1(t)),\\ x_2'(t) &= x_2(t)(1 - x_2(t)), \end{aligned}\right\} \quad t \neq k\\ &\left.\begin{aligned} x_1(t^+) &= e^{-1}x_1(t_k),\\ x_2(t^+) &= e x_2(t_k), \end{aligned}\right\} \quad t = k. \end{aligned} \tag{4.2}$$

According to the formula in [1], we can obtain that subsystem (4.2) has a unique globally asymptotically stable positive 2-periodic solution $(x_{10}(t), x_{20}(t))$, which can be expressed in following form:

$$\begin{aligned} x_{10}(t) &= \frac{2x_{10}}{x_{10} + (2 - x_{10})e^{-2(t-k)}}, \quad t \in [k, k+1),\ k = 0, 1, 2, \ldots,\\ x_{20}(t) &= \frac{x_{20}}{x_{20} + (1 - x_{20})e^{-(t-k)}}, \quad t \in [k, k+1),\ k = 0, 1, 2, \ldots, \end{aligned} \tag{4.3}$$

where $x_{10} = (2(e^{-0.2} - e^{-2})/(1 - e^{-2}))$ and $x_{20} = (e - e^{-1})/(1 - e^{-1})$. Since

$$\begin{aligned} &\int_0^{\omega} (a_1(t) - a_{12}(t)x_{20}(t - \tau_{12}(t)))\mathrm{d}t + \sum_{k=1}^{q} \ln h_{1k}\\ &\quad = 2\int_0^1 \left(2 - \frac{x_{20}}{x_{20} + (1 - x_{20})e^{-(t-2)}}\right)\mathrm{d}t + \sum_{k=1}^{2} \ln h_{1k}\\ &\quad \approx 1.5244,\\ &\int_0^{\omega} (a_2(t) - a_{21}(t)x_{10}(t - \tau_{21}(t)))\mathrm{d}t + \sum_{k=1}^{q} \ln h_{2k}\\ &\quad = 2\int_0^1 \left(1 - \left(1 - \sin\frac{\pi}{2}t\right)\frac{2x_{10}}{x_{10} + (2 - x_{10})e^{-2(t-2)}}\right)\mathrm{d}t + \sum_{k=1}^{2} \ln h_{2k}\\ &\quad \approx 3.8398, \end{aligned} \tag{4.4}$$

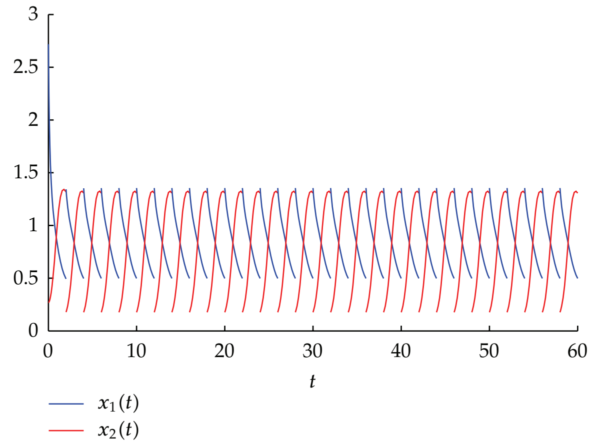

Figure 1: Time series of $x_1(t)$ and $x_2(t)$.

we obtain that all conditions in Corollary 3.2 for system (1.1) holds. Therefore, from Theorem 3.1. we see that system (1.1) is permanent (see Figure 1).

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