Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2012, Article ID 184534, 9 pages
doi:10.1155/2012/184534

Research Article

Coupled Fixed Point Theorems under Weak Contractions

Y. J. Cho,[1] Z. Kadelburg,[2] R. Saadati,[3] and W. Shatanawi[4]

[1] *Department of Mathematics Education and RINS, Gyeongsang National University, Jinju 660-701, Republic of Korea*
[2] *Faculty of Mathematics, University of Belgrade, 11000 Beograd, Serbia*
[3] *Department of Mathematics, Iran University of Science and Technology, Behshahr, Iran*
[4] *Department of Mathematics, Hashemite University, P.O. Box 150459, Zarqa 13115, Jordan*

Correspondence should be addressed to R. Saadati, rsaadati@eml.cc

Received 6 October 2011; Accepted 28 December 2011

Academic Editor: Binggen Zhang

Cho et al. [Comput. Math. Appl. 61(2011), 1254–1260] studied common fixed point theorems on cone metric spaces by using the concept of c-distance. In this paper, we prove some coupled fixed point theorems in ordered cone metric spaces by using the concept of c-distance in cone metric spaces.

1. Introduction

Many fixed point theorems have been proved for mappings on cone metric spaces in the sense of Huang and Zhang [1]. For some more results on fixed point theory and applications in cone metric spaces, we refer the readers to [2–15]. Recently, Bhaskar and Lakshmikantham [16] introduced the concept of a coupled coincidence point of a mapping F from $X \times X$ into X and a mapping g from X into X and studied fixed point theorems in partially ordered metric spaces. For some more results on couple fixed point theorems, refer to [17–23].

Recently, Cho et al. [7] introduced a new concept of c-distance in cone metric spaces, which is a cone version of w-distance of Kada et al. [24] (see also [25]) and proved some fixed point theorems for some contractive type mappings in partially ordered cone metric spaces using the c-distance.

In this paper, we prove some coupled fixed point theorems in ordered cone metric spaces by using the concept of c-distance.

2. Preliminaries

In this paper, assume that E is a real Banach space. Let P be a subset of E with int $(P) \neq \emptyset$. Then P is called a *cone* if the following conditions are satisfied:

(1) P is closed and $P \neq \{\theta\}$;

(2) $a, b \in \mathbf{R}^+$, $x, y \in P$ implies $ax + by \in P$;

(3) $x \in P \cap -P$ implies $x = \theta$.

For a cone P, define the *partial ordering* $\preceq$ with respect to P by $x \preceq y$ if and only if $y - x \in P$. We write $x \prec y$ to indicate that $x \preceq y$ but $x \neq y$, while $x \ll y$ stand for $y - x \in \operatorname{int} P$.

It can be easily shown that λ int $(P) \subseteq \operatorname{int}(P)$ for all positive scalars λ.

Definition 2.1 (see [1]). Let X be a nonempty set. Suppose that the mapping $d : X \times X \to E$ satisfies the following conditions:

(1) $\theta \preceq d(x, y)$ for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$;

(2) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(3) $d(x, y) \preceq d(x, z) + d(y, z)$ for all $x, y, z \in X$.

Then d is called a *cone metric* on X, and (X, d) is called a *cone metric space*.

Definition 2.2 (see [1]). Let (X, d) be a cone metric space. Let (x_n) be a sequence in X and $x \in X$.

(1) If, for any $c \in X$ with $\theta \ll c$, there exists $N \in \mathbf{N}$ such that $d(x_n, x) \ll c$ for all $n \geq N$, then (x_n) is said to be *convergent* to a point $x \in X$ and x is the *limit* of (x_n). We denote this by $\lim_{n \to \infty} x_n = x$ or $x_n \to x$ as $n \to \infty$.

(2) If, for any $c \in E$ with $\theta \ll c$, there exists $N \in \mathbf{N}$ such that $d(x_n, x_m) \ll c$ for all $n, m \geq N$, then (x_n) is called a *Cauchy sequence* in X.

(3) The space (X, d) is called a complete cone metric space if every Cauchy sequence is convergent.

Definition 2.3 (see [7]). Let $(X, \sqsubseteq)$ be a partially ordered set, and let $F : X \times X \to X$ be a function. Then the mapping F is said to have the *mixed monotone property* if $F(x, y)$ is monotone nondecreasing in x and is monotone nonincreasing in y; that is,

$$x_1 \sqsubseteq x_2 \text{ implies } F(x_1, y) \sqsubseteq F(x_2, y) \tag{2.1}$$

for all $y \in X$ and

$$y_1 \sqsubseteq y_2 \text{ implies } F(x, y_2) \sqsubseteq F(x, y_1) \tag{2.2}$$

for all $x \in X$.

Definition 2.4 (see [7]). An element $(x, y) \in X \times X$ is called a *coupled fixed point* of a mapping $F : X \times X \to X$ if $F(x, y) = x$ and $F(y, x) = y$.

Recently, Cho et al. [7] introduced the concept of c-distance on cone metric space (X, d) which is a generalization of w-distance of Kada et al. [24].

Definition 2.5 (see [7]). Let (X, d) be a cone metric space. Then a function $q : X \times X \to E$ is called a *c-distance* on X if the following are satisfied:

(q1) $\theta \preceq q(x, y)$ for all $x, y \in X$;

(q2) $q(x, z) \preceq q(x, y) + q(y, z)$ for all $x, y, z \in X$;

(q3) for any $x \in X$, if there exists $u = u_x \in P$ such that $q(x, y_n) \preceq u$ for each $n \geq 1$, then $q(x, y) \preceq u$ whenever (y_n) is a sequence in X converging to a point $y \in X$;

(q4) for any $c \in E$ with $\theta \ll c$, there exists $e \in E$ with $0 \preceq e$ such that $q(z, x) \ll e$ and $q(z, y) \ll c$ imply $d(x, y) \ll c$.

Cho et al. [7] noticed the following important remark in the concept of c-distance on cone metric spaces.

Remark 2.6 (see [7]). Let q be a c-distance on a cone metric space (X, d). Then

(1) $q(x, y) = q(y, x)$ does not necessarily hold for all $x, y \in X$,

(2) $q(x, y) = \theta$ is not necessarily equivalent to $x = y$ for all $x, y \in X$.

The following lemma is crucial in proving our results.

Lemma 2.7 (see [7]). *Let (X, d) be a cone metric space, and let q be a c-distance on X. Let (x_n) and (y_n) be sequences in X and $x, y, z \in X$. Suppose that (u_n) is a sequence in P converging to θ. Then the following hold:*

(1) *if $q(x_n, y) \preceq u_n$ and $q(x_n, z) \preceq u_n$, then $y = z$;*

(2) *if $q(x_n, y_n) \preceq u_n$ and $q(x_n, z) \preceq u_n$, then (y_n) converges to a point $z \in X$;*

(3) *if $q(x_n, x_m) \preceq u_n$ for each $m > n$, then (x_n) is a Cauchy sequence in X;*

(4) *If $q(y, x_n) \preceq u_n$, then (x_n) is a Cauchy sequence in X.*

3. Main Results

In this section, we prove some coupled fixed point theorems by using c-distance in partially ordered cone metric spaces.

Theorem 3.1. *Let $(X, \sqsubseteq)$ be a partially ordered set, and suppose that (X, d) is a complete cone metric space. Let q be a c-distance on X, and let $F : X \times X \to X$ be a continuous function having the mixed monotone property such that*

$$q(F(x, y), F(x^*, y^*)) \preceq \frac{k}{2}(q(x, x^*) + q(y, y^*)) \tag{3.1}$$

for some $k \in [0, 1)$ and all $x, y, x^*, y^* \in X$ with $(x \sqsubseteq x^*) \wedge (y \sqsupseteq y^*)$ or $(x \sqsupseteq x^*) \wedge (y \sqsubseteq y^*)$. If there exist $x_0, y_0 \in X$ such that $x_0 \sqsubseteq F(x_0, y_0)$ and $F(y_0, x_0) \sqsubseteq y_0$, then F has a coupled fixed point (u, v). Moreover, one has $q(v, v) = \theta$ and $q(u, u) = \theta$.

Proof. Let $x_0, y_0 \in X$ be such that $x_0 \sqsubseteq F(x_0, y_0)$ and $F(y_0, x_0) \sqsubseteq y_0$. Let $x_1 = F(x_0, y_0)$ and $y_1 = F(y_0, x_0)$. Since F has the mixed monotone property, we have $x_0 \sqsubseteq x_1$ and $y_1 \sqsubseteq y_0$. Continuing this process, we can construct two sequences (x_n) and (y_n) in X such that

$$\begin{aligned} x_n &= F(x_{n-1}, y_{n-1}) \sqsubseteq x_{n+1} = F(x_n, y_n), \\ y_{n+1} &= F(y_n, x_n) \sqsubseteq y_n = F(y_{n-1}, x_{n-1}). \end{aligned} \tag{3.2}$$

Let $n \in \mathbf{N}$. Now, by (3.1), we have

$$\begin{aligned} q(x_n, x_{n+1}) &= q(F(x_{n-1}, y_{n-1}), F(x_n, y_n)) \\ &\leq \frac{k}{2}(q(x_{n-1}, x_n) + q(y_{n-1}, y_n)), \\ q(x_{n+1}, x_n) &= q(F(x_n, y_n), F(x_{n-1}, y_{n-1})) \\ &\leq \frac{k}{2}(q(x_n, x_{n-1}) + q(y_n, y_{n-1})). \end{aligned} \tag{3.3}$$

From (3.3), it follows that

$$q(x_n, x_{n+1}) + q(x_{n+1}, x_n) \leq \frac{k}{2}(q(x_{n-1}, x_n) + q(y_{n-1}, y_n) + q(x_n, x_{n-1}) + q(y_n, y_{n-1})). \tag{3.4}$$

Similarly, we have

$$q(y_n, y_{n+1}) + q(y_{n+1}, y_n) \leq \frac{k}{2}(q(x_{n-1}, x_n) + q(y_{n-1}, y_n) + q(x_n, x_{n-1}) + q(y_n, y_{n-1})). \tag{3.5}$$

Thus it follows from (3.4) and (3.5) that

$$\begin{aligned} &q(x_n, x_{n+1}) + q(x_{n+1}, x_n) + q(y_n, y_{n+1}) + q(y_{n+1}, y_n) \\ &\quad \leq k(q(x_{n-1}, x_n) + q(y_{n-1}, y_n) + q(x_n, x_{n-1}) + q(y_n, y_{n-1})). \end{aligned} \tag{3.6}$$

Repeating (3.6) n-times, we get

$$\begin{aligned} &q(x_n, x_{n+1}) + q(x_{n+1}, x_n) + q(y_n, y_{n+1}) + d(y_{n+1}, y_n) \\ &\quad \leq k^n(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)). \end{aligned} \tag{3.7}$$

Thus we have

$$\begin{aligned} q(x_n, x_{n+1}) &\leq k^n(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)), \\ q(y_n, y_{n+1}) &\leq k^n(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)). \end{aligned} \tag{3.8}$$

Let $m, n \in \mathbf{N}$ with $m > n$. Since

$$q(x_n, x_m) \preceq \sum_{i=n}^{m-1} q(x_i, x_{i+1}),$$
$$q(y_n, y_m) \preceq \sum_{i=n}^{m-1} q(y_i, y_{i+1}), \tag{3.9}$$

and $k < 1$, we have

$$q(x_n, x_m) \preceq \frac{k^n}{1-k}\left(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\right),$$
$$q(y_n, y_m) \preceq \frac{k^n}{1-k}\left(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\right). \tag{3.10}$$

From Lemma 2.7 (3), it follows that (x_n) and (y_n) are Cauchy sequences in (X, d). Since X is complete, there exist $u, v \in X$ such that $x_n \to u$ and $y_n \to v$. Since F is continuous, we have

$$x_{n+1} = F(x_n, y_n) \longrightarrow F(u, v),$$
$$y_{n+1} = F(y_n, x_n) \longrightarrow F(v, u). \tag{3.11}$$

By the uniqueness of the limits, we get $u = f(u, v)$ and $v = F(v, u)$. Thus (u, v) is a coupled fixed point of F.

Moreover, by (3.1), we have

$$q(u, u) = q(F(u, v), F(u, v)) \preceq \frac{k}{2}\left(q(u, u) + q(v, v)\right),$$
$$q(v, v) = q(F(v, u), F(v, u)) \preceq \frac{k}{2}\left(q(v, v) + q(u, u)\right). \tag{3.12}$$

Therefore, we get

$$q(u, u) + q(v, v) \preceq k\left(q(v, v) + q(u, u)\right). \tag{3.13}$$

Since $k < 1$, we conclude that $q(u, u) + q(v, v) = \theta$, and hence $q(u, u) = \theta$ and $q(v, v) = \theta$. This completes the proof. □

Theorem 3.2. *In addition to the hypotheses of Theorem 3.1, suppose that any two elements x and y in X are comparable. Then the coupled fixed point has the form (u, u), where $u \in X$.*

Proof. As in the proof of Theorem 3.1, there exists a coupled fixed point $(u, v) \in X \times X$. Here $u = F(u, v)$ and $v = F(v, u)$. By the additional assumption and (3.1), we have

$$q(u, v) = q(F(u, v), F(v, u)) \preceq \frac{k}{2}\left(q(u, v) + q(v, u)\right),$$
$$q(v, u) = q(F(v, u), F(u, v)) \preceq \frac{k}{2}\left(q(v, u) + q(u, v)\right). \tag{3.14}$$

Thus we have

$$q(u,v)+q(v,u)\preceq k\big(q(v,u)+q(u,v)\big). \tag{3.15}$$

Since $k<1$, we get $q(u,v)+q(v,u)=\theta$. Hence $q(u,v)=\theta$ and $q(v,u)=\theta$. Let $u_n=\theta$ and $x_n=u$. Then

$$\begin{aligned} q(x_n,u)&\preceq u_n,\\ q(x_n,v)&\preceq u_n. \end{aligned} \tag{3.16}$$

From Lemma 2.7 (1), we have $u=v$. Hence the coupled fixed point of F has the form (u,u). This completes the proof. □

Theorem 3.3. *Let $(X,\sqsubseteq)$ be a partially ordered set, and suppose that (X,d) is a complete cone metric space. Let q be a c-distance on X, and let $F:X\times X\to X$ be a function having the mixed monotone property such that*

$$q\big(F(x,y),F(x^*,y^*)\big)\preceq\frac{k}{4}\big(q(x,x^*)+q(y,y^*)\big) \tag{3.17}$$

for some $k\in(0,1)$ and all $x,y,x^*,y^*\in X$ with $(x\sqsubseteq x^*)\wedge(y\sqsupseteq y^*)$ or $(x\sqsupseteq x^*)\wedge(y\sqsubseteq y^*)$. Also, suppose that X has the following properties:

(a) *if (x_n) is a nondecreasing sequence in X with $x_n\to x$, then $x_n\sqsubseteq x$ for all $n\geq 1$;*

(b) *if (x_n) is a nonincreasing sequence in X with $x_n\to x$, then $x\sqsubseteq x_n$ for all $n\geq 1$.*

Assume there exist $x_0,y_0\in X$ such that $x_0\sqsubseteq F(x_0,y_0)$ and $F(y_0,x_0)\sqsubseteq y_0$. If $y_0\sqsubseteq x_0$, then F has a coupled fixed point.

Proof. As in the proof of Theorem 3.1, we can construct two Cauchy sequences (x_n) and (y_n) in X such that

$$\begin{aligned} &x_0\sqsubseteq x_1\sqsubseteq\cdots\sqsubseteq x_n\sqsubseteq\cdots,\\ &y_0\sqsupseteq y_1\sqsupseteq\cdots y_n\sqsupseteq\cdots. \end{aligned} \tag{3.18}$$

Moreover, we have that (x_n) converges to a point $u\in X$ and (y_n) converges to $v\in X$,

$$\begin{aligned} q(x_n,x_m)&\preceq\frac{k^n}{1-k}\big(q(x_1,x_0)+q(y_1,y_0)+q(x_0,x_1)+q(y_0,y_1)\big),\\ q(y_n,y_m)&\preceq\frac{k^n}{1-k}\big(q(x_1,x_0)+q(y_1,y_0)+q(x_0,x_1)+q(y_0,y_1)\big) \end{aligned} \tag{3.19}$$

for each $n > m \geq 1$. By (q3), we have

$$
\begin{aligned}
q(x_n, u) &\preceq \frac{k^n}{1-k}\left(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\right),\\
q(y_n, v) &\preceq \frac{k^n}{1-k}\left(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\right),
\end{aligned}
\tag{3.20}
$$

and so

$$
q(x_n, u) + q(y_n, v) \preceq \frac{2k^n}{1-k}\left(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\right).
\tag{3.21}
$$

By the properties (a) and (b), we have

$$
v \sqsubseteq y_n \sqsubseteq y_0 \sqsubseteq x_0 \sqsubseteq x_n \sqsubseteq u.
\tag{3.22}
$$

By (3.17), we have

$$
\begin{aligned}
q(x_n, F(u, v)) &= q\left(F(x_{n-1}, y_{n-1}), F(u, v)\right)\\
&\preceq \frac{k}{4}\left(q(x_{n-1}, u) + q(y_{n-1}, v)\right),\\
q(y_n, F(v, u)) &= q\left(F(y_{n-1}, x_{n-1}), F(v, u)\right)\\
&\preceq \frac{k}{4}\left(q(y_{n-1}, v) + q(x_{n-1}, u)\right).
\end{aligned}
\tag{3.23}
$$

Thus we have

$$
q(x_n, F(u, v)) + q(y_n, F(v, u)) \preceq \frac{k}{2}\left(q(x_{n-1}, u) + q(y_{n-1}, v)\right).
\tag{3.24}
$$

By (3.21), we get

$$
\begin{aligned}
q(x_n, F(u, v)) + q(y_n, F(v, u)) &\preceq \frac{k}{2} \cdot \frac{2k^{n-1}}{1-k}\left(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\right)\\
&= \frac{k^n}{1-k}\left(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\right).
\end{aligned}
\tag{3.25}
$$

Therefore, we have

$$
\begin{aligned}
q(x_n, F(u, v)) &\preceq \frac{k^n}{1-k}\left(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\right),\\
q(y_n, F(v, u)) &\preceq \frac{k^n}{1-k}\left(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\right).
\end{aligned}
\tag{3.26}
$$

By using (3.20) and (3.26), Lemma 2.7 (1) shows that $u = F(u,v)$ and $v = F(v,u)$. Therefore, (u,v) is a coupled fixed point of F. This completes the proof. □

Example 3.4. Let $E = C^1_{\mathbf{R}}[0,1]$ with $\|x\| = \|x\|_\infty + \|x'\|_\infty$ and $P = \{x \in E : x(t) \geq 0, t \in [0,1]\}$. Let $X = [0,+\infty)$ (with usual order), and let $d : X \times X \to E$ be defined by $d(x,y)(t) = |x-y|e^t$. Then (X,d) is an ordered cone metric space (see [7, Example 2.9]). Further, let $q : X \times X \to E$ be defined by $q(x,y)(t) = ye^t$. It is easy to check that q is a c-distance. Consider now the function $F : X \times X \to X$ defined by

$$F(x,y) = \begin{cases} \dfrac{1}{8}(x-y), & x \geq y, \\ 0, & x < y. \end{cases} \tag{3.27}$$

Then it is easy to see that

$$q(F(x,y),F(u,v)) \preceq \frac{1}{6}(q(x,u) + q(y,v)) \tag{3.28}$$

for all $x,y,u,v \in X$ with $(x \leq u) \wedge (y \geq v)$ or $(x \geq u) \wedge (y \leq v)$. Note that $0 \leq F(0,1)$ and $1 \geq F(1,0)$. Thus, by Theorem 3.1, it follows that F has a coupled fixed point in E. Here $(0,0)$ is a coupled fixed point of F.

Acknowledgments

The first author was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (Grant No.: 2011–0021821).

References

[1] L. G. Huang and X. Zhang, "Cone metric spaces and fixed point theorems of contractive mappings," *Journal of Mathematical Analysis and Applications*, vol. 332, no. 2, pp. 1468–1476, 2007.

[2] M. Abbas, Y. J. Cho, and T. Nazir, "Common fixed point theorems for four mappings in TVS-valued cone metric spaces," *Journal of Mathematical Inequalities*, vol. 5, no. 2, pp. 287–299, 2011.

[3] M. Abbas and G. Jungck, "Common fixed point results for noncommuting mappings without continuity in cone metric spaces," *Journal of Mathematical Analysis and Applications*, vol. 341, no. 1, pp. 416–420, 2008.

[4] M. Abbas and B. E. Rhoades, "Fixed and periodic point results in cone metric spaces," *Applied Mathematics Letters*, vol. 22, no. 4, pp. 511–515, 2009.

[5] I. Altun and V. Rakočević, "Ordered cone metric spaces and fixed point results," *Computers & Mathematics with Applications*, vol. 60, no. 5, pp. 1145–1151, 2010.

[6] A. Amini-Harandi and M. Fakhar, "Fixed point theory in cone metric spaces obtained via the scalarization method," *Computers & Mathematics with Applications*, vol. 59, no. 11, pp. 3529–3534, 2010.

[7] Y. J. Cho, R. Saadati, and S. Wang, "Common fixed point theorems on generalized distance in ordered cone metric spaces," *Computers & Mathematics with Applications*, vol. 61, no. 4, pp. 1254–1260, 2011.

[8] E. Graily, S. M. Vaezpour, R. Saadati, and Y. J. Cho, "Generalization of fixed point theorems in ordered metric spaces concerning generalized distance," *Fixed Point Theory and Applications*, vol. 2011, p. 30, 2011.

[9] D. Ilić and V. Rakočević, "Common fixed points for maps on cone metric space," *Journal of Mathematical Analysis and Applications*, vol. 341, no. 2, pp. 876–882, 2008.

[10] S. Janković, Z. Kadelburg, and S. Radenović, "On cone metric spaces: a survey," *Nonlinear Analysis. Theory, Methods & Applications*, vol. 74, no. 7, pp. 2591–2601, 2011.
[11] Z. Kadelburg, M. Pavlović, and S. Radenović, "Common fixed point theorems for ordered contractions and quasicontractions in ordered cone metric spaces," *Computers & Mathematics with Applications*, vol. 59, no. 9, pp. 3148–3159, 2010.
[12] S. Radenović and B. E. Rhoades, "Fixed point theorem for two non-self mappings in cone metric spaces," *Computers & Mathematics with Applications*, vol. 57, no. 10, pp. 1701–1707, 2009.
[13] W. Sintunavarat, Y. J. Cho, and P. Kumam, "Common fixed point theorems for c-distance in ordered cone metric spaces," *Computers and Mathematics with Applications*, vol. 62, no. 4, pp. 1969–1978, 2011.
[14] D. Turkoglu and M. Abuloha, "Cone metric spaces and fixed point theorems in diametrically contractive mappings," *Acta Mathematica Sinica*, vol. 26, no. 3, pp. 489–496, 2010.
[15] D. Turkoglu, M. Abuloha, and T. Abdeljawad, "KKM mappings in cone metric spaces and some fixed point theorems," *Nonlinear Analysis. Theory, Methods & Applications*, vol. 72, no. 1, pp. 348–353, 2010.
[16] T. G. Bhaskar and V. Lakshmikantham, "Fixed point theorems in partially ordered metric spaces and applications," *Nonlinear Analysis. Theory, Methods & Applications*, vol. 65, no. 7, pp. 1379–1393, 2006.
[17] Y. J. Cho, G. He, and N.-j. Huang, "The existence results of coupled quasi-solutions for a class of operator equations," *Bulletin of the Korean Mathematical Society*, vol. 47, no. 3, pp. 455–465, 2010.
[18] Y. J. Cho, M. H. Shah, and N. Hussain, "Coupled fixed points of weakly F-contractive mappings in topological spaces," *Applied Mathematics Letters*, vol. 24, no. 7, pp. 1185–1190, 2011.
[19] M. E. Gordji, Y. J. Cho, and H. Baghani, "Coupled fixed point theorems for contractions in intuitionistic fuzzy normed spaces," *Mathematical and Computer Modelling*, vol. 54, no. 9-10, pp. 1897–1906, 2011.
[20] W. Sintunavarat, Y. J. Cho, and P. Kumam, "Coupled coincidence point theorems for contractions without commutative condition in intuitionistic fuzzy normed spaces," *Fixed Point Theory and Applications*, vol. 2011, p. 81, 2011.
[21] V. Lakshmikantham and L. Cirić, "Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces," *Nonlinear Analysis. Theory, Methods & Applications*, vol. 70, no. 12, pp. 4341–4349, 2009.
[22] W. Shatanawi, "Some common coupled fixed point results in cone metric spaces," *International Journal of Mathematical Analysis*, vol. 4, no. 45–48, pp. 2381–2388, 2010.
[23] W. Shatanawi, "Partially ordered cone metric spaces and coupled fixed point results," *Computers & Mathematics with Applications*, vol. 60, no. 8, pp. 2508–2515, 2010.
[24] O. Kada, T. Suzuki, and W. Takahashi, "Nonconvex minimization theorems and fixed point theorems in complete metric spaces," *Mathematica Japonica*, vol. 44, no. 2, pp. 381–391, 1996.
[25] D. Ilić and V. Rakočević, "Common fixed points for maps on metric space with w-distance," *Applied Mathematics and Computation*, vol. 199, no. 2, pp. 599–610, 2008.