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Research Article

Stability of Various Functional Equations in Non-Archimedean Intuitionistic Fuzzy Normed Spaces

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We define and study the concept of non-Archimedean intuitionistic fuzzy normed space by using the idea of t-norm and t-conorm. Furthermore, by using the non-Archimedean intuitionistic fuzzy normed space, we investigate the stability of various functional equations. That is, we determine some stability results concerning the Cauchy, Jensen and its Pexiderized functional equations in the framework of non-Archimedean IFN spaces.

1. Introduction

The study of stability problem of functional equations originated from a question of Ulam [1] concerning the stability of group homomorphisms.

Let $(G, *)$ be a group and let $(G', \circ, d)$ be a metric group with the metric $d(\cdot, \cdot)$. Given $\epsilon > 0$, does there exist a $\delta(\epsilon) > 0$ such that if a mapping $h : G \to G'$ satisfies the inequality $d(h(x * y), h(x) \circ h(y)) < \delta$ for all $x, y \in G$, then there exists a homomorphism $H : G \to G'$ with $d(h(x), H(x)) < \epsilon$ for all $x \in G$?

If the answer is affirmative, we would say that the equation of homomorphism $H(x*y) = H(x) \circ H(y)$ is stable. The concept of stability for a functional equation arises when we replace the functional equation by an inequality which acts as a perturbation of the equation. Hyers [2] gave a first affirmative partial answer to the question of Ulam for Banach spaces. Hyers theorem was generalized by Aoki [3] for additive mappings and by Rassias [4] for linear mappings by considering an unbounded Cauchy difference. The paper of Rassias has significantly influenced the development of what we now call the Hyers-Ulam-Rassias stability of functional equations. Since then several stability problems for various functional equations have been investigated in [5–20]. Quite recently, the stability problem for Pexiderized quadratic functional equation, Jensen functional equation, cubic functional equation, functional equations associated with inner product spaces, and additive functional equation was considered in [21–26], respectively, in the intuitionistic fuzzy normed spaces; while the idea of intuitionistic fuzzy normed space was introduced in [27] and further studied in [28–34] to deal with some summability problems. Quite recently, Alotaibi and Mohiuddine [35] established the stability of a cubic functional equation in random 2-normed spaces, while the notion of random 2-normed spaces was introduced by Goleţ [36] and further studied in [37–39].

By modifying the definition of intuitionistic fuzzy normed space [27], in this paper, we introduce the notion of non-Archimedean intuitionistic fuzzy normed space and also establish Hyers-Ulam-Rassias-type stability results concerning the Cauchy, Pexiderized Cauchy, Jensen, and Pexiderized Jension functional equations in this new setup. This work indeed presents a relationship between four various disciplines: the theory of fuzzy spaces, the theory of non-Archimedean spaces, the theory of Hyers-Ulam-Rassias stability, and the theory of functional equations.

2. Non-Archimedean Intuitionistic Fuzzy Normed Space

In this section, we introduce the concept of non-Archimedean intuitionistic fuzzy normed space and further define the notions of convergence and Cauchy sequences in this new framework. We will assume throughout this paper that the symbols $\mathbb{N}$, $\mathbb{R}$, $\mathbb{C}$, and $\mathbb{Q}$ will denote the set of all natural, real, complex, and rational numbers, respectively.

A *valuation* is a map $|\cdot|$ from a field $\mathbb{K}$ into $[0,\infty)$ such that 0 is the unique element having the 0 valuation, $|k_1k_2| = |k_1||k_2|$, and the triangle inequality holds, that is, $|k_1 + k_2| \le |k_1| + |k_2|$, for all $k_1, k_2 \in \mathbb{K}$. We say that a field $\mathbb{K}$ is *valued* if $\mathbb{K}$ carries a valuation. The usual absolute values of $\mathbb{R}$ and $\mathbb{C}$ are examples of valuations.

Let us consider a valuation which satisfies stronger condition than the triangle inequality. If the triangle inequality is replaced by $|k_1 + k_2| \le \max\{|k_1|, |k_2|\}$, for all $k_1, k_2 \in \mathbb{K}$ then, a map $|\cdot|$ is called *non-Archimedean* or *ultrametric valuation*, and field is called a *non-Archimedean field*. Clearly $|1| = |-1| = 1$ and $|n| \le 1$, for all $n \in \mathbb{N}$. A trivial example of a non-Archimedean valuation is the map $|\cdot|$ taking everything but 0 into 1 and $|0| = 0$.

Let X be a vector space over a field $\mathbb{K}$ with a non-Archimedean valuation $|\cdot|$. A non-Archimedean normed space is a pair $(X, \|\cdot\|)$, where $\|\cdot\| : X \to [0,\infty)$ is such that

(i) $\|x\| = 0$ if and only if $x = 0$,

(ii) $\|\alpha x\| = |\alpha|\|x\|$ for $\alpha \in \mathbb{K}$, and

(iii) the strong triangle inequality, $\|x + y\| \le \max\{\|x\|, \|y\|\}$, for $x, y \in X$.

In [40], Hensel discovered the p-adic numbers as a number theoretical analogue of power series in complex analysis. The most interesting example of non-Archimedean spaces is p-adic numbers.

Example 2.1. Let p be a prime number. For any nonzero rational number $a = p^r m/n$ such that m and n are coprime to the prime number p, define the p-adic absolute value $|a|_p = p^{-r}$. Then $|\cdot|$ is a non-Archimedean norm on $\mathbb{Q}$. The completion of $\mathbb{Q}$ with respect to $|\cdot|$ is denoted by $\mathbb{Q}_p$ and is called the p-adic number field.

A binary operation $*$: $[0,1] \times [0,1] \to [0,1]$ is said to be a *continuous t-norm* if it satisfies the following conditions.

(a) $*$ is associative and commutative, (b) $*$ is continuous, (c) $a * 1 = a$ for all $a \in [0,1]$, and (d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for each a, b, c, $d \in [0,1]$.

A binary operation $\Diamond$: $[0,1] \times [0,1] \to [0,1]$ is said to be a *continuous t-conorm* if it satisfies the following conditions.

(a') $\Diamond$ is associative and commutative, (b') $\Diamond$ is continuous, (c') $a\Diamond 0 = a$ for all $a \in [0,1]$, and (d') $a\Diamond b \leq c\Diamond d$ whenever $a \leq c$ and $b \leq d$ for each $a, b, c, d \in [0,1]$.

Definition 2.2. The five-tuple $(X, \mathcal{E}, \mathcal{F}, *, \Diamond)$ is said to be an *non-Archimedean intuitionistic fuzzy normed space* (for short, non-Archimedean IFN space) if X is a vector space over a non-Archimedean field $\mathbb{K}$, $*$ is a *continuous t-norm*, $\Diamond$ is a *continuous t-conorm*, and $\mathcal{E}, \mathcal{F}$ are functions from $X \times \mathbb{R}$ to $[0,1]$ satisfying the following conditions. For every $x, y \in X$ and $s, t \in \mathbb{K}$ (i) $\mathcal{E}(x,t) + \mathcal{F}(x,t) \leq 1$, (ii) $\mathcal{E}(x,t) > 0$, (iii) $\mathcal{E}(x,t) = 1$ if and only if $x = 0$, (iv) $\mathcal{E}(\alpha x, t) = \mathcal{E}(x, t/|\alpha|)$ for each $\alpha \neq 0$, (v) $\mathcal{E}(x,t) * \mathcal{E}(y,s) \leq \mathcal{E}(x+y, \max\{t,s\})$, (vi) $\mathcal{E}(x,\cdot) : (0,\infty) \to [0,1]$ is continuous, (vii) $\lim_{t\to\infty}\mathcal{E}(x,t) = 1$ and $\lim_{t\to 0}\mathcal{E}(x,t) = 0$, (viii) $\mathcal{F}(x,t) < 1$, (ix) $\mathcal{F}(x,t) = 0$ if and only if $x = 0$, (x) $\mathcal{F}(\alpha x, t) = \mathcal{F}(x, t/|\alpha|)$ for each $\alpha \neq 0$, (xi) $\mathcal{F}(x,t)\Diamond\mathcal{F}(y,s) \geq \mathcal{F}(x+y, \max\{t,s\})$, (xii) $\mathcal{F}(x,\cdot) : (0,\infty) \to [0,1]$ is continuous, and (xiii) $\lim_{t\to\infty}\mathcal{F}(x,t) = 0$ and $\lim_{t\to 0}\mathcal{F}(x,t) = 1$.

In this case $(\mathcal{E}, \mathcal{F})$ is called a non-Archimedean intuitionistic fuzzy norm.

Example 2.3. Let $(X, \|\cdot\|)$ be a non-Archimedean normed space, $a * b = ab$ and $a\Diamond b = \min\{a + b, 1\}$ for all $a, b \in [0,1]$. For all $x \in X$, every $t > 0$ and $k = 1, 2$, consider the following:

$$\mathcal{E}_k(x,t) = \begin{cases} \dfrac{t}{t + k\|x\|} & \text{if } t > 0, \\ 0 & \text{if } t \leq 0; \end{cases} \qquad \mathcal{F}_k(x,t) = \begin{cases} \dfrac{k\|x\|}{t + k\|x\|} & \text{if } t > 0, \\ 1 & \text{if } t \leq 0. \end{cases} \tag{2.1}$$

Then $(X, \mathcal{E}_k, \mathcal{F}_k, *, \Diamond)$ is a non-Archimedean intuitionistic fuzzy normed space.

Definition 2.4. Let $(X, \mathcal{E}, \mathcal{F}, *, \Diamond)$ be a non-Archimedean intuitionistic fuzzy normed space. Then, a sequence $s = (s_n)$ is said to be

(i) *convergent* in $(X, \mathcal{E}, \mathcal{F}, *, \Diamond)$ or simply $(\mathcal{E}, \mathcal{F})$-*convergent* to $\xi \in X$ if for every $\epsilon > 0$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $\mathcal{E}(s_n - \xi, t) > 1 - \epsilon$ and $\mathcal{F}(s_n - \xi, t) < \epsilon$ for all $n \geq n_0$. In this case we write $(\mathcal{E}, \mathcal{F})$-$\lim_n s_n = \xi$ and ξ is called the $(\mathcal{E}, \mathcal{F})$-limit of $s = (s_n)$.

(ii) Cauchy in $(X, \mathcal{E}, \mathcal{F}, *, \Diamond)$ or simply $(\mathcal{E}, \mathcal{F})$-*Cauchy* if for every $\epsilon > 0$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $\mathcal{E}(s_n - s_m, t) > 1 - \epsilon$ and $\mathcal{F}(s_n - s_m, t) < \epsilon$ for all $n, m \geq n_0$. A non-Archimedean IFN-space $(X, \mathcal{E}, \mathcal{F}, *, \Diamond)$ is said to be *complete* if every $(\mathcal{E}, \mathcal{F})$-Cauchy is $(\mathcal{E}, \mathcal{F})$-*convergent*. In this case $(X, \mathcal{E}, \mathcal{F}, *, \Diamond)$ is called *non-Archimedean intuitionistic fuzzy Banach space*.

3. Stability of Cauchy Functional Equation

In this section, we determine stability result concerning the Cauchy functional equation $f(x + y) = f(x) + f(y)$ in non-Archimedean intuitionistic fuzzy normed space.

Theorem 3.1. *Let X be a linear space over a non-Archimedean field $\mathbb{K}$ and let $(Z, \mathcal{E}', \mathcal{F}')$ be a non-Archimedean IFN space. Suppose that $\varphi : X \times X \to Z$ is a function such that for some $\alpha > 0$ and some positive integer k with $|k| < \alpha$*

$$\begin{aligned} &\mathcal{E}'\left(\varphi\left(k^{-1}x, k^{-1}y\right), t\right) \geq \mathcal{E}'(\varphi(x, y), \alpha t), \\ &\mathcal{F}'\left(\varphi\left(k^{-1}x, k^{-1}y\right), t\right) \leq \mathcal{F}'(\varphi(x, y), \alpha t), \end{aligned} \tag{3.1}$$

for all $x, y \in X$ and $t > 0$. Let $(Y, \mathcal{E}, \mathcal{F})$ be a non-Archimedean intuitionistic fuzzy Banach space over $\mathbb{K}$ and let $f : X \to Y$ be a φ-approximately Cauchy mapping in the sense that

$$\begin{aligned} &\mathcal{E}(f(x+y) - f(x) - f(y), t) \geq \mathcal{E}'(\varphi(x, y), t), \\ &\mathcal{F}(f(x+y) - f(x) - f(y), t) \leq \mathcal{F}'(\varphi(x, y), t), \end{aligned} \tag{3.2}$$

for all $x, y \in X$ and $t > 0$. Then there exists a unique additive mapping $C : X \to Y$ such that

$$\mathcal{E}(f(x) - C(x), t) \geq \mathcal{M}(x, \alpha t), \qquad \mathcal{F}(f(x) - C(x), t) \leq \mathcal{N}(x, \alpha t), \tag{3.3}$$

for all $x \in X$ and $t > 0$, where

$$\begin{aligned} &\mathcal{M}(x, t) = \mathcal{E}'(\varphi(x, x), t) * \mathcal{E}'(\varphi(x, 2x), t) * \cdots * \mathcal{E}'(\varphi(x, (k-1)x), t), \\ &\mathcal{N}(x, t) = \mathcal{F}'(\varphi(x, x), t) \diamond \mathcal{F}'(\varphi(x, 2x), t) \diamond \cdots \diamond \mathcal{F}'(\varphi(x, (k-1)x), t). \end{aligned} \tag{3.4}$$

Proof. By induction on j we will show that for each $x \in X$, $t > 0$ and $j \geq 2$

$$\begin{aligned} &\mathcal{E}(f(jx) - jf(x), t) \geq \mathcal{M}_j(x, t) = \mathcal{E}'(\varphi(x, x), t) * \cdots * \mathcal{E}'(\varphi(x, (j-1)x), t), \\ &\mathcal{F}(f(jx) - jf(x), t) \leq \mathcal{N}_j(x, t) = \mathcal{F}'(\varphi(x, x), t) \diamond \cdots \diamond \mathcal{F}'(\varphi(x, (j-1)x), t). \end{aligned} \tag{3.5}$$

Putting $x = y$ in (3.2), we obtain

$$\mathcal{E}(f(2x) - 2f(x), t) \geq \mathcal{E}'(\varphi(x, x), t), \qquad \mathcal{F}(f(2x) - 2f(x), t) \leq \mathcal{F}'(\varphi(x, x), t), \tag{3.6}$$

for all $x \in X$ and $t > 0$. This proves (3.5) for $j = 2$. Let (3.5) hold for some $j > 2$. Replacing y by jx in (3.2), we get

$$\begin{aligned} &\mathcal{E}(f((j+1)x) - f(x) - f(jx), t) \geq \mathcal{E}'(\varphi(x, jx), t), \\ &\mathcal{F}(f((j+1)x) - f(x) - f(jx), t) \leq \mathcal{F}'(\varphi(x, jx), t), \end{aligned} \tag{3.7}$$

for each $x \in X$ and $t > 0$. Thus

$$\begin{aligned}
&\mathcal{E}(f((j+1)x) - (j+1)f(x), t) \\
&\quad = \mathcal{E}(f((j+1)x) - f(x) - f(jx) + f(jx) - jf(x), t) \\
&\quad \geq \mathcal{E}(f((j+1)x) - f(x) - f(jx), t) * \mathcal{E}(f(jx) - jf(x), t) \\
&\quad \geq \mathcal{E}'(\varphi(x, jx), t) * \mathcal{M}_j(x, t) = \mathcal{M}_{j+1}(x, t), \\
&\mathcal{F}(f((j+1)x) - (j+1)f(x), t) \\
&\quad = \mathcal{F}(f((j+1)x) - f(x) - f(jx) + f(jx) - jf(x), t) \\
&\quad \leq \mathcal{F}(f((j+1)x) - f(x) - f(jx), t) \diamond \mathcal{F}(f(jx) - jf(x), t) \\
&\quad \leq \mathcal{F}'(\varphi(x, jx), t) \diamond \mathcal{N}_j(x, t) = \mathcal{N}_{j+1}(x, t),
\end{aligned} \tag{3.8}$$

for each $x \in X$ and $t > 0$. Hence (3.5) holds for all $j \geq 2$. In particular

$$\mathcal{E}(f(kx) - kf(x), t) \geq \mathcal{M}(x, t), \qquad \mathcal{F}(f(kx) - kf(x), t) \leq \mathcal{N}(x, t). \tag{3.9}$$

Replacing x by $k^{-n-1}x$ in (3.9) and using (3.1), we get

$$\begin{aligned}
\mathcal{E}\left(f(k^{-n}x) - kf\left(k^{-(n+1)}x\right), t\right) \geq \mathcal{M}\left(x, \alpha^{n+1}t\right), \\
\mathcal{F}\left(f(k^{-n}x) - kf\left(k^{-(n+1)}x\right), t\right) \leq \mathcal{N}\left(x, \alpha^{n+1}t\right),
\end{aligned} \tag{3.10}$$

for all $x \in X$, $t > 0$ and $n = 0, 1, 2, \ldots$. Therefore

$$\begin{aligned}
\mathcal{E}\left(k^n f(k^{-n}x) - k^{n+1} f\left(k^{-(n+1)}x\right), t\right) \geq \mathcal{M}\left(x, \frac{\alpha^{n+1}t}{|k|^n}\right), \\
\mathcal{F}\left(k^n f(k^{-n}x) - k^{n+1} f\left(k^{-(n+1)}x\right), t\right) \leq \mathcal{N}\left(x, \frac{\alpha^{n+1}t}{|k|^n}\right),
\end{aligned} \tag{3.11}$$

for all $x \in X$, $t > 0$ and $n = 0, 1, 2, \ldots$. Since

$$\lim_{m \to \infty} \mathcal{M}\left(x, \frac{\alpha^{m+1}t}{|k|^m}\right) = 1, \qquad \lim_{m \to \infty} \mathcal{N}\left(x, \frac{\alpha^{m+1}t}{|k|^m}\right) = 0, \tag{3.12}$$

so (3.11) shows that $(k^n f(k^{-n}x))$ is a Cauchy sequence in non-Archimedean intuitionistic fuzzy Banach space $(Y, \mathcal{E}, \mathcal{F})$. Therefore, we can define a mapping $C : X \to Y$ by $Cx = (\mathcal{E}, \mathcal{F}) - \lim_{n \to \infty} k^n f(k^{-n}x)$. Hence

$$\lim_{n \to \infty} \mathcal{E}(k^n f(k^{-n}x) - C(x), t) = 1, \qquad \lim_{n \to \infty} \mathcal{F}(k^n f(k^{-n}x) - C(x), t) = 0. \tag{3.13}$$

For each $n \geq 1$, $x \in X$ and $t > 0$

$$\begin{aligned}
\mathcal{E}(f(x) - k^n f(k^{-n}x), t) &= \mathcal{E}\left(\sum_{i=0}^{n-1} k^i f\left(k^{-i}x\right) - k^{i+1} f\left(k^{-(i+1)}x\right), t\right) \\
&\geq \prod_{i=0}^{n-1} \mathcal{E}\left(k^i f\left(k^{-i}x\right) - k^{i+1} f\left(k^{-(i+1)}x\right), t\right) \\
&= \mathcal{M}(x, \alpha t), \\
\mathcal{F}(f(x) - k^n f(k^{-n}x), t) &\leq \coprod_{i=0}^{n-1} \mathcal{F}\left(k^i f\left(k^{-i}x\right) - k^{i+1} f\left(k^{-(i+1)}x\right), t\right) \\
&= \mathcal{N}(x, \alpha t),
\end{aligned} \tag{3.14}$$

where $\prod_{j=1}^{n} a_j = a_1 * a_2 * \cdots * a_n$ and $\coprod_{j=1}^{n} a_j = a_1 \Diamond a_2 \Diamond \cdots \Diamond a_n$. It follows from (3.13) and (3.14) that

$$\begin{aligned}
\mathcal{E}(f(x) - C(x), t) &\geq \mathcal{E}(f(x) - k^n f(k^{-n}x), t) * \mathcal{E}(k^n f(k^{-n}x) - C(x), t) \geq \mathcal{M}(x, \alpha t), \\
\mathcal{F}(f(x) - C(x), t) &\leq \mathcal{F}(f(x) - k^n f(k^{-n}x), t) \Diamond \mathcal{F}(k^n f(k^{-n}x) - C(x), t) \leq \mathcal{N}(x, \alpha t),
\end{aligned} \tag{3.15}$$

for each $x \in X$, $t > 0$ and for sufficiently large n; that is, (3.3) holds. Also, from (3.1), (3.2), and (3.13), we have

$$\begin{aligned}
&\mathcal{E}(C(x+y) - C(x) - C(y), t) \\
&\quad \geq \mathcal{E}(C(x+y) - k^n f(k^{-n}(x+y)), t) * \mathcal{E}(k^n f(k^{-n}x) - C(x), t) \\
&\qquad * \mathcal{E}(k^n f(k^{-n}y) - C(y), t) * \mathcal{E}(k^n f(k^{-n}(x+y)) - k^n f(k^{-n}x) - k^n f(k^{-n}y), t) \\
&\quad \geq \mathcal{E}'\left(\varphi(k^{-n}x, k^{-n}y), \frac{t}{|k|^n}\right) \geq \mathcal{E}'\left(\varphi(x, y), \frac{\alpha^n t}{|k|^n}\right), \\
&\mathcal{F}(C(x+y) - C(x) - C(y), t) \\
&\quad \leq \mathcal{F}(C(x+y) - k^n f(k^{-n}(x+y)), t) \Diamond \mathcal{F}(k^n f(k^{-n}x) - C(x), t) \\
&\qquad \Diamond \mathcal{F}(k^n f(k^{-n}y) - C(y), t) \Diamond \mathcal{F}(k^n f(k^{-n}(x+y)) - k^n f(k^{-n}x) - k^n f(k^{-n}y), t) \\
&\quad \leq \mathcal{F}'\left(\varphi(k^{-n}x, k^{-n}y), \frac{t}{|k|^n}\right) \geq \mathcal{F}'\left(\varphi(x, y), \frac{\alpha^n t}{|k|^n}\right),
\end{aligned} \tag{3.16}$$

for all $x, y \in X$, $t > 0$ and for large n. Since

$$\lim_{n \to \infty} \mathcal{E}'\left(\varphi(x, y), \frac{\alpha^n t}{|k|^n}\right) = 1, \qquad \lim_{n \to \infty} \mathcal{F}'\left(\varphi(x, y), \frac{\alpha^n t}{|k|^n}\right) = 0, \tag{3.17}$$

which shows that C is additive. Now if $C' : X \to Y$ is another additive mapping such that

$$\mathcal{E}(C'(x) - f(x), t) \geq \mathcal{M}(x, t), \qquad \mathcal{F}(C'(x) - f(x), t) \geq \mathcal{N}(x, t), \tag{3.18}$$

for all $x \in X$ and $t > 0$. Then, for all $x \in X$, $t > 0$ and $n \in \mathbb{N}$, we have

$$\begin{aligned}
\mathcal{E}(C(x) - C'(x), t) &\geq \mathcal{E}(C(x) - k^n f(k^{-n}x), t) * \mathcal{E}(k^n f(k^{-n}x) - C'(x), t) \\
&\geq \mathcal{E}\left(C(k^{-n}x) - f(k^{-n}x), \frac{t}{|k|^n}\right) * \mathcal{E}\left(f(k^{-n}x) - C'(k^{-n}x), \frac{t}{|k|^n}\right) \\
&\geq \mathcal{M}\left(k^{-n}x, \frac{\alpha t}{|k|^n}\right) \geq \mathcal{M}\left(x, \frac{\alpha^{n+1}t}{|k|^n}\right), \\
\mathcal{F}(C(x) - C'(x), t) &\leq \mathcal{F}(C(x) - k^n f(k^{-n}x), t) \diamond \mathcal{F}(k^n f(k^{-n}x) - C'(x), t) \\
&\leq \mathcal{F}\left(C(k^{-n}x) - f(k^{-n}x), \frac{t}{|k|^n}\right) \diamond \mathcal{F}\left(f(k^{-n}x) - C'(k^{-n}x), \frac{t}{|k|^n}\right) \\
&\geq \mathcal{N}\left(k^{-n}x, \frac{\alpha t}{|k|^n}\right) \leq \mathcal{N}\left(x, \frac{\alpha^{n+1}t}{|k|^n}\right).
\end{aligned} \tag{3.19}$$

Therefore

$$\lim_{n\to\infty} \mathcal{M}\left(x, \frac{\alpha^{n+1}t}{|k|^n}\right) = 1, \qquad \lim_{n\to\infty} \mathcal{N}\left(x, \frac{\alpha^{n+1}t}{|k|^n}\right) = 0. \tag{3.20}$$

Hence $C(x) = C'(x)$ for all $x \in X$. □

Corollary 3.2. *Let X be a linear space over non-Archimedean field $\mathbb{K}$ and let $(Y, \|\cdot\|)$ be a non-Archimedean normed space. Suppose that a function $\varphi : X \times X \to \mathbb{R}^+$ satisfies*

$$\varphi\left(k^{-1}x, k^{-1}y\right) \leq \alpha^{-1}\varphi(x, y), \tag{3.21}$$

for all $x, y \in X$, where $\alpha > 0$ and k is an integer with $|k| < \alpha$. If a map $f : X \to Y$ satisfies

$$\|f(x+y) - f(x) - f(y)\| \leq \varphi(x, y), \tag{3.22}$$

for all $x, y \in X$, then there exists a unique additive mapping $C : X \to Y$ satisfies

$$\|f(x) - C(x)\| \leq \frac{1}{\alpha}\max\{\varphi(x, x) * \varphi(x, 2x) * \cdots * \varphi(x, (k-1)x)\}. \tag{3.23}$$

Proof. Consider the non-Archimedean intuitionistic fuzzy norm

$$\mathcal{E}(y,t) = \begin{cases} \dfrac{t}{t+\|y\|} & \text{if } t>0, \\ 0 & \text{if } t\le 0; \end{cases} \qquad \mathcal{F}(x,t) = \begin{cases} \dfrac{\|y\|}{t+\|y\|} & \text{if } t>0, \\ 1 & \text{if } t\le 0, \end{cases} \tag{3.24}$$

on Y. Let $Z = \mathbb{R}$ and let the function $\mathcal{E}', \mathcal{F}' : \mathbb{R}\times\mathbb{R} \to [0,1]$ be defined by

$$\mathcal{E}'(z,t) = \begin{cases} \dfrac{t}{t+|z|} & \text{if } t>0, \\ 0 & \text{if } t\le 0; \end{cases} \qquad \mathcal{F}'(z,t) = \begin{cases} \dfrac{|z|}{t+|z|} & \text{if } t>0, \\ 1 & \text{if } t\le 0. \end{cases} \tag{3.25}$$

Then $(\mathcal{E}', \mathcal{F}')$ is a non-Archimedean intuitionistic fuzzy norm on $\mathbb{R}$. The result follows from the fact that (3.21), (3.22), and (3.23) are equivalent to (3.1), (3.2), and (3.3), respectively. □

Example 3.3. Let X be a linear space over non-Archimedean field $\mathbb{K}$ and let $(Y, \|\cdot\|)$ be a non-Archimedean normed space. Suppose that a function $f : X \to Y$ satisfies

$$\|f(x+y) - f(x) - f(y)\| \le \|x\|^p + \|y\|^p, \tag{3.26}$$

for all $x, y \in X$ and $p \in [0,1)$. Suppose that there exists an integer k such that $|k| < 1$. Since $p < 1$, by applying Corollary 3.2 for $\varphi(x,y) = \|x\|^p + \|y\|^p$, we observe that (3.21) holds for $\alpha = |k|^p$. Inequality (3.23) assures the existence of a unique additive mapping $C : X \to Y$ such that

$$\|f(x) - C(x)\| \le \frac{1+(k-1)^p}{|k|^p}\|x\|^p, \tag{3.27}$$

for all $x \in X$.

4. Stability of Pexiderized Cauchy Functional Equation

The functional equation $f(x+y) = g(x) + h(y)$ is said to be Pexiderized Cauchy, where f, g, and h are mappings between linear spaces. In the case $f = g = h$, it is called Cauchy functional equation.

Theorem 4.1. *Let X be a linear space over a non-Archimedean field $\mathbb{K}$ and let $(Y, \mathcal{E}, \mathcal{F})$ be a non-Archimedean intuitionistic fuzzy Banach space. Suppose that f, g, and h are mappings from X to Y with $f(0) = g(0) = h(0) = 0$. Suppose that φ is a function from $X \times X$ to a non-Archimedean IFN space $(Z, \mathcal{E}', \mathcal{F}')$ such that*

$$\begin{aligned} &\mathcal{E}(f(x+y) - g(x) - h(y), t) \ge \mathcal{E}'(\varphi(x,y), t), \\ &\mathcal{F}(f(x+y) - g(x) - h(y), t) \le \mathcal{F}'(\varphi(x,y), t), \end{aligned} \tag{4.1}$$

for all $x, y \in X$ and $t > 0$. If

$$\mathcal{E}'\left(\varphi\left(k^{-1}x, k^{-1}y\right), t\right) \ge \mathcal{E}'(\varphi(x,y), \alpha t), \qquad \mathcal{F}'\left(\varphi\left(k^{-1}x, k^{-1}y\right), t\right) \le \mathcal{F}'(\varphi(x,y), \alpha t), \tag{4.2}$$

for some positive real number $\alpha > 0$ and some positive integer k with $|k| < \alpha$, then there exists a unique additive mapping $C : X \to Y$ such that

$$\mathcal{E}(f(x) - C(x), t) \geq \mathcal{M}(x, \alpha t), \qquad \mathcal{F}(f(x) - C(x), t) \leq \mathcal{N}(x, \alpha t), \tag{4.3}$$

$$\mathcal{E}(g(x) - C(x), t) \geq \mathcal{M}(x, \min\{1, \alpha\} t), \qquad \mathcal{F}(g(x) - C(x), t) \leq \mathcal{N}(x, \min\{1, \alpha\} t), \tag{4.4}$$

$$\mathcal{E}(h(x) - C(x), t) \geq \mathcal{M}(x, \min\{1, \alpha\} t), \qquad \mathcal{F}(h(x) - C(x), t) \leq \mathcal{N}(x, \min\{1, \alpha\} t), \tag{4.5}$$

for all $x \in X$ and $t > 0$, where

$$\begin{aligned} \mathcal{M}(x,t) = {} & \mathcal{E}'(\varphi(x,x),t) * \cdots * \mathcal{E}'(\varphi(x,(k-1)x),t) * \mathcal{E}'(\varphi(0,x),t) * \cdots * \mathcal{E}'(\varphi(0,(k-1)x),t) \\ & * \mathcal{E}'(\varphi(x,0),t) * \cdots * \mathcal{E}'(\varphi((k-1)x,0),t), \\ \mathcal{N}(x,t) = {} & \mathcal{F}'(\varphi(x,x),t) \diamond \cdots \diamond \mathcal{F}'(\varphi(x,(k-1)x),t) \diamond \mathcal{F}'(\varphi(0,x),t) \diamond \cdots \diamond \mathcal{F}'(\varphi(0,(k-1)x),t) \\ & \diamond \mathcal{F}'(\varphi(x,0),t) \diamond \cdots \diamond \mathcal{F}'(\varphi((k-1)x,0),t). \end{aligned} \tag{4.6}$$

Proof. Put $y = 0$ in (4.1). Then, for all $x \in X$ and $t > 0$

$$\mathcal{E}(f(x) - g(x), t) \geq \mathcal{E}'(\varphi(x,0), t), \qquad \mathcal{F}(f(x) - g(x), t) \leq \mathcal{F}'(\varphi(x,0), t). \tag{4.7}$$

For $x = 0$, (4.1) becomes

$$\mathcal{E}(f(y) - h(y), t) \geq \mathcal{E}'(\varphi(0,y), t), \qquad \mathcal{F}(f(y) - h(y), t) \leq \mathcal{F}'(\varphi(0,y), t), \tag{4.8}$$

for all $y \in X$ and $t > 0$. Combining (4.1), (4.7), and (4.8), we obtain

$$\begin{aligned} & \mathcal{E}(f(x+y) - f(x) - f(y), t) \geq \mathcal{E}'(\varphi(x,y),t) * \mathcal{E}'(\varphi(x,0),t) * \mathcal{E}'(\varphi(0,y),t), \\ & \mathcal{F}(f(x+y) - f(x) - f(y), t) \leq \mathcal{F}'(\varphi(x,y),t) \diamond \mathcal{F}'(\varphi(x,0),t) \diamond \mathcal{F}'(\varphi(0,y),t), \end{aligned} \tag{4.9}$$

for each $x, y \in X$ and $t > 0$. Replacing $\mathcal{E}'(\varphi(x,y),t)$ and $\mathcal{F}'(\varphi(x,y),t)$ by $\mathcal{E}'(\varphi(x,y),t) * \mathcal{E}'(\varphi(x,0),t) * \mathcal{E}'(\varphi(0,y),t)$ and $\mathcal{F}'(\varphi(x,y),t) \diamond \mathcal{F}'(\varphi(x,0),t) \diamond \mathcal{F}'(\varphi(0,y),t)$, respectively, in Theorem 3.1, we can find that there exists a unique additive mapping $C : X \to Y$ that satisfies (4.3). From (4.3) and (4.7), we see that

$$\begin{aligned} & \mathcal{E}(g(x) - T(x), t) \geq \mathcal{E}(g(x) - f(x), t) * \mathcal{E}(f(x) - T(x), t) \geq \mathcal{M}(x,t), \\ & \mathcal{F}(g(x) - T(x), t) \leq \mathcal{F}(g(x) - f(x), t) \diamond \mathcal{F}(f(x) - T(x), t) \leq \mathcal{N}(x,t), \end{aligned} \tag{4.10}$$

for all $x, y \in X$ and $t > 0$, which proves (4.4). Similarly, we can prove (4.5). □

Corollary 4.2. *Let X be a linear space over a non-Archimedean field $\mathbb{K}$ and let $(Z, \mathcal{E}', \mathcal{F}')$ be a non-Archimedean IFN space. Let $(Y, \mathcal{E}, \mathcal{F})$ be a non-Archimedean intuitionistic fuzzy Banach space.*

Suppose that f, g and h are functions from X to Y such that $f(0) = g(0) = h(0) = 0$, and there is an integer k with $|k| < 1$ and satisfies

$$\mathcal{E}(f(x+y) - g(x) - h(y), t) \geq \mathcal{E}'(\|x\|^r \|y\|^s z_\circ, t),$$
$$\mathcal{F}(f(x+y) - g(x) - h(y), t) \leq \mathcal{F}'(\|x\|^r \|y\|^s z_\circ, t), \tag{4.11}$$

for all $x, y \in X$, $t > 0$ and for some fixed $z_\circ \in Z$ and $r, s \geq 0$ with $r + s < 1$. Then there exists a unique additive mapping $T : X \to Y$ such that

$$\mathcal{E}(f(x) - T(x), t) \geq \mathcal{E}'((k-1)^s \|x\|^{r+s} z_\circ, |k|^{r+s} t),$$
$$\mathcal{F}(f(x) - T(x), t) \leq \mathcal{F}'((k-1)^s \|x\|^{r+s} z_\circ, |k|^{r+s} t),$$
$$\mathcal{E}(g(x) - T(x), t) \geq \mathcal{E}'((k-1)^s \|x\|^{r+s} z_\circ, |k|^{r+s} t),$$
$$\mathcal{F}(g(x) - T(x), t) \leq \mathcal{F}'((k-1)^s \|x\|^{r+s} z_\circ, |k|^{r+s} t), \tag{4.12}$$
$$\mathcal{E}(h(x) - T(x), t) \geq \mathcal{E}'((k-1)^s \|x\|^{r+s} z_\circ, |k|^{r+s} t),$$
$$\mathcal{F}(h(x) - T(x), t) \leq \mathcal{F}'((k-1)^s \|x\|^{r+s} z_\circ, |k|^{r+s} t),$$

for all $x \in X$ and $t > 0$.

Proof. Let the function $\varphi : X \times X \to Z$ be defined by $\varphi(x, y) = \|x\|^r \|y\|^s z_0$ for all $x, y \in X$ and $z_\circ$ is a fixed unit vector in Z. Then (4.1) holds. Since

$$\mathcal{E}'\left(\varphi\left(k^{-1}x, k^{-1}y\right), t\right) = \mathcal{E}'\left(\left(\left\|k^{-1}x\right\|^r \left\|k^{-1}y\right\|^s\right) z_\circ, t\right) = \mathcal{E}'((\|x\|^r \|y\|^s) z_\circ, |k|^{r+s} t),$$
$$\mathcal{F}'\left(\varphi\left(k^{-1}x, k^{-1}y\right), t\right) = \mathcal{F}'((\|x\|^r \|y\|^s) z_\circ, |k|^{r+s} t), \tag{4.13}$$

for each $x, y \in X$ and $t > 0$. If $\alpha = |k|^{r+s}$ and $r + s < 1$, then $\alpha > |k|$ holds. It follows from Theorem 4.1 that there exists a unique additive mapping $C : X \to Y$ such that (4.3)–(4.5) hold. □

5. Stability of Jensen Functional Equation

The stability problem for the Jensen functional equation was first proved by Kominek [13] and since then several generalizations and applications of this notion have been investigated by various authors, namely, Jung [12], Mohiuddine [23], Parnami and Vasudeva [41], and many others. The Jensen functional equation is $2f((x+y)/2) = f(x) + f(y)$, where f is a mapping between linear spaces. It is easy to see that a mapping $f : X \to Y$ between linear spaces with $f(0) = 0$ satisfies the Jensen equation if and only if it is additive (cf. [41]).

Theorem 5.1. *Let X be a linear space over a non-Archimedean field $\mathbb{K}$ and let $(Z, \mathcal{E}', \mathcal{F}')$ be a non-Archimedean IFN space. Suppose that $\varphi : X \times X \to Z$ is a function such that for some $\alpha > 0$ and* *some positive integer k with $|k| < \alpha$ satisfies (3.1). Suppose that $(Y, \mathcal{E}, \mathcal{F})$ is a non-Archimedean intuitionistic fuzzy Banach space. If a map $f : X \to Y$ satisfies*

$$\begin{aligned}\mathcal{E}\left(2f\left(\frac{x+y}{2}\right) - f(x) - f(y), t\right) &\geq \mathcal{E}'(\varphi(x,y), t),\\ \mathcal{F}\left(2f\left(\frac{x+y}{2}\right) - f(x) - f(y), t\right) &\leq \mathcal{F}'(\varphi(x,y), t),\end{aligned} \tag{5.1}$$

for all $x, y \in X$ and $t > 0$, then there exists a unique additive mapping $C : X \to Y$ such that

$$\mathcal{E}(f(x) - f(0) - C(x), t) \geq \mathcal{M}(x, \alpha t), \qquad \mathcal{F}(f(x) - f(0) - C(x), t) \leq \mathcal{N}(x, \alpha t), \tag{5.2}$$

for all $x \in X$ and $t > 0$, where

$$\begin{aligned}\mathcal{M}(x,t) = {}& \mathcal{E}'(\varphi(x,x),t) * \mathcal{E}'(\varphi(x,2x),t) * \cdots * \mathcal{E}'(\varphi(x,(k-1)x),t) * \mathcal{E}'(\varphi(2x,0),t)\\ & * \mathcal{E}'(\varphi(3x,0),t) * \cdots * \mathcal{E}'(\varphi(kx,0),t),\end{aligned} \tag{5.3}$$

$$\begin{aligned}\mathcal{N}(x,t) = {}& \mathcal{F}'(\varphi(x,x),t) \Diamond \mathcal{F}'(\varphi(x,2x),t) \Diamond \cdots \Diamond \mathcal{F}'(\varphi(x,(k-1)x),t) \Diamond \mathcal{F}'(\varphi(2x,0),t)\\ & \Diamond \mathcal{F}'(\varphi(3x,0),t) \Diamond \cdots \Diamond \mathcal{F}'(\varphi(kx,0),t).\end{aligned} \tag{5.4}$$

Proof. Suppose that $g(x) = f(x) - f(0)$ for all $x \in X$. Then

$$\begin{aligned}\mathcal{E}\left(2g\left(\frac{x+y}{2}\right) - g(x) - g(y), t\right) &\geq \mathcal{E}'(\varphi(x,y), t),\\ \mathcal{F}\left(2g\left(\frac{x+y}{2}\right) - g(x) - g(y), t\right) &\leq \mathcal{F}'(\varphi(x,y), t),\end{aligned} \tag{5.5}$$

for all $x, y \in X$ and $t > 0$. Replacing x by $x + y$ and y by 0 in (5.5), then, for all $x, y \in X$ and $t > 0$, we have

$$\begin{aligned}\mathcal{E}\left(2g\left(\frac{x+y}{2}\right) - g(x+y), t\right) &\geq \mathcal{E}'(\varphi(x+y,0), t),\\ \mathcal{F}\left(2g\left(\frac{x+y}{2}\right) - g(x+y), t\right) &\leq \mathcal{F}'(\varphi(x+y,0), t).\end{aligned} \tag{5.6}$$

From (5.5) and (5.6), we conclude that

$$\begin{aligned}\mathcal{E}(g(x+y) - g(x) - g(y), t) &\geq \mathcal{E}'(\varphi(x,y), t) * \mathcal{E}'(\varphi(x+y,0), t),\\ \mathcal{F}(g(x+y) - g(x) - g(y), t) &\leq \mathcal{F}'(\varphi(x,y), t) \Diamond F'(\varphi(x+y,0), t),\end{aligned} \tag{5.7}$$

for all $x, y \in X$ and $t > 0$. Proceeding the same lines as in the proof of Theorem 3.1, one can show that there exists a unique additive mapping $C : X \to Y$ such that

$$\begin{aligned}\mathcal{E}(f(x) - f(0) - C(x), t) &= \mathcal{E}(g(x) - T(x), \alpha t) \geq \mathcal{M}(x, t),\\ \mathcal{F}(f(x) - f(0) - C(x), t) &= \mathcal{F}(g(x) - T(x), \alpha t) \leq \mathcal{N}(x, t),\end{aligned} \tag{5.8}$$

for all $x \in X$ and $t > 0$. □

6. Stability of Pexiderized Jensen Functional Equation

The functional equation $2f((x+y)/2) = g(x) + h(y)$ is said to be Pexiderized Jensen, where f, g, and h are mappings between linear spaces. In the case $f = g = h$, it is called Jensen functional equation.

Theorem 6.1. *Let X be a linear space over a non-Archimedean field $\mathbb{K}$ and let $(Y, \mathcal{E}, \mathcal{F})$ be a non-Archimedean intuitionistic fuzzy Banach space. Suppose that f, g, and h are mappings from X to Y with $f(0) = g(0) = h(0) = 0$. Let $(Z, \mathcal{E}', \mathcal{F}')$ be non-Archimedean IFN space. Suppose that $\varphi : X \times X \to Z$ is a function such that for some $\alpha > 0$, and some positive integer k with $|k| < \alpha$ satisfies* (3.1) *and inequality*

$$\begin{aligned}\mathcal{E}\left(2f\left(\frac{x+y}{2}\right) - g(x) - h(y), t\right) &\geq \mathcal{E}'(\varphi(x, y), t),\\ \mathcal{F}\left(2f\left(\frac{x+y}{2}\right) - g(x) - h(y), t\right) &\leq \mathcal{F}'(\varphi(x, y), t),\end{aligned} \tag{6.1}$$

for all $x, y \in X$ and $t > 0$. Then there exists a unique additive mapping $C : X \to Y$ such that

$$\mathcal{E}(f(x) - C(x), t) \geq \mathcal{M}(x, \alpha t), \qquad \mathcal{F}(f(x) - C(x), t) \leq \mathcal{N}(x, \alpha t), \tag{6.2}$$

$$\begin{aligned}\mathcal{E}(g(x) - C(x), t) &\geq \mathcal{M}\left(\frac{x}{2}, \frac{\alpha t}{2}\right) * \mathcal{E}'(\varphi(x, 0), t),\\ \mathcal{F}(g(x) - C(x), t) &\leq \mathcal{N}\left(\frac{x}{2}, \frac{\alpha t}{2}\right) \diamond \mathcal{F}'(\varphi(x, 0), t),\end{aligned} \tag{6.3}$$

$$\begin{aligned}\mathcal{E}(h(x) - C(x), t) &\geq \mathcal{M}\left(\frac{x}{2}, \frac{\alpha t}{2}\right) * \mathcal{E}'(\varphi(0, x), t),\\ \mathcal{F}(h(x) - C(x), t) &\leq \mathcal{N}\left(\frac{x}{2}, \frac{\alpha t}{2}\right) \diamond \mathcal{F}'(\varphi(0, x), t),\end{aligned} \tag{6.4}$$

for all $x \in X$ *and* $t > 0$, *where*

$$\begin{aligned}\mathcal{M}(x,t) &= \prod_{m=1}^{k-1}\left\{\mathcal{E}'(\varphi(x,mx),|2|t) * \mathcal{E}'(\varphi(mx,mx),|2|t)\right\}\\&\quad * \prod_{m=0}^{k}\left\{\mathcal{E}'(\varphi(mx,0),|2|t) * \mathcal{E}'(\varphi(0,mx),|2|t)\right\},\\\mathcal{N}(x,t) &= \coprod_{m=1}^{k-1}\left\{\mathcal{E}'(\varphi(x,mx),|2|t)\diamond\mathcal{E}'(\varphi(mx,mx),|2|t)\right\}\\&\quad \diamond\coprod_{m=0}^{k}\left\{\mathcal{E}'(\varphi(mx,0),|2|t)\diamond\mathcal{E}'(\varphi(0,mx),|2|t)\right\}.\end{aligned} \tag{6.5}$$

Proof. Put $y = x$ in (6.1). Then, for all $x \in X$ and $t > 0$

$$\begin{aligned}\mathcal{E}(2f(x) - g(x) - h(x), t) &\geq \mathcal{E}'(\varphi(x,x),t),\\\mathcal{F}(2f(x) - g(x) - h(x), t) &\leq \mathcal{F}'(\varphi(x,x),t).\end{aligned} \tag{6.6}$$

Replacing x by y in (6.1), we get

$$\begin{aligned}\mathcal{E}(2f(y) - g(y) - h(y), t) &\geq \mathcal{E}'(\varphi(y,y),t),\\\mathcal{F}(2f(y) - g(y) - h(y), t) &\leq \mathcal{F}'(\varphi(y,y),t),\end{aligned} \tag{6.7}$$

for all $y \in X$ and $t > 0$. Again replacing x by y as well as y by x in (6.1), we get

$$\begin{aligned}\mathcal{E}\left(2f\left(\frac{x+y}{2}\right) - g(y) - h(x), t\right) &\geq \mathcal{E}'(\varphi(y,x),t),\\\mathcal{F}\left(2f\left(\frac{x+y}{2}\right) - g(y) - h(x), t\right) &\leq \mathcal{F}'(\varphi(y,x),t),\end{aligned} \tag{6.8}$$

for all $x, y \in X$ and $t > 0$. It follows from (6.1) and (6.6)–(6.8) that

$$\begin{aligned}&\mathcal{E}\left(4f\left(\frac{x+y}{2}\right) - 2f(x) - 2f(y), t\right)\\&\quad\geq \mathcal{E}'(\varphi(x,x),t) * \mathcal{E}'(\varphi(x,y),t) * \mathcal{E}'(\varphi(y,y),t) * \mathcal{E}'(\varphi(y,x),t),\\&\mathcal{F}\left(4f\left(\frac{x+y}{2}\right) - 2f(x) - 2f(y), t\right)\\&\quad\leq \mathcal{F}'(\varphi(x,x),t)\diamond\mathcal{F}'(\varphi(x,y),t)\diamond\mathcal{F}'(\varphi(y,y),t)\diamond\mathcal{F}'(\varphi(y,x),t).\end{aligned} \tag{6.9}$$

Thus, for all $x, y \in X$ and $t > 0$,

$$\begin{aligned}
&\mathcal{E}\left(2f\left(\frac{x+y}{2}\right) - f(x) - f(y), t\right) \\
&\quad \geq \mathcal{E}'(\varphi(x,x), |2|t) * \mathcal{E}'(\varphi(x,y), |2|t) * \mathcal{E}'(\varphi(y,y), |2|t) * \mathcal{E}'(\varphi(y,x), |2|t), \\
&\mathcal{F}\left(2f\left(\frac{x+y}{2}\right) - f(x) - f(y), t\right) \\
&\quad \leq \mathcal{F}'(\varphi(x,x), |2|t) \diamond \mathcal{F}'(\varphi(x,y), |2|t) \diamond \mathcal{F}'(\varphi(y,y), |2|t) \diamond \mathcal{F}'(\varphi(y,x), |2|t).
\end{aligned} \tag{6.10}$$

Proceeding the same argument used in Theorem 5.1 shows that there exists a unique additive mapping $C : X \to Y$ such that (6.2) holds. Therefore

$$\mathcal{E}\left(2f\left(\frac{x}{2}\right) - C(x), t\right) \geq \mathcal{M}\left(\frac{x}{2}, \frac{\alpha t}{2}\right), \qquad \mathcal{F}\left(2f\left(\frac{x}{2}\right) - C(x), t\right) \leq \mathcal{N}\left(\frac{x}{2}, \frac{\alpha t}{2}\right), \tag{6.11}$$

for all $x \in X$ and $t > 0$. Put $y = 0$ in (6.1), we get

$$\mathcal{E}\left(2f\left(\frac{x}{2}\right) - g(x), t\right) \geq \mathcal{E}'(\varphi(x,0), t), \qquad \mathcal{F}\left(2f\left(\frac{x}{2}\right) - g(x), t\right) \leq \mathcal{F}'(\varphi(x,0), t), \tag{6.12}$$

for all $x \in X$ and $t > 0$. It follows from (6.11) and (6.12) that (6.3) holds. Similarly we can show that (6.4) holds. □

Corollary 6.2. *Let X be a non-Archimedean normed space. Suppose that $f, g, h : X \to Y$ such that $f(0) = g(0) = h(0) = 0$, and there is an integer k with $|k| < 1$ and satisfies*

$$\left\| 2f\left(\frac{x+y}{2}\right) - g(x) - h(y) \right\| \leq \epsilon, \tag{6.13}$$

for all $x, y \in X$. Then there exists a unique additive mapping $C : X \to Y$ such that

$$\| f(x) - C(x) \| \leq \epsilon, \qquad \| g(x) - C(x) \| \leq \epsilon, \qquad \| h(x) - C(x) \| \leq \epsilon, \tag{6.14}$$

for all $x \in X$.

Proof. Let the function $\mathcal{E}, \mathcal{F} : \mathbb{Y} \times \mathbb{R} \to [0,1]$ be defined by

$$\mathcal{E}(x,t) = \begin{cases} \dfrac{t}{t + \|x\|} & \text{if } t > 0, \\ 0 & \text{if } t \leq 0; \end{cases} \qquad \mathcal{F}(x,t) = \begin{cases} \dfrac{\|x\|}{t + \|x\|} & \text{if } t > 0, \\ 1 & \text{if } t \leq 0, \end{cases} \tag{6.15}$$

on Y. It is easy to see that $(Y, \mathcal{E}, \mathcal{F})$ is a non-Archimedean intuitionistic fuzzy Banach space. Consider the non-Archimedean intuitionistic fuzzy norm

$$\mathcal{E}'(z,t) = \begin{cases} \dfrac{t}{t+|z|} & \text{if } t > 0, \\ 0 & \text{if } t \leq 0; \end{cases} \qquad \mathcal{F}'(z,t) = \begin{cases} \dfrac{|z|}{t+|z|} & \text{if } t > 0, \\ 1 & \text{if } t \leq 0. \end{cases} \tag{6.16}$$

Then $(\mathcal{E}', \mathcal{F}')$ is a non-Archimedean intuitionistic fuzzy norm on $\mathbb{R}$. It is easy to see that (4.1) holds for $\varphi(x,y) = \epsilon$ and $\alpha = 1$ satisfies (3.1). Therefore the condition of Theorem 6.1 is fulfilled. Hence there exists a unique additive mapping $C : X \to Y$ such that (6.14) holds. □

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