Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2012, Article ID 373904, 9 pages
doi:10.1155/2012/373904

Research Article

Approximation of Homomorphisms and Derivations on non-Archimedean Lie C^*-Algebras via Fixed Point Method

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Received 19 October 2011; Accepted 12 January 2012

Academic Editor: Yong Zhou

Using fixed point methods, we prove the generalized Hyers-Ulam stability of homomorphisms in C^*-algebras and Lie C^*-algebras and of derivations on non-Archimedean C^*-algebras and Non-Archimedean Lie C^*-algebras for an m-variable additive functional equation.

1. Introduction and Preliminaries

By a *non-Archimedean field* we mean a field K equipped with a function (valuation) $|\cdot|$ from K into $[0,\infty)$ such that $|r| = 0$ if and only if $r = 0$, $|rs| = |r||s|$, and $|r+s| \le \max\{|r|,|s|\}$ for all $r,s \in K$. Clearly $|1| = |-1| = 1$ and $|n| \le 1$ for all $n \in \mathbb{N}$. By the trivial valuation we mean the mapping $|\cdot|$ taking everything but 0 into 1 and $|0| = 0$. Let X be a vector space over a field K with a non-Archimedean nontrivial valuation $|\cdot|$. A function $\|\cdot\| : X \to [0,\infty)$ is called a *non-Archimedean norm* if it satisfies the following conditions:

(i) $\|x\| = 0$ if and only if $x = 0$;

(ii) for any $r \in K$, and $x \in X$, $\|rx\| = |r|\|x\|$;

(iii) the strong triangle inequality (ultrametric) holds; namely,

$$\|x+y\| \le \max\{\|x\|,\|y\|\}. \tag{1.1}$$

for all $x,y \in X$.

Then $(X, \|\cdot\|)$ is called a *non-Archimedean normed space*. From the fact that

$$\|x_n - x_m\| \leq \max\{\|x_{j+1} - x_j\| : m \leq j \leq n-1\} \quad (n > m) \tag{1.2}$$

holds, a sequence $\{x_n\}$ is a Cauchy sequence if and only if $\{x_{n+1} - x_n\}$ converges to zero in a non-Archimedean normed space. By a complete non-Archimedean normed space we mean one in which every Cauchy sequence is convergent.

For any nonzero rational number x, there exists a unique integer $n_x \in \mathbb{Z}$ such that $x = (a/b)p^{n_x}$, where a and b are integers not divisible by p. Then $|x|_p := p^{-n_x}$ defines a non-Archimedean norm on $\mathbb{Q}$. The completion of $\mathbb{Q}$ with respect to the metric $d(x, y) = |x - y|_p$ is denoted by $\mathbb{Q}_p$, which is called the *p-adic number field*.

A non-Archimedean Banach algebra is a *complete non-Archimedean algebra* $\mathcal{A}$ which satisfies $\|ab\| \leq \|a\| \cdot \|b\|$ for all $a, b \in \mathcal{A}$. For more detailed definitions of non-Archimedean Banach algebras, we refer the reader to [1, 2].

If $\mathcal{U}$ is a non-Archimedean Banach algebra, then an *involution* on $\mathcal{U}$ is a mapping $t \to t^*$ from $\mathcal{U}$ into $\mathcal{U}$ which satisfies

(i) $t^{**} = t$ for $t \in \mathcal{U}$;

(ii) $(\alpha s + \beta t)^* = \overline{\alpha} s^* + \overline{\beta} t^*$;

(iii) $(st)^* = t^* s^*$ for $s, t \in \mathcal{U}$.

If, in addition, $\|t^* t\| = \|t\|^2$ for $t \in \mathcal{U}$, then $\mathcal{U}$ is a *non-Archimedean C^*-algebra*.

The stability problem of functional equations was originated from a question of Ulam [3] concerning the stability of group homomorphisms: let $(G_1, *)$ be a group and let $(G_2, \diamond, d)$ be a metric group (a metric which is defined on a set with group property) with the metric $d(\cdot, \cdot)$. Given $\epsilon > 0$, does there exist a $\delta(\epsilon) > 0$ such that, if a mapping $h : G_1 \to G_2$ satisfies the inequality $d(h(x * y), h(x) \diamond h(y)) < \delta$ for all $x, y \in G_1$, then there is a homomorphism $H : G_1 \to G_2$ with $d(h(x), H(x)) < \epsilon$ for all $x \in G_1$? If the answer is affirmative, we would say that the equation of homomorphism $H(x * y) = H(x) \diamond H(y)$ is stable (see also [4–6]).

We recall a fundamental result in fixed point theory. Let Ω be a set. A function $d : \Omega \times \Omega \to [0, \infty]$ is called a *generalized metric* on Ω if d satisfies

(1) $d(x, y) = 0$ if and only if $x = y$;

(2) $d(x, y) = d(y, x)$ for all $x, y \in \Omega$;

(3) $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in \Omega$.

Theorem 1.1 (see [7]). *Let (Ω, d) be a complete generalized metric space and let $J : \Omega \to \Omega$ be a contractive mapping with Lipschitz constant $L < 1$. Then for each given element $x \in \Omega$, either $d(J^n x, J^{n+1} x) = \infty$ for all nonnegative integers n or there exists a positive integer n_0 such that*

(1) *$d(J^n x, J^{n+1} x) < \infty$ for all $n \geq n_0$;*

(2) *the sequence $\{J^n x\}$ converges to a fixed point y^* of J;*

(3) *y^* is the unique fixed point of J in the set $\Gamma = \{y \in \Omega \mid d(J^{n_0} x, y) < \infty\}$;*

(4) *$d(y, y^*) \leq (1/(1-L)) d(y, Jy)$ for all $y \in \Gamma$.*

In this paper, using the fixed point method, we prove the generalized Hyers-Ulam stability of homomorphisms and derivations in non-Archimedean C^*-algebras and non-Archimedean Lie C^*-algebras for the following additive functional equation (see [8]):

$$\sum_{i=1}^{m} f\left(mx_i + \sum_{j=1, j\neq i}^{m} x_j \right) + f\left(\sum_{i=1}^{m} x_i \right) = 2f\left(\sum_{i=1}^{m} mx_i \right) \quad (m \in \mathbb{N},\ m \geq 2). \tag{1.3}$$

2. Stability of Homomorphisms and Derivations in C^*-Algebras

Throughout this section, assume that $\mathcal{A}$ is a non-Archimedean C^*-algebra with norm $\|\cdot\|_{\mathcal{A}}$ and that $\mathcal{B}$ is a non-Archimedean C^*-algebra with norm $\|\cdot\|_{\mathcal{B}}$.

For a given mapping $f : \mathcal{A} \to \mathcal{B}$, we define

$$D_\mu f(x_1, \ldots, x_m) := \sum_{i=1}^{m} \mu f\left(mx_i + \sum_{j=1, j\neq i}^{m} x_j \right) + f\left(\mu \sum_{i=1}^{m} x_i \right) - 2f\left(\mu \sum_{i=1}^{m} mx_i \right) \tag{2.1}$$

for all $\mu \in \mathbb{T}^1 := \{\nu \in \mathbb{C} : |\nu| = 1\}$ and all $x_1, \ldots, x_m \in \mathcal{A}$.

Note that a $\mathbb{C}$-linear mapping $H : \mathcal{A} \to \mathcal{B}$ is called a *homomorphism* in non-Archimedean C^*-algebras if H satisfies $H(xy) = H(x)H(y)$ and $H(x^*) = H(x)^*$ for all $x, y \in \mathcal{A}$.

We prove the generalized Hyers-Ulam stability of homomorphisms in non-Archimedean C^*-algebras for the functional equation $D_\mu f(x_1, \ldots, x_m) = 0$.

Theorem 2.1. *Let $f : \mathcal{A} \to \mathcal{B}$ be a mapping for which there are functions $\varphi : \mathcal{A}^m \to [0, \infty)$, $\psi : \mathcal{A}^2 \to [0, \infty)$ and $\eta : \mathcal{A} \to [0, \infty)$ such that $|m| < 1$ is far from zero and*

$$\left\| D_\mu f(x_1, \ldots, x_m) \right\|_{\mathcal{B}} \leq \varphi(x_1, \ldots, x_m), \tag{2.2}$$

$$\left\| f(xy) - f(x)f(y) \right\|_{\mathcal{B}} \leq \psi(x, y), \tag{2.3}$$

$$\left\| f(x^*) - f(x)^* \right\|_{\mathcal{B}} \leq \eta(x) \tag{2.4}$$

for all $\mu \in \mathbb{T}^1$ and $x_1, \ldots, x_m, x, y \in A$. If there exists an $L < 1$ such that

$$\varphi(mx_1, \ldots, mx_m) \leq |m|\ L\varphi(x_1, \ldots, x_m), \tag{2.5}$$

$$\psi(mx, my) \leq |m|^2 L\psi(x, y), \tag{2.6}$$

$$\eta(mx) \leq |m| L\eta(x) \tag{2.7}$$

for all $x, y, x_1, \ldots, x_m \in \mathcal{A}$, then there exists a unique homomorphism $H : \mathcal{A} \to \mathcal{B}$ such that

$$\left\| f(x) - H(x) \right\|_{\mathcal{B}} \leq \frac{1}{|m| - |m|L} \varphi(x, 0, \ldots, 0) \tag{2.8}$$

for all $x \in \mathcal{A}$.

Proof. It follows from (2.5), (2.6), (2.7) and $L < 1$ that

$$\lim_{n\to\infty} \frac{1}{|m|^n} \varphi(m^n x_1, \ldots, m^n x_m) = 0, \tag{2.9}$$

$$\lim_{n\to\infty} \frac{1}{|m|^{2n}} \psi(m^n x, m^n y) = 0, \tag{2.10}$$

$$\lim_{n\to\infty} \frac{1}{|m|^n} \eta(m^n x) = 0 \tag{2.11}$$

for all $x, y, x_1, \ldots, x_m \in \mathcal{A}$.

Let us define Ω to be the set of all mappings $g : \mathcal{A} \to \mathcal{B}$ and introduce a generalized metric on Ω as follows

$$d(g,h) = \inf\{k \in (0,\infty) : \|g(x) - h(x)\|_{\mathcal{B}} < k\phi(x,0,\ldots,0), \ \forall x \in \mathcal{A}\}. \tag{2.12}$$

It is easy to show that (Ω, d) is a generalized complete metric space (see [9]).

Now we consider the function $J : \Omega \to \Omega$ defined by $Jg(x) = (1/m)g(mx)$ for all $x \in \mathcal{A}$ and $g \in \Omega$. Note that for all $g, h \in \Omega$ we have

$$\begin{aligned} d(g,h) < k &\Longrightarrow \|g(x) - h(x)\|_{\mathcal{B}} < k\phi(x,0,\ldots,0) \\ &\Longrightarrow \left\| \frac{1}{m} g(mx) - \frac{1}{m} h(mx) \right\|_{\mathcal{B}} < \frac{k}{|m|} \phi(mx,0,\ldots,0) \\ &\Longrightarrow \left\| \frac{1}{m} g(mx) - \frac{1}{m} h(mx) \right\|_{\mathcal{B}} < kL\phi(mx,0,\ldots,0) \\ &\Longrightarrow d(Jg, Jh) < kL. \end{aligned} \tag{2.13}$$

From this it is easy to see that $d(Jg, Jk) \le Ld(g,h)$ for all $g, h \in \Omega$, that is, J is a self-function of Ω with the Lipschitz constant L.

Putting $\mu = 1$, $x = x_1$ and $x_2 = x_3 = \cdots = x_m = 0$ in (2.2), we have

$$\|f(mx) - mf(x)\|_{\mathcal{B}} \le \phi(x,0,\ldots,0) \tag{2.14}$$

for all $x \in \mathcal{A}$. Then

$$\left\| f(x) - \frac{1}{m} f(mx) \right\|_{\mathcal{B}} \le \frac{1}{|m|} \phi(x,0,\ldots,0) \tag{2.15}$$

for all $x \in \mathcal{A}$, that is, $d(Jf, f) \le 1/|m| < \infty$. Now, from the fixed point alternative, it follows that there exists a fixed point H of J in Ω such that

$$H(x) = \lim_{n\to\infty} \frac{1}{|m|^n} f(m^n x) \tag{2.16}$$

for all $x \in \mathcal{A}$ since $\lim_{n\to\infty} d(J^n f, H) = 0$.

On the other hand, it follows from (2.2), (2.9), and (2.16) that

$$\begin{aligned}\left\|D_\mu H(x_1,\ldots,x_m)\right\|_{\mathcal{B}} &= \lim_{n\to\infty}\left\|\frac{1}{m^n}Df(m^n x_1,\ldots,m^n x_m)\right\|_{\mathcal{B}} \\ &\le \lim_{n\to\infty}\frac{1}{|m|^n}\phi(m^n x_1,\ldots,m^n x_m) = 0.\end{aligned} \tag{2.17}$$

By a similar method to the above, we get $\mu H(mx) = H(m\mu x)$ for all $\mu \in \mathbb{T}^1$ and $x \in \mathcal{A}$. Thus one can show that the mapping $H : \mathcal{A} \to \mathcal{B}$ is $\mathbb{C}$-linear.

It follows from (2.3), (2.10) and (2.16) that

$$\begin{aligned}\left\|H(xy) - H(x)H(y)\right\|_{\mathcal{B}} &= \lim_{n\to\infty}\frac{1}{|m|^{2n}}\left\|f\left(m^{2n}xy\right) - f(m^n x)f(m^n y)\right\|_{\mathcal{B}} \\ &\le \lim_{n\to\infty}\frac{1}{|m|^{2n}}\psi(m^n x, m^n y) = 0\end{aligned} \tag{2.18}$$

for all $x, y \in \mathcal{A}$. So $H(xy) = H(x)H(y)$ for all $x, y \in \mathcal{A}$. Thus $H : \mathcal{A} \to \mathcal{B}$ is a homomorphism, satisfying (2.8), as desired.

Also, by (2.4), (2.11), (2.16) and by a similar method, we have $H(x^*) = H(x)^*$. □

Corollary 2.2. *Let $r > 1$ and θ be nonnegative real numbers, and let $f : \mathcal{A} \to \mathcal{B}$ be a mapping such that*

$$\begin{aligned}\left\|D_\mu f(x_1,\ldots,x_m)\right\|_{\mathcal{B}} &\le \theta\cdot\left(\|x_1\|_{\mathcal{A}}^r + \|x_2\|_{\mathcal{A}}^r + \cdots + \|x_m\|_{\mathcal{A}}^r\right), \\ \left\|f(xy) - f(x)f(y)\right\|_{\mathcal{B}} &\le \theta\cdot\left(\|x\|_{\mathcal{A}}^r\cdot\|y\|_{\mathcal{A}}^r\right), \\ \left\|f(x^*) - f(x)^*\right\|_{\mathcal{B}} &\le \theta\cdot\|x\|_{\mathcal{A}}^r,\end{aligned} \tag{2.19}$$

for all $\mu \in \mathbb{T}^1$ and $x_1,\ldots,x_m, x, y \in \mathcal{A}$. Then there exists a unique homomorphism $H : \mathcal{A} \to \mathcal{B}$ such that

$$\left\|f(x) - H(x)\right\|_{\mathcal{B}} \le \frac{\theta}{|m| - |m|^r}\|x\|_{\mathcal{A}}^r \tag{2.20}$$

for all $x \in \mathcal{A}$.

Proof. The proof follows from Theorem 2.1 by taking

$$\begin{aligned}\varphi(x_1,\ldots,x_m) &= \theta\cdot\left(\|x_1\|_{\mathcal{A}}^r + \|x_2\|_{\mathcal{A}}^r + \cdots + \|x_m\|_{\mathcal{A}}^r\right), \\ \psi(x,y) &:= \theta\cdot\left(\|x\|_{\mathcal{A}}^r\cdot\|y\|_{\mathcal{A}}^r\right), \\ \eta(x) &= \theta\cdot\|x\|_{\mathcal{A}}^r\end{aligned} \tag{2.21}$$

for all $x_1,\ldots,x_m$, $x, y \in A$, $L = |m|^{r-1}$ and so we get the desired result. □

Note that a $\mathbb{C}$-linear mapping $\delta : \mathcal{A} \to \mathcal{A}$ is called a *derivation* on $\mathcal{A}$ if δ satisfies $\delta(xy) = \delta(x)y + x\delta(y)$ for all $x, y \in \mathcal{A}$.

We prove the generalized Hyers-Ulam stability of derivations on non-Archimedean C^*-algebras for the functional equation $D_\mu f(x_1, \ldots, x_m) = 0$.

Theorem 2.3. *Let $f : \mathcal{A} \to \mathcal{A}$ be a mapping for which there are functions $\varphi : \mathcal{A}^m \to [0,\infty)$, $\psi : \mathcal{A}^2 \to [0,\infty)$ and $\eta : \mathcal{A} \to [0,\infty)$ such that $|m| < 1$ is far from zero and*

$$\begin{gathered}\left\|D_\mu f(x_1, \ldots, x_m)\right\|_{\mathcal{A}} \le \varphi(x_1, \ldots, x_m), \\ \left\|f(xy) - f(x)y - xf(y)\right\|_{\mathcal{A}} \le \psi(x,y), \left\|f(x^*) - f(x)^*\right\|_{\mathcal{A}} \le \eta(x)\end{gathered} \tag{2.22}$$

for all $\mu \in \mathbb{T}^1$ and $x_1, \ldots, x_m, x, y \in A$. If there exists an $L < 1$ such that (2.5), (2.6) *and* (2.7) *hold, then there exists a unique derivation $\delta : \mathcal{A} \to \mathcal{A}$ such that*

$$\|f(x) - \delta(x)\|_{\mathcal{A}} \le \frac{1}{(|m| - |m|L)}\varphi(x, 0, \ldots, 0) \tag{2.23}$$

for all $x \in \mathcal{A}$.

3. Stability of Homomorphisms and Derivations in Non-Archimedean Lie C^*-Algebras

A non-Archimedean C^*-algebra $\mathcal{C}$, endowed with the Lie product

$$[x, y] := \frac{xy - yx}{2} \tag{3.1}$$

on $\mathcal{C}$, is called a *Lie non-Archimedean C^*-algebra*.

Definition 3.1. Let $\mathcal{A}$ and $\mathcal{B}$ be Lie C^*-algebras. A $\mathbb{C}$-linear mapping $H : \mathcal{A} \to \mathcal{B}$ is called a *non-Archimedean Lie C^*-algebra homomorphism* if $H([x,y]) = [H(x), H(y)]$ for all $x, y \in \mathcal{A}$.

Throughout this section, assume that $\mathcal{A}$ is a non-Archimedean Lie C^*-algebra with norm $\|\cdot\|_{\mathcal{A}}$ and $\mathcal{B}$ is a non-Archimedean Lie C^*-algebra with norm $\|\cdot\|_{\mathcal{B}}$.

We prove the generalized Hyers-Ulam stability of homomorphisms in non-Archimedean Lie C^*-algebras for the functional equation $D_\mu f(x_1, \ldots, x_m) = 0$.

Theorem 3.2. *Let $f : \mathcal{A} \to \mathcal{B}$ be a mapping for which there are functions $\varphi : \mathcal{A}^m \to [0,\infty)$ and $\psi : \mathcal{A}^2 \to [0,\infty)$ such that* (2.2) *and* (2.4) *hold and*

$$\left\|f([x,y]) - [f(x), f(y)]\right\|_{\mathcal{B}} \le \psi(x,y) \tag{3.2}$$

for all $\mu \in \mathbb{T}^1$ and $x, y \in \mathcal{A}$. If there exists an $L < 1$ and (2.5), (2.6), *and* (2.7) *hold, then there exists a unique homomorphism $H : \mathcal{A} \to \mathcal{B}$ such that* (2.8) *holds.*

Proof. By the same reasoning as in the proof of Theorem 2.1, we can find the mapping $H : \mathcal{A} \to \mathcal{B}$ given by

$$H(x) = \lim_{n\to\infty} \frac{f(m^n x)}{|m|^n} \tag{3.3}$$

for all $x \in \mathcal{A}$. It follows from (2.6) and (3.3) that

$$\begin{aligned} \|H([x,y]) - [H(x), H(y)]\|_{\mathcal{B}} &= \lim_{n\to\infty} \frac{1}{|m|^{2n}} \left\| f\left(m^{2n}[x,y]\right) - [f(m^n x), f(m^n y)\right\|_{\mathcal{B}} \\ &\leq \lim_{n\to\infty} \frac{1}{|m|^{2n}} \varphi(m^n x, m^n y) = 0 \end{aligned} \tag{3.4}$$

for all $x, y \in \mathcal{A}$ and so

$$H([x,y]) = [H(x), H(y)], \tag{3.5}$$

for all $x, y \in \mathcal{A}$. Thus $H : \mathcal{A} \to \mathcal{B}$ is a Lie C^*-algebra homomorphism satisfying (2.8), as desired. □

Corollary 3.3. *Let $r > 1$ and θ be nonnegative real numbers, and let $f : \mathcal{A} \to \mathcal{B}$ be a mapping such that*

$$\begin{gathered} \|D_\mu f(x_1, \ldots, x_m)\|_{\mathcal{B}} \leq \theta(\|x_1\|_A^r + \|x_2\|_A^r + \cdots + \|x_m\|_A^r), \\ \|f([x,y]) - [f(x), f(y)]\|_{\mathcal{B}} \leq \theta \cdot \|x\|_A^r \cdot \|y\|_A^r, \\ \|f(x^*) - f(x)^*\|_{\mathcal{B}} \leq \theta \cdot \|x\|_{\mathcal{A}}^r \end{gathered} \tag{3.6}$$

all $\mu \in \mathbb{T}^1$ and $x_1, \ldots, x_m, x, y \in \mathcal{A}$. Then there exists a unique homomorphism $H : \mathcal{A} \to \mathcal{B}$ such that

$$\|f(x) - H(x)\|_{\mathcal{B}} \leq \frac{\theta}{|m| - |m|^r} \|x\|_{\mathcal{A}}^r \tag{3.7}$$

for all $x \in \mathcal{A}$.

Proof. The proof follows from Theorem 3.2 and a method similar to Corollary 3.3. □

Definition 3.4. Let $\mathcal{A}$ be a non-Archimedean Lie C^*-algebra. A $\mathbb{C}$-linear mapping $\delta : \mathcal{A} \to \mathcal{A}$ is called a *Lie derivation* if $\delta([x,y]) = [\delta(x), y] + [x, \delta(y)]$ for all $x, y \in \mathcal{A}$.

We prove the generalized Hyers-Ulam stability of derivations on non-Archimedean Lie C^*-algebras for the functional equation $D_\mu f(x_1, \ldots, x_m) = 0$.

Theorem 3.5. *Let $f : \mathcal{A} \to \mathcal{A}$ be a mapping for which there are functions $\varphi : A^m \to [0,\infty)$ and $\psi : A^2 \to [0,\infty)$ such that* (2.2) *and* (2.4) *hold and*

$$\|f([x,y]) - [f(x),y] - [x,f(y)]\|_{\mathcal{A}} \le \psi(x,y) \tag{3.8}$$

for all $x,y \in \mathcal{A}$. If there exists an $L < 1$ and (2.5), (2.6) *and* (2.7) *hold, then there exists a unique Lie derivation $\delta : \mathcal{A} \to \mathcal{A}$ such that such that* (2.8) *holds.*

Proof. By the same reasoning as the proof of Theorem 2.3, there exists a unique $\mathbb{C}$-linear mapping $\delta : \mathcal{A} \to \mathcal{A}$ satisfying (2.8) and the mapping $\delta : \mathcal{A} \to \mathcal{A}$ is given by

$$\delta(x) = \lim_{n\to\infty} \frac{f(m^n x)}{|m|^n} \tag{3.9}$$

for all $x \in \mathcal{A}$.

It follows from (2.6) and (3.9) that

$$\begin{aligned} &\|\delta([x,y]) - [\delta(x),y] - [x,\delta(y)]\|_{\mathcal{A}} \\ &\quad = \lim_{n\to\infty} \frac{1}{|m|^{2n}} \left\| f\left(m^{2n}[x,y]\right) - [f(m^n x), m^n y] - [m^n x, f(m^n y)] \right\|_{\mathcal{A}} \\ &\quad \le \lim_{n\to\infty} \frac{1}{|m|^{2n}} \psi(m^n x, m^n y) = 0, \end{aligned} \tag{3.10}$$

for all $x,y \in \mathcal{A}$ and so

$$\delta([x,y]) = [\delta(x),y] + [x,\delta(y)] \tag{3.11}$$

for all $x,y \in \mathcal{A}$. Thus $\delta : \mathcal{A} \to \mathcal{A}$ is a Lie derivation satisfying (2.8). □

Acknowledgment

This paper was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (No. 2011-0021821).

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