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Research Article

Robust Finite-Time $\mathscr{H}_\infty$ Control for Uncertain Systems Subject to Intermittent Measurements

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This paper investigates the robust finite-time $\mathscr{H}_\infty$ controller design problem of discrete-time systems with intermittent measurements. It is assumed that the system is subject to the norm-bounded uncertainties and the measurements are intermittent. The Bernoulli process is used to describe the phenomenon of intermittent measurements. By substituting the state-feedback controller into the system, a stochastic closed-loop system is obtained. Based on the analysis of the robust stochastic finite-time stability and the $\mathscr{H}_\infty$ performance, the controller design method is proposed. The controller gain can be calculated by solving a sequence of linear matrix inequalities. Finally, a numerical example is used to show the design procedure and the effectiveness of the proposed design methodology.

1. Introduction

In the real world, system models are unavoidable to contain uncertainties which can result from the modeling error or variations of the system parameters. During the past 20 years, the norm-bounded uncertainties have been widely used in the system modeling and control for practical plants [1–10]. In [11], the authors studied the time-delay linear systems with the norm-bounded uncertainties. In [10], the robust memoryless $\mathscr{H}_\infty$ controller design for linear time-delay systems with norm-bounded time-varying uncertainty was studied. In [3], the robust stability of neutral systems with time-varying discrete delay and norm-bounded uncertainty was explored.

Since the late 1980s, the $\mathscr{H}_\infty$ strategy has attracted a lot of attention due to the fact that this control strategy can be easily utilized to deal with the uncertainties and attenuation the effect from the external input to the controlled output [12–16]. The strategy was original from [17]. From then on, the useful tool has been applied to different kinds of systems. In [18], the $\mathscr{H}_\infty$ controller of linear parameter-varying systems with parameter-varying delays was exploited. In [19], the strategy was used to the filter design for uncertain Markovian jump systems with mode-dependent time delays. While in [20], the strategy was used in the robust controller design of discrete-time Markovian jump linear systems with mode-dependent time-delays. Gain-scheduled $\mathscr{H}_\infty$ controller design problem for time-varying systems was investigated in [21].

In the literature, the $\mathscr{H}_\infty$ strategy was always based on the Lyapunov asymptotic stability which is with an infinite interval. However, in some practical applications, the asymptotic stability is inadequate if large values of the state are not acceptable and there exists saturation [15, 22–37]. Although the finite-time stability was proposed in 1960s [22], it only attracted the researchers' attention very recently. In [38], observer-based finite-time stabilization for extended Markov jump systems was studied. The observer-based finite-time control of time-delayed jump systems was studied in [39]. By considering the partially known transition jump rates, the finite-time filtering for non-linear stochastic systems was explored in [40]. For time-varying singular impulsive systems, the finite-time stability conditions were obtained in [41]. At the application side, the finite-time stability has been used in [25].

On another research frontier, the intermittent measurements have been paid a great number of efforts. In an ideal sampling, the measurements are consecutive. But, in a harsh sampling environment, the sampling may not be consecutive but intermittent [42–47]. If the phenomenon of intermittent measurements is not considered during the controller and filter design period, the actual missing measurements may deteriorate the designed systems. Although, there are many results on the $\mathscr{H}_\infty$ control, uncertain systems, and finite-time stability, there are few results on the $\mathscr{H}_\infty$ control for uncertain systems subject to intermittent measurements. This fact motivates the research. In this paper, the contributions of this work are summarized as follows: (1) The intermittent measurements are considered the finite-time framework. Due to the induced stochastic system, the robust stochastic finite-time boundedness is studied. (2) The $\mathscr{H}_\infty$ performance with the robust stochastic finite-time stability is investigated.

Notation. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ represents the set of all $m\times n$ real matrices. $\mathbb{E}\{\cdot\}$ is the expectation operator with respect to some probability measure. $\lambda_{\max}\{\cdot\}$ and $\lambda_{\min}\{\cdot\}$ are the maximum eigenvalue and the minimum eigenvalue of the matrix, respectively.

2. Problem Formulation

In this paper, the following uncertain discrete-time linear system is considered:

$$\begin{aligned} x_{k+1} &= (A+\Delta A)x_k + B_1 u_k + (B_2+\Delta B_2)\omega_k, \\ z_k &= E x_k + F_1 u_k + F_2 \omega_k, \end{aligned} \tag{2.1}$$

where $x_k \in \mathbb{R}^n$ denotes the state vector, $u_k \in \mathbb{R}^m$ is the control input, $z_k \in \mathbb{R}^p$ is the controlled output, and $\omega_k \in \mathbb{R}^r$ is the time-varying disturbance which satisfies

$$\sum_{k=1}^{\infty} \omega_k^{\mathrm{T}} \omega_k \le d^2, \quad (k \in \mathbb{N}_0), \tag{2.2}$$

where $d > 0$ is a given scalar.

The matrices $A, B_1, B_2, E, F_1,$ and F_2 are constant matrices with appropriate dimensions. ΔA and ΔB_2 are real time-varying matrix functions representing the time-varying parameter uncertainties. It is assumed that the uncertainties are normbounded and admissible, which can be modeled as

$$\begin{bmatrix}\Delta A & \Delta B_2\end{bmatrix} = HG_k\begin{bmatrix}M_1 & M_2\end{bmatrix}, \tag{2.3}$$

where H, M_1, and M_2 are known real constant matrices, which characterize how the uncertain parameters in G_k enter the nominal matrices A and B_1, and B_2 is an unknown time-varying matrix function satisfying

$$\|G_k\| \leq I, \quad \forall k \in \mathbb{N}_0. \tag{2.4}$$

Consider an ideal state feedback controller as follows:

$$\hat{u}_k = Kx_k, \tag{2.5}$$

where $\hat{u}_k$ is the ideal control signal which is obtained with the ideal state measuring, K is the state feedback gain to be designed, and x_k is the system state. In an ideal sensing environment, it is always assumed that x_k is available for all the time instants k. However, in many practical applications, such as the networked control systems (NCSs), the measurements may not be consecutive but intermittent. In order to get the general case, it is assumed that the state measurements are intermittent and the actual control law is governed by

$$\begin{aligned} &u_k = Kx_k, \quad \text{the measurement is available,} \\ &u_k = 0, \quad \text{the measurement is missing.} \end{aligned} \tag{2.6}$$

To better describe the intermittent measurements, a Bernoulli process α_k is used to represent the intermittent measurements such that the actual control signal is

$$u_k = \alpha_k K x_k, \tag{2.7}$$

where α_k takes values in the set $\{0, 1\}$, $\alpha_k = 0$ refers to that the measurement is missing, and $\alpha_k = 1$ means that the measurement is available. In addition, it is assumed that the probability of $\alpha_k = 1$ is β.

By substituting the actual control signal into the state-space model of (2.1), one gets

$$\begin{aligned} x_{k+1} &= \left(A + \Delta A + \beta B_1 K + (\alpha_k - \beta) B_1 K\right) x_k + (B_2 + \Delta B_2)\omega_k \\ &= \overline{A}(\Delta A, \alpha_k) x_k + \overline{B}(\Delta B_2)\omega_k, \\ z_k &= \left(E + \beta F_1 K + (\alpha_k - \beta) F_1 K\right) x_k + F_2 \omega_k \\ &= \overline{E}(\alpha_k) x_k + \overline{F}\omega_k. \end{aligned} \tag{2.8}$$

It is obvious that there is a stochastic variable α_k in the closed-loop system in (2.8). Therefore, the objective of this paper is to find some sufficient conditions which can guarantee that the closed-loop system in (2.8) is robustly stochastically finite-time boundedness and reduces the effect of the disturbance input to the controlled output to a prescribed level.

Firstly, some useful definitions and lemmas are introduced, which will be used throughout the rest of the paper.

Definition 2.1 (Finite-Time Stable (FTS) [23]). For a class of discrete-time linear systems

$$x_{k+1} = Ax_k, \quad k \in \mathbb{N}_0, \tag{2.9}$$

is said to be FTS with respect to (c_1, c_2, R, N), where R is a positive definite matrix, $0 < c_1 < c_2$ and $N \in \mathbb{N}_0$, if $x_0^{\mathrm{T}} R x_0 \le c_1^2$, then $x_k^{\mathrm{T}} R x_k \le c_2^2$, for all $k \in \{1, 2, \ldots, N\}$.

Definition 2.2 (Robustly Finite-Time Stable (RFTS) [23]). For a class of discrete-time linear uncertain systems

$$x_{k+1} = (A + \Delta A)x_k, \quad k \in \mathbb{N}_0, \tag{2.10}$$

is said to be RFTS with respect to (c_1, c_2, R, N), where R is a positive definite matrix, $0 < c_1 < c_2$, and $N \in \mathbb{N}_0$; if for all admissible uncertainties ΔA, $x_0^{\mathrm{T}} R x_0 \le c_1^2$, then $x_k^{\mathrm{T}} R x_k \le c_2^2$, for all $k \in \{1, 2, \ldots, N\}$.

Definition 2.3 (Robustly Stochastically Finite-Time Stable (RSFTS)). For a class of discrete-time linear uncertain systems

$$x_{k+1} = \overline{A}(\Delta A, \alpha_k)x_k, \quad k \in \mathbb{N}_0, \tag{2.11}$$

is said to be RSFTS with respect to (c_1, c_2, R, N), where the system matrix $\overline{A}(\Delta A, \alpha_k)$ has the uncertainty and the stochastic variable, R is a positive definite matrix, $0 < c_1 < c_2$, and $N \in \mathbb{N}_0$; if for all admissible uncertainties ΔA, stochastic variable α_k, $x_0^{\mathrm{T}} R x_0 \le c_1^2$, then $\mathbb{E}\{x_k^{\mathrm{T}} R x_k\} \le c_2^2$, for all $k \in \{1, 2, \ldots, N\}$.

Definition 2.4 (Robustly Stochastically Finite-Time Bounded (RSFTB)). For a class of discrete-time linear uncertain systems

$$x_{k+1} = \overline{A}(\Delta A, \alpha_k)x_k + \overline{B}(\Delta B)\omega_k, \quad k \in \mathbb{N}_0 \tag{2.12}$$

is said to be RSFTB with respect to (c_1, c_2, d, R, N), where the system matrix $\overline{A}(\Delta A, \alpha_k)$ has the uncertainty and the stochastic variable, the input matrix contains the norm-bounded uncertainty, R is a positive definite matrix, $0 < c_1 < c_2$, and $N \in \mathbb{N}_0$; if for all admissible uncertainties ΔA and ΔB, stochastic variable α_k, $x_0^{\mathrm{T}} R x_0 \le c_1^2$, then $\mathbb{E}\{x_k^{\mathrm{T}} R x_k\} \le c_2^2$, for all $k \in \{1, 2, \ldots, N\}$.

With the above definitions, the robust finite-time $\mathscr{H}_\infty$ control problem in this paper can be summarized as follows: for the uncertain system in (2.1), the objective is to design a state feedback controller in (2.7) such that for all the admissible uncertainties and the intermittent measurements:

(i) the closed-loop system (2.8) is RSFTS;

(ii) under the zero-initial condition, the controlled output z_k satisfies

$$\mathbb{E}\left\{\sum_{i=1}^{N} z_k^{\mathrm{T}} z_k\right\} < \gamma^2 \sum_{i=1}^{N} \omega_k^{\mathrm{T}} \omega_k, \tag{2.13}$$

for all l_2-bounded ω_k, where prescribed value γ is the $\mathscr{H}_\infty$ attenuation level.

If the above conditions are both satisfied, the designed controller is called a RSFTB state-feedback controller. To achieve the design objectives, the following lemmas are introduced.

Lemma 2.5 (Schur complement). *Given a symmetric matrix* $\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ \Xi_{21} & \Xi_{22} \end{bmatrix}$, *the following three conditions are equivalent to each other:*

(i) $\Xi < 0$;

(ii) $\Xi_{11} < 0$, $\Xi_{22} - \Xi_{12}^{\mathrm{T}} \Xi_{11}^{-1} \Xi_{12} < 0$;

(iii) $\Xi_{22} < 0$, $\Xi_{11} - \Xi_{12} \Xi_{22}^{-1} \Xi_{12}^{\mathrm{T}} < 0$.

Lemma 2.6 (see [48, 49]). *Let* $\Theta = \Theta^{\mathrm{T}}$, $\overline{H}$ *and* $\overline{M}$ *be real matrices with compatible dimensions, and* G_k *be time varying and satisfy* (2.4). *Then it concludes that the following condition:*

$$\Theta + \overline{H} G_k \overline{M} + \left(\overline{H} G_k \overline{M}\right)^{\mathrm{T}} < 0 \tag{2.14}$$

holds if and only if there exists a positive scaler $\varepsilon > 0$ *such that*

$$\begin{bmatrix} \Theta & \varepsilon \overline{H} & \overline{M}^{\mathrm{T}} \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0 \tag{2.15}$$

is satisfied.

3. Main Results

3.1. Stability and $\mathscr{H}_\infty$ Performance Analysis

In this section, the finite-time stability, robust finite-time stability, and robust stochastic finite-time stability will be analyzed by assuming the controller gain is given.

Lemma 3.1 (sufficient conditions for finite-time stability [23]). *For a class of discrete-time linear systems*

$$x_{k+1} = A x_k, \quad k \in \mathbb{N}_0, \tag{3.1}$$

they are finite-time stable with respect to (c_1, c_2, R, N) *if there exist a positive-definite matrix* P *and a scalar* $\theta \geq 1$ *such that the following conditions hold:*

$$A^{\mathrm{T}} PA - \theta P < 0, \tag{3.2}$$

$$\operatorname{cond}\left(\widetilde{P}\right) < \frac{c_2^2}{\theta^N c_1^2}, \tag{3.3}$$

where $\widetilde{P} = R^{-1/2} P R^{-1/2}$ *and* $\operatorname{cond}(\widetilde{P}) = \lambda_{\max}(\widetilde{P})/\lambda_{\min}(\widetilde{P})$.

Lemma 3.2 (sufficient conditions for robust finite-time stability). *For a class of discrete-time linear systems*

$$x_{k+1} = (A + \Delta A)x_k, \quad k \in \mathbb{N}_0, \tag{3.4}$$

they are robustly finite-time stable with respect to (c_1, c_2, R, N) *if there exist a positive-definite matrix* P*, a scalar* $\varepsilon > 0$*, and a scalar* $\theta \geq 1$ *such that* (3.3) *and the following condition hold:*

$$\begin{bmatrix} -P & PA & \varepsilon PH & 0 \\ * & -\theta P & 0 & M_1^{\mathrm{T}} \\ * & * & -\varepsilon I & 0 \\ * & * & * & -\varepsilon I \end{bmatrix} < 0. \tag{3.5}$$

Proof. According to Lemma 3.1, the uncertain system is RFTS if the following condition holds:

$$(A + \Delta A)^{\mathrm{T}} P (A + \Delta A) - \theta P < 0. \tag{3.6}$$

Using the Schur complement, the above condition is equivalent with

$$\begin{bmatrix} -P & P(A + \Delta A) \\ * & -\theta P \end{bmatrix} < 0. \tag{3.7}$$

Since $\Delta A = HG_k M_1$, by using Lemma 2.6, (3.7) is equivalent with (3.5). □

Now, let us study the RSFTB of the closed-loop system in (2.8) and deal with the uncertainty and the stochastic variable by using the skills mentioned above.

Theorem 3.3. *The closed-loop system in* (2.8) *is RSFTB with respect to* (c_1, c_2, d, R, N), *if there exist positive-definite matrices* $P_1 = P_1^{\mathrm{T}}$, $P_2 = P_2^{\mathrm{T}}$, *and two scalars* $\theta \geq 1$, *and* $\varepsilon > 0$ *such that the following conditions hold:*

$$\begin{bmatrix} -P_1 & 0 & hP_1B_1K & 0 & 0 & 0 \\ * & -P_1 & P_1(A+\beta B_1K) & P_1B_2 & \varepsilon PH & 0 \\ * & * & -\theta P_1 & 0 & 0 & M_1^{\mathrm{T}} \\ * & * & * & -\theta P_2 & 0 & M_2^{\mathrm{T}} \\ * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & 0 & -\varepsilon I \end{bmatrix} < 0, \tag{3.8}$$

$$\lambda_{\max}\left(\widetilde{P}_1\right)c_1^2 + \lambda_{\max}(P_2)d^2 < \frac{c_2^2\lambda_{\min}\left(\widetilde{P}_1\right)}{\theta^N}, \tag{3.9}$$

where $\widetilde{P}_1 = R^{-1/2}P_1R^{-1/2}$ *and* $h = \sqrt{\beta(1-\beta)}$.

Proof. Consider the following Lyapunov function:

$$V(k) = x_k^{\mathrm{T}}P_1x_k, \tag{3.10}$$

where P_1 is a symmetric positive-definite matrix. For the closed-loop system in (2.8), the expectation of one step advance of the Lyapunov function can be obtained as

$$\begin{aligned} \mathbb{E}\{V(k+1) \mid x_k\} &= x_k^{\mathrm{T}}(A + \Delta A + (\beta+h)B_1K)^{\mathrm{T}}P_1(A + \Delta A + (\beta+h)B_1K)x_k \\ &\quad + 2x_k^{\mathrm{T}}(A + \Delta A + \beta B_1K)^{\mathrm{T}}P_1(B_2 + \Delta B_2)\omega_k \\ &\quad + \omega_k^{\mathrm{T}}(B_2 + \Delta B_2)^{\mathrm{T}}P_1(B_2 + \Delta B_2)\omega_k \\ &= \begin{bmatrix} x_k \\ \omega_k \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ * & \Omega_{22} \end{bmatrix} \begin{bmatrix} x_k \\ \omega_k \end{bmatrix} \\ &= \begin{bmatrix} x_k \\ \omega_k \end{bmatrix}^{\mathrm{T}} \Omega \begin{bmatrix} x_k \\ \omega_k \end{bmatrix}, \end{aligned} \tag{3.11}$$

where

$$\begin{aligned} \Omega_{11} &= (A + \Delta A + (\beta+h)B_1K)^{\mathrm{T}}P_1(A + \Delta A + (\beta+h)B_1K), \\ \Omega_{12} &= (A + \Delta A + \beta B_1K)^{\mathrm{T}}P_1(B_2 + \Delta B_2), \\ \Omega_{22} &= (B_2 + \Delta B_2)^{\mathrm{T}}P_1(B_2 + \Delta B_2). \end{aligned} \tag{3.12}$$

On the other hand, by using Schur complement, the condition in (3.8) implies that

$$\Omega < \begin{bmatrix} \theta P_1 & 0 \\ 0 & \theta P_2 \end{bmatrix}, \tag{3.13}$$

since the condition in (3.8) is equivalent with

$$\Theta + \overline{H}G_k\overline{M} + \left(\overline{H}G_k\overline{M}\right)^{\mathrm{T}} < 0, \tag{3.14}$$

where

$$\begin{gathered}
\Theta = \begin{bmatrix} -P_1 & 0 & hPB_1K & 0 \\ * & -P_1 & P(A+\beta B_1K) & PB_2 \\ * & * & -\theta P_1 & 0 \\ * & * & * & -\theta P_2 \end{bmatrix}, \\
\overline{H} = \begin{bmatrix} 0 \\ PH \\ 0 \\ 0 \end{bmatrix}, \\
\overline{M} = \begin{bmatrix} 0 & 0 & M_1 & M_2 \end{bmatrix}.
\end{gathered} \tag{3.15}$$

With the condition (3.13), one gets

$$\mathbb{E}\{V(k+1) \mid x_k\} < \theta V(k) + \theta\omega_k^{\mathrm{T}} P_2 \omega_k. \tag{3.16}$$

Taking the iterative operation with respect to the time instant k, one obtains

$$\begin{aligned}
\mathbb{E}\{V(k) \mid x_0\} &< \theta^k V(0) + \sum_{i=1}^{k} \theta^{k-i+1} \omega_{j-1}^{\mathrm{T}} P_2 \omega_{j-1} \\
&< \theta^N \left(\lambda_{\max}\left(\widetilde{P}_1\right)c_1^2 + \lambda_{\max}(P_2)d^2\right).
\end{aligned} \tag{3.17}$$

Recalling the Lyapunov function, there is

$$\mathbb{E}\{V(k) \mid x_0\} > \lambda_{\min}\left(\widetilde{P}_1\right) x_k^{\mathrm{T}} R x_k. \tag{3.18}$$

Combing (3.17) and (3.19), one gets

$$\mathbb{E}\left\{x_k^{\mathrm{T}} R x_k\right\} < \frac{\theta^N}{\lambda_{\min}\left(\widetilde{P}_1\right)} \left(\lambda_{\max}\left(\widetilde{P}_1\right)c_1^2 + \lambda_{\max}(P_2)d^2\right). \tag{3.19}$$

With the condition (3.9), it concludes that

$$\mathbb{E}\left\{x_k^{\mathrm{T}} R x_k\right\} < c_2^2. \tag{3.20}$$

Therefore, if the conditions in (3.8) and (3.9) are satisfied, the closed-loop system (2.8) is RSFTB. The proof is completed. □

In order to incorporate the $\mathscr{H}_\infty$ performance γ, the following theorem provides other sufficient conditions for the RSFTB of the closed-loop system (2.8).

Theorem 3.4. *The closed-loop system in* (2.8) *is RSFTB with respect to* (c_1, c_2, d, R, N), *if there exist positive-definite matrix* $P = P^{\mathrm{T}}$, *and three scalars* $\theta \geq 1$, $\varepsilon > 0$, *and* $\gamma > 0$ *such that the following conditions hold:*

$$\begin{bmatrix} -P & 0 & hPB_1K & 0 & 0 & 0 \\ * & -P & P(A+\beta B_1 K) & PB_2 & \varepsilon PH & 0 \\ * & * & -\theta P_1 & 0 & 0 & M_1^{\mathrm{T}} \\ * & * & * & -\gamma^2 I & 0 & M_2^{\mathrm{T}} \\ * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & 0 & -\varepsilon I \end{bmatrix} < 0, \tag{3.21}$$

$$\lambda_{\max}\left(\widetilde{P}\right)c_1^2 + \gamma^2 d^2 < \frac{c_2^2 \lambda_{\min}\left(\widetilde{P}\right)}{\theta^N}, \tag{3.22}$$

where $\widetilde{P} = R^{-1/2}PR^{-1/2}$ *and* $h = \sqrt{\beta(1-\beta)}$.

Proof. Suppose that P_1 and P_2 in Theorem 3.3 are substituted by P and $\gamma^2 I/\theta$, respectively. Then, the condition (3.8) turns to (3.21). The maximum eigenvalue of $\gamma^2 I/\theta$ is no more than γ^2. Therefore, (3.22) can guarantee the holdness of (3.9). The proof is completed. □

Now, consider the controlled output and the $\mathscr{H}_\infty$ attenuation level γ.

Theorem 3.5. *The closed-loop system in* (2.8) *is RSFTB with respect to* $(0, c_2, d, R, N)$ *and with an* $\mathscr{H}_\infty$ *attenuation level* γ, *if there exist positive-definite matrix* $P = P^{\mathrm{T}}$, *and three scalars* $\theta \geq 1$, $\varepsilon > 0$, *and* $\gamma > 0$ *such that the following conditions hold:*

$$\begin{bmatrix} -I & 0 & 0 & 0 & hF_1K & 0 & 0 & 0 \\ * & -P & 0 & 0 & hPB_1K & 0 & 0 & 0 \\ * & * & -P & 0 & P(A+\beta B_1K) & PB_2 & \varepsilon PH & 0 \\ * & * & * & -I & E+\beta F_1 K & F_2 & 0 & 0 \\ * & * & * & * & -\theta P & 0 & 0 & M_1^{\mathrm{T}} \\ * & * & * & * & * & -\gamma^2 I & 0 & M_2^{\mathrm{T}} \\ * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & 0 & -\varepsilon I \end{bmatrix} < 0, \tag{3.23}$$

$$\gamma^2 d^2 < \frac{c_2^2 \lambda_{\min}\left(\widetilde{P}\right)}{\theta^N}, \tag{3.24}$$

where $\widetilde{P} = R^{-1/2}PR^{-1/2}$ *and* $h = \sqrt{\beta(1-\beta)}$.

Proof. The $\mathscr{H}_\infty$ attenuation level γ refers to the zero-initial value of the state. Therefore, c_1 is set to be zero. Consider the following cost function:

$$J = \mathbb{E}\{V(k+1) \mid x_k\} + \mathbb{E}\left\{z_k^{\mathrm{T}} z_k\right\} - \gamma^2 I. \tag{3.25}$$

The cost function can be revaluated with similar lines in Theorem 3.3. The proof is omitted. □

3.2. Controller Design

The robust stochastic finite-time stability and the $\mathscr{H}_\infty$ performance have been investigated in the above subsection. In this subsection, the controller design will be proposed.

Theorem 3.6. *Given a positive constant γ, the closed-loop system in* (2.8) *is RSFTB with respect to* $(0, c_2, d, R, N)$ *and with a prescribed $\mathscr{H}_\infty$ attenuation level γ, if there exist positive-definite matrix $Q = Q^{\mathrm{T}}$, two scalars $\theta \geq 1$, and $\varepsilon > 0$ and L such that the following conditions hold:*

$$\begin{bmatrix} -I & 0 & 0 & 0 & hF_1L & 0 & 0 & 0 \\ * & -Q & 0 & 0 & hB_1L & 0 & 0 & 0 \\ * & * & -Q & 0 & AQ+\beta B_1L & B_2 & \varepsilon H & 0 \\ * & * & * & -I & EQ+\beta F_1L & F_2 & 0 & 0 \\ * & * & * & * & -\theta Q & 0 & 0 & QM_1^{\mathrm{T}} \\ * & * & * & * & * & -\gamma^2 I & 0 & M_2^{\mathrm{T}} \\ * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & 0 & -\varepsilon I \end{bmatrix} < 0, \tag{3.26}$$

$$\gamma^2 d^2 < \frac{c_2^2}{\theta^N \lambda_{\max}\left(\tilde{Q}\right)}, \tag{3.27}$$

where $\tilde{Q} = R^{1/2} Q R^{1/2}$ and $h = \sqrt{\beta(1-\beta)}$. Moreover, the controller gain can be calculated as $K = LQ^{-1}$.

Proof. In Theorem 3.5, pre- and postmultiplying (3.25) by $\operatorname{diag}\{I, P^{-1}, P^{-1}, I, I, I, I, I\}$, the following equivalent condition is obtained:

$$\begin{bmatrix} -I & 0 & 0 & 0 & hF_1K & 0 & 0 & 0 \\ * & -P^{-1} & 0 & 0 & hB_1K & 0 & 0 & 0 \\ * & * & -P^{-1} & 0 & A+\beta B_1K & B_2 & \varepsilon H & 0 \\ * & * & * & -I & E+\beta F_1K & F_2 & 0 & 0 \\ * & * & * & * & -\theta P & 0 & 0 & M_1^{\mathrm{T}} \\ * & * & * & * & * & -\gamma^2 I & 0 & M_2^{\mathrm{T}} \\ * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & 0 & -\varepsilon I \end{bmatrix} < 0. \tag{3.28}$$

Letting Q denote P^{-1}, the condition (3.28) is equivalent with

$$\begin{bmatrix} -I & 0 & 0 & 0 & hF_1K & 0 & 0 & 0 \\ * & -Q & 0 & 0 & hB_1K & 0 & 0 & 0 \\ * & * & -Q & 0 & A+\beta B_1K & B_2 & \varepsilon H & 0 \\ * & * & * & -I & E+\beta F_1K & F_2 & 0 & 0 \\ * & * & * & * & -\theta Q^{-1} & 0 & 0 & M_1^{\mathrm{T}} \\ * & * & * & * & * & -\gamma^2 I & 0 & M_2^{\mathrm{T}} \\ * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & 0 & -\varepsilon I \end{bmatrix} < 0. \tag{3.29}$$

Pre- and post-multiplying (3.29) by the symmetric matrix $\operatorname{diag}\{I, I, I, I, Q, I, I, I\}$, the following equivalent condition is obtained:

$$\begin{bmatrix} -I & 0 & 0 & 0 & hF_1KQ & 0 & 0 & 0 \\ * & -Q & 0 & 0 & hB_1KQ & 0 & 0 & 0 \\ * & * & -Q & 0 & AQ+\beta B_1KQ & B_2 & \varepsilon H & 0 \\ * & * & * & -I & EQ+\beta F_1KQ & F_2 & 0 & 0 \\ * & * & * & * & -\theta Q & 0 & 0 & QM_1^{\mathrm{T}} \\ * & * & * & * & * & -\gamma^2 I & 0 & M_2^{\mathrm{T}} \\ * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & 0 & -\varepsilon I \end{bmatrix} < 0. \tag{3.30}$$

Defining $L = KQ$, the condition (3.30) is equivalent with (3.26). With the fact that

$$\lambda_{\min}\tilde{P} = \frac{1}{\lambda_{\max}\tilde{Q}}, \tag{3.31}$$

the condition (3.27) is equivalent with (3.24). □

It is noted that the condition (3.27) is not an linear matrix inequality. However, it is easy to check that the condition (3.27) is guaranteed by imposing the following conditions [50]:

$$\begin{gathered} 0 < \tilde{Q} < I, \\ \gamma^2 d^2 < \frac{c_2^2}{\theta^N}. \end{gathered} \tag{3.32}$$

In addition, the $\mathcal{H}_\infty$ performance γ refers to the attenuation level from the external noise to the controlled output. Therefore, it is desired that the performance γ should be as smaller as possible. The controller with the optimal γ is called the optimal $\mathcal{H}_\infty$ controller. For fixed θ and c_2, the optimal γ can be obtained by

$$\begin{aligned} &\min \quad \gamma^2, \\ &\text{s.t.} \quad (3.26) \text{ and } (3.32). \end{aligned} \tag{3.33}$$

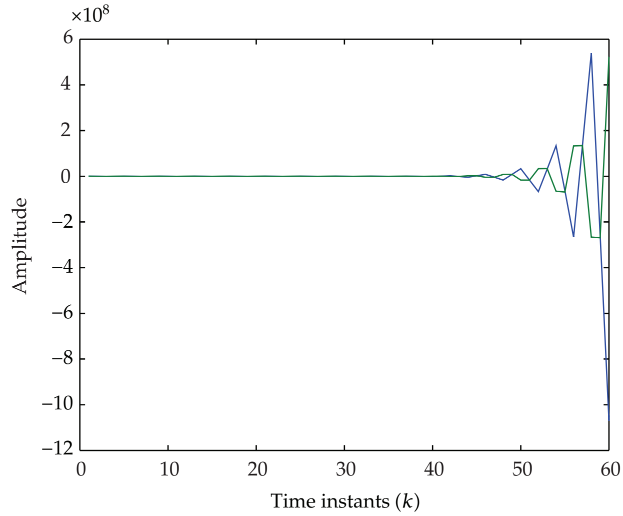

Figure 1: Trajectories of the system state in the simulation without the designed controller.

Remark 3.7. The results in this paper are obtained by using the Lyapunov method and only sufficient. In the future work, more techniques will be used to reduce the possible conservativeness of the results.

4. Numerical Example

Consider the system in (2.1) with the following matrix:

$$A = \begin{bmatrix} 1 & 3 \\ -1 & -1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}$$
$$E = \begin{bmatrix} 1 & 0 \end{bmatrix}, \qquad F_1 = 1, \qquad F_2 = 0, \tag{4.1}$$
$$H = \begin{bmatrix} 0.02 \\ 0.01 \end{bmatrix}, \qquad M_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}, \qquad M_2 = 0.5.$$

For the finite-time stability test, it is assumed that

$$R = I, \qquad N = 5, \qquad c2 = 5, \qquad d = 0.1, \qquad \theta = 1.2. \tag{4.2}$$

The probability of the available measurements is 0.95. With the proposed optimization problem in (3.33), the obtained minimum performance index is $\gamma = 0.5034$ and the corresponding controller gain is

$$K = \begin{bmatrix} 1.0341 & 1.5465 \end{bmatrix}. \tag{4.3}$$

In the simulation, the initial system state is chosen as $[0.5;\ \ 0.5]$. Without the designed controller, Figure 1 depicts the trajectories of the system state. It is obvious that the open-loop

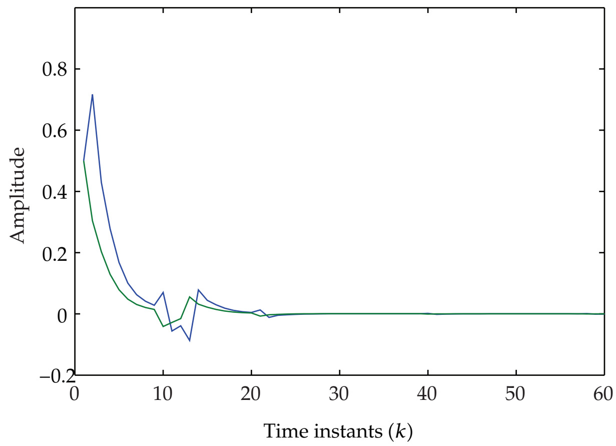

Figure 2: Trajectories of the system state in the simulation with the designed controller.

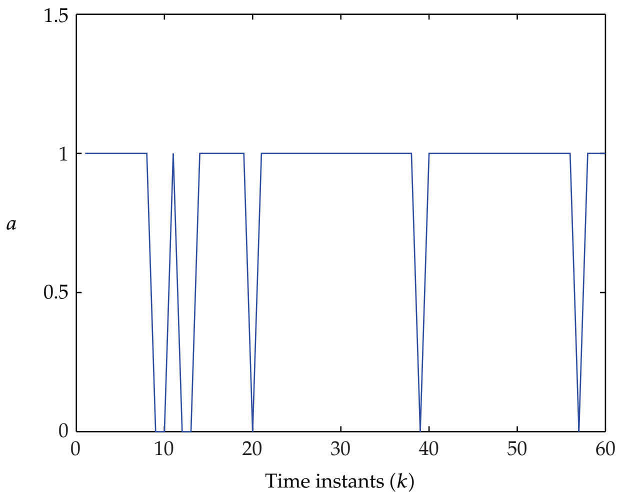

Figure 3: Stochastic values of the intermittent measurements in the simulation.

system is unstable. However, with the designed controller, Figure 2 shows the trajectories of the system state with the intermittent measurements in Figure 3. The trajectories converge to zeros even though the system is subject to uncertainties, external disturbance and intermittent measurements.

5. Conclusion

In this paper, the robust finite-time $\mathscr{H}_\infty$ controller design problem of discrete-time systems with intermittent measurements has been investigated. The uncertainties are assumed to be norm bounded. The measurements of the system state are intermittent and Bernoulli process is used to describe the phenomenon of intermittent measurements. Based on the results of the robust stochastic finite-time stability and the $\mathscr{H}_\infty$ performance, the controller design approach was proposed. Finally, an illustrative example was used to show the design procedure and the effectiveness of the proposed design methodology.

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