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Research Article

Dynamic Behaviors of a Nonautonomous Discrete Predator-Prey System Incorporating a Prey Refuge and Holling Type II Functional Response

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A nonautonomous discrete predator-prey system incorporating a prey refuge and Holling type II functional response is studied in this paper. A set of sufficient conditions which guarantee the persistence and global stability of the system are obtained, respectively. Our results show that if refuge is large enough then predator species will be driven to extinction due to the lack of enough food. Two examples together with their numerical simulations show the feasibility of the main results.

1. Introduction

As was pointed out by Berryman [1], the dynamic relationship between predator and prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. Furthermore, the study of the consequences of the hiding behavior of the prey on the dynamics of predator-prey interactions can be recognized as a major issue in both applied mathematics and theoretical ecology [2]. In general, the effects of prey refuges on the population dynamics are very complex in nature, but for modeling purposes, it can be considered as constituted by two components [2]. The first one, which affects positively the growth of prey and negatively that of predators, comprises the reduction of prey mortality due to the decrease in predation success. The second one may be the tradeoffs and by-products of the hiding behavior of prey which could be advantageous or detrimental for all the interacting populations [3].

Sih [4] obtained a set of general conditions which ensure that the refuge use has a stabilizing effect on Lotka-Volterra-type predator-prey systems; he also examined the effect of the cost of refuge use in decreased prey feeding or reproductive rate. In [5], González-Olivares and Ramos-Jiliberto investigated the dynamic behaviors of predator-prey system incorporating Holling type II functional response and a constant refuge:

$$\begin{aligned}\dot{x}(t) &= rx\left(1-\frac{x}{K}\right)-\frac{\beta(1-m)xy}{(1-m)x+a},\\ \dot{y}(t) &= -dy+\frac{c\beta(1-m)xy}{(1-m)x+a},\end{aligned} \tag{1.1}$$

where $x(t), y(t)$ denote the densities of prey and predator population at any time t, respectively; c, d, k, r, β, a, and m are positive constants; here r is the intrinsic per capita growth rate of prey; K is the prey environmental carrying capacity; β is the maximal per capita consumption rate of predators; a is the amount of prey needed to achieve one-half of β; c is the conversion factor denoting the number of newly born predators for each captured prey; d is the death rate of the predator; xm is the number of prey that refuge can protect at time t. Kar [6] also studies the dynamic behaviors of system (1.1). He obtained the conditions for the existence and stability of the equilibria and persistent criteria for the system. He also shows that the system admits a unique limit cycle when the positive equilibria is unstable. In these papers, all their finds indicate that the refuge influencing the dynamic behavior of predator-prey system greatly and increasing the amount of refuge could increase prey density and lead to population outbreaks. Kar [7] also studied the influence of harvesting on a system with prey refuge.

Some scholars argued that the nonautonomous case is more realistic, because many biological or environmental parameters do subject to fluctuate with time; thus more complex equations should be introduced. Already, many scholars [8–15] studied the dynamic behaviors of nonautonomous predator-prey system incorporating prey refuge. Recently, Xu and Jia [11] proposed and studied the nonautonomous predator-prey system incorporating prey refuge and Holling type II functional response, that is,

$$\begin{aligned}\dot{x}(t) &= (a(t)-b(t)x(t))x(t)-\frac{\beta(t)(1-m(t))x(t)y(t)}{1+\alpha(t)(1-m(t))x(t)},\\ \dot{y}(t) &= \left(-\gamma(t)-d(t)y(t)\right)y(t)+\frac{c(t)\beta(t)(1-m(t))x(t)y(t)}{1+\alpha(t)(1-m(t))x(t)},\end{aligned} \tag{1.2}$$

where $x(t)$ and $y(t)$ denote the density of prey and predator populations at time t, respectively; $x(t)m(t)$ denotes the number of prey that the refuge can protect at time t; $a(t), b(t), c(t), d(t), \alpha(t), \beta(t), \gamma(t)$, and $m(t)$ $(0 \leq m(t) < 1)$ are nonnegative continuous function that have the upper and lower bounds.

Though most dynamic behaviors of population models are based on the continuous models governed by differential equations, the discrete time models are more appropriate than the continuous ones when the size of the population is rarely small or the population has nonoverlapping generations [12]. It has been found that the dynamic behaviors of the discrete system is rather complex and contains more rich dynamics than the continuous ones [16]. Though the influence of prey refuge for continuous model has been extensively investigated, seldom did scholars investigated the influence of prey refuge for discrete predator-prey system. To the best of the authors' knowledge, to this day, only Zhuang and Wen [17] studied the local property stability of the fixed points of the discrete Leslie-Gower predator-prey systems with and without Allee effect. In this paper, we study the corresponding discrete prey-predator system of (1.2):

$$\begin{aligned} x(n+1) &= x(n)\exp\left[a(n) - b(n)x(n) - \frac{\beta(n)(1-m(n))y(n)}{1+\alpha(n)(1-m(n))x(n)}\right], \\ y(n+1) &= y(n)\exp\left[-\gamma(n) - d(n)y(n) + \frac{c(n)\beta(n)(1-m(n))x(n)}{1+\alpha(n)(1-m(n))x(n)}\right]. \end{aligned} \tag{1.3}$$

Here, we assume that $a(n), b(n), c(n), d(n), \alpha(n), \beta(n),$ and $m(n)$ $(0 \le m(n) < 1), \gamma(n)$ are all bounded nonnegative sequences. Noting that

$$\begin{aligned} x(n) &= x(0)\exp\sum_{k=0}^{n-1}\left[a(k) - b(k)x(k) - \frac{\beta(k)(1-m(k))y(k)}{1+\alpha(k)(1-m(k))x(k)}\right], \\ y(n) &= y(0)\exp\sum_{k=0}^{n-1}\left[-\gamma(k) - d(k)y(k) + \frac{c(k)\beta(k)(1-m(k))x(k)}{1+\alpha(k)(1-m(k))x(k)}\right]. \end{aligned} \tag{1.4}$$

For the point of view of biology, in the sequel, we assume that $x(0) > 0, y(0) > 0$, then from (1.4), we know that the solutions of system (1.3) are positive. From now on, for any bounded sequence $x(n)$,

$$x^u = \sup_{n\in N} x(n), \qquad x_l = \inf_{n\in N} x(n). \tag{1.5}$$

2. Permanence

We will investigate the persistent property of the system in this section.

Lemma 2.1 (see [12]). *Assume that* $\{x(k)\} > 0$ *and*

$$x(k+1) \le x(k)\exp\{a(k) - b(k)x(k)\} \tag{2.1}$$

for $k \in N$, *where* $a(k)$ *and* $b(k)$ *are nonnegative sequences bounded above and below by positive constants. Then*

$$\limsup_{k\to\infty} x(k) \le \frac{1}{b_l}\exp(a^u - 1). \tag{2.2}$$

Lemma 2.2 (see [12]). *Assume that* $\{x(k)\}$ *satisfies*

$$x(k+1) \ge x(k)\exp\{a(k) - b(k)x(k)\}, \quad k \ge N_0, \tag{2.3}$$

$\limsup_{k\to\infty} x(k) \le x^*$, *and* $x(N_0) > 0$, *where* $a(k)$ *and* $b(k)$ *are nonnegative sequences bounded above and below by positive constants and* $N_0 \in N$. *Then*

$$\liminf_{k\to\infty} x(k) \ge \min\left\{\frac{a_l}{b^u}\exp\{a_l - b^u x^*\}, \frac{a_l}{b^u}\right\}. \tag{2.4}$$

Theorem 2.3. *Every positive solution* $(x(n), y(n))$ *of system* (1.3) *satisfies*

$$\limsup_{n\to\infty} x(n) \le M_1, \qquad \limsup_{n\to\infty} y(n) \le M_2. \tag{2.5}$$

Here $M_1 = (1/b_l)\exp(a^u - 1)$, $M_2 = (1/d_l)\exp(-\gamma_l + (c^u\beta^u/\alpha_l) - 1)$.

Proof. Let $(x(n), y(n))$ be any positive solution of system (1.3). From the first equation of system (1.3) it follows that

$$x(n+1) \le x(n)\exp[a(n) - b(n)x(n)]. \tag{2.6}$$

Applying Lemma 2.1 to (2.6) leads to

$$\limsup_{n\to\infty} x(n) \le \frac{1}{b_l}\exp(a^u - 1) \overset{\text{def}}{=} M_1. \tag{2.7}$$

From the second equation of system (1.3), similarly to the analysis of (2.6)-(2.7), we can obtain

$$\limsup_{n\to\infty} y(n) \le \frac{1}{d_l}\exp\left(-\gamma_l + \frac{c^u\beta^u}{\alpha_l} - 1\right) \overset{\text{def}}{=} M_2. \tag{2.8}$$

This ends the proof of Theorem 2.3. □

Theorem 2.4. *Assume that inequalities*

$$a_l > \beta^u M_2, \qquad \gamma^u < \frac{c_l\beta_l(1-m^u)m_1}{1+\alpha^u(1-m_l)M_1} \tag{H_1}$$

hold. Let $(x(n), y(n))$ *be any positive solution of system of* (1.3), *then*

$$\liminf_{n\to\infty} x(n) \ge m_1, \qquad \liminf_{n\to\infty} y(n) \ge m_2. \tag{2.9}$$

Here

$$m_1 = \frac{a_l - \beta^u M_2}{b^u}\exp[a_l - b^u M_1 - \beta^u M_2], \qquad m_2 = \frac{-\gamma^u + B}{d^u}\exp[-\gamma^u + B - d^u M_2],$$
$$B = \frac{c_l\beta_l(1-m^u)m_1}{1+\alpha^u(1-m_l)M_1}. \tag{2.10}$$

Proof. According to the first inequality of (H_1), one could choose $\varepsilon > 0$ small enough, such that the inequality

$$a_l - \beta^u(M_2 + \varepsilon) > 0. \tag{2.11}$$

holds. For the above $\varepsilon > 0$, according to Theorem 2.3, there exists an integer $n^* \in N$ such that for all $n \geq n^*$,

$$x(n) \leq M_1 + \varepsilon, \qquad y(n) \leq M_2 + \varepsilon. \tag{2.12}$$

For $n \geq n^*$, from (2.12) and the first equation of system (1.3), we have

$$\begin{aligned} x(n+1) &= x(n)\exp\left[a(n) - b(n)x(n) - \frac{\beta(n)(1-m(n))y(n)}{1+\alpha(n)(1-m(n))x(n)}\right] \\ &\geq x(n)\exp\left[a(n) - b(n)x(n) - \frac{\beta(n)(1-m(n))(M_2+\varepsilon)}{1+\alpha(n)(1-m(n))x(n)}\right] \\ &\geq x(n)\exp\left[a(n) - b(n)x(n) - \beta(n)(1-m(n))(M_2+\varepsilon)\right] \\ &\geq x(n)\exp\left[a_l - b^u x(n) - \beta^u(1-m(n))(M_2+\varepsilon)\right] \\ &\geq x(n)\exp\left[a_l - b^u x(n) - \beta^u(M_2+\varepsilon)\right]. \end{aligned} \tag{2.13}$$

As a direct corollary of Lemma 2.2, according to (2.7) and (2.13), one has

$$\liminf_{n\to\infty} x(n) \geq \min\{A_{1\varepsilon}, A_{2\varepsilon}\}, \tag{2.14}$$

where

$$\begin{aligned} A_{1\varepsilon} &= \frac{a_l - \beta^u(M_2+\varepsilon)}{b^u}, \\ A_{2\varepsilon} &= A_{1\varepsilon}\exp\left[a_l - b^u(M_1+\varepsilon) - \beta^u(M_2+\varepsilon)\right]. \end{aligned} \tag{2.15}$$

Noting that

$$M_1 = \frac{\exp(a^u - 1)}{b_l} \geq \frac{a^u}{b_l} \geq \frac{a_l}{b^u}, \tag{2.16}$$

and so $a_l - b^u M_1 \leq 0$, consequently, for arbitrary ε,

$$a_l - b^u(M_1+\varepsilon) - \beta^u(M_2+\varepsilon) \leq 0. \tag{2.17}$$

The above inequality leads to

$$A_{1\varepsilon} \geq A_{2\varepsilon}. \tag{2.18}$$

Thus,

$$\liminf_{n\to\infty} x(n) \geq A_{2\varepsilon}. \tag{2.19}$$

Letting $\varepsilon \to \infty$, it follows that

$$\liminf_{n\to\infty} x(n) \geq A_1 = \frac{a_l - \beta^u M_2}{b^u} \exp\left[a_l - b^u M_1 - \beta^u M_2\right] \overset{\text{def}}{=} m_1. \tag{2.20}$$

According to (2.7), (2.8), and (2.20), for any $\varepsilon > 0$, there exists $n_1 \in N$, such that for all $n \geq n_1$,

$$m_1 - \varepsilon \leq x(n) \leq M_1 + \varepsilon, \qquad y(n) \leq M_2 + \varepsilon. \tag{2.21}$$

Similarly to the analysis of (2.13)–(2.20), by applying (2.21), from the second equation of system (1.3), it follows that

$$\liminf_{n\to\infty} y(n) \geq \frac{-\gamma^u + B}{d^u} \exp\left[-\gamma^u + B - d^u M_2\right] \overset{\text{def}}{=} m_2. \tag{2.22}$$

Here $B \overset{\text{def}}{=} c_l \beta_l (1 - m^u) m_1 / (1 + \alpha^u (1 - m_l) M_1)$.

This completes the proof of Theorem 2.4. □

3. Global Stability

In this section, by developing the analysis technique of [18], we obtain the conditions which guarantee the global stability of the system (1.3).

Theorem 3.1. *Assume that* (H_1) *holds, assume further that*

$$\begin{aligned}
\lambda_1 &= \max\left\{\left|1 - M_1 p_1\right|, \left|1 - m_1 p_2\right|\right\} + \frac{\beta^u (1 - m_l) M_2}{1 + \alpha_l (1 - m^u) m_1} < 1, \\
\lambda_2 &= \max\{|1 - d^u M_2|, |1 - d_l m_2|\} + \frac{c^u \beta^u (1 - m_l) M_1}{[1 + \alpha_l (1 - m^u) m_1]^2} < 1.
\end{aligned} \tag{3.1}$$

Then for any two positive solutions $(x(n), y(n))$ *and* $(\tilde{x}(n), \tilde{y}(n))$ *of system* (1.3), *one has*

$$\lim_{n\to\infty} (x(n) - \tilde{x}(n)) = 0, \qquad \lim_{n\to\infty} \left(y(n) - \tilde{y}(n)\right) = 0. \tag{3.2}$$

Here, $p_1 \overset{\text{def}}{=} b^u - (\beta_l \alpha_l m_2 (1 - m^u)^2 / [1 + \alpha^u M_1 (1 - m_l)]^2)$, $p_2 \overset{\text{def}}{=} b_l - (\beta^u \alpha^u M_2 (1 - m_l)^2 / [1 + \alpha_l m_1 (1 - m^u)]^2)$.

Proof. Let

$$x(n) = \tilde{x}(n) \exp(u(n)), \qquad y(n) = \tilde{y}(n) \exp(v(n)). \tag{3.3}$$

then system (1.3) is equivalent to

$$
\begin{aligned}
u(n+1)-u(n) = {} & \left[b(n)-\frac{\beta(n)\alpha(n)(1-m(n))^2 y(n)}{\left(1+\tilde{\delta}(n)\right)(1+\delta(n))}\right]\tilde{x}(n)(1-\exp(u(n))) \\
& + \frac{\beta(n)(1-m(n))}{1+\tilde{\delta}(n)}\tilde{y}(n)(1-\exp(v(n))), \\
v(n+1)-v(n) = {} & -\frac{c(n)\beta(n)(1-m(n))}{\left(1+\tilde{\delta}(n)\right)(1+\delta(n))}\tilde{x}(n)(1-\exp(u(n))) \\
& + d(n)\tilde{y}(n)(1-\exp(v(n))).
\end{aligned}
\tag{3.4}
$$

Here $\tilde{\delta}(n) \stackrel{\text{def}}{=} \alpha(n)(1-m(n))\tilde{x}(n)$, $\delta(n) \stackrel{\text{def}}{=} \alpha(n)(1-m(n))x(n)$. By using the mean-value theorem, it follows that

$$
\begin{aligned}
u(n+1) = {} & u(n)\left[1-\left(b(n)-\frac{\beta(n)\alpha(n)(1-m(n))^2 y(n)}{\left(1+\tilde{\delta}(n)\right)(1+\delta(n))}\right)\tilde{x}(n)\exp(\theta_1(n)u(n))\right] \\
& - \frac{\beta(n)(1-m(n))}{\left(1+\tilde{\delta}(n)\right)}v(n)\tilde{y}(n)\exp(\theta_2(n)v(n)), \\
v(n+1) = {} & u(n)\frac{c(n)\beta(n)(1-m(n))\tilde{x}(n)\exp(\theta_4(n)u(n))}{\left(1+\tilde{\delta}(n)\right)(1+\delta(n))} \\
& + v(n)(1-d(n)\tilde{y}(n)\exp(\theta_3(n)v(n))),
\end{aligned}
\tag{3.5}
$$

where $\theta_i(n) \in (0,1)$ $(i = 1,2,3,4)$. To complete the proof, it suffices to show that

$$
\lim_{n\to\infty} u(n) = \lim_{n\to\infty} v(n) = 0. \tag{3.6}
$$

In view of (3.1), we can choose $\varepsilon > 0$ small enough such that

$$
\begin{aligned}
\lambda_1^{\varepsilon} &= \max\left\{\left|1-(M_1+\varepsilon)p_1^{\varepsilon}\right|, \left|1-(m_1-\varepsilon)p_2^{\varepsilon}\right|\right\} + \frac{\beta^u(1-m_l)(M_2+\varepsilon)}{1+\alpha_l(1-m^u)(m_1-\varepsilon)} < 1, \\
\lambda_2^{\varepsilon} &= \max\{|1-d^u(M_2+\varepsilon)|, |1-d_l(m_2-\varepsilon)|\} + \frac{c^u\beta^u(1-m_l)(M_1+\varepsilon)}{[1+\alpha_l(1-m^u)(m_1-\varepsilon)]^2} < 1.
\end{aligned}
\tag{3.7}
$$

Here, $p_1^{\varepsilon} \stackrel{\text{def}}{=} b^u - (\beta_l\alpha_l(m_2-\varepsilon)(1-m^u)^2/[1+\alpha^u(M_1+\varepsilon)(1-m_l)]^2)$, $p_2^{\varepsilon} \stackrel{\text{def}}{=} b_l - (\beta^u\alpha^u(M_2+\varepsilon)(1-m_l)^2/[1+\alpha_l(m_1-\varepsilon)(1-m^u)]^2)$.

For the above $\varepsilon > 0$, according to Theorems 2.3 and 2.4, there exists a $k^* \in N$, such that

$$m_1 - \varepsilon \le x(n), \qquad \tilde{x}(n) \le M_1 + \varepsilon, \qquad m_2 - \varepsilon \le y(n), \qquad \tilde{y}(n) \le M_2 + \varepsilon, \tag{3.8}$$

for all $n \ge k^*$.

Noticing that $\theta_i(n) \in (0,1)$ $(i = 1,2,3,4)$ implies that $\tilde{x}(n)\exp(\theta_1(n)u(n))$, $\tilde{x}(n)\exp(\theta_4(n)u(n))$ lie between $\tilde{x}(n)$ and $x(n)$, $\tilde{y}(n)\exp(\theta_2(n)v(n))$, $\tilde{y}(n)\exp(\theta_3(n)v(n))$ lie between $\tilde{y}(n)$ and $y(n)$. From (3.5), it follows that

$$\begin{aligned}
|u(n+1)| &\le |u(n)|\max\left\{\left|1-(M_1+\varepsilon)p_1^{\varepsilon}\right|, \left|1-(m_1-\varepsilon)p_2^{\varepsilon}\right|\right\} \\
&\quad + |v(n)|\frac{\beta^u(1-m_l)(M_2+\varepsilon)}{1+\alpha_l(1-m^u)(m_1-\varepsilon)} \\
&\le \lambda_1^{\varepsilon}\max\{|u(n)|,|v(n)|\}, \\
|v(n+1)| &\le |v(n)|\max\{|1-d^u(M_2+\varepsilon)|, |1-d_l(m_2-\varepsilon)|\} \\
&\quad + |u(n)|\frac{c^u\beta^u(1-m_l)(M_1+\varepsilon)}{[1+\alpha_l(1-m^u)(m_1-\varepsilon)]^2} \\
&\le \lambda_2^{\varepsilon}\max\{|u(n)|,|v(n)|\}.
\end{aligned} \tag{3.9}$$

Let $\lambda = \max\{\lambda_1^{\varepsilon}, \lambda_2^{\varepsilon}\}$, then $\lambda < 1$. In view of (3.9), we have

$$\max\{|u(n+1)|,|v(n+1)|\} \le \lambda\max\{|u(n)|,|v(n)|\} \le \lambda^{n-k^*}\max\{|u(k^*)|,|v(k^*)|\}. \tag{3.10}$$

Therefore (3.6) holds and the proof is complete. □

4. Extinction of Predator Species and Stability of Prey Species

In this section, by developing the analysis technique of [16], we show that under some suitable assumptions, the predator will be driven to extinction while prey will be globally attractive to a certain solution of a logistic equation.

We consider a discrete logistic equation:

$$x(n+1) = x(n)\exp(a(n) - b(n)x(n)), \quad n \in N. \tag{4.1}$$

For the above equation, we have the following lemma.

Lemma 4.1 (see [17]). *For any positive solution $x^*(n)$ of (4.1), one has*

$$m \le \liminf_{n\to\infty} x^*(n) \le \limsup_{n\to\infty} x^*(n) \le M_1, \tag{4.2}$$

where $m = (a_l/b^u)\exp(a_l - b^u M_1)$ and M_1 is defined by Theorem 2.3.

Theorem 4.2. *Assume that the inequality*

$$m_l > 1 - \frac{\gamma_l}{c^u \beta^u M_1} \tag{H_2}$$

holds. Let $(x(n), y(n))$ *be any positive solution of system* (1.3), *then* $y(n) \to 0$ *as* $n \to +\infty$.

Proof. (H_2) is equivalent to the following inequality:

$$-\gamma_l + c^u \beta^u (1 - m_l) M_1 < 0. \tag{4.3}$$

From (4.3) we can choose positive constant $\varepsilon > 0$ small enough such that inequality

$$-\gamma_l + c^u \beta^u (1 - m_l)(M_1 + \varepsilon) < 0 \tag{4.4}$$

holds. Thus, there exists a $\delta > 0$,

$$-\gamma_l + c^u \beta^u (1 - m_l)(M_1 + \varepsilon) < -\delta < 0. \tag{4.5}$$

Let $(x(n), y(n))$ be any positive solution of system (1.3). For any $q \in N$, according to the equation of system (1.3), we obtain

$$\begin{aligned}
\ln \frac{y(q+1)}{y(q)} &= -\gamma(q) - d(q)y(q) + \frac{c(q)\beta(q)(1 - m(q))x(q)}{1 + \alpha(q)(1 - m(q))x(q)} \\
&\le -\gamma(q) + \frac{c(q)\beta(q)(1 - m(q))x(q)}{1 + \alpha(q)(1 - m(q))x(q)} \\
&\le -\gamma_l + c^u \beta^u (1 - m(q))(M_1 + \varepsilon) \\
&\le -\gamma_l + c^u \beta^u (1 - m_l)(M_1 + \varepsilon) \\
&< -\delta < 0.
\end{aligned} \tag{4.6}$$

Summating both sides of the above inequations from 0 to $n - 1$, we obtain

$$\ln \frac{y(n)}{y(0)} < -\delta n, \tag{4.7}$$

then

$$y(n) < y(0) \exp(-\delta n). \tag{4.8}$$

Theorem 2.3 implies that $x(n)$ are bounded eventually, which together with the above inequality shows that $y(n) \to 0$, exponentially, as $n \to +\infty$. This completes the proof of Theorem 4.2. □

Theorem 4.3. *Assume that $a_l > \beta^u M_2$ and (H_2) holds, also*

$$\frac{b^u}{b_l}\exp(a^u - 1) < 2. \tag{H_3}$$

Then for any positive solution $(x(n), y(n))$ of system (1.3) *and any positive solution $x^*(n)$ of system* (4.1), *one has*

$$\lim_{n\to\infty}(x(n) - x^*(n)) = 0, \qquad \lim_{n\to\infty} y(n) = 0. \tag{4.9}$$

Proof. Since (H_2) holds, it follows from Theorem 4.2 that

$$\lim_{n\to\infty} y(n) = 0. \tag{4.10}$$

To prove $\lim_{n\to\infty}(x(n) - x^*(n)) = 0$, let

$$x(n) = x^*(n)\exp(u(n)), \tag{4.11}$$

then from the first equation of system (1.3) and (4.11),

$$u(n+1) = u(n) - b(n)x^*(n)\bigl(\exp(u(n)) - 1\bigr) - \frac{\beta(n)(1 - m(n))y(n)}{1 + \alpha(n)(1 - m(n))x(n)}. \tag{4.12}$$

Using the mean-value Theorem, one has

$$\exp(u(n) - 1) = \exp(\theta(n)u(n))u(n), \quad \theta(n) \in (0, 1). \tag{4.13}$$

Then the first equation of system (1.3) is equivalent to

$$u(n+1) = u(n)\bigl(1 - b(n)x^*(n)\exp(\theta(n)u(n))\bigr) - \frac{\beta(n)(1 - m(n))y(n)}{1 + \alpha(n)(1 - m(n))x(n)}. \tag{4.14}$$

To complete the proof, it suffices to show that

$$\lim_{n\to\infty} u(n) = 0. \tag{4.15}$$

We first assume that

$$\lambda = \max\{|1 - b^u M_1|, |1 - b_l m_1|\} < 1, \tag{4.16}$$

then we can choose positive constant $\varepsilon > 0$ small enough such that

$$\lambda_\varepsilon = \max\{|1 - b^u(M_1 + \varepsilon)|, |1 - b_l(m_1 - \varepsilon)|\} < 1. \tag{4.17}$$

For the above ε, according to Theorems 2.3 and 2.4, Lemma 4.1, and (4.10), there exists an integer $n_2 \in N$ such that

$$m_1 - \varepsilon \le x(n) \le M_1 + \varepsilon, \quad m - \varepsilon \le x^*(n) \le M_1 + \varepsilon, \quad y(n) \le \varepsilon, \quad n \ge n_2. \tag{4.18}$$

Noting that $m_1 \le m$, then

$$m_1 - \varepsilon \le x(n), \quad x^*(n) \le M_1 + \varepsilon, \quad y(n) \le \varepsilon, \quad n \ge n_2. \tag{4.19}$$

It follows from (4.19) that

$$\frac{\beta(n)(1-m(n))}{1+\alpha(n)(1-m(n))x(n)} \le \frac{\beta^u(1-m_l)}{1+\alpha_l(1-m^u\ )(m_1-\varepsilon)} \overset{\text{def}}{=} M_\varepsilon, \quad n \ge n_2. \tag{4.20}$$

Noting that $\theta(n) \in (0,1)$, it implies that $x^*(n)\exp(\theta(n)u(n))$ lies between $x^*(n)$ and $x(n)$. From (4.14), (4.17)–(4.20), we get

$$\begin{aligned} |u(n+1)| &\le |u(n)| \max\{|1-b^u(M_1+\varepsilon)|, |1-b_l(m_1-\varepsilon)|\} + \frac{\beta^u(1-m_l)}{1+\alpha_l(1-m^u)(m_1-\varepsilon)}\varepsilon \\ &= \lambda_\varepsilon |u(n)| + M_\varepsilon \varepsilon, \quad n \ge n_2. \end{aligned} \tag{4.21}$$

This implies that

$$|u(n)| \le \lambda_\varepsilon^{n-n_2} |u(n_2)| + \frac{1-\lambda_\varepsilon^{n-n_2}}{1-\lambda_\varepsilon} M_\varepsilon \varepsilon, \quad n \ge n_2. \tag{4.22}$$

Since $\lambda_\varepsilon < 1$ and ε is arbitrary small, we obtain $\lim_{n\to\infty} u(n) = 0$; it means that (4.15) holds when $\lambda < 1$.

Note that

$$1 - b^u M_1 \le 1 - b_l m_1 < 1. \tag{4.23}$$

Thus, $\lambda < 1$ is equivalent to

$$1 - b^u M_1 > -1, \tag{4.24}$$

or

$$b^u M_1 = \frac{b^u}{b_l} \exp(a^u - 1) < 2. \tag{4.25}$$

Now, we can conclude that (4.15) is satisfied as (H_3) holds, and so $\lim_{n\to\infty}(x(n) - x^*(n)) = 0$. This completes the proof of Theorem 4.3. □

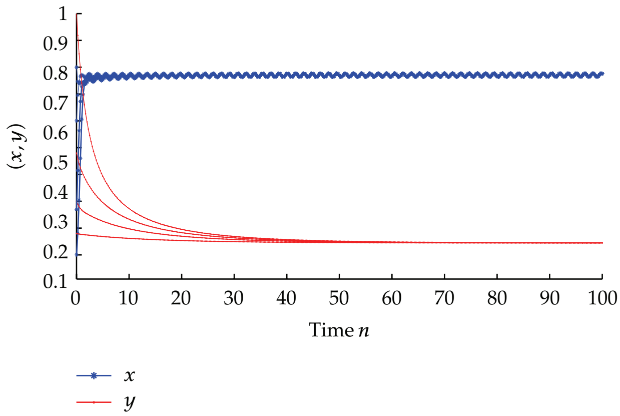

Figure 1: Dynamic behaviors of system (5.1) with the initial values $(0.14, 0.3)$, $(1.0, 0.24)$, $(0.6, 0.5)$, and $(0.4, 0.8)$.

5. Examples and Numeric Simulations

In this section, we will give two examples to show the feasibility of our results.

Example 5.1. Consider the following system:

$$x(n+1) = x(n)\exp\left[0.7 - (0.9 + 0.1\cos(n))x(n) - \frac{0.1(1-0.4)y(n)}{1+0.7(1-0.4)x(n)}\right],$$
$$y(n+1) = y(n)\exp\left[-0.01 - 0.1y(n) + \frac{0.7\times 0.1(1-0.4)x(n)}{1+0.7(1-0.4)x(n)}\right]. \tag{5.1}$$

One could easily see that $a_l - \beta^u M_2 \approx 0.2975 > 0$, $(c_l\beta_l(1-m^u)m_1/(1+\alpha^u(1-m_l)M_1)) - \gamma^u \approx 0.002 > 0$, then, condition (H_1) is satisfied. According to Theorem 2.3, system (1.3) is permanent. Numerical simulation (see Figure 1) indicates the permanence of system (5.1).

Example 5.2. Consider the following system:

$$x(n+1) = x(n)\exp\left[0.7 - (0.9 + 0.1\cos(n))x(n) - \frac{0.1(1-0.85)y(n)}{1+0.7(1-0.85)x(n)}\right],$$
$$y(n+1) = y(n)\exp\left[-0.01 - 0.1y(n) + \frac{0.7\times 0.1(1-0.85)x(n)}{1+0.7(1-0.85)x(n)}\right]. \tag{5.2}$$

We could easily see that $1-(\gamma_l/c^u\beta^u M_1) = 0.8457 < m_l = 0.85, a_l - \beta^u M_2 \approx 0.2975 > 0$, $(b^u/b_l)\exp(a^u - 1) \approx 0.926 < 2$. Clearly, conditions of Theorems 4.2 and 4.3 are satisfied. And so, $\lim_{n\to\infty}(x(n) - x^*(n)) = 0$ and $\lim_{n\to\infty} y(n) = 0$, where $x^*(n)$ is any positive solution of system (4.1). Figure 2 shows the dynamic behaviors of system (5.2).

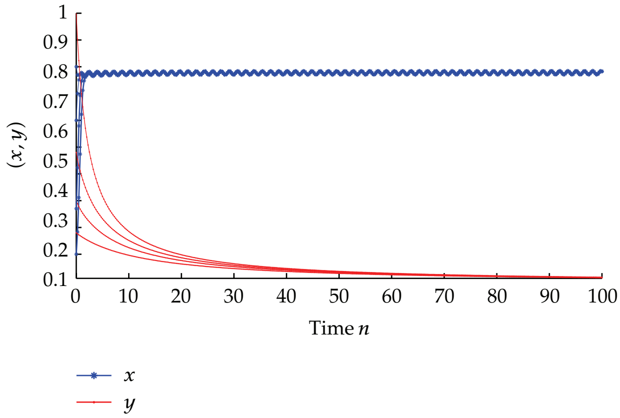

Figure 2: Dynamic behaviors of system (5.2) with the initial values $(0.1, 0.3)$, $(0.27, 0.48)$, $(0.6, 0.18)$, and $(0.8, 1.0)$.

6. Discussion

We proposed a nonautonomous discrete predator-prey system incorporating a prey refuge and Holling type II functional responses. It is well known that prey species makes use of refuges to decrease predation risk and refuge plays an important role on the dynamic behaviors of predator-prey populations. For system (1.3), we showed that the predator and prey will be coexistent in a globally stable state under some suitable conditions. However, in Section 4, we found that if the refuge is enough large, the predator species will be driven to extinction due to the fewer chances of predation. Obviously, increasing the amount of refuge can increase prey densities and lead to population outbreaks; such kind of finding is consistent with the continuous ones as shown by Kar [6]. In [11], Xu and Jia studied the continuous system (1.2). Sufficient conditions which guarantee the persistence and global stability of positive periodic solution of the system are obtained. Comparing the results of [11] with ours, we found that the conditions which guarantee the persistence of continuous system were similar to the discrete. However, for the conditions that guarantee the global stability of system, the discrete system is more complicated than that of the continuous system. Maybe the reason lies in that for the discrete population dynamics, the constructing and computing of Lyapunov function are relatively complicated than the continuous ones. Unlike the work of [11], we argued that it is an important topic to study the extinction of the species; since more and more species are driven to extinction with the development of modern society, this motivated us to study the extinction of the predator species.

At the end of the paper we would like to mention that one of the referees pointed out that “the nonautonomous character of the model is introduced to simulate the time dependent fluctuating properties of the environment. But it seems to me that a more realistic description of this dependence should be done in terms of stochastic variables rather than deterministic ones.” Indeed, recently, many excellent works concerned with the continuous population model with stochastic variables had been done, see [19–21] and the references cited therein. However, to the best of the authors’ knowledge, to this day, for discrete population dynamics, no similar work has been done. We leave this problem to future research.

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References

[1] A. A. Berryman, "The origins and evolution of predator-prey theory," *Ecology*, vol. 75, no. 5, pp. 1530–1535, 1992.

[2] Y. Wang and J. Wang, "Influence of prey refuge on predator-prey dynamics," *Nonlinear Dynamics*, vol. 67, no. 1, pp. 191–201, 2012.

[3] H. I. Freedman, *Deterministic Mathematical Models in Population Ecology*, vol. 57, Marcel Dekker, New York, NY, USA, 1980.

[4] A. Sih, "Prey refuges and predator-prey stability," *Theoretical Population Biology*, vol. 31, no. 1, pp. 1–12, 1987.

[5] E. González-Olivares and R. Ramos-Jiliberto, "Dynamic consequences of prey refuges in a simple model system: more prey, fewer predators and enhanced stability," *Ecological Modelling*, vol. 166, no. 1-2, pp. 135–146, 2003.

[6] T. K. Kar, "Stability analysis of a prey-predator model incorporating a prey refuge," *Communications in Nonlinear Science and Numerical Simulation*, vol. 10, no. 6, pp. 681–691, 2005.

[7] T. K. Kar, "Modelling and analysis of a harvested prey-predator system incorporating a prey refuge," *Journal of Computational and Applied Mathematics*, vol. 185, no. 1, pp. 19–33, 2006.

[8] Z. S. Shuai, C. M. Miao, W. P. Zhang, D. Ye, and K. Wang, "A three-species prey-predator model with refuges," *Journal of Biomathematics*, vol. 19, no. 1, pp. 65–71, 2004 (Chinese).

[9] J. Zhu and H. M. Liu, "Permanence of the two interacting prey-predator with refuges," *Journal of Northwest University For Nationalities*, vol. 27, no. 62, pp. 1–3, 2006 (Chinese).

[10] G. M. Xu and X. H. Chen, "Persistence and periodic solution for three interacting predator-prey system with refuges," *Yinshan Academic Journal*, vol. 23, no. 1, pp. 14–17, 2009 (Chinese).

[11] G. M. Xu and J. W. Jia, "Stability analysis of a predator-prey system with refuges," *Journal of Shanxi Normal University*, vol. 21, no. 4, pp. 4–7, 2007 (Chinese).

[12] J. D. Murry, *Mathematical Biology*, Springer, New York, NY, USA, 1989.

[13] F. Chen, "Permanence for the discrete mutualism model with time delays," *Mathematical and Computer Modelling*, vol. 47, no. 3-4, pp. 431–435, 2008.

[14] S. P. Yu, W. T. Xiong, and H. Qi, "A ratio-dependent one-predator two-competing-prey model with delays and refuges," *Mathematica Applicata*, vol. 23, no. 1, pp. 198–203, 2010.

[15] Y. B. Zhang, W. X. Wang, and Y. H. Duan, "Analysis of prey-predator with Holling III functional response and prey refuge," *Mathematics in Practice and Theory*, vol. 40, no. 24, pp. 149–154, 2010 (Chinese).

[16] Z. Li and F. Chen, "Extinction in two dimensional discrete Lotka-Volterra competitive system with the effect of toxic substances," *Dynamics of Continuous, Discrete and Impulsive Systems B*, vol. 15, no. 2, pp. 165–178, 2008.

[17] K. Zhuang and Z. Wen, "Dynamical behaviors in a discrete predator-prey model with a prey refuge," *International Journal of Computational and Mathematical Sciences*, vol. 5, no. 4, pp. 195–197, 2011.

[18] Y. Chen and Z. Zhou, "Stable periodic solution of a discrete periodic Lotka-Volterra competition system," *Journal of Mathematical Analysis and Applications*, vol. 277, no. 1, pp. 358–366, 2003.

[19] C. Ji, D. Jiang, and X. Li, "Qualitative analysis of a stochastic ratio-dependent predator-prey system," *Journal of Computational and Applied Mathematics*, vol. 235, no. 5, pp. 1326–1341, 2011.

[20] A. Yagi and T. V. Ton, "Dynamic of a stochastic predator-prey population," *Applied Mathematics and Computation*, vol. 218, no. 7, pp. 3100–3109, 2011.

[21] J. Lv and K. Wang, "Asymptotic properties of a stochastic predator-prey system with Holling II functional response," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 10, pp. 4037–4048, 2011.


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