Research Article

On Stochastic Finite-Time Control of Discrete-Time Fuzzy Systems with Packet Dropout

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This paper is concerned with the stochastic finite-time stability and stochastic finite-time boundedness problems for one family of fuzzy discrete-time systems over networks with packet dropout, parametric uncertainties, and time-varying norm-bounded disturbance. Firstly, we present the dynamic model description studied, in which the discrete-time fuzzy T-S systems with packet loss can be described by one class of fuzzy Markovian jump systems. Then, the concepts of stochastic finite-time stability and stochastic finite-time boundedness and problem formulation are given. Based on Lyapunov function approach, sufficient conditions on stochastic finite-time stability and stochastic finite-time boundedness are established for the resulting closed-loop fuzzy discrete-time system with Markovian jumps, and state-feedback controllers are designed to ensure stochastic finite-time stability and stochastic finite-time boundedness of the class of fuzzy systems. The stochastic finite-time stability and stochastic finite-time boundedness criteria can be tackled in the form of linear matrix inequalities with a fixed parameter. As an auxiliary result, we also give sufficient conditions on the stochastic stability of the class of fuzzy T-S systems with packet loss. Finally, two illustrative examples are presented to show the validity of the developed methodology.

1. Introduction

Systems where feedback loops are closed via digital communication channel are often regarded as networked control systems (NCSs). Compared with traditional point-to-point controller architectures, the advantages of NCSs include low cost, easy maintenance, and increased system flexibility, and so on. However, the insertion of the communication channels creates discrepancies between the data records to be transmitted and their associated remotely transmitted images and hence raises new interesting problems such as quantization, time delays, and data packet loss, see the references [1–4] and the references cited therein. Among

a number of issues arising from such a framework, packet loss of NCSs is an important issue to be addressed and has been receiving great attentions. For instance, Xiong and Lam [5] studied the problem of stability and stabilization of linear systems over networks with bounded packet loss. Bakule and De La Sen [6] tackled the problem of decentralized stabilization of networked complex composite systems with nonlinear perturbations. Wang and Yang [7] investigated the problem of state-feedback control synthesis for networked control systems with packet dropout. For more details of the literature related to networked problems with packet dropout, the reader is referred to [8–16] and the references therein. It is worth noting that Takagi-Sugeno (T-S) fuzzy model has been recognized as a popular and powerful tool in approximating and describing complex nonlinear systems [17] in the sense that it can be able to achieve any degree of accuracy in any convex compact set. The T-S fuzzy model is appealing since it combines the flexible fuzzy logic theory and fruitful linear multivariable system theory into a unified framework and then carries out the control design and analysis via Lyapunov function-based approaches. By choosing appropriate Lyapunov functions including a common Lyapunov function, piecewise Lyapunov function, or fuzzy (nonquadratic) Lyapunov function, the stability and stabilization of the class of fuzzy systems can be determined by solving a set of linear matrix inequalities (LMIs), for instance, see [18–24] and the references cited therein.

It is worth pointing out that classical control theory focuses mainly on the asymptotic behavior of the systems, which, just as was mentioned above, deals with the asymptotic property of system trajectories over an infinite time interval and does not usually specify bounds on the trajectories. In many practical applications, however, many concerned problems are the practical ones which described system state that does not exceed some bound during some time interval, for example, large values of the state are not acceptable in the presence of saturations [25]. In order to deal with these transient performance of control systems, finite-time stability or short-time stability was presented in [26]. Applying linear matrix inequality theory, some appealing results were obtained to ensure finite-time stability, finite-time boundedness, and finite-time stabilization of various systems including linear systems, nonlinear systems, fuzzy systems, and stochastic systems. For instance, Zhang and An [27] addressed the problem of finite-time stability and stabilization of linear stochastic systems. Ambrosino et al. [28] investigated finite-time stability of impulsive dynamic systems. For more details of the literature related to finite-time stability, the reader is referred to [29–35], and the references therein. However, to date and to the best of our knowledge, the problems of stochastic finite-time stability and stabilization of fuzzy systems with packet loss have not been investigated, although results related to fuzzy control systems over networks with packet loss are available in the existing literature, see the references [14–16], which motivates the main purpose of our study.

In this paper, we tackle the stochastic finite-time stability (SFTS) and stochastic finite-time boundedness (SFTB) problems for one family of fuzzy discrete-time systems over networks with packet dropout, parametric uncertainties, and time-varying norm-bounded disturbance. Firstly, we present dynamic model description studied, which, if the data packet loss is assumed to be a time-homogenous Markov process, the discrete-time fuzzy T-S systems with packet loss can be regarded as one class of fuzzy Markovian jump systems. Markov jump systems are referred to as a special family of hybrid systems and stochastic systems, which are very appropriate to model plants whose structure is subject to random abrupt changes, see the papers [36–41]. Thus, the class of fuzzy systems studied can be disposed by the theoretical framework of Markov jumps systems. Then, the concepts of SFTS and SFTB and problem formulation are given. The main contribution of this paper is to design

a state-feedback controller which guarantees the resulting closed-loop fuzzy discrete-time system with Markovian jumps SFTS and SFTB. The SFTS and SFTB criteria can be tackled in the form of LMIs with a fixed parameter. As an auxiliary result, sufficient conditions on the stochastic stability of the class of fuzzy T-S systems with packet loss are also presented.

The rest of this paper is organized as follows. Section 2 devotes to the dynamic model description and problem formulation. The results on the SFTS and SFTB are given in Section 3. Section 4 presents numerical examples to demonstrate the validity of the proposed methodology. In Section 5, some conclusions are drawn.

Notations 1. The notation used throughout the paper is fairly standard, $\mathbb{R}^n$, $\mathbb{R}^{n \times m}$, and $\mathbb{Z}^+$ denoting the sets of n component real vectors, $n \times m$ real matrices, and the set of nonnegative integers, respectively. The superscript T stands for matrix transposition or vector and $\mathbb{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure $\mathbb{P}$. In addition, the symbol $*$ denotes the transposed elements in the symmetric positions of a matrix. $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ denote the smallest and the largest eigenvalue of matrix P, respectively. Notations sup and inf denote the supremum and infimum, respectively. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulation and Preliminaries

2.1. Physical Plant

The fuzzy model proposed by Takagi and Sugeno [17] is described by fuzzy IF-THEN rules, which represents local near input-output relations of a nonlinear system. The rules of the T-S fuzzy models are of the following forms, here DFS denotes the the discrete-time fuzzy system.

Fuzzy Rule i

IF $\theta_1(k)$ is M_{i1} and $\theta_2(k)$ is $M_{i2} \cdots \theta_c(k)$ is M_{ic}, THEN

$$x(k+1) = [A_i + \Delta A_i(k)]x(k) + [B_i + \Delta B_i(k)]u(k) + [G_i + \Delta G_i(k)]w(k), \qquad (2.1)$$

where $i \in \mathbb{M} := \{1, 2, \ldots, r\}$, $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^m$ is the control input, r is the number of IF-THEN rule, $M_{i1}, M_{i2}, \ldots, M_{ic}$ are fuzzy variable, and $\theta(k) = [\theta_1(k), \theta_2(k), \ldots, \theta_c(k)]$ is the premise variables vector. It is assumed the premise variable does not depend on the input variable $u(k)$, which is employed to avoid a complicated defuzzification process of fuzzy controllers [22]. The noise signal $w(k) \in \mathbb{R}^l$ satisfies

$$\sum_{k=0}^{\infty} w^T(k)w(k) \leq d^2, \quad d \geq 0. \qquad (2.2)$$

The matrices $\Delta A_i(k)$, $\Delta B_i(k)$, and $\Delta G_i(k)$ are uncertain matrices and satisfy

$$[\Delta A_i(k), \Delta B_i(k), \Delta G_i(k)] = F_i \Delta_i(k)[E_{1i}, E_{2i}, E_{3i}], \qquad (2.3)$$

where $\Delta_i(k)$ is an unknown, time-varying matrix function and satisfies

$$\Delta_i^T(k)\Delta_i(k) \leq I, \quad \forall k \in \mathbb{Z}^+. \tag{2.4}$$

Given a pair of $(x(k), u(k))$, the final outputs of the fuzzy systems are inferred as follows:

$$x(k+1) = \sum_{i=1}^{r} h_i(\theta(k))[(A_i + \Delta A_i(k))x(k) + (B_i + \Delta B_i(k))u(k) + [G_i + \Delta G_i(k))w(k)], \tag{2.5}$$

where $h_i(\theta(k))$ are fuzzy basis function given by

$$h_i(\theta(k)) = \frac{w_i(\theta(k))}{\sum_{i=1}^{r} w_i(\theta(k))}, \qquad w_i(\theta(k)) = \prod_{j=1}^{c} M_{ij}(\theta_j(k)), \tag{2.6}$$

where $M_{ij}(\theta_j(k))$ represents the grade of membership of $\theta_j(k)$ in M_{ij}. Then, it can be seen that

$$w_i(\theta(k)) \geq 0, \quad i \in \mathbb{M}, \qquad \sum_{i=1}^{r} w_i(\theta(k)) > 0, \quad \forall k \geq 0. \tag{2.7}$$

Therefore, we can obtain

$$h_i(\theta(k)) \geq 0, \quad i \in \mathbb{M}, \qquad \sum_{i=1}^{r} h_i(\theta(k)) = 1, \quad \forall k \geq 0. \tag{2.8}$$

2.2. Communication Links and Controller

Owing to the existence of the packet dropout of the communication during the transmission, the packet dropout process of the network can be regarded as a time-homogenous Markov process $\{\gamma(k), k \geq 0\}$. Let $\gamma(k) = 1$ mean that the packet has been successfully delivered to the decoder while $\gamma(k) = 0$ corresponds to the dropout of the packet. The Markov chain has a transition probability matrix defined by

$$\mathbb{P}\{\gamma(k+1) = j \mid \gamma(k) = i\} = \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix}, \tag{2.9}$$

where $i, j \in \mathbb{W} := \{0, 1\}$ are the state of the Markov chain. Without loss of generality, let $\gamma(0) = 1$ and the failure rate p and the recovery rate q of the channel satisfy $p, q \in (0, 1)$. It is worth noting that a smaller value of p and a larger value of q indicate a more reliable channel.

Remark 2.1. When the above transition probability matrix is $\begin{bmatrix} p & 1-p \\ p & 1-p \end{bmatrix}$ with $0 \leq p \leq 1$, the above two-state Markov process is reduced to a Bernoulli process [42].

In this paper, we consider the following fuzzy control law for the DFS (2.5).

Controller Rule i

IF $\theta_1(k)$ is M_{i1} and $\theta_2(k)$ is $M_{i2} \cdots \theta_c(k)$ is M_{ic}, THEN

$$u(k) = \gamma(k)L_i x(k), \tag{2.10}$$

where L_i are to be designed the control gain matrices. $\{\gamma(k), k \geq 0\}$ is a Markov packet drop-out process satisfying (2.9). Then, we have the controller of DFS as

$$u(k) = \sum_{i=1}^{r} h_i(\theta(k))\gamma(k)L_i x(k). \tag{2.11}$$

2.3. Closed-Loop System

The following closed-loop DFS can be obtained from (2.5) and (2.11):

$$x(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k))h_j(\theta(k))\left[\left(\overline{A}_i + \gamma(k)\overline{B}_i L_j\right)x(k) + \overline{G}_i w(k)\right], \tag{2.12}$$

where $\overline{A}_i = A_i + \Delta A_i(k), \overline{B}_i = B_i + \Delta B_i(k)$, and $\overline{G}_i = G_i + \Delta G_i(k)$.

Now, we define two models according to the value of $\gamma(k)$. If $\gamma(k) = 1$, we define the Model 1 at time $k + 1$ as follows:

$$x(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k))h_j(\theta(k))\left[\left(\overline{A}_i + \overline{B}_i L_{j\zeta}\right)x(k) + \overline{G}_i w(k)\right]. \tag{2.13}$$

If $\gamma(k) = 0$, we define the Model 2 at time $k + 1$ as follows:

$$x(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k))h_j(\theta(k))\left[\overline{A}_i x(k) + \overline{G}_i w(k)\right], \tag{2.14}$$

where the selection of $L_{j\zeta}$ in (2.13) is according to the model of $x(k)$ for all $\zeta \in \{1,0\}$, that is to say, if $x(k)$ is at Model 1, which is $\gamma(k-1) = 1, L_{j\zeta} = L_{j1}$, otherwise, if $x(k)$ is at Model 2, which is $\gamma(k-1) = 0, L_{j\zeta} = L_{j0}$.

Then, (2.12) can be regarded as a DFS with Markovian jumps described by

$$
\begin{aligned}
x(k+1) = \mathbf{1}(1)\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k))h_j(\theta(k))\left[\left(\overline{A}_i + \overline{B}_i L_{j\zeta}\right)x(k) + \overline{G}_i w(k)\right] \\
+ \mathbf{1}(2)\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k))h_j(\theta(k))\left[\overline{A}_i x(k) + \overline{G}_i w(k)\right],
\end{aligned}
\tag{2.15}
$$

where $\mathbf{1}(a), a \in \{1,2\}$ denotes the mode indicator function. $\mathbf{1}(1)$ corresponds to a mode with feedback and $\mathbf{1}(2)$ corresponds to a mode without feedback. It is noted that it yields $\mathbf{1}(a) = 1$

when at time $k + 1$ be $a \in \{1, 2\}$ and $\mathbf{1}(b) = 0$ for $b \neq a$. The mode transition probabilities of Markovian jump DFS (2.15) is given by

$$\mathbb{P}\left\{\eta_v(k + 1) = v \mid \eta_u(k) = u\right\} = \pi_{uv}, \tag{2.16}$$

where $\pi_{uv} \geq 0$ for all $u, v \in \{1, 2\}$ and $\sum_{v=1}^{2} \pi_{uv} = 1$. $\eta_v(k) = 1$ implies $\mathbf{1}(1) = 1, \mathbf{1}(2) = 0$, which the communication transmission succeeds, and $\eta_v(k) = 2$ implies $\mathbf{1}(1) = 0, \mathbf{1}(2) = 1$, in which the communication dropout occurs. Thus, compared with (2.9), it follows that $\pi_{11} = 1 - p$, $\pi_{12} = p, \pi_{21} = q, \pi_{22} = 1 - q$.

2.4. Definitions and Lemmas

Definition 2.2 (stochastic finite-time stability (SFTS)). The closed-loop DFS with Markov jumps (2.15) with $w(k) = 0$ is said to be SFTS with respect to $(\delta_x, \epsilon, R, N)$, where $0 < \delta_x < \epsilon$, R is a symmetric positive-definite matrix, and $N \in \mathbb{Z}^+$, if

$$\mathbb{E}\left\{x^T(0)Rx(0)\right\} \leq \delta_x^2 \Longrightarrow \mathbb{E}\left\{x^T(k)Rx(k)\right\} < \epsilon^2, \quad \forall k \in \{1, 2, \ldots, N\}. \tag{2.17}$$

Definition 2.3 (stochastic finite-time bounded (SFTB)). The closed-loop DFS with Markov jumps (2.15) is said to be SFTB with respect to $(\delta_x, \epsilon, R, N, d)$, where $0 < \delta_x < \epsilon$, R is a symmetric positive-definite matrix, and $N \in \mathbb{Z}^+$, if

$$\mathbb{E}\left\{x^T(0)Rx(0)\right\} \leq \delta_x^2 \Longrightarrow \mathbb{E}\left\{x^T(k)Rx(k)\right\} < \epsilon^2, \quad \forall k \in \{1, 2, \ldots, N\}. \tag{2.18}$$

Lemma 2.4 (see [23]). *For any matrices M_{ij}, N_{ij} for $1 \leq i, j \leq r$, and $S > 0$ with appropriate dimensions, one has*

$$2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r}h_i h_j h_s h_t M_{ij} S N_{st} \leq \sum_{i=1}^{r}\sum_{j=1}^{r}h_i h_j \left(M_{ij}^T S M_{ij} + N_{ij}^T S N_{ij}\right). \tag{2.19}$$

In particular, when $M_{ij} = N_{ij}$ for all $1 \leq i, j \leq r$, one has

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r}h_i h_j h_s h_t M_{ij} S M_{st} \leq \sum_{i=1}^{r}\sum_{j=1}^{r}h_i h_j M_{ij}^T S M_{ij}. \tag{2.20}$$

Lemma 2.5 (Schur complement lemma, see [43]). *The linear matrix inequality $\begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} < 0$ is equivalent to $X_{22} < 0$ and $X_{11} - X_{12}X_{22}^{-1}X_{12}^T < 0$, where $X_{11} = X_{11}^T$ and $X_{22} = X_{22}^T$.*

Lemma 2.6 (see [43]). *For matrices X, Y and Z of appropriate dimensions, where X is a symmetric matrix, then*

$$X + YF(t)Z + [YF(t)Z]^T < 0 \tag{2.21}$$

holds for all matrix $F(t)$ satisfying $F^T(t)F(t) \leq I$ for all $t \in \mathbb{R}$, if and only if there exists a positive constant κ, such that the following inequality holds:

$$X + \kappa Y Y^T + \kappa^{-1} Z^T Z < 0. \tag{2.22}$$

In this paper, the feedback gain matrices L_{j1} and L_{j0} with Markov packet dropout of failure rate p and recovery rate q will be designed to guarantee the states of the closed-loop Markovian jump DFS (2.15) SFTB and SFTS.

3. Main Results

In this section, for the given failure rate p and recovery rate q with $p, q \in (0,1)$, we start by developing results that can be used to design a state-feedback controller that assures that the nominal Markovian jump DFS (2.15) is SFTB and SFTS. Then, these results will be extended to the case of uncertain systems. LMI conditions are established to design a state-feedback controller that guarantees that either a nominal Markovian jump DFS or an uncertain Markovian jump DFS of the class we are considering is SFTB and SFTS.

3.1. SFTB and SFTS of the Nominal Markov Jump System

Theorem 3.1. *For the given failure rate p and recovery rate q with $p, q \in (0,1)$, the closed-loop nominal Markovian jump DFS (2.15) is SFTB with respect to $(\delta_x, \epsilon, R, N, d)$, if there exist a scalar $\mu \geq 1$, four symmetric positive-definite matrices P_1, P_2, Q_1, Q_2, and a set of feedback control matrices $\{L_{j\zeta}, j \in \mathbb{M}, \zeta \in \{1,0\}\}$, such that the following inequalities hold for all $i, j \in \mathbb{S}$:*

$$\begin{bmatrix} -\mu P_1 & 0 & \left(A_i + B_i L_{j1}\right)^T & A_i^T \\ * & -Q_1 & G_i^T & G_i^T \\ * & * & -\dfrac{1}{1-p}P_1^{-1} & 0 \\ * & * & * & -\dfrac{1}{p}P_2^{-1} \end{bmatrix} < 0, \tag{3.1}$$

$$\begin{bmatrix} -\mu P_2 & 0 & \left(A_i + B_i L_{j0}\right)^T & A_i^T \\ * & -Q_2 & G_i^T & G_i^T \\ * & * & -\dfrac{1}{q}P_1^{-1} & 0 \\ * & * & * & -\dfrac{1}{1-q}P_2^{-1} \end{bmatrix} < 0, \tag{3.2}$$

$$\sup_{a\in\{1,2\}} \left\{\lambda_{\max}\left(\widetilde{P}_a\right)\right\}\delta_x^2 + \sup_{a\in\{1,2\}} \{\lambda_{\max}(Q_a)\}d^2 < \inf_{a\in\{1,2\}} \left\{\lambda_{\min}\left(\widetilde{P}_a\right)\right\}\mu^{-N}\epsilon^2, \tag{3.3}$$

where $\widetilde{P}_a = R^{-1/2} P_a R^{-1/2}$ for all $a \in \{1,2\}$.

Proof. Assume the mode at time k be $a \in \{1,2\}$. Taking into account that if $a = 1$, then we have $\gamma(k-1) = 1$ and $L_{j\zeta} = L_{j1}$, otherwise, if $a = 2$, then $\gamma(k-1) = 0$ and $L_{j\zeta} = L_{j0}$. Let us consider

the following quadratic Lyapunov-Krasovskii functional candidate for the Markov jump DFS (2.15):

$$V(x(k), \eta_v(k) = a) = x^T(k)P_a x(k). \tag{3.4}$$

We compute

$$
\begin{aligned}
\mathbb{E}\{V(k+1)\} &= \mathbb{E}\left\{\sum_{v=1}^{2}\mathbb{P}\{\eta_v(k+1) = v \mid \eta_v(k) = a\} \times x^T(k+1)P_v x(k+1)\right\} \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r}h_i(\theta(k))h_j(\theta(k))h_s(\theta(k))h_t(\theta(k)) \\
&\quad \times \Big\{\pi_{a1}\big[(A_i + B_i L_{j\zeta})x(k) + G_i w(k)\big]^T P_1\big[(A_s + B_s L_{t\zeta})x(k) + G_s w(k)\big] \\
&\quad + \pi_{a2}[A_i x(k) + G_i w(k)]^T P_2[A_s x(k) + G_s w(k)]\Big\}.
\end{aligned}
\tag{3.5}
$$

According to Lemma 2.4, we obtain

$$\mathbb{E}\{V(k+1)\} - \mu V(k) - w^T(k)Q_a w(k) \leq \Xi_{ij}(x(k), w(k), a), \tag{3.6}$$

where

$$
\begin{aligned}
\Xi_{ij}&(x(k), w(k), a) \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(k))h_j(\theta(k)) \\
&\quad \times \Big\{\pi_{a1}\big[(A_i + B_i L_{j\zeta})x(k) + G_i w(k)\big]^T P_1\big[(A_i + B_i L_{j\zeta})x(k) + G_i w(k)\big] \\
&\quad + \pi_{a2}[A_i x(k) + G_i w(k)]^T P_2[A_i x(k) + G_i w(k)] \\
&\quad - \mu x^T(k)P_a x(k) - w^T(k)Q_a w(k)\Big\}.
\end{aligned}
\tag{3.7}
$$

Taking into account that if $a = 1$, then $L_{j\zeta} = L_{j1}$, otherwise, $a = 2$, then $L_{j\zeta} = L_{j0}$. Noting that $\pi_{11} = 1 - p, \pi_{12} = p, \pi_{21} = q, \pi_{22} = 1 - q$. Thus, when $a = 1$, we have

$$
\begin{aligned}
\Xi_{ij}&(x(k), w(k), 1) \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(k))h_j(\theta(k)) \\
&\quad \times \Big\{(1-p)\big[(A_i + B_i L_{j1})x(k) + G_i w(k)\big]^T P_1\big[(A_i + B_i L_{j1})x(k) + G_i w(k)\big] \\
&\quad + p[A_i x(k) + G_i w(k)]^T P_2[A_i x(k) + G_i w(k)] \\
&\quad - \mu x^T(k)P_1 x(k) - w^T(k)Q_1 w(k)\Big\} \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(k))h_j(\theta(k))\begin{bmatrix} x(k) \\ w(k) \end{bmatrix}^T \left\{\Gamma_{ij1}^T\begin{bmatrix}(1-p)P_1 & 0 \\ * & pP_2\end{bmatrix}\Gamma_{ij1} - \begin{bmatrix}\mu P_1 & 0 \\ * & Q_1\end{bmatrix}\right\}\begin{bmatrix} x(k) \\ w(k) \end{bmatrix},
\end{aligned}
\tag{3.8}
$$

where

$$\Gamma_{ij1} = \begin{bmatrix} A_i + B_i L_{j1} & G_i \\ A_i & G_i \end{bmatrix}. \tag{3.9}$$

By Lemma 2.5, we can obtain from (3.1) that

$$\Xi_{ij}(x(k), w(k), 1) < 0. \tag{3.10}$$

Thus, when $a = 1$, from (3.6)–(3.10), we can obtain

$$\mathbb{E}\{V(k+1)\} - \mu V(k) < w^T(k) Q_1 w(k). \tag{3.11}$$

When $a = 2$, taking into account condition (3.2), the similar as the above discussion, we can obtain the following inequality:

$$\mathbb{E}\{V(k+1)\} - \mu V(k) < w^T(k) Q_2 w(k). \tag{3.12}$$

Thus, for all $a \in \{1, 2\}$, it follows from (3.11) and (3.12) that

$$\begin{aligned} \mathbb{E}\{V(k+1)\} - \mu V(k) &< w^T(k) Q_a w(k) \\ &\leq \sup_{a \in \{1,2\}} \{\lambda_{\max}(Q_a)\} w^T(k) w(k). \end{aligned} \tag{3.13}$$

By (3.13), it is obvious that

$$\mathbb{E}\{V(k+1)\} < \mu \mathbb{E}\{V(k)\} + \sup_{a \in \{1,2\}} \{\lambda_{\max}(Q_a)\} w^T(k) w(k). \tag{3.14}$$

From (2.2) and (3.14) and noting that $\mu \geq 1$, we have

$$\begin{aligned} \mathbb{E}\{V(k)\} &< \mu^k \mathbb{E}\{V(0)\} + \sup_{a \in \{1,2\}} \{\lambda_{\max}(Q_a)\} \sum_{j=0}^{k-1} \mu^{k-j-1} w^T(j) w(j) \\ &\leq \mu^k \mathbb{E}\{V(0)\} + \sup_{a \in \{1,2\}} \{\lambda_{\max}(Q_a)\} \sum_{j=0}^{k-1} \mu^k w^T(j) w(j) \\ &\leq \mu^k \mathbb{E}\{V(0)\} + \sup_{a \in \{1,2\}} \{\lambda_{\max}(Q_a)\} \mu^k d^2. \end{aligned} \tag{3.15}$$

Let $\widetilde{P}_a = R^{-1/2} P_a R^{-1/2}$ and noting that $\mathbb{E}\{x^T(0)Rx(0)\} \le \delta_x^2$, we have

$$
\begin{aligned}
\mathbb{E}\{V(0)\} &= \mathbb{E}\left\{x^T(0)P_a x(0)\right\} \\
&= \mathbb{E}\left\{x^T(0)R^{1/2}\widetilde{P}_a R^{1/2}x(0)\right\} \\
&\le \sup_{a\in\{1,2\}}\left\{\lambda_{\max}\left(\widetilde{P}_a\right)\right\}\mathbb{E}\left\{x^T(0)Rx(0)\right\} \\
&\le \sup_{a\in\{1,2\}}\left\{\lambda_{\max}\left(\widetilde{P}_a\right)\right\}\delta_x^2.
\end{aligned}
\tag{3.16}
$$

On the other hand, for all $a \in \{1,2\}$, we have

$$
\begin{aligned}
\mathbb{E}\{V(k)\} &= \mathbb{E}\left\{x^T(k)P_a x(k)\right\} \\
&= \mathbb{E}\left\{x^T(k)R^{1/2}\widetilde{P}_a R^{1/2}x(k)\right\} \\
&\ge \inf_{a\in\{1,2\}}\left\{\lambda_{\min}\left(\widetilde{P}_a\right)\right\}\mathbb{E}\left\{x^T(k)Rx(k)\right\}.
\end{aligned}
\tag{3.17}
$$

Combine (3.15)–(3.17) together, we can obtain

$$
\mathbb{E}\left\{x^T(k)Rx(k)\right\} < \frac{\sup_{a\in\{1,2\}}\left\{\lambda_{\max}\left(\widetilde{P}_a\right)\right\}\mu^k\delta_x^2 + \sup_{a\in\{1,2\}}\{\lambda_{\max}(Q_a)\}\mu^k d^2}{\inf_{a\in\{1,2\}}\left\{\lambda_{\min}\left(\widetilde{P}_a\right)\right\}}.
\tag{3.18}
$$

Noting condition (3.3), it follows that $\mathbb{E}\{x^T(k)Rx(k)\} < \epsilon^2$ for all $k \in \{1,2,\dots,N\}$. This completes the proof of this Theorem. $\qquad\square$

The similar proof is as the above Theorem 3.1, if $w(k) = 0$, we can obtain the results on finite-time stability as follows.

Corollary 3.2. *For the given failure rate p and recovery rate q with $p, q \in (0,1)$, the closed-loop nominal Markovian jump DFS (2.15) with $w(k) = 0$ is SFTS with respect to $(\delta_x, \epsilon, R, N)$, if there exist a scalar $\mu \ge 1$, two symmetric positive-definite matrices P_1, P_2, and a set of feedback control matrices $\{L_{j\zeta}, j \in \mathbb{M}, \zeta \in \{1,0\}\}$, such that the following inequalities hold for all $i, j \in \mathbb{S}$:*

$$
\begin{bmatrix}
-\mu P_1 & \left(A_i + B_i L_{j1}\right)^T & A_i^T \\
* & -\dfrac{1}{1-p}P_1^{-1} & 0 \\
* & * & -\dfrac{1}{p}P_2^{-1}
\end{bmatrix} < 0,
$$

$$\begin{bmatrix} -\mu P_2 & (A_i + B_i L_{j0})^T & A_i^T \\ * & -\dfrac{1}{q}P_1^{-1} & 0 \\ * & * & -\dfrac{1}{1-q}P_2^{-1} \end{bmatrix} < 0,$$

$$\sup_{a\in\{1,2\}} \left\{ \lambda_{\max}\left(\widetilde{P}_a\right) \right\} \delta_x^2 < \inf_{a\in\{1,2\}} \left\{ \lambda_{\min}\left(\widetilde{P}_a\right) \right\} \mu^{-N}\epsilon^2,$$

(3.19)

where $\widetilde{P}_a = R^{-1/2}P_a R^{-1/2}$ for all $a \in \{1,2\}$.

Remark 3.3. If $\mu = 1$ is a solution of feasibility problem (3.19), then the closed-loop Markovian jumps DFS (2.15) with $w(t) = 0$ is SFTS with respect to $(\delta_x, \epsilon, R, N)$ and is also stochastically stable.

Denoting $X_1 = P_1^{-1}, X_2 = P_2^{-1}, L_{j1} = Y_{j1}X_1^{-1}, L_{j0} = Y_{j0}X_2^{-1}$, one can obtain the following results on the stochastic finite-time stabilization.

Theorem 3.4. *For the given failure rate p and recovery rate q with $p, q \in (0,1)$, there exists a state-feedback controller $u(t) = L_{j\zeta}x(t), \zeta \in \{1,0\}$ with $L_{j1} = Y_{j1}X_1^{-1}, L_{j0} = Y_{j0}X_2^{-1}$ such that the closed-loop nominal DFS with Markovian jump (2.15) is SFTB with respect to $(\delta_x, \epsilon, R, N, d)$, if there exist a scalar $\mu \geq 1$, four symmetric positive-definite matrices X_1, X_2, Q_1, Q_2, and a set of feedback control matrices $\{Y_{j\zeta}, j \in \mathbb{M}, \zeta \in \{1,0\}\}$, such that the following inequalities hold for all $i, j \in \mathbb{S}$:*

$$\begin{bmatrix} -\mu X_1 & 0 & (A_i X_1 + B_i Y_{j1})^T & (A_i X_1)^T \\ * & -Q_1 & G_i^T & G_i^T \\ * & * & -\dfrac{1}{1-p}X_1 & 0 \\ * & * & * & -\dfrac{1}{p}X_2 \end{bmatrix} < 0,$$

(3.20)

$$\begin{bmatrix} -\mu X_2 & 0 & (A_i X_2 + B_i Y_{j0})^T & (A_i X_2)^T \\ * & -Q_2 & G_i^T & G_i^T \\ * & * & -\dfrac{1}{q}X_1 & 0 \\ * & * & * & -\dfrac{1}{1-q}X_2 \end{bmatrix} < 0,$$

(3.21)

$$\sup_{a\in\{1,2\}} \left\{ \lambda_{\max}\left(\widetilde{X}_a\right) \right\} \delta_x^2 + \sup_{a\in\{1,2\}} \left\{ \lambda_{\max}(Q_a) \right\} d^2 < \inf_{a\in\{1,2\}} \left\{ \lambda_{\min}\left(\widetilde{X}_a\right) \right\} \mu^{-N}\epsilon^2,$$

(3.22)

where $\widetilde{X}_a = R^{-1/2}X_a^{-1}R^{-1/2}$ for all $a \in \{1,2\}$.

Corollary 3.5. *For the given failure rate p and recovery rate q with $p, q \in (0,1)$, there exists a state-feedback controller $u(t) = L_{j\zeta}x(t), \zeta \in \{1,0\}$ with $L_{j1} = Y_{j1}X_1^{-1}, L_{j0} = Y_{j0}X_2^{-1}$ such that the closed-loop nominal Markovian jump DFS (2.15) with $w(k) = 0$ is SFTS with respect to $(\delta_x, \epsilon, R, N)$, if*

there exist a scalar $\mu \geq 1$, two symmetric positive-definite matrices X_1, X_2, and a set of feedback control matrices $\{Y_{j\zeta}, j \in \mathbb{M}, \zeta \in \{0,1\}\}$, such that the following inequalities hold for all $i, j \in \mathbb{S}$:

$$\begin{bmatrix} -\mu X_1 & (A_i X_1 + B_i Y_{j1})^T & (A_i X_1)^T \\ * & -\dfrac{1}{1-p} X_1 & 0 \\ * & * & -\dfrac{1}{p} X_2 \end{bmatrix} < 0, \tag{3.23}$$

$$\begin{bmatrix} -\mu X_2 & (A_i X_2 + B_i Y_{j0})^T & (A_i X_2)^T \\ * & -\dfrac{1}{q} X_1 & 0 \\ * & * & -\dfrac{1}{1-q} X_2 \end{bmatrix} < 0, \tag{3.24}$$

$$\sup_{a\in\{1,2\}} \left\{ \lambda_{\max}\left(\widetilde{X}_a\right) \right\} \delta_x^2 < \inf_{a\in\{1,2\}} \left\{ \lambda_{\min}\left(\widetilde{X}_a\right) \right\} \mu^{-N} \epsilon^2, \tag{3.25}$$

where $\widetilde{X}_a = R^{-1/2} X_a^{-1} R^{-1/2}$ for all $a \in \{1,2\}$.

3.2. Extension to SFTB and SFTS of the Uncertain Markov Jump DFS

By Theorems 3.1 and 3.4 and applying Lemmas 2.5 and 2.6, one can obtain the following results stated.

Theorem 3.6. *For the given failure rate p and recovery rate q with $p, q \in (0,1)$, there exists a state-feedback controller $u(t) = L_{j\zeta} x(t), \zeta \in \{1,0\}$ with $L_{j1} = Y_{j1} X_1^{-1}, L_{j0} = Y_{j0} X_2^{-1}$ such that the closed-loop uncertain DFS with Markovian jump (2.15) is SFTB with respect to $(\delta_x, \epsilon, R, N, d)$, if there exist a scalar $\mu \geq 1$, two sets of positive scalars $\{\epsilon_{1i}, i \in \mathbb{M}\}$, $\{\epsilon_{2i}, i \in \mathbb{M}\}$, four symmetric positive-definite matrices X_1, X_2, Q_1, Q_2, and a set of feedback control matrices $\{Y_{j\zeta}, j \in \mathbb{M}, \zeta \in \{1,0\}\}$, such that (3.22) and the following inequalities hold for all $i, j \in \mathbb{S}$:*

$$\begin{bmatrix} -\mu X_1 & 0 & (A_i X_1 + B_i Y_{j1})^T & (A_i X_1)^T & (E_{1i} X_1 + E_{2i} Y_{j1})^T & (E_{1i} X_1)^T \\ * & -Q_1 & G_i^T & G_i^T & E_{3i}^T & E_{3i}^T \\ * & * & -\dfrac{1}{(1-p)} X_1 + \epsilon_{1i} F_i F_i^T & 0 & 0 & 0 \\ * & * & * & -\dfrac{1}{p} X_2 + \epsilon_{1i} F_i F_i^T & 0 & 0 \\ * & * & * & * & -\epsilon_{1i} I & 0 \\ * & * & * & * & * & -\epsilon_{1i} I \end{bmatrix} < 0, \tag{3.26}$$

$$
\begin{bmatrix}
-\mu X_2 & 0 & (A_i X_2 + B_i Y_{j0})^T & (A_i X_2)^T & (E_{1i}X_2 + E_{2i}Y_{j0})^T & (E_{1i}X_2)^T \\
* & -Q_2 & G_i^T & G_i^T & E_{3i}^T & E_{3i}^T \\
* & * & -\dfrac{1}{q}X_1 + \epsilon_{2i}F_i F_i^T & 0 & 0 & 0 \\
* & * & * & -\dfrac{1}{1-q}X_2 + \epsilon_{2i}F_i F_i^T & 0 & 0 \\
* & * & * & * & -\epsilon_{2i}I & 0 \\
* & * & * & * & * & -\epsilon_{2i}I
\end{bmatrix} < 0, \quad (3.27)
$$

where $\widetilde{X}_a = R^{-1/2}X_a^{-1}R^{-1/2}$ for all $a \in \{1,2\}$.

Corollary 3.7. *For the given failure rate p and recovery rate q with $p,q \in (0,1)$, there exists a state-feedback controller $u(t) = L_{j\zeta}x(t), \zeta \in \{1,0\}$ with $L_{j1} = Y_{j1}X_1^{-1}, L_{j0} = Y_{j0}X_2^{-1}$ such that the closed-loop uncertain DFS with Markovian jump (2.15) with $w(k) = 0$ is SFTS with respect to $(\delta_x, \epsilon, R, N)$, if there exist a scalar $\mu \geq 1$, two sets of positive scalars $\{\epsilon_{1i}, i \in \mathbb{M}\}, \{\epsilon_{2i}, i \in \mathbb{M}\}$, two symmetric positive-definite matrices X_1, X_2, and a set of feedback control matrices $\{Y_{j\zeta}, j \in \mathbb{M}, \zeta \in \{0,1\}\}$, such that (3.25) and the following inequalities hold for all $i,j \in \mathbb{S}$:*

$$
\begin{bmatrix}
-\mu X_1 & (A_i X_1 + B_i Y_{j1})^T & (A_i X_1)^T & (E_{1i}X_1 + E_{2i}Y_{j1})^T & (E_{1i}X_1)^T \\
* & -\dfrac{1}{(1-p)}X_1 + \epsilon_{1i}F_i F_i^T & 0 & 0 & 0 \\
* & * & -\dfrac{1}{p}X_2 + \epsilon_{1i}F_i F_i^T & 0 & 0 \\
* & * & * & -\epsilon_{1i}I & 0 \\
* & * & * & * & -\epsilon_{1i}I
\end{bmatrix} < 0, \quad (3.28)
$$

$$
\begin{bmatrix}
-\mu X_2 & (A_i X_2 + B_i Y_{j0})^T & (A_i X_2)^T & (E_{1i}X_2 + E_{2i}Y_{j0})^T & (E_{1i}X_2)^T \\
* & -\dfrac{1}{q}X_1 + \epsilon_{2i}F_i F_i^T & 0 & 0 & 0 \\
* & * & -\dfrac{1}{1-q}X_2 + \epsilon_{2i}F_i F_i^T & 0 & 0 \\
* & * & * & -\epsilon_{2i}I & 0 \\
* & * & * & * & -\epsilon_{2i}I
\end{bmatrix} < 0, \quad (3.29)
$$

where $\widetilde{X}_a = R^{-1/2}X_a^{-1}R^{-1/2}$ for all $a \in \{1,2\}$.

Remark 3.8. It is easy to check that conditions (3.22) and (3.25) are guaranteed by imposing the conditions for all $a \in \{1,2\}$, respectively:

$$
\lambda_1 R^{-1} < X_a < R^{-1}, \qquad 0 < Q_a < \lambda_2 I, \qquad \begin{bmatrix} \lambda_2 d^2 - \mu^{-N}\epsilon^2 & \delta_x \\ * & -\lambda_1 \end{bmatrix} < 0, \qquad (3.30)
$$

$$
\lambda R^{-1} < X_a < R^{-1}, \qquad \begin{bmatrix} -\mu^{-N}\epsilon^2 & \delta_x \\ * & -\lambda \end{bmatrix} < 0. \qquad (3.31)
$$

Conditions (3.26)–(3.31) are not strict LMIs, however, once we fix the parameter μ, the conditions can be turned into LMIs based feasibility problem.

Remark 3.9. From the above discussion, we can obtain that the feasibility of conditions stated inz Theorem 3.6 and Corollary 3.7 can be turned into the following LMIs based feasibility problem with a fixed parameter μ, respectively:

$$
\begin{aligned}
&\min \quad \epsilon^2 \\
&\qquad X_1, X_2, Y_{j1}, Y_{j0}, \epsilon_{1i}, \epsilon_{2i}, Q_1, Q_2, \lambda_1, \lambda_2 \\
&\text{s.t.} \quad (3.26), (3.27), \text{ and } (3.30) \\
&\min \quad \epsilon^2 \\
&\qquad X_1, X_2, Y_{j1}, Y_{j0}, \epsilon_{1i}, \epsilon_{2i}, \lambda \\
&\text{s.t.} \quad (3.28), (3.29), \text{ and } (3.31).
\end{aligned}
\tag{3.32}
$$

Furthermore, we can also find the parameter μ by an unconstrained nonlinear optimization approach, in which a locally convergent solution can be obtained by using the program *fminsearch* in the optimization toolbox of MATLAB.

4. Illustrative Examples

In this section, we present two examples to illustrate the proposed methods.

Example 4.1. Consider a DFS (2.15) with parameters as

$$
A_1 = \begin{bmatrix} 2 & 0.4 & 0 \\ 0 & 0.7 & 0.3 \\ 0 & 0 & -0.6 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \\ 0 & 0.1 \end{bmatrix},
$$

$$
F_1 = \begin{bmatrix} 0.01 & 0 & 0.02 \\ 0 & 0.2 & 0 \\ 0 & 0.01 & 0 \end{bmatrix}, \quad E_{11} = \begin{bmatrix} 0.01 & 0 & 0.02 \\ 0 & 0.02 & 0 \\ 0 & 0.1 & 0 \end{bmatrix}, \quad E_{21} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \\ 0 & 0.2 \end{bmatrix}, \quad E_{31} = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0 \\ 0 & 0.1 \end{bmatrix},
$$

$$
A_2 = \begin{bmatrix} 0.8 & 3 & 0 \\ -0.2 & 0 & 0.75 \\ 0.2 & 0 & -0.4 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.5 & 0.1 \\ 0 & 1 \\ 0 & 0.1 \end{bmatrix},
$$

$$
F_2 = \begin{bmatrix} 0.1 & 0 & 0.01 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \quad E_{12} = \begin{bmatrix} 0.1 & 0 & 0.02 \\ 0 & 0.2 & 0 \\ 0 & 0.1 & 0 \end{bmatrix}, \quad E_{22} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \\ 0 & 0.3 \end{bmatrix}, \quad E_{32} = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.1 \\ 0 & 0.1 \end{bmatrix},
$$

$$
\tag{4.1}
$$

and $d = 2$, $\Delta_i(k) = \text{diag}\{\Delta_{1i}(k), \Delta_{2i}(k), \Delta_{3i}(k)\}$, where Δ_{ji} satisfies $|\Delta_{ji}(k)| \leq 1$ for all $i = 1, 2$, $j = 1, 2, 3$ and $k \in \mathbb{Z}^+$. The h_1 and h_2 are fuzzy sets defined as

$$
h_1(x(k)) = \begin{cases} \dfrac{\sin x(k)}{x(k)}, & x(k) \neq 0; \\ 1, & x(k) = 0, \end{cases}
$$

$$
h_2(x(k)) = 1 - h_1(x(k)).
\tag{4.2}
$$

Moreover, we assume the failure rate $p = 0.4$ and the recovery rate $q = 0.8$. Then, we choose $R = I_3, \delta_x = 1, N = 5, \mu = 2$, Theorem 3.6 yields $\epsilon = 21.8239$ and

$$
X_1 = \begin{bmatrix} 0.9837 & -0.0671 & 0.0587 \\ -0.0671 & 0.4318 & -0.0351 \\ 0.0587 & -0.0351 & 0.5113 \end{bmatrix}, \qquad X_2 = \begin{bmatrix} 0.9994 & 0.0078 & 0.0129 \\ 0.0078 & 0.4969 & -0.0715 \\ 0.0129 & -0.0715 & 0.5220 \end{bmatrix},
$$

$$
Y_{11} = Y_{21} = \begin{bmatrix} -1.8795 & -0.7470 & -0.1754 \\ 0.0142 & -0.0668 & -0.2793 \end{bmatrix},
$$

$$
Y_{10} = Y_{20} = \begin{bmatrix} -1.6200 & -0.8361 & 0.0592 \\ 0.0464 & -0.0911 & -0.1958 \end{bmatrix}, \tag{4.3}
$$

$$
Q_1 = \begin{bmatrix} 3.1164 & 0 \\ 0 & 3.1164 \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} 3.1139 & 0 \\ 0 & 3.1139 \end{bmatrix},
$$

$$
\epsilon_{11} = 0.6146, \qquad \epsilon_{12} = 2.3435, \qquad \epsilon_{21} = 0.6014, \qquad \epsilon_{22} = 2.1335,
$$

$$
\lambda_1 = 0.4147, \qquad \lambda_2 = 3.1177.
$$

Thus, we can obtain the following state-feedback controller gains:

$$
L_{11} = L_{21} = \begin{bmatrix} -2.0379 & -2.0938 & -0.2325 \\ 0.0404 & -0.2016 & -0.5563 \end{bmatrix},
$$

$$
L_{10} = L_{20} = \begin{bmatrix} -1.6030 & -1.6832 & -0.0717 \\ 0.0559 & -0.2491 & -0.4119 \end{bmatrix}. \tag{4.4}
$$

Furthermore, let $R = I_3, \delta_x = 1, N = 5$, by Theorem 3.6, the optimal bound with minimum value of ϵ^2 relies on the parameter μ. We can find feasible solution when $1.20 \leq \mu \leq 44.43$. Figure 1 shows the optimal value with different value of μ. Noting that when $\mu = 1.4$, it yields the optimal value $\epsilon = 13.7246$. Then, by using the program *fminsearch* in the optimization toolbox of MATLAB starting at $\mu = 1.4$, the locally convergent solution can be derived as

$$
L_{11} = L_{21} = \begin{bmatrix} -1.8847 & -1.8653 & -0.6149 \\ 0.0512 & -0.1196 & -0.5385 \end{bmatrix},
$$

$$
L_{10} = L_{20} = \begin{bmatrix} -1.5200 & -1.6018 & -0.3826 \\ 0.0426 & -0.1279 & -0.4005 \end{bmatrix}. \tag{4.5}
$$

with $\mu = 1.3591$ and the optimal value $\epsilon = 13.6304$.

Example 4.2. Consider a DFS (2.15) with $w(t) = 0$ and

$$
A_1 = \begin{bmatrix} 1.5 & 0.4 & 0 \\ 0 & 0.7 & 0.3 \\ 0 & 0.5 & -1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 1 & 0.4 & 0 \\ -0.3 & 0 & 0.75 \\ 0.6 & 0.8 & -0.4 \end{bmatrix}. \tag{4.6}
$$

In addition, the other matrices parameters, fuzzy basis functions, and the failure rate p and the recovery rate q are the same as Example 4.1.
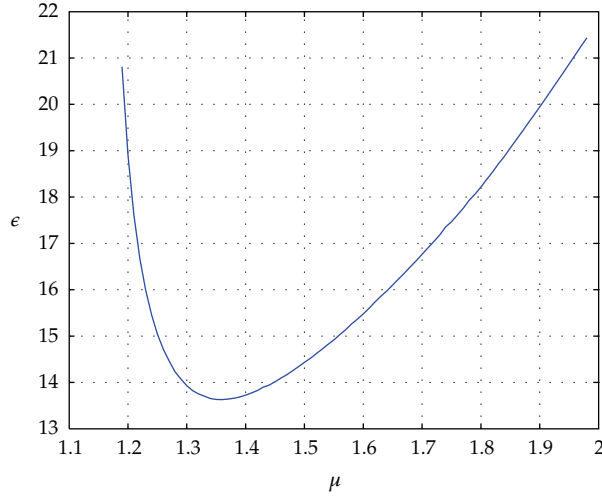
Figure 1: The local optimal bound of ϵ.

Then, let $R = I_3, \delta_x = 1, N = 6$, by Corollary 3.7, the optimal bound with minimum value of ϵ^2 relies on the parameter μ. We can find feasible solution when $1 \leq \mu \leq 23.61$. Thus, the above system is stochastically stable and when $\mu = 1$, it yields the optimal value $\epsilon = 4.5131$ and the following optimized state-feedback controller gains:

$$
\begin{aligned}
L_{11} = L_{21} &= \begin{bmatrix} -1.6149 & -0.3843 & -0.8106 \\ -0.0447 & -0.4269 & 0.2344 \end{bmatrix}, \\
L_{10} = L_{20} &= \begin{bmatrix} -1.3138 & -0.2433 & -0.7052 \\ -0.0960 & -0.3308 & 0.2755 \end{bmatrix}.
\end{aligned}
\tag{4.7}
$$

5. Conclusions

This paper addresses the stochastic finite-time stability and stochastic finite-time boundedness problems for one family of fuzzy systems over networks with packet dropout. Taking into account the data packet drop being a time homogenous Markov process, thus the discrete-time DFS with packet loss can be described by one class of fuzzy Markovian jump systems. Then, state-feedback controllers are designed to guarantee the resulting closed-loop fuzzy discrete-time system with Markovian jumps SFTS and SFTB. The SFTS and SFTB criteria can be tackled in the form of linear matrix inequalities with a fixed parameter. As an auxiliary result, we also give sufficient conditions on the stochastic stability of the class of fuzzy systems with data packet dropout. Finally, simulation results are also given to demonstrate the validity of the proposed methodology.

Acknowledgments

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