Research Article

A Fuzzy Optimization Model for High-Speed Railway Timetable Rescheduling

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A fuzzy optimization model based on improved symmetric tolerance approach is introduced, which allows for rescheduling high-speed railway timetable under unexpected interferences. The model nests different parameters of the soft constraints with uncertainty margin to describe their importance to the optimization purpose and treats the objective in the same manner. Thus a new optimal instrument is expected to achieve a new timetable subject to little slack of constraints. The section between Nanjing and Shanghai, which is the busiest, of Beijing-Shanghai high-speed rail line in China is used as the simulated measurement. The fuzzy optimization model provides an accurate approximation on train running time and headway time, and hence the results suggest that the number of seriously impacted trains and total delay time can be reduced significantly subject to little cost and risk.

1. Introduction

The infrastructures of high-speed railway have been extensively developed in China for the past several years. The network topology structure and operation mode of the railway are changing profoundly. The target is to cover its major economic areas with a high-speed railway network, which consists of four horizontal and four vertical lines [1], in the following several years. The network scale is much larger than any existing ones in the world. However, passengers have to face huge challenging variables generated by unexpectedness, for example, weather, equipment failure. On June 20, 2010, for example, the southern part of the existing Beijing-Shanghai line experienced speed restriction for a heavy rain storm. As announced, 18 trains departing from Shanghai were canceled, thereby more than 20,000 passengers being affected. Also, the rest in-service trains were more or less delayed. Considering the high speed and frequency of bullet train, the impact of train delay and

speed restriction would be more serious than that on existing lines (in this paper, the existing nonhigh-speed railway line is called existing line for short). On the other hand, high-speed lines are more passenger-oriented than existing lines, where punctuality is extraordinarily important. Thus, train timetable rescheduling is a focal point to improve the operation mode with the characteristics of high train speed, high train frequency, and mixed train speed (HHM), which is of great theoretical and practical significance for safe and efficient operation of China high speed railway network.

Train rescheduling in China is mostly manually developed by the operator with the support of computer in the existing line or the high speed railway, which depends on individual's experience to dominate single line or some sections in one line with fixed three-hour period. In future, China will establish six comprehensive operation centers of the railway network, each of which will dominate several lines covering one thousand kilometers overall. The scale of rescheduling object is far larger, and the relationship between lines is far more complex than the existing situation, so the inefficient manual rescheduling, which seriously affects the use of the ability of high-speed rail and brings risk to railway safety, cannot support the advanced operation command mode.

The research on the train operation automatic adjustment has been a controversial topic for many years in literature. Variable academic algorithms for optimal rescheduling have been put forward. Some traditional optimization model, such as integer programming (branch and bound method [2–5]) and linear programming [6–9], are usually used for the single transportation organization mode (only high-speed or medium-speed train running). In China, there are usually mixed high-speed and medium-speed trains running in one line, so there are great differences on the variable size, constraints, and target in the actual situation of the Chinese high-speed railway, and these methods cannot be directly applied to the train rescheduling problems in China.

Since the railway network has been extensively developed, and the operation mode is becoming more complex in recent years, many heuristic methods, such as DEDS-based simulation method [10], expert system [11, 12], tabu search [13–15], and some computational intelligence methods, such as genetic algorithm [16–18], particle swarm optimization algorithm [19–21], and other composite algorithms [22–26], were used to solve the large-scale combinatorial optimization problem by many scholars. To some extent, these mathematical models and optimization algorithms have given feasible solutions for train operation automatic adjustment problem. However, the decision variables, parameters, and constraints are far more complex, as they affect on the goals for high-speed railway timetable rescheduling optimization. For example, typical uncertain variables, such as the running time at a section and the dwell time at a station of the train, are easily affected by railway and equipment status, driving behavior of drivers, and environment condition [27, 28]. As a result, these methods are limited on applying to the actual operation environment of high-speed railway under the significant influence of uncertainty, and hence they cannot be considered as an appreciative approach to either achieve the purpose of automatic rescheduling or play the role of decision support in practical applications. Therefore, the uncertainty character of high-speed train automatic adjustment model should be considered in substance.

Uncertainty optimization theory is to solve the optimization decision problem with all kinds of uncertainties, which involves stochastic optimization, fuzzy optimization, rough set, and so on. Nowadays, some papers studied uncertainty in train scheduling problem. Jia and Zhang [29] proposed a distributed intelligent railway traffic control system based on fuzzy decision making. Yang et al. [30] investigates a passenger train timetable problem

with fuzzy passenger demand on a single line railway. Since the number of passengers getting on/off the train at each station is assumed to be a fuzzy variable, the total passengers time is also a fuzzy variable. An expected value goal-programming model is constructed to minimize the total passenger time and the total delay. A branch-and-bound algorithm based on the fuzzy simulation is designed in order to obtain an optimal solution. Then they extend the uncertainty in [31] by considering stochastic and fuzzy parameters synchronously to solve the railway freight transportation planning problem. Based on the chance measure and critical values of the random fuzzy variable, three chance-constrained programming models are constructed for the problem with respect to different criteria. A fuzzy periodic job shop scheduling model is introduced to address the framework of the periodic robust train scheduling problem in [32]. Fuzzy approach is used to reach a tradeoff among the total train delays, the robustness of schedules, and the time interval between departures of trains from the same origins in this paper. Cucala et al. [33] proposes a fuzzy linear programming model to minimize energy consumption with uncertain delays and drivers behavioral response. The method is applied to a real Spanish high speed line to optimize the operation, and comparing to the current commercial service evaluates the potential energy savings. Wang et al. [34, 35] and Guo et al. [36] analyzed the driver's safety approaching behaviour and pedestrian safety crossing behaviour in the urban traffic environment. The traffic participants revealed different behavioral decisions with various personal characteristics and can be described as fuzzy parameters. Overall, none of the previous papers analyzed the uncertainty of all factors in train rescheduling, although they considered some constraints as fuzzy member, such as passenger time and delay time. Furthermore, the fuzzy constraints are defined as triangular or trapezoidal fuzzy numbers, which are very difficult to determine the distribution.

In this paper, we attempt to achieve optimal timetable rescheduling under the uncertainties, for example, constraints and/or unexpected parameters, by means of proposing the fuzzy optimization model as discussed in the above. In the rest of this paper, typical rescheduling model will be discussed in the section of Timetable Rescheduling Problem. In the following section, we will describe in details of the fuzzy optimization model based on improved tolerance approach to timetable rescheduling, including the fuzzy membership functions of the original objective and soft constraints. A case study on the busiest section of Beijing-Shanghai high speed line will be illustrated in the section of Case Study. The final section concludes the results of the paper and suggests for further research.

2. Timetable Rescheduling Problem

The aim of train rescheduling is to get a new timetable that adjusts the train movements to be consistent with the planned schedule as much as possible under some interference [37]. The following model focuses on minimizing the total delay as well as the number of seriously impacted trains.

2.1. Input Data

Take a rail line with n trains and m stations for example. The numerical inputs are described as follows:

S_j: the station j according to original timetable.

$x_{i,j}^*$: the departure time of train i at station j according to original timetable.

$y_{i,j}^*$: the arrival time of train i at station j according to original timetable.

$a_{i,j}$: the minimum running time of train i on section $[j, j + 1]$.

τ_j^f: the time interval between two adjacent trains at station j.

τ_j^d: the headway of section $[j, j + 1]$.

τ_j^{tr}: the track number of station j.

$T_{i,j}$: the minimum dwell time of the train i at station j.

$h_{i,j}$: if train i stop at station j according to original timetable, then $h_{i,j} = 1$; otherwise, $h_{i,j} = 0$.

c_i^{delay}: the cost per time unit delay for train i.

T_i^D: delay tolerance for train i.

θ: weight for objective.

M: a very big integer, for example, 100000.

2.2. Decision Variables

The decision variables are described as follows:

$x_{i,j}$: the new departure time of train i at station j after adjustment.

$y_{i,j}$: the new arrival time of train i at station j after adjustment.

$d_{i,j}$: delay of train i at station j, which is defined as the difference between the arrival time after adjustment and the planned arrival time in the original timetable.

b_i: if train reaches its final considered stop with a delay larger than w_i, then $b_i = 1$; otherwise, $b_i = 0$.

$\eta_{i,j,l}$: if train i use track l at station j, then $\eta_{i,j,l} = 1$; otherwise, $\eta_{i,j,l} = 0$.

$\alpha_{i,j,i',j'}$: if the departure time of train i at station j is earlier than the departure time of train i' at station j' to original timetable, then $\alpha_{i,j,i',j'} = 1$; otherwise, $\alpha_{i,j,i',j'} = 0$.

$\beta_{i,j,i',j'}$: if the departure time of train i at station j is changed to occur after the departure time of train i' at station j', then $\beta_{i,j,i',j'} = 1$; otherwise, $\beta_{i,j,i',j'} = 0$.

2.3. Objective Functions

(i) To minimize the delay cost:

$$\text{Minimize} \sum_{i=1}^{n} \left(c_i^{delay} \cdot \sum_{j=1}^{m} d_{i,j} \right). \tag{2.1}$$

(ii) To minimize the number of seriously impacted trains:

$$\text{Minimize} \sum_{i=1}^{n} b_i. \tag{2.2}$$

We set the final objective function as:

$$S = \theta \sum_{i=1}^{n} \left(c_i^{\text{delay}} \cdot \sum_{j=1}^{m} d_{i,j} \right) + (1 - \theta) \sum_{i=1}^{n} b_i. \tag{2.3}$$

2.4. Constraints

(i) Section running time restrictions:

$$y_{i,j+1} \geqslant x_{i,j} + a_{ij}. \tag{2.4}$$

The real departure time cannot be earlier than the original departure time:

$$\begin{aligned} x_{i,j} &\geqslant x_{i,j}^*, \qquad h_{i,j} = 1, \\ x_{i,j} - x_{i,j}^* &= d_{i,j}. \end{aligned} \tag{2.5}$$

(ii) Station dwell time restrictions:

$$x_{i,j} - y_{i,j} \geqslant T_{i,j}. \tag{2.6}$$

(iii) Track restrictions:

$$\sum_{l=1}^{\tau_j^{\text{tr}}} \eta_{i,j,l} = 1. \tag{2.7}$$

(iv) Station headway restrictions.

For each station, if two trains use the same track, at least one of α and β is forced to be 1:

$$\eta_{i,j,l} + \eta_{i',j,l} - 1 < \alpha_{i,j,i',j'} + \beta_{i,j,i',j'}, \tag{2.8}$$

$$y_{i',j} - y_{i,j} \geqslant \tau_j^f \alpha_{i,j,i',j'} - M(1 - \alpha_{i,j,i',j'}), \tag{2.9}$$

$$y_{i,j} - y_{i',j} \geqslant \tau_j^f \beta_{i,j,i',j'} - M(1 - \beta_{i,j,i',j'}). \tag{2.10}$$

(v) Section headway restrictions.

For each section, at least one of α and β is forced to be 1 because there is only one track:

$$\alpha_{i,j,i',j'} + \beta_{i,j,i',j'} = 1, \tag{2.11}$$

$$x_{i',j} - x_{i,j} \geqslant \tau_j^d \alpha_{i,j,i',j'} - M(1 - \alpha_{i,j,i',j'}), \tag{2.12}$$

$$x_{i,j} - x_{i',j} \geqslant \tau_j^d \beta_{i,j,i',j'} - M(1 - \beta_{i,j,i',j'}). \tag{2.13}$$

(vi) Auxiliary restrictions:

$$d_{\text{lost}(i)} - T_i^D \leqslant M b_i. \tag{2.14}$$

In practice, some constraints of the model are not strictly satisfied due to the inexact operation time. Thus four of the above constraints need to be changed as below.

(i) Section Running Time Restrictions

$a_{i,j}$ is the minimum running time of train i in section $[j, j+1]$, including the pure running time in section and the additional time for train stop or departure at the station. The minimum running time is decided by the length of the section, the infrastructure characters of the section, and the train type. In the actual operation environment, the train running speed is not a constant value because of the various infrastructure characters of the railway, that is, bridges, tunnels, and culverts. Furthermore, some factors (i.e., railway equipment statuses, technological level of crew, and weather condition) also increase the uncertainty of running time. Thus it is very important to find a safe and reasonable average speed that is far below the limited speed and full utilization of railway capacity, which will greatly improve the optimization result especially when some trains are delayed due to some interference. Since the minimum running time of the train in the section usually changes within a certain range, there should be a tolerance for $a_{i,j}$. Thus (2.4) does not need to be strictly satisfied and can be changed as below:

$$g_1 = y_{i,j+1} - x_{i,j} \gtrless a_{i,j}. \tag{2.15}$$

(ii) Station Dwell Time Restrictions

$T_{i,j}$ is the minimum dwell time of the train i at station j, including the pure operation time at station, passengers on and off time, crew setup time, and some additional time, like waiting for other trains. The minimum dwell time is decided by station operation type, station level, and the train type. In the actual operation environment, railway equipment statuses, driving behavior of drivers, and environment condition may increase the uncertainty of running time. Thus it is also important to find a safe and reasonable average dwell time that can be full

utilization of station capacity. Similar to section running time, there also should be a tolerance for $T_{i,j}$, and the dwell time constraint can be changed as below:

$$g_2 = x_{i,j} - y_{i,j} \gtrapprox T_{i,j}. \tag{2.16}$$

(iii) Station Headway Restrictions

The headway time for each station is decided by the number of receiving-departure track, the operation time of turnout, and holding time of the track, which also faces uncertainty problems due to the factors of equipment status and human technological level. So there should be a tolerance for τ_j^f. Equations (2.9) and (2.10) do not need to be strictly satisfied and can be changed as below:

$$g_5 = y_{i',j} - y_{i,j} \gtrapprox \tau_j^f \alpha_{i,j,i',j'} - M(1 - \alpha_{i,j,i',j'}),$$
$$g_6 = y_{i,j} - y_{i',j} \gtrapprox \tau_j^f \beta_{i,j,i',j'} - M(1 - \beta_{i,j,i',j'}). \tag{2.17}$$

(iv) Section Headway Restrictions

The headway time for each section is decided by the number and length of the block between two tracking trains and the speeds of the trains, which ranges from 2.4 minutes to 3 minutes according to Shi [38]. Similar to station headway constraint, (2.12) and (2.13) can be changed as below:

$$g_3 = x_{i',j} - x_{i,j} \gtrapprox \tau_j^d \alpha_{i,j,i',j'} - M(1 - \alpha_{i,j,i',j'}),$$
$$g_4 = x_{i,j} - x_{i',j} \gtrapprox \tau_j^d \beta_{i,j,i',j'} - M(1 - \beta_{i,j,i',j'}). \tag{2.18}$$

Since the four constraints are changed as above, the model turns to be not representative in the sense of mathematical viewpoints. To construct a reasonable mathematical model under the uncertain environment, the tolerance approach based timetable rescheduling model will be introduced in the next section.

3. Fuzzy Optimization Model for Timetable Rescheduling

In the paper, we use the fuzzy optimization based improved tolerance approach to solve the uncertainty program, and some necessary backgrounds and notions of the approach are reviewed.

3.1. Improved Tolerance Approach

Tolerances are indicated in any technical process, that is, the admissible limit of variation around the object value and the deviations allowed from the specified parameters [39].

A general model of a fuzzy linear programming problem is presented by the following system [40]. $\tilde{A}_{i,j}$, $\tilde{B}_i$, and $\tilde{C}_j$ ($i = 1 \cdots m; j = 1 \cdots n$) are fuzzy set in R. The symbol $\oplus$ represents the extended addition. Each real number can be modeled as a fuzzy number.

$$
\begin{aligned}
\text{Minimize} \quad & \tilde{C}_1 x_1 \oplus \tilde{C}_2 x_2 \oplus \cdots \oplus \tilde{C}_n x_n, \\
\text{Subject to} \quad & \tilde{A}_{i,1} x_1 \oplus \tilde{A}_{i,2} x_2 \oplus \cdots \oplus \tilde{A}_{i,n} x_n \geqslant \tilde{B}_i, \quad i = 1 \ldots m.
\end{aligned}
\tag{3.1}
$$

Here we only discuss the special case (3.2) where the objective function is crisp, some constraints have the soft form, and the rest constraints are crisp.

$$
\begin{aligned}
\text{Minimize} \quad & z(x) = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n, \\
& g_i(x) = a_{i,1} x_1 + a_{i,2} x_2 + \cdots + a_{i,n} x_n \gtrsim \tilde{B}_i, \quad i = 1 \cdots m_1, \\
& a_{i,1} x_1 + a_{i,2} x_2 + \cdots + a_{i,n} x_n \geqslant b_i, \quad i = m_1 + 1 \cdots m, \\
& x_1, \ldots, x_n \geqslant 0.
\end{aligned}
\tag{3.2}
$$

The m_1 soft constraints may be described more precisely by the fuzzy set with the support $[b_i - d_i, b_i]$, and d_i is the tolerance according to b_i. Moreover the membership function of $g_i(x)$ can be specified as (3.3), and its graphics is Figure 1(a). Then we can directly assigns a measure of the satisfaction of the ith constraint to the solution $X = (x_1, x_2, \ldots, x_n)$.

$$
\mu(g_i) = \begin{cases}
0, & g_i \leqslant b_i - d_i, \\
\dfrac{g_i - (b_i - d_i)}{d_i}, & b_i - d_i < g_i \leqslant b_i, \ i = 1 \cdots m_1, \\
1, & b_i \leqslant g_i.
\end{cases}
\tag{3.3}
$$

Fuzzy constraints will inevitably lead to the fuzzy objective based on the ideology of symmetric model. Thus, for the objective function $z(x)$, there is a fuzzy set $\tilde{Z} = \{(z, \mu_z(z)) \mid z \in R\}$. Let $X_l = \{x \in R \mid a_{i,1} x_1 + \cdots + a_{i,n} x_n \geqslant b_i - d_i, \text{ for all } i = 1 \cdots m_1 \text{ and } a_{i,1} x_1 + \cdots + a_{i,n} x_n \geqslant b_i, \text{ for all } i = m_1 + 1 \cdots m\}$, $X_U = \{x \in R \mid a_{i,1} x_1 + a_{i,2} x_2 + \cdots + a_{i,n} x_n \geqslant b_i, \text{ for all } i = 1 \cdots m\}$, $\bar{z} = \text{Min}_{x \in X_U} z(x)$, and $\underline{z} = \text{Min}_{x \in X_L} z(x)$. So the membership function $\mu_z(z)$ of $\tilde{Z}$ is given by (3.4), and its graphics is Figure 1(b).

$$
\mu_z(z) = \begin{cases}
0, & \bar{z} \leqslant z, \\
1 - \dfrac{z - \underline{z}}{\bar{z} - \underline{z}}, & \underline{z} < z \leqslant \bar{z}, \\
1, & z \leqslant \underline{z}.
\end{cases}
\tag{3.4}
$$

In order to determine a compromise solution, it is usually assumed that the total satisfaction of a decision maker may be described by $\lambda(x) = \min(\mu_z(x), \mu_1(x), \mu_{m_1}(x))$. Since not all the constraints are equally important to the solution in practice, we take λ_i to describe the priority of the ith soft constraint to the solution, and $\lambda(x)$ can be described by $\lambda(x) = \sum_1^{m_1} w_i \lambda_i(x)$. w_i is the weight of λ_i, which shows the importance of different soft
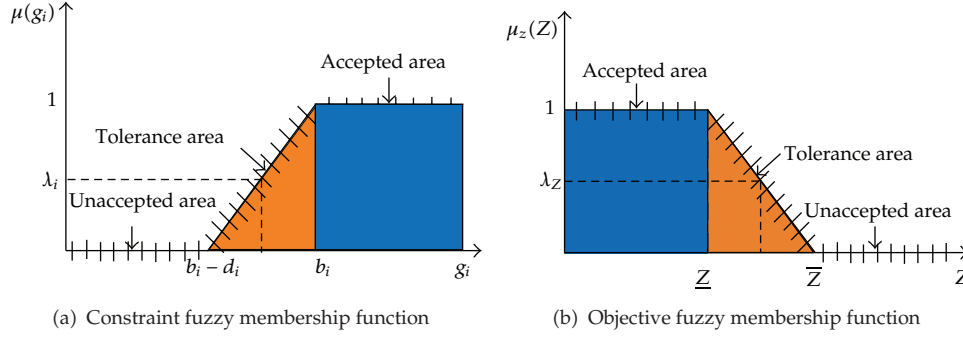
(a) Constraint fuzzy membership function

(b) Objective fuzzy membership function

Figure 1: Fuzzy membership functions.

constraints to the optimization purpose. Generally, there are many complex constraints for different optimization systems in practice. It is very hard to handle all constraints during the system optimization, and therefore the systems always treat the constraints with different priorities. Moreover the decision makers also hold different views on the importance of various factors about the same issue. The timetable rescheduling problem, for example, will take different optimization objectives and constraint priorities in different emergencies. The dispatcher will focus on the reducing of train delay time and train headway time if few trains are disturbed by equipment failure. However, minimizing the number of seriously impacted trains and the train interval running time as much as possible are more effective measures to return to the normal states, when some accidents or natural disasters lead to speed restriction on lots of sections for a long time.

Since we treat the objective in the same manner as the soft constraints, this is also a symmetric model [41]. More information about symmetric fuzzy model can refer to [42]. The optimization program is clearly equivalent to (3.5). Using linear membership functions or piecewise linear, concave membership function, the system can easily be solved by well-known algorithms.

$$\text{Maximize } \lambda,$$
$$\mu_z(x) \geqslant \lambda,$$
$$\mu_i(x) \geqslant \lambda_i, \quad i = 1 \cdots m_1, \tag{3.5}$$
$$a_{i,1}x_1 + a_{i,2}x_2 + \cdots + a_{i,n}x_n \geqslant b_i, \quad i = m_1 + 1 \cdots m,$$
$$x \in X, \quad \lambda_i \in [0,1].$$

Figure 2 shows the principle of the tolerance based symmetric fuzzy model. When the constraint relaxes to b^1, the objective value minimized to Z^1 (a significant improvement). However, when the constraint relaxes to b^2, the objective value improves little ($Z^1 - Z^2$) compare to the slack of constraint ($b^2 - b^1$). The tolerance-based symmetric fuzzy model is to get the maximum value of the fuzzy member λ, such that the system can obtains a great improvement of the objective on the conditions of less relaxation of the constraints. This means, for the train adjustment problem, we can get a new timetable to eliminate interference as much as possible with little slack of train rescheduling constraints.
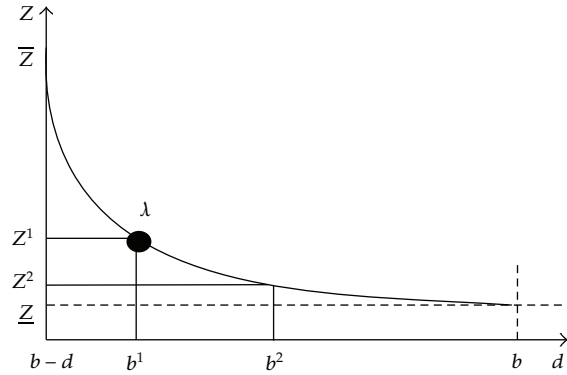
Figure 2: The principle of the fuzzy model.

3.2. A New Timetable Rescheduling Model

In the rescheduling model mentioned above, four additional inputs are involved to describe the tolerances of minimum running time, dwell time, and headway time for section and station, respectively. An additional decision variable λ is used to describe the fuzzy number for the objective and constraints. All the additional parameters are listed as below:

$c_{a_{i,j}}$: tolerance of $a_{i,j}$.

$c_{T_{i,j}}$: tolerance of $T_{i,j}$.

$c_{\tau_j^d}$: tolerance of τ_j^d.

$c_{\tau_j^f}$: tolerance of τ_j^f.

λ_i: decision variable for the fuzzy number.

s^0: objective value of the original model based on original constraints.

s^*: objective value of the original model based on relaxed constraints.

Fuzzy membership functions of objective and soft constraints are given by μ_0 to μ_6 (3.6); their graphics are similar to Figure 1. The timetable rescheduling model is changed as (3.7). Some principles for the relaxation extent of the constraints are summarized as follows. (1) Collect statistics of train running states in all kinds of emergencies in reality. (2) Consult with the train drivers and dispatchers, who have a wealth of experience about train driving

and operation. (3) Refer to the train safety operation specifications under different emergency conditions.

$$\mu_0 = \begin{cases} 1, & S < s^*, \\ 1 - \dfrac{S - s^*}{s_0 - s^*}, & s^* \leqslant S \leqslant s_0, \\ 0, & S > s_0, \end{cases} \qquad \mu_1 = \begin{cases} 1, & g_1 > a_{i,j}, \\ 1 - \dfrac{a_{i,j} - g_1}{c_{a_{i,j}}}, & a_{i,j} - c_{a_{i,j}} \leqslant g_1 \leqslant a_{i,j}, \\ 0, & g_1 < a_{i,j} - c_{a_{i,j}}, \end{cases}$$

$$\mu_2 = \begin{cases} 1, & g_2 > T_{i,j}, \\ 1 - \dfrac{T_{i,j} - g_2}{c_{T_{i,j}}}, & T_{i,j} - c_{T_{i,j}} \leqslant g_2 \leqslant T_{i,j}, \\ 0, & g_2 < T_{i,j} - c_{T_{i,j}}, \end{cases} \quad \mu_3 = \begin{cases} 1, & g_3 > \tau_j^d, \\ 1 - \dfrac{\tau_j^d - g_3}{c_{\tau_j^d}}, & T_{i,j} - c_{\tau_j^d} \leqslant g_3 \leqslant \tau_j^d, \\ 0, & g_3 < \tau_j^d - c_{\tau_j^d}, \end{cases}$$

$$\mu_4 = \begin{cases} 1, & g_4 > \tau_j^d, \\ 1 - \dfrac{\tau_j^d - g_4}{c_{\tau_j^d}}, & T_{i,j} - c_{\tau_j^d} \leqslant g_4 \leqslant \tau_j^d, \\ 0, & g_4 < \tau_j^d - c_{\tau_j^d}, \end{cases} \quad \mu_5 = \begin{cases} 1, & g_5 > \tau_j^f, \\ 1 - \dfrac{\tau_j^f - g_5}{c_{\tau_j^f}}, & T_{i,j} - c_{\tau_j^f} \leqslant g_5 \leqslant \tau_j^f, \\ 0, & g_5 < \tau_j^f - c_{\tau_j^f}, \end{cases}$$

$$\mu_6 = \begin{cases} 1, & g_6 > \tau_j^f, \\ 1 - \dfrac{\tau_j^f - g_6}{c_{\tau_j^f}}, & T_{i,j} - c_{\tau_j^f} \leqslant g_6 \leqslant \tau_j^f, \\ 0, & g_6 < \tau_j^f - c_{\tau_j^f}, \end{cases}$$

$$(3.6)$$

Maximize λ,

$$\theta \sum_{i=1}^{n} \left(c_i^{\text{delay}} \cdot \sum_{j=1}^{m} d_{i,j} \right) + (1 - \theta) \sum_{i=1}^{n} b_i \leqslant s^* + (1 - \lambda)\left(s^0 - s^* \right),$$

$$\lambda = \sum_{1}^{4} w_i \lambda_i,$$

$$y_{i,j+1} - x_{i,j} \geqslant a_{ij} + (\lambda_1 - 1)c_{a_{ij}},$$

$$x_{i,j} \geqslant x_{i,j}^*, \quad h_{i,j} = 1,$$

$$x_{i,j} - x_{i,j}^* = d_{i,j},$$

$$x_{i,j} - y_{i,j} \geqslant T_{i,j} + (\lambda_2 - 1)c_{T_{i,j}},$$

$$\sum_{l=1}^{\tau_j^{\text{tr}}} \eta_{i,j,l} = 1,$$

$$\eta_{i,j,l} + \eta_{i',j,l} - 1 < \alpha_{i,j,i',j'} + \beta_{i,j,i',j'},$$

$$y_{i',j} - y_{i,j} \geqslant \left(\tau_j^f + (\lambda_3 - 1)c_{\tau_j^f} \right)\alpha_{i,j,i',j} - M(1 - \alpha_{i,j,i',j}),$$

$$y_{i',j} - y_{i,j} \geqslant \left(\tau_j^f + (\lambda_3 - 1)c_{\tau_j^f} \right)\beta_{i,j,i',j} - M(1 - \beta_{i,j,i',j}),$$

$$\alpha_{i,j,i',j} + \beta_{i,j,i',j} = 1,$$

$$x_{i',j} - x_{i,j} \geqslant \left(\tau_j^d + (\lambda_4 - 1)c_{\tau_j^d}\right)\alpha_{i,j,i',j} - M\left(1 - \alpha_{i,j,i',j}\right),$$

$$x_{i',j} - x_{i,j} \geqslant \left(\tau_j^d + (\lambda_4 - 1)c_{\tau_j^d}\right)\beta_{i,j,i',j} - M\left(1 - \beta_{i,j,i',j}\right),$$

$$d_{\text{lost}(i)} - T_i^D \leqslant Mb_i.$$

$$(3.7)$$

Since M is a very integer, the headway time constraints can be changed as below:

$$y_{i',j} - y_{i,j} \geqslant \tau_j^f + (\lambda_3 - 1)c_{\tau_j^f} - M\left(1 - \alpha_{i,j,i',j}\right),$$

$$y_{i',j} - y_{i,j} \geqslant \tau_j^f + (\lambda_3 - 1)c_{\tau_j^f} - M\left(1 - \beta_{i,j,i',j}\right),$$

$$x_{i',j} - x_{i,j} \geqslant \tau_j^d + (\lambda_4 - 1)c_{\tau_j^d} - M\left(1 - \alpha_{i,j,i',j}\right),$$

$$x_{i',j} - x_{i,j} \geqslant \tau_j^d + (\lambda_4 - 1)c_{\tau_j^d} - M\left(1 - \beta_{i,j,i',j}\right).$$

$$(3.8)$$

4. Case Study

This model is simulated on the busiest part of Beijing to Shanghai high speed line, between Nanjing (Ning for short) and Shanghai (Hu for short). In the rest of this paper, "Hu-Ning Section" is used to represent this part of the Beijing to Shanghai high speed line. There are seven stations on the Hu-Ning Section, thereby there are 6 sections whose lengths are 65110 m, 61050 m, 56400 m, 26810 m, 31350 m, and 43570 m. Its daily service starts at 6:30 am and ends at 11:30 pm. As currently planned, there are 52 trains (14 high speed trains and 38 medium speed trains) from Beijing to Shanghai line and 8 extra medium speed trains from Riverside line that go from Ning to Hu by the Hu-Ning Section. The trains from Beijing to Shanghai line are called self-line trains, while those from Riverside line are called cross-line trains. In reality, self-line trains have higher priority than cross-line trains. As for self-line trains, the high speed trains have higher priority than the quasi-high speed trains. Since Beijing to Shanghai high speed line is double track, we only consider the direction from Ning to Hu without loss of generality. The experimental procedure is divided into two stages for the full proof of the validity of the fuzzy model. Firstly, we do research in six aspects with different fuzzy constraints of the same weight, which proves the effectiveness of the tolerance-based fuzzy model in different trains operation conditions. Then a sensitivity analysis of the weighing factors is realized based on the previous operation circumstances. All the models are solved by Ilog Cplex 12.2.

4.1. Same Weight

4.1.1. Delay of One Train

In the simulation, we assume that the train G103 is late for 20 minutes in the section from Nanjing to Zhenjiang, and the trains can run at the normal speed in all the sections;
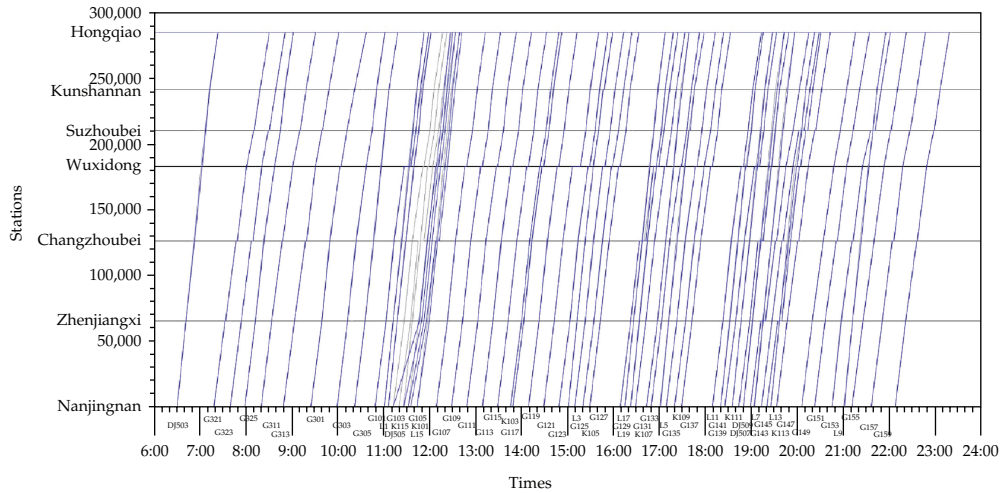
Figure 3: Timetable of one train delay.

the headway is set from 3 minutes to 2.5 minutes, and the minimum separation time on track possession in each station is set to 1 min.

First we use the original model to solve the problem and get the follow results. When the speed of high-speed train is set to 360 km/h, the speed of medium high-speed train is set to 320 km/h, the headway time is defined as 3 minutes, and the solution value is 368784; there are 3 trains late with the delay time between 10 minutes to 20 minutes and 1 train late with the delay time between 20 minutes to 30 minutes; and the total delay time is 47.5 minutes. When the speed of high-speed train is set to 380 km/h, the speed of medium high-speed train is set to 350 km/h, the headway time is defined as 2.5 minutes, and the solution value is 350449; there are one train late with the delay time between 0 minutes to 10 minutes and 2 trains late with the delay time between 10 minutes to 20 minutes; and the total delay time is 31.85 minutes. Then we use the above results as the inputs of the fuzzy model the paper described before and get the follow results. The fuzzy member is 0.86392, which means the average high speed is 365 km/h, the average medium high speed is 335 km/h, and headway time is 2.9319 minutes; there are also only one train late with the delay time between 0 minutes to 10 minutes and 2 trains late with the delay time between 10 minutes to 20 minutes; and the total delay time is 33.5 minutes. The adjustment strategy is to extend the dwell time for the train K101 and then make the train G105 overtake the train K101 at Changzhou North Station, which reflects the adjustment priority for high-grade train. Figure 3 shows detail rescheduling result by fuzzy model. The gray lines mean the initial timetable, and blue lines mean the rescheduling timetable.

4.1.2. Two Trains Delay

In the simulation, we assume that the train G305 is late for 30 minutes in the section from Nanjing to Zhenjiang, then the train G103 gets further delay for 20 minutes in the section from Zhenjiang to Changzhou under the condition of the existing delay, and other conditions are identical to Section 4.1.1.
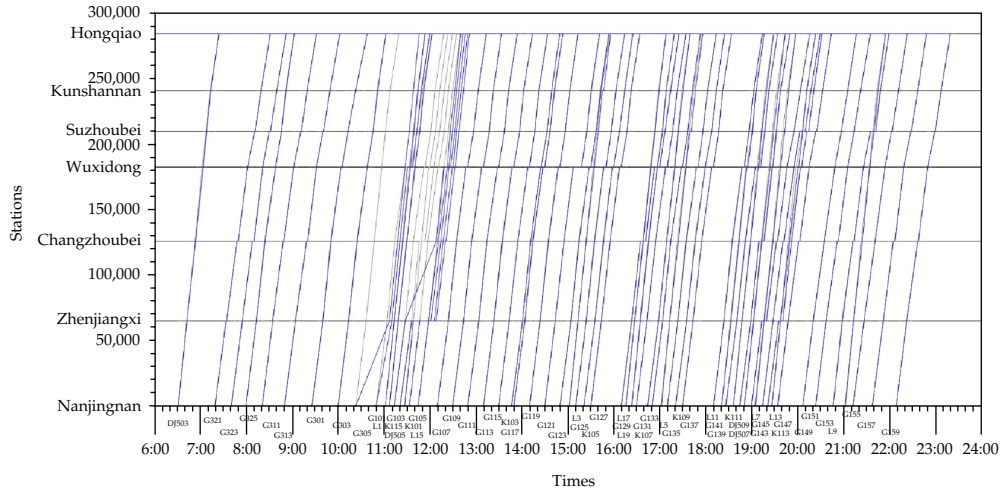
Figure 4: Timetable of two trains delay.

First we use the original model to solve the problem and get the follow results. When we take the strict constraints, the solution value is 860720; there are 2 trains late with the delay time between 0 minutes to 10 minutes, 3 trains late with the delay time between 10 minutes to 20 minutes, and 3 trains late with the delay time between 20 minutes to 30 minutes; the total delay time is 122.43 minutes. When we relax the constraints, the solution value is 848871; there are 2 trains late with the delay time between 0 minutes to 10 minutes, 2 trains late with the delay time between 10 minutes to 20 minutes, and also 3 trains late with the delay time between 20 minutes to 30 minutes; and the total delay time is 109.18 minutes. Then we solve the fuzzy model and get the follow results. The fuzzy member is 0.89425, which means the average high speed is 362 km/h, the average medium high speed is 332 km/h, and headway time is 2.9512 minutes; there are also 7 delay trains, 2 trains of which late with the delay time between 0 minutes to 10 minutes, 2 trains late with the delay time between 10 minutes to 20 minutes, and 3 trains late with the delay time between 20 minutes to 30 minutes; and the total delay time is 114.29 minutes. The adjustment strategy is to make the high level train G105 subsequently overtakes the train K115 L15 and K101 at Zhenjiang West Station, and train L15 overtakes the train G103 at the Wuxi East Station, which effectively avoid the high level train G105 being late and reduce the delay time of the train L15. Figure 4 shows detail rescheduling result by fuzzy model.

4.1.3. Speed Restriction in All Sections

When some natural hazards happened, like heavy storm and strong wind, railway will be greatly affected in a large area. In the simulation, we assume there is a speed restriction in all sections and the average limited speed ranges from 170 km/h to 150 km/h; other conditions are identical to Section 4.1.1.

First we use the original model to solve the problem and get the follow results. When we set the speed as 150 km/h and headway time as 3 minutes, the solution value is 7969213; there are 2 trains late for the time between 30 and 40 minutes, 18 trains late for the time between 40 and 50 minutes, 34 trains late for the time between 50 and 60 minutes, and

6 trains late over one hour; and the total delay time is 3052.95 minutes. When the speed is set to 170 km/h, and headway time is set to 2.5 minutes, the solution value is 7473849; there are 5 trains late for the time between 20 and 30 minutes, 38 trains late for the time between 30 and 40 minutes, 17 trains late for the time between 40 and 50 minutes, and no train late over one hour; and the total delay time is 2248.26 minutes. Although the latter result is very exciting, it is a significant risk to take the speed of 170 km/h, as the highest speed must bring the high operation cost and may arouse some new delays. Then we use the above results as the inputs of the fuzzy model the paper described before and get the follow results. The fuzzy member is 0.689117, which means the average speed is 156 km/h, and headway time is 2.8445 minutes; there are 19 trains late for the time between 30 and 40 minutes, 25 trains late for the time between 40 and 50 minutes, 16 trains late for the time between 50 and 60 minutes, and no train late over one hour; and the total delay time is 2655.16 minutes. We can see that the fuzzy optimization result improved greatly with little slack of constraints that means little risk and operation cost. Figure 5 is the rescheduling timetable by fuzzy model. In addition, the result is a parallel diagram since all trains must comply with the same speed.

4.1.4. Speed Restriction in One Section

Sometimes equipment failure, like train signal failure, happens in some but not all the sections. In the simulation, we assume there is a speed restriction in first section, and the limited speed ranges from 60 km/h to 50 km/h; other conditions are identical to Section 4.1.1.

First we use the original model to solve the problem and get the follow result. When we take the strict constraints, the solution value is 9294339; there are 19 trains late for the time between 40 and 50 minutes, 30 trains late for the time between 50 and 60 minutes, and 11 trains late over one hour; and the total delay time is 3269.32 minutes. When the speed is 60 km/h, and headway time is 2.5 minutes, the solution value is 8577661; there are 20 trains late for the time between 30 and 40 minutes, 37 trains late for the time between 40 and 50 minutes, 3 trains late for the time between 50 and 60 minutes, and no train late over one hour; and the total delay time is 2486.32 minutes. Then we get the follow result using the fuzzy model the paper described before. The fuzzy member is 0.70606, which means the average speed is 53 km/h, and headway time is 2.85303 minutes; there are 5 trains late for the time between 30 and 40 minutes, 39 trains late for the time between 40 and 50 minutes, 16 trains late for the time between 50 and 60, minutes and no train late over one hour; the total delay time is 2753.17 minutes. The adjustment strategy is to make the train G117 overtakes the train K105 at Wuxi East Station, and then let the train L7 overtake the train G143 at the Wuxi East Station. Figure 6 is the rescheduling timetable with speed restriction in the first section.

4.1.5. Speed Restriction in One Section with One Train Delay

In the simulation, we assume that there is a speed restriction in first section, and the limited speed ranges from 60 km/h to 50 km/h; the train G103 gets delay in the section from Zhenjiang to Changzhou. Other conditions are identical to Section 4.1.1.

First we use the original model to solve the problem and get the following results. When the speed in the first section is 50 km/h; the trains can run at the normal speed; the headway time is set to 3 minutes, the solution value is 9367637; there are 19 trains late with the delay time between 40 minutes to 50 minutes, 24 trains late with the delay time between 50 minutes to 60 minutes, and 17 trains late for more than one hour; and the total delay time
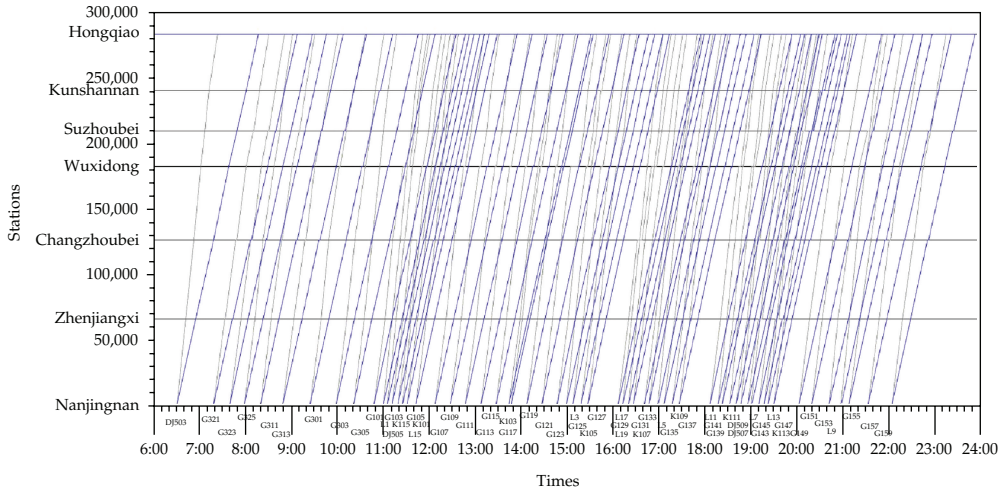
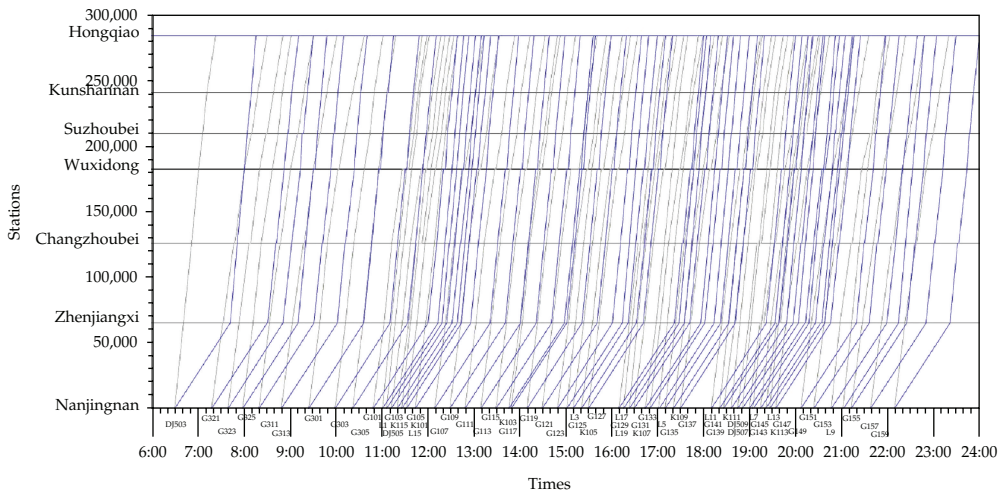**Figure 5:** Timetable of speed restriction in all sections.

Figure 6: Timetable of speed restriction in first section.

is 3305.19 minutes. When the speed in the first section is 60 km/h; the headway time is set to 2.5 minutes; then the solution value is 8671072; 3 trains late with the delay time between 20 minutes to 30 minutes, 34 trains late with the delay time between 30 minutes to 40 minutes, 17 trains late with the delay time between 40 minutes to 50 minutes, 1 train late with the delay time between 50 minutes to 60 minutes, and 5 train late with the delay time more than one hour; and the total delay time is 2557.32 minutes. Then we use the above results as the inputs of the fuzzy model the paper described before and get the follow results. The fuzzy member is 0.740856, which means the average speed in the first section is 52 km/h, and headway time is 2.86 minutes; there are 17 trains late with the delay time between 30 minutes to 40 minutes, 23 trains late with the delay time between 40 minutes to 50 minutes, 15 trains late with the delay time between 50 minutes to 60 minutes, and 5 trains late with the delay time more than one hour; and the total delay time is 2823.34 minutes. The adjustment strategy is to make the
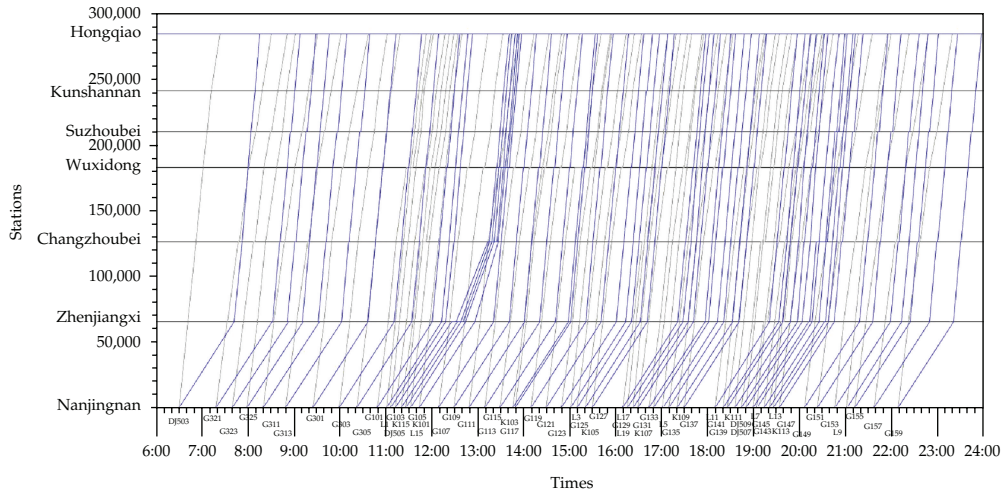
Figure 7: Timetable of speed restriction in one section and one train delay.

trains G105, K115, and L15 overtake the train K115 at Changzhou North Station, and then let the train G105 overtakes the train K101 at the Wuxi East Station. Figure 7 is the rescheduling timetable by fuzzy model.

4.1.6. Speed Restriction in All Sections with Two Trains Delay

In the simulation, we assume that the train G305 is late for 40 minutes in the section between Nanjing to Zhenjiang and then get further delay for 15 minutes in the section between Zhenjiang to Changzhou; the train G103 gets a delay for 30 minutes in the section from Zhenjiang to Changzhou; also there is a speed restriction in all sections, and the average limited speed ranges from 170 km/h to 150 km/h for all the trains; other conditions are identical to Section 4.1.1.

First we use the original model to solve the problem and get the follow results. When we set the speed as 150 km/h and headway time as 3 minutes, the solution value is 8042152; there are 1 train late with the delay time between 30 minutes to 40 minutes, 17 trains late with the delay time between 40 minutes to 50 minutes, 27 trains late with the delay time between 50 minutes to 60 minutes, and 15 trains late with the delay time more than one hour; and the total delay time is 3197.86 minutes. When the speed is 170 km/h, and headway time is 2.5 minutes, the solution value is 7588658; there are 5 trains late with the delay time between 20 minutes to 30 minutes, 35 trains late with the delay time between 30 minutes to 40 minutes, 14 trains late with the delay time between 40 minutes to 50 minutes, 3 trains late with the delay time between 50 minutes to 60 minutes, and 3 trains late with the delay time more than one hour; and the total delay time is 2397.15 minutes. Then we use the above results as the inputs of the fuzzy model the paper described before and get the follow results. The fuzzy member is 0.792877, which means the average speed is 153 km/h, and headway time is 2.89 minutes; there are 17 trains late with the delay time between 30 minutes to 40 minutes, 23 trains late with the delay time between 40 minutes to 50 minutes, 15 trains late with the delay time between 50 minutes to 60 minutes, and 5 trains late with the delay time more than one hour; and the total delay time is 2704.43 minutes. The adjustment strategy is to make the train
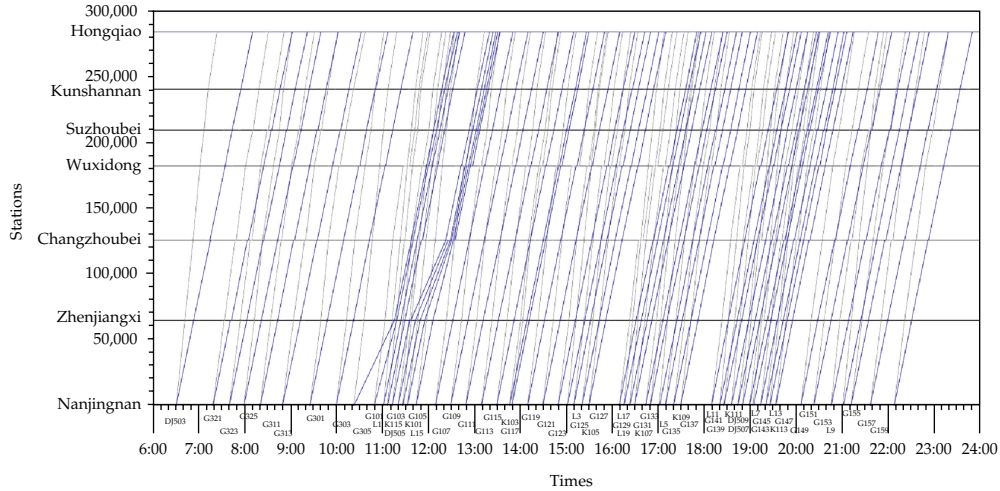
Figure 8: Timetable of speed restriction in all sections and two trains delay.

L1 overtakes the train G101 at the Wuxi East Station, the train L15 and the train K101 overtake K115, and then extend the dwell time of K115 at the Wuxi East Station for the stopover by the train G101, so the importance of the high level trains is also shown from the steps above. Figure 8 is the rescheduling timetable by fuzzy model.

These simulations are realized on the computer with Intel Core 2 Duo CPU E7500 and 2 sG Memory. All the optimization results for the 6 cases are given in Table 1, where "R1" denotes original optimization subject to high speed, "R2" denotes original optimization subject to low speed, "R3" denotes fuzzy optimization, "D1" denotes the number of trains with a delay less than 10 mins, "D2" denotes the number of trains with a delay between 10 and 20 mins, "D3" denotes the number of trains with a delay between 20 and 30 mins, "D4" denotes the number of trains with a delay between 30 and 40 mins, "D5" denotes the number of trains with a delay between 40 and 50 mins, "D6" denotes the number of trains with a delay between 50 and 60 mins, "D7" denotes the number of trains with a delay not less than 60 mins, "D8" denotes the objective value, "D9" denotes the total delay time, "D10" denotes the number of stopover trains, and "D11" denotes the calculation time.

4.2. Different Weights

There are four kinds of fuzzy constraints as described in Sections 2.4 and 3.2. Since the minimum separation time on track possession in each station is set to 1 min in the previous experiments, we can ignore the uncertainty of station dwell time constraints. Furthermore, all the station and section headway times are set to the same value that from 3 minutes to 2.5 minutes; thereby the two constraints are considered equally important for the optimal objective and can use the same fuzzy member. Therefore two kinds of fuzzy constraints are analyzed in this section, section running time and headway time, and the weights of their fuzzy member are w_1 and w_2.

Table 2 and Figure 9 are the comparison of the optimal objectives for the six cases. It can be seen from Figure 9 that the objective increases obviously as w_1 is set from 0.1 to 0.9 in the last four cases; however, the objective curves tend to flat in the former two cases,

Table 1: Rescheduling results of different cases.

Case	R	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11
	R1	0	3	1	0	0	0	0	368784	47.5	—	5.38
Case 1	R2	1	2	0	0	0	0	0	350449	31.5	—	3.39
	R3	1	2	0	0	0	0	0	0.86392	33.5	1	178.17
	R1	2	3	3	0	0	0	0	860720	122.43	—	6.22
Case 2	R2	2	2	3	0	0	0	0	848871	109.18	—	3.36
	R3	2	2	3	0	0	0	0	0.89425	114.29	2	142.58
	R1	0	0	0	2	18	34	6	7969213	3052.95	—	3.38
Case 3	R2	0	0	5	38	17	0	0	7473849	2248.26	—	3.32
	R3	0	0	0	19	25	16	0	0.689117	2655.16	—	23.13
	R1	0	0	0	0	19	30	11	9294339	3269.32	—	3.36
Case 4	R2	0	0	0	20	37	3	0	8577661	2486.32	—	3.39
	R3	0	0	0	5	39	16	0	0.70606	2753.17	2	76.75
	R1	0	0	0	0	19	24	17	9367637	3305.19	—	8.05
Case 5	R2	0	0	3	34	17	1	5	8671072	2557.32	—	7.97
	R3	0	0	0	17	23	15	5	0.740856	2823.34	3	280.23
	R1	0	0	0	1	17	27	15	8086249	3197.86	—	7.74
Case 6	R2	0	0	5	35	14	3	3	7588658	2397.15	—	7.75
	R3	0	0	0	17	23	15	5	0.792877	2704.43	4	260.56

Table 2: Comparison of the optimal objectives.

Case	w_1								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Case 1	0.860180	0.860920	0.861390	0.862250	0.863920	0.864470	0.865731	0.865020	0.864170
Case 2	0.889860	0.890440	0.891050	0.892280	0.894240	0.896030	0.895490	0.892661	0.890040
Case 3	0.671015	0.674356	0.678921	0.684357	0.689117	0.693258	0.698091	0.702655	0.705186
Case 4	0.697540	0.698920	0.700130	0.702480	0.706060	0.709138	0.712240	0.714521	0.715390
Case 5	0.730050	0.732160	0.734875	0.737280	0.740856	0.744030	0.747980	0.749721	0.751060
Case 6	0.779810	0.781530	0.784010	0.787903	0.792877	0.797460	0.801130	0.804655	0.806515

especially the curves decline slightly when w_1 is greater than 0.6. This may explain that the same constraints contribute differently for the optimal objective in different emergencies. Section running time constraint has a higher priority than headway time constraint when some accidents or natural disasters lead to speed restriction for a long time, and adjusting the train headway time is more effective if few trains delay by some interference.

5. Conclusion and Future Work

This paper presents a fuzzy optimization model based on improved tolerance approach for train rescheduling in case of train delay and speed restriction, which deals with the train running time at sections, the headway time at sections, and station as the fuzzy parameters. The simulations on Beijing to Shanghai high speed line reveal that the fuzzy optimization result improved greatly with little slack of constraints. This means we can get a new timetable with less total delay time as well as the number of seriously impacted trains in safe and lower average speed, little dwell time, and enough headway time. In addition, the sensitivity
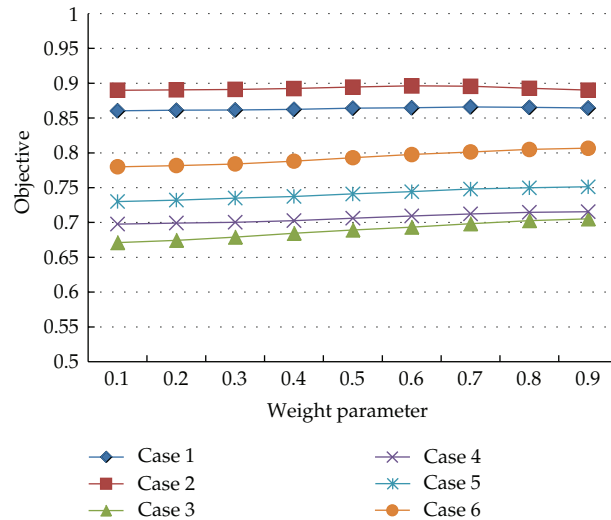
Figure 9: Optimal objectives with different weights.

analysis of the weighing factors shows that the same constraints contribute differently to the optimal objective in different emergencies, thus the dispatchers should take different trains adjustment strategies to eliminate interference as much as possible.

There remain many interesting areas to explore around the uncertainty in timetable rescheduling problem. Firstly, the membership functions of fuzzy parameters used in the paper may be more complex form in practice than the linear function in the paper, so we can take some genetic functions, like gauss membership function, to model the fuzzy programming. The more accurate the membership function is, the better result the fuzzy optimization model gets. Secondly, the tolerance can also be described by fuzzy set-based schemes. Finally, fuzzy operators in the model can be improved to adapt well to the information processing mechanism of despatchers in dealing with rescheduling problems. Our ultimate goal is to develop a real-time rescheduling system to significantly improve operation management and scheduling efficiency in the future.

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