Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2012, Article ID 948126, 11 pages
doi:10.1155/2012/948126

Research Article

Design and Synchronization of Master-Slave Electronic Horizontal Platform System

Hang-Hong Kuo,[1] Teh-Lu Liao,[1] and Jun-Juh Yan[2]

[1] *Department of Engineering Science, National Cheng Kung University, Tainan 701, Taiwan*
[2] *Department of Computer and Communication, Shu-Te University, Kaohsiung 824, Taiwan*

Correspondence should be addressed to Jun-Juh Yan, jjyan@mail.stu.edu.tw

Received 22 October 2011; Accepted 16 December 2011

Academic Editor: Mingshu Peng

Horizontal platform system (HPS) is one of the mechanical systems with rich behavior and has extensively been applied in offshore and earthquake engineering. A corresponding electronic HPS is proposed in this paper to reduce the research cost and time when studying dynamics of the mechanical HPS. Furthermore, an output feedback controller is proposed for global synchronization between coupled electronic HPS systems and its stability condition is also derived by employing the Lyapunov stability theory. The experimental simulations verify the dynamics of the proposed electronic HPS and the synchronization effectiveness of the proposed control scheme.

1. Introduction

In recent years, the study for the dynamics analysis of chaotic systems has become a very popular research field [1–3]. A chaotic system is with several potential properties, such as highly complex dynamics, broad-band Fourier power spectrums, and strange attractors. Chaos synchronization has been found to be useful in many areas of physics and engineering systems such as those in power converters, flow dynamics and liquid mixing, biological systems, and information processing, especially in encryption and communication [4–8]. Beside that, many effective methods based on the experimental implementation for synchronization problems have been thoroughly investigated [1–10].

Horizontal platform system (HPS) is a mechanical system with rich chaos behavior, which can freely rotate around the horizontal axis and extensively applied in offshore and earthquake engineering [11] however, not as electronic circuits, to establish a mechanical system for research costs much time and money. In the present paper, we propose a corresponding electronic HPS model which has chaotic dynamics as the mechanical HPS owns. After establishing the electronic HPS, a new sufficient criterion for global chaos synchronization of two electronic HPS coupled by an output feedback controller is deduced. This method makes it possible to construct all of the state information from only a transmitted signal.

This paper is organized as follows. The original mechanical HPS is illustrated in Section 2. In Section 3, the corresponding electronic HPS is introduced and the corresponding chaotic behavior is verified. Section 4 formulates the design of an output feedback controller for synchronization of two identical electronic HPSs. The experimental simulations are given to verify the effectiveness of the proposed approach in Section 5. Finally, a concise conclusion is made in Section 6.

2. Mechanical Horizontal Platform System

Figure 1 depicts the system structure of mechanical HPS. The platform can freely rotate around the horizontal axis, which penetrates its mass center. An accelerometer is located on the platform to detect the position. The accelerometer will give an output signal to the actuator, which generates a torque to inverse the rotation of the platform to balance the HPS if the platform deviates from horizon. The dynamics of HPS can be described as

$$A\ddot{x}(t) + D\dot{x}(t) + rg\sin x(t) - \frac{3g}{R}(B - C)\cos x(t)\sin x(t) = F\cos\omega t, \tag{2.1}$$

where A, B, and C are the inertia moments of the platform for axes 1, 2, and 3, respectively. D is the damping coefficient. R is the radius of the earth, r is the proportional constant of the accelerometer, and g is the constant of gravity. x indicates the rotation of the platform relative to the earth, and $F\ \cos\,\omega t$ is the harmonic torque. A more detailed analysis of this system can be found in [12]. As shown in [11, 12], this mechanical HPS has rich chaos behavior.

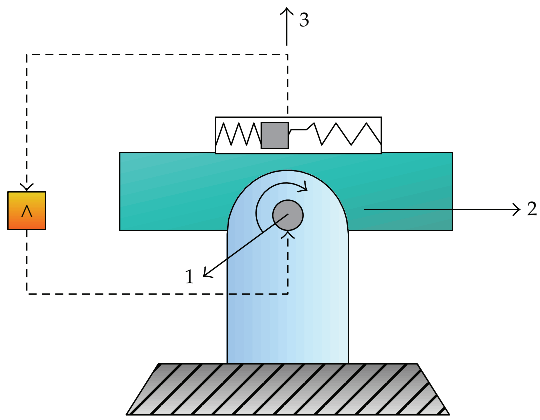
3. The Corresponding Electronic HPS Model and Its Chaotic Behavior

By denoting $x_1(t) = x(t)$ and $x_2(t) = \dot{x}(t)$ as the state variables, the HPS model (2.1) can be rewritten as follows:

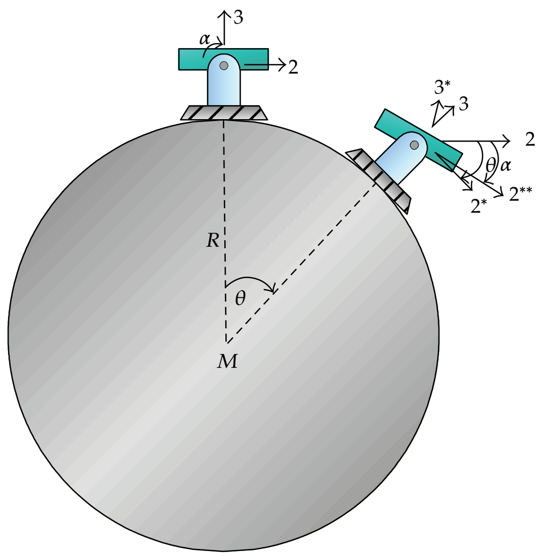
$$\begin{gathered}\dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = -\frac{D}{A}x_2(t) - \frac{rg}{A}\sin x_1(t) + \frac{3g}{RA}(B - C)\cos x_1(t)\sin x_1(t) + \frac{F}{A}\cos\omega t.\end{gathered} \tag{3.1}$$

The typical parameters of HPS dynamics are given as $A = 0.3$, $B = 0.5$, $C = 0.2$, $D = 0.4$, $r = 0.11559633$, $R = 6378000$, $g = 9.8$, $F = 3.4$, and $\omega = 1.8$, respectively. The phase portrait is illustrated in Figure 2. The largest Lyapunov exponent of electronic HPS is 0.3424; therefore, the chaotic behavior of the corresponding electronic HPS is guaranteed [13, 14].

The electronic HPS circuit corresponding to the mechanical one is shown in Figure 2 which consists of one AC voltage (v4), two integral circuits (U1A, U6B), one addition circuit (U7A), and two nonlinear circuits (cos, sin). The state portrait of the proposed circuit is shown in Figure 3.



(a)



(b)

Figure 1: (a) The horizontal platform system. (b) The horizontal platform system on the earth.

4. Output Feedback Design for Synchronizing the Coupled Electronic HPSs

In this section, we further propose a sufficient criterion for global chaos synchronization of two identical electronic HPSs coupled by an output feedback controller. Several works utilizing all states to fulfil the synchronization problem have been developed in the literature [4–8, 15–17]. In contrast to the previous works, this paper only uses the available output to construct the control scheme for achieving global synchronization. The main steps for designing the output feedback controller are stated as follows.

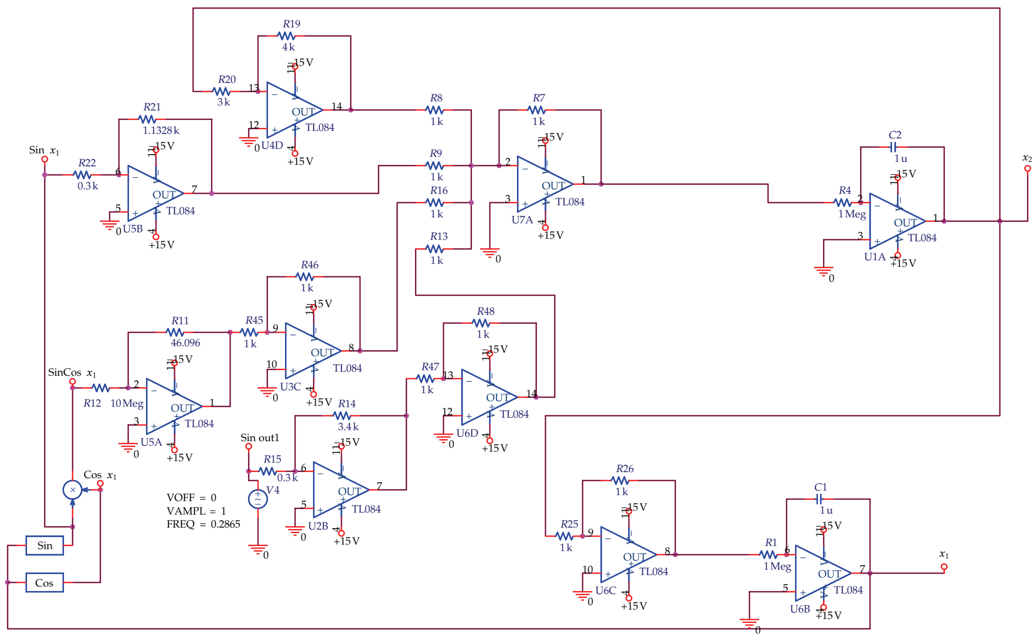


Figure 2: The electronic HPS circuit.

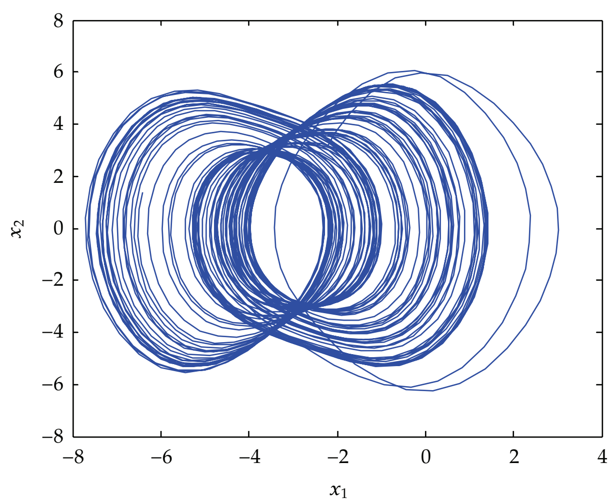


Figure 3: The simulation of HPS in phase plane.

Step 1. Rewrite the HPS system (3.1) in the state space as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -ax_2 - b\sin x_1 + l\cos x_1 \sin x_1 + h\cos\omega t, \end{aligned} \tag{4.1}$$

where

$$a = \frac{D}{A} > 0, \qquad b = \frac{rg}{A} > 0, \qquad l = \frac{3g}{RA}(B - C), \qquad h = \frac{F}{A} > 0. \tag{4.2}$$

Let $x = (x_1, x_2)^T \in R^{2\times 1}$, the vector form of the system (4.1) is obtained as

$$\dot{x} = \overline{A}x + f(x) + \overline{B}h\cos\omega t, \tag{4.3}$$

with

$$\overline{A} = \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix}, \qquad \overline{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad f(x) = \begin{bmatrix} 0 \\ -b\sin x_1 + l\cos x_1 \sin x_1 \end{bmatrix}. \tag{4.4}$$

Step 2. Define the master-slave systems as below, respectively.

Master system

$$\begin{gathered} \dot{x} = \overline{A}x + f(x) + \overline{B}h\cos\omega t, \\ y = \overline{C}x, \end{gathered} \tag{4.5}$$

where $y \in R$ denotes the output variable, $x \in R^{2\times 1}$ represents the state vector, and $\overline{C} = [1 \;\; 0]$.

Slave system

$$\begin{gathered} \dot{\hat{x}} = \overline{A}\hat{x} + f(\hat{x}) + \overline{B}h\cos\omega t + L\left(y - \hat{y}\right), \\ \hat{y} = \overline{C}\hat{x}, \end{gathered} \tag{4.6}$$

where $\hat{x} \in R^{2\times 1}$ represents the state vector of slave system, $\hat{y} \in R$ denotes the slave system output, and $L = [l_1 \;\; l_2]^T$ denotes the gain matrix of output feedback control.

Defining the error vector $e = x - \hat{x}$, we have the following error dynamics:

$$\dot{e} = \dot{x} - \dot{\hat{x}} = \overline{A}x + f(x) - \overline{A}\hat{x} - f(\hat{x}) - L\left(y - \hat{y}\right) = \left(\overline{A} - L\overline{C}\right)e + f(x) - f(\hat{x}). \tag{4.7}$$

The error dynamics (4.7) is further described as

$$\dot{e} = \left(\overline{A} - L\overline{C} + Q(x)\right)e, \tag{4.8}$$

where

$$Q(x) = \begin{bmatrix} 0 & 0 \\ q(x) & 0 \end{bmatrix}, \quad q(x) = \frac{-b(\sin x_1 - \sin\hat{x}_1) + l(\sin x_1 \cos x_1 - \sin\hat{x}_1 \cos\hat{x}_1)}{x_1 - \hat{x}_1}. \tag{4.9}$$

Our goal is to select the coupling matrix L in (4.6) such that the orbits $x(t)$ and $\hat{x}(t)$ of master-slave systems satisfy

$$\lim_{t\to\infty} \|x(t) - \hat{x}(t)\| = 0, \tag{4.10}$$

where $\|\cdot\|$ denotes the 2-norm of the vector.

Theorem 4.1. *Suppose the feedback gain matrix L is selected such that*

(a) $(\overline{A} - L\overline{C}, M)$ *and* $(\overline{A} - L\overline{C})$ *are observable and stable, respectively, where*

$$M = \begin{bmatrix} 0 & 0 \\ b + |l| & 0 \end{bmatrix}; \tag{4.11}$$

(b) *the following Hamiltonian matrix H with some*

$$\gamma > 1 H = \begin{bmatrix} \overline{A} - L\overline{C} & \gamma I_2 \\ -M^T M & -\left(\overline{A} - L\overline{C}\right)^T \end{bmatrix} \tag{4.12}$$

has no eigenvalues on the imaginary axis.

Then the master-slave system defined in (4.5) *and* (4.6) *achieves global chaos synchronization.*

The following lemmas will be needed for the main theorem.

Lemma 4.2 (see [18]). *For any $\gamma > 1$, define the Hamiltonian matrix as* (4.12)*; assume that (i)* $(\overline{A} - L\overline{C}, M)$ *and* $(\overline{A} - L\overline{C})$ *are observable and stable, respectively, and (ii) H has no eigenvalues on the imaginary axis. Then the algebraic Riccati equation (ARE)*

$$\left(\overline{A} - L\overline{C}\right)^T P + P\left(\overline{A} - L\overline{C}\right) + \gamma PP + M^T M = 0 \tag{4.13}$$

has a positive definite solution P.

Proof. It is an immediate result of the work of Doyle et al. [18] and hence is omitted. □

Lemma 4.3. *For matrix $Q(x)$ defined in* (4.9)*, the following inequality holds:*

$$\|Q(x)\| \le b + |l|. \tag{4.14}$$

Proof. By the differential mean-value theorem, we have

$$\begin{gathered} \sin x_1 - \sin \hat{x}_1 = \cos\xi(x_1 - \hat{x}_1), \quad \xi \in [x_1, \hat{x}_1] \text{ or } [\hat{x}_1, x_1], \\ \sin 2x_1 - \sin 2\hat{x}_1 = 2\cos(2\eta)(x_1 - \hat{x}_1), \quad \eta \in [x_1, \hat{x}_1] \text{ or } [\hat{x}_1, x_1], \\ q(x) = \frac{-b(\sin x_1 - \sin \hat{x}_1) + l(\sin x_1 \cos x_1 - \sin \hat{x}_1 \cos \hat{x}_1)}{x_1 - \hat{x}_1} = -b\cos\xi + l\cos(2\eta). \end{gathered} \tag{4.15}$$

Thus, we have

$$\|Q(x)\| = |q(x)| \leq b + |l|. \tag{4.16}$$

□

Proof of Theorem 4.1. Suppose the feedback gain matrix L is selected such that $(\overline{A}-L\overline{C}, M)$ and $(\overline{A}-L\overline{C})$ are observable and stable, respectively. According to Lemma 4.2, if the Hamiltonian matrix (4.12) has no eigenvalue on the imaginary axis, then we have the following ARE:

$$\left(\overline{A}-L\overline{C}\right)^T P + P\left(\overline{A}-L\overline{C}\right) + \gamma PP + M^T M = 0, \tag{4.17}$$

where P is a positive definition solution. Now, introducing a Lyapunov function $V(t)$ as $V(t) = e^T(t)Pe(t) \geq 0$, it is easily verified that $V(t)$ is a nonnegative function over $[0,+\infty)$ and unbounded; that is, $V(t) \to \infty$ as $e(t) \to \infty$. Subsequently, evaluating the time derivative of V along the trajectory of (4.8), we have

$$\begin{aligned}
\dot{V} &= \dot{e}^T Pe + e^T P\dot{e} \\
&= \left[\left(\overline{A}-L\overline{C}+Q(x)\right)e\right]^T Pe + e^T P\left[\left(\overline{A}-L\overline{C}+Q(x)\right)e\right] \\
&= e^T\left[\left(\overline{A}-L\overline{C}\right)^T P + P\left(\overline{A}-L\overline{C}\right)\right]e + 2e^T PQ(x)e \\
&\leq e^T\left[\left(\overline{A}-L\overline{C}\right)^T P + P\left(\overline{A}-L\overline{C}\right)\right]e + 2\left\|e^T P\right\|\|Q(x)e\|.
\end{aligned} \tag{4.18}$$

Since

$$\begin{gathered}
2\left\|e^T P\right\|\|Q(x)e\| \leq \left\|e^T P\right\|^2 + \|Q(x)e\|^2 = e^T PPe + e^T Q^T(x)Q(x)e, \\
e^T Q^T(x)Q(x)e = q^2(x)e_1^2 \leq (b+|l|)^2 e_1^2 = e^T M^T Me,
\end{gathered} \tag{4.19}$$

we have

$$\begin{aligned}
\dot{V} &\leq e^T\left[(A-LC)^T P + P(A-LC) + PP + M^T M\right]e \\
&< e^T\left[(A-LC)^T P + P(A-LC) + \gamma PP + M^T M\right]e,
\end{aligned} \tag{4.20}$$

where $\gamma > 1$. According to the Lyapunov stability theory, the last inequality $\dot{V}(t) < 0$ indicates $V(t)$ as well as $e(t)$ converges to zero asymptotically. This completes the proof. □

5. Numerical Simulation

In this section, simulation results are presented to demonstrate the effectiveness of the proposed synchronization scheme. All the simulation procedures are coded and executed using the software of OrCAD. The system parameters are chosen as follows: $A = 0.3$, $B = 0.5$, $C = 0.2$, $D = 0.4$, $r = 0.11559633$, $R = 6378000$, $g = 9.8$, $F = 3.4$, $\omega = 1.8$ such

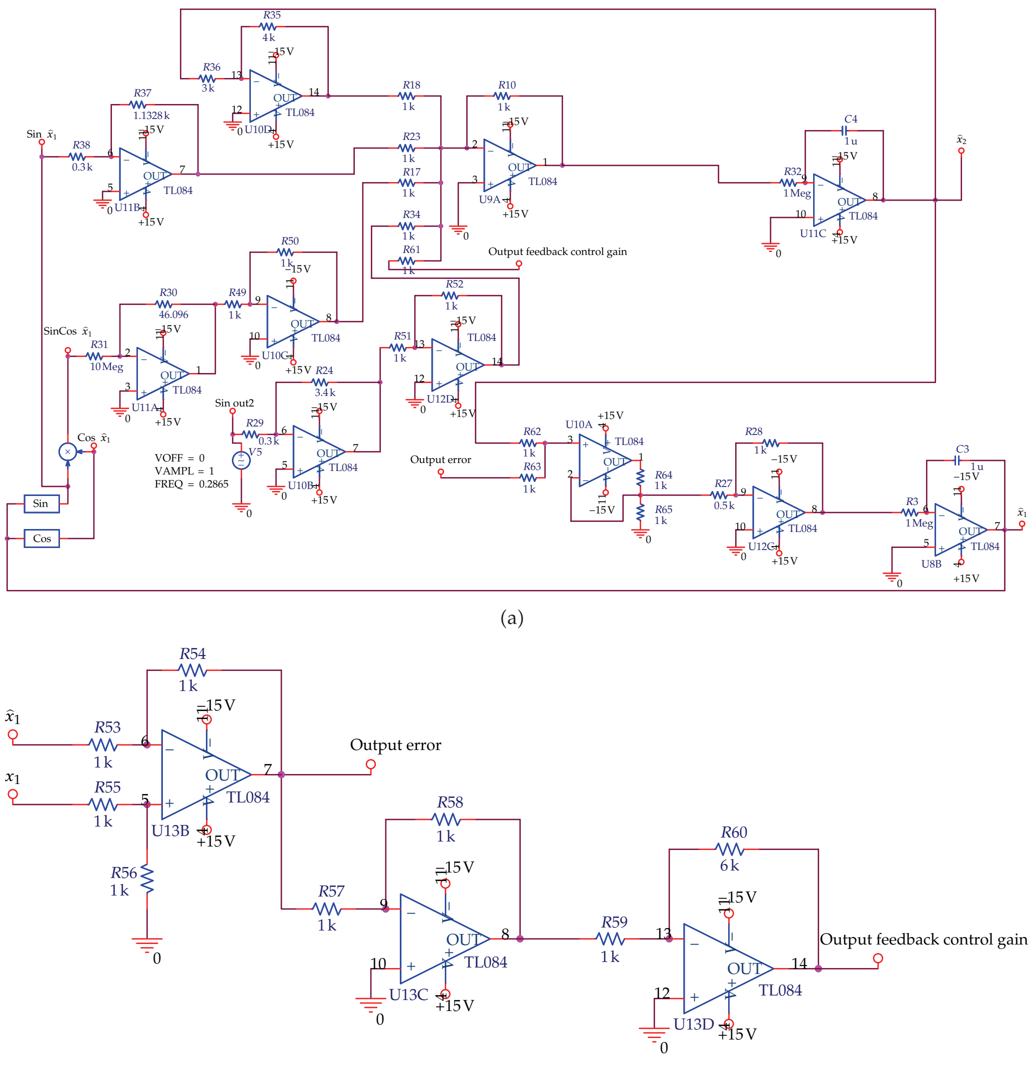


Figure 4: (a) The slave electronic HPS system. (b) The output error state and output error feedback control gain.

that $b = rg/A = 3.7761$, $l = (3g/RA)(B - C) = 4.6096 * 10^{-6}$. The initial states of the master system (4.5) are $(x_1(0), x_2(0)) = (-2.2, 3.2)$, and initial states of the slave system (4.6) are $(\hat{x}_1(0), \hat{x}_2(0)) = (-5, 5)$. The output feedback control gain matrix is selected as $L = [l_1 \ \ l_2]^T = [4.3333 \ \ 3.1111]^T$. Obviously, the resulting $(\overline{A} - L\overline{C}, M)$ and $(\overline{A} - L\overline{C})$ are observable and stable, respectively. Then according to [11], we construct the Hamiltonian matrix with $\gamma = 1.1$, which is shown as

$$H = \begin{bmatrix} -4.3333 & 1 & 1.1 & 0 \\ -3.1111 & -0.6667 & 0 & 1 \\ -14.2589 & 0 & 4.3333 & 3.1111 \\ 0 & 0 & -1 & 0.6667 \end{bmatrix}. \tag{5.1}$$

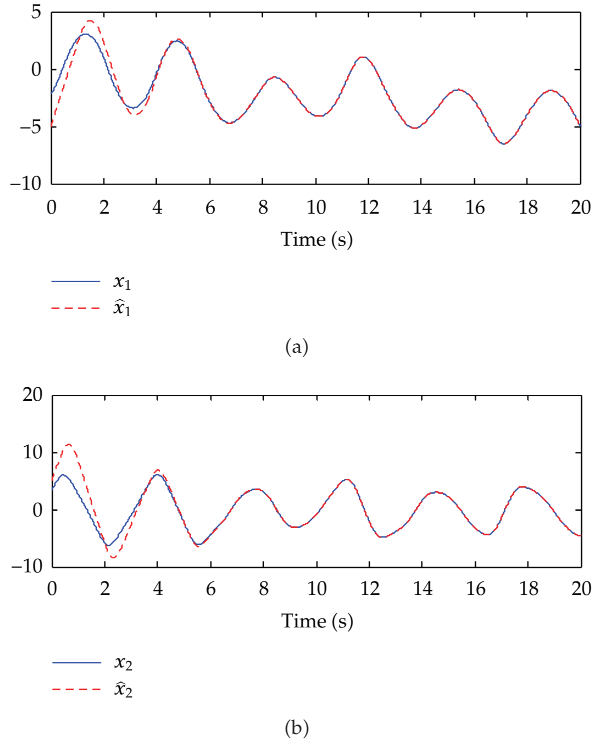


Figure 5: The time responses of x_1 and $\hat{x}_1$, x_2 and $\hat{x}_2$.

It is easy to check that the eigenvalues of H are $[-1.042 + 1.5899i \quad -1.042 - 1.5899i \quad 1.042 + 1.5899i \quad 1.042 - 1.5899i]$ and obviously no eigenvalues are on the imaginary axis. Thus according to Theorem 4.1, the master-slave system defined in (4.5) and (4.6) achieves global chaos synchronization. The controlled slave electronic HPS system is shown in Figure 4.

The simulation results are shown in Figures 5–7. The time responses of master and slave systems are shown in Figure 5. Figure 6 showing the time responses of output errors between master and slave systems. Finally, the output feedback control of $L(y - \hat{y})$ is shown in Figure 7. The above simulation results show that the trajectories of master-slave systems are synchronized and the synchronization error surely converges to zero.

6. Conclusion

In this paper, two main results have been proposed. First, we have presented an electronic HPS model to reduce the cost and time when studying the mechanical one. Second, we have investigated the global chaos synchronization of two identical electronic horizontal platform systems only coupled by an output feedback control. A new sufficient criterion has been proposed based on the Lyapunov stability theory. Numerical simulations have verified the effectiveness of the proposed method.

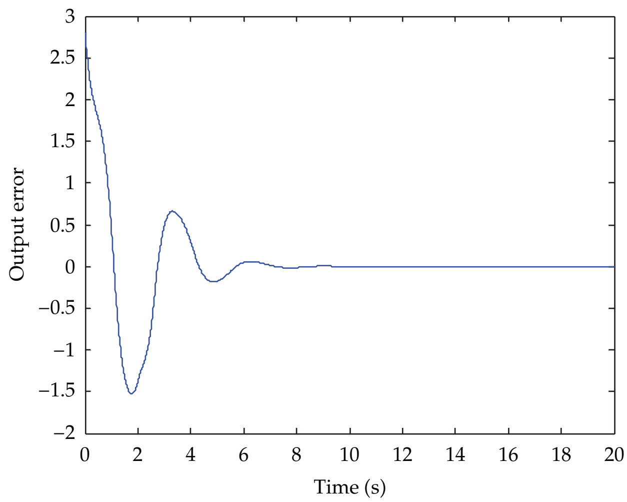


Figure 6: The error output state between master and slave systems.

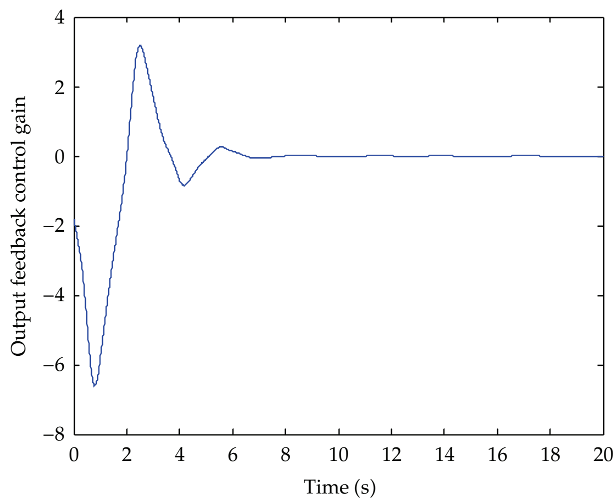


Figure 7: The output feedback control gain of $L(y - \hat{y})$.

Acknowledgment

This work was supported by the National Science Council of Republic of China under Contract 98-2221-E-006-192-MY3 and 100-2221-E-366-001.

References

[1] A. N. Miliou, I. P. Antoniades, S. G. Stavrinides, and A. N. Anagnostopoulos, "Secure communication by chaotic synchronization: robustness under noisy conditions," *Nonlinear Analysis: Real World Applications*, vol. 8, no. 3, pp. 1003–1012, 2007.

[2] M. Chen and W. Min, "Unknown input observer based chaotic secure communication," *Physics Letters A*, vol. 372, no. 10, pp. 1595–1600, 2008.

[3] C. Zhang and H. Zheng, “Dynamic properties of coupled maps,” *Discrete Dynamics in Nature and Society*, vol. 2010, Article ID 905102, 10 pages, 2010.
[4] X.-Y. Wang and J.-M. Song, “Synchronization of the unified chaotic system,” *Nonlinear Analysis: Theory, Methods & Applications*, vol. 69, no. 10, pp. 3409–3416, 2008.
[5] Z. M. Ge, C. H. Li, S. Y. Li, and C. M. Chang, “Chaos synchronization of double Duffing systems with parameters excited by a chaotic signal,” *Journal of Sound and Vibration*, vol. 317, no. 3-5, pp. 449–455, 2008.
[6] H. Sun and H. Cao, “Chaos control and synchronization of a modified chaotic system,” *Chaos, Solitons and Fractals*, vol. 37, no. 5, pp. 1442–1455, 2008.
[7] C. K. Huang, S. C. Tsay, and Y. R. Wu, “Implementation of chaotic secure communication systems based on OPA circuits,” *Chaos, Solitons and Fractals*, vol. 23, no. 2, pp. 589–600, 2005.
[8] V. Astakhov, A. Shabunin, A. Klimshin, and V. Anishchenko, “In-phase and antiphase complete chaotic synchronization in symmetrically coupled discrete maps,” *Discrete Dynamics in Nature and Society*, vol. 7, no. 4, pp. 215–229, 2002.
[9] S. C. Tsay, C. K. Huang, D. L. Qiu, and W. T. Chen, “Implementation of bidirectional chaotic communication systems based on Lorenz circuits,” *Chaos, Solitons and Fractals*, vol. 20, no. 3, pp. 567–579, 2004.
[10] M. Itoh, “Synthesis of electronic circuits for simulating nonlinear dynamics,” *International Journal of Bifurcation and Chaos in Applied Sciences and Engineering*, vol. 11, no. 3, pp. 605–653, 2001.
[11] C. L. Huang, *Nonlinear Dynamics of the Horizontal Platform*, National Chiao Tung University Press, 1996.
[12] J. Y. Lee, “The corresponding phenomena of mechanical and electronic impact oscillator,” *Journal of Sound and Vibration*, vol. 311, no. 1-2, pp. 579–587, 2008.
[13] A. Wolf, J. B. Swift, H. L. Swinney, and J. A. Vastano, “Determining Lyapunov exponents from a time series,” *Physica D*, vol. 16, no. 3, pp. 285–317, 1985.
[14] C. Knudsen, *Lyapunov Exponents*, Technical University of Denmark Press, 1999.
[15] X. Wu, J. Cai, and M. Wang, “Master-slave chaos synchronization criteria for the horizontal platform systems via linear state error feedback control,” *Journal of Sound and Vibration*, vol. 295, no. 1-2, pp. 378–387, 2006.
[16] D. I. R. Almeida, J. Alvarez, and J. G. Barajas, “Robust synchronization of Sprott circuits using sliding mode control,” *Chaos, Solitons and Fractals*, vol. 30, no. 1, pp. 11–18, 2006.
[17] G. H. Li, S. P. Zhou, and K. Yang, “Controlling chaos in Colpitts oscillator,” *Chaos, Solitons and Fractals*, vol. 33, no. 2, pp. 582–587, 2007.
[18] J. C. Doyle, K. Glover, P. P. Khargonekar, and B. A. Francis, “State-space solutions to standard H_2 and H_∞ control problems,” *IEEE Transactions on Automatic Control*, vol. 34, no. 8, pp. 831–847, 1989.