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Research Article

Qualitative Behavior of Rational Difference Equation of Big Order

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We investigate the global convergence, boundedness, and periodicity of solutions of the recursive sequence $x_{n+1} = (ax_{n-l} + bx_{n-x})/(c + dx_{n-l}x_{n-k})$, $n = 0, 1, \ldots$, where the parameters a, b, c, and d are positive real numbers, and the initial conditions $x_{-t}, x_{-t+1}, \ldots, x_{-1}$ and x_0 are positive real numbers where $t = \max\{k, l\}$.

1. Introduction

Recently, there has been a lot of interest in studying the global attractivity, the boundedness character, and the periodicity nature of nonlinear difference equations see for example, [1–22].

The study of the nonlinear rational difference equations of a higher order is quite challenging and rewarding, and the results about these equations offer prototypes towards the development of the basic theory of the global behavior of nonlinear difference equations of a big order; recently, many researchers have investigated the behavior of the solution of difference equations. For example, in [8]. Elabbasy et al. investigated the global stability and periodicity character and gave the solution of special case of the following recursive sequence:

$$x_{n+1} = ax_n - \frac{bx_n}{cx_n - dx_{n-1}}. \tag{1}$$

Elabbasy et al. [9] investigated the global stability, boundedness, and periodicity character and gave the solution of some special cases of the difference equation

$$x_{n+1} = \frac{\alpha x_{n-k}}{\beta + \gamma \prod_{i=0}^{k} x_{n-i}}. \tag{2}$$

Elabbasy et al. [10] investigated the global stability and periodicity character and gave the solution of some special cases of the difference equation

$$x_{n+1} = \frac{dx_{n-l}x_{n-k}}{cx_{n-s} - b} + a. \tag{3}$$

Saleh and Aloqeili [23] investigated the difference equation

$$y_{n+1} = A + \frac{y_n}{y_{n-k}}, \quad \text{with } A < 0. \tag{4}$$

Wang et al. [24] studied the global attractivity of the equilibrium point and the asymptotic behavior of the solutions of the difference equation

$$x_{n+1} = \frac{ax_{n-l}x_{n-k}}{\alpha + bx_{n-s} + cx_{n-t}}. \tag{5}$$

In [25], Wang et al. investigated the asymptotic behavior of equilibrium point for a family of rational difference equation

$$x_{n+1} = \frac{\sum_{i=1}^{t} A_{s_i} x_{n-s_i}}{B + C\prod_{j=1}^{k} x_{n-t_j}} + Dx_n. \tag{6}$$

Yalçinkaya [26] considered the dynamics of the difference equation

$$x_{n+1} = \alpha + \frac{x_{n-m}}{x_n^k}. \tag{7}$$

Zayed and El-Moneam [27, 28] studied the behavior of the following rational recursive sequences:

$$x_{n+1} = ax_n - \frac{bx_n}{cx_n - dx_{n-k}}, \qquad x_{n+1} = \frac{\alpha + \beta x_n + \gamma x_{n-1}}{A + Bx_n + Cx_{n-1}}. \tag{8}$$

For some related works see [29–39].

Our goal in this paper is to investigate the global stability character and the periodicity of solutions of the recursive sequence

$$x_{n+1} = \frac{ax_{n-l} + bx_{n-k}}{c + dx_{n-l}x_{n-k}}, \quad n = 0, 1, \ldots, \tag{9}$$

where the parameters a, b, c, and d are positive real numbers and the initial conditions $x_{-t}, x_{-t+1}, \ldots, x_{-1}$ and x_0 are positive real numbers where $t = \max\{k, l\}$.

2. Local Stability of the Equilibrium Point of (9)

This section deals with the local stability character of the equilibrium point of (9)

Equation (9) has equilibrium points given by

$$\overline{x} = \frac{(a+b)\,\overline{x}}{c + d\overline{x}^2}, \tag{10}$$

then

$$\overline{x}\left\{d\overline{x}^2 + c - a - b\right\} = 0. \tag{11}$$

Then the equilibrium points of (9) are given by

$$\overline{x} = 0 \quad \text{or} \quad \overline{x} = \sqrt{\frac{a+b-c}{d}} \quad \text{when } a + b > c. \tag{12}$$

Let $f : (0, \infty)^2 \to (0, \infty)$ be a continuously differentiable function defined by

$$f(u, v) = \frac{au + bv}{c + duv}. \tag{13}$$

Therefore, it follows that

$$\frac{\partial f(u,v)}{\partial u} = \frac{ac - bdv^2}{(c + duv)^2}, \qquad \frac{\partial f(u,v)}{\partial v} = \frac{bc - adu^2}{(c + duv)^2}. \tag{14}$$

Theorem 1. *The following statements are true.*

(1) *If $a + b \le c$, then the only equilibrium point $\overline{x} = 0$ of (9) is locally stable.*

(2) *If $a + b > c$, then the positive equilibrium point $\overline{x} = \sqrt{(a+b-c)/d}$ of (9) is locally stable if $|c-b| + |c-a| < a + b$.*

Proof. (1) If $a + b \le c$, then we see from (14) that

$$\frac{\partial f(0,0)}{\partial x_{n-l}} = \frac{a}{c}, \qquad \frac{\partial f(0,0)}{\partial x_{n-k}} = \frac{b}{c}. \tag{15}$$

Then, the linearized equation associated with (9) about $\overline{x} = 0$ is

$$y_{n+1} - \frac{a}{c}y_{n-l} - \frac{b}{c}y_{n-k} = 0, \tag{16}$$

whose characteristic equation is

$$\lambda^{k+1} - \frac{a}{c}\lambda^{k-l} - \frac{b}{c} = 0. \tag{17}$$

Then, (16) is asymptotically stable if $a + b < c$, and then the equilibrium point $\overline{x} = 0$ of (9) is locally stable.

(2) If $a + b > c$, then we see from (14) that

$$\begin{aligned} \frac{\partial f(\overline{x}, \overline{x})}{\partial x_{n-l}} &= \frac{ac - bd\left((a+b-c)/d\right)}{\left(c + d\left((a+b-c)/d\right)\right)^2} = \frac{c-b}{a+b}, \\ \frac{\partial f(\overline{x}, \overline{x})}{\partial x_{n-k}} &= \frac{bc - ad\left((a+b-c)/d\right)}{\left(c + d\left((a+b-c)/d\right)\right)^2} = \frac{c-a}{a+b}. \end{aligned} \tag{18}$$

Then, the linearized equation of (9) about $\overline{x}$ is

$$y_{n+1} - \frac{c-b}{a+b}y_{n-l} - \frac{c-a}{a+b}y_{n-k} = 0, \tag{19}$$

whose characteristic equation is

$$\lambda^{k+1} - \frac{c-b}{a+b}\lambda^{k-l} - \frac{c-a}{a+b} = 0. \tag{20}$$

Then, (19) is asymptotically stable if all roots of (20) lie in the open disc $|\lambda| < 1$, that is, if

$$\left|\frac{c-b}{a+b}\right| + \left|\frac{c-a}{a+b}\right| < 1, \tag{21}$$

which is true if

$$|c - b| + |c - a| < a + b. \tag{22}$$

The proof is complete. □

3. Boundedness of the Solutions of (9)

Here, we study the boundedness nature of the solutions of (9).

Theorem 2. *Every solution of (9) is bounded if $c > a + b$.*

Proof. Let $\{x_n\}_{n=-t}^{\infty}$ be a solution of (9). It follows from (9) that

$$x_{n+1} = \frac{ax_{n-l} + bx_{n-k}}{c + dx_{n-l}x_{n-k}} \le \frac{ax_{n-l} + bx_{n-k}}{c}. \tag{23}$$

By using a comparison, we can write the right-hand side as follows:

$$y_{n+1} = \frac{ay_{n-l}}{c} + \frac{by_{n-k}}{c}, \tag{24}$$

and this equation is locally asymptotically stable if $a + b < c$ and converges to the equilibrium point $\overline{y} = 0$.

Therefore,

$$\limsup_{n \to \infty} x_n = 0. \tag{25}$$

Thus, the solution is bounded. □

4. Existence of Periodic Solutions

In this section, we study the existence of periodic solutions of (9). The following theorem states the necessary and sufficient conditions that this equation has periodic solutions of prime period two.

Theorem 3. *Equation* (9) *has a prime period two solutions if and only if one of the following statements holds:*

(1) *$a+b-c>0$, and l, k—odd,*

(2) *$a+c-b>0$, and k—odd, l—even,*

(3) *$b+c-a>0$, and l—odd, k—even.*

Proof. We will prove the theorem when condition (1) is true, and the proof of the other cases is similar and so we will be omit it.

First suppose that there exists a prime period two solution

$$\ldots, p, q, p, q, \ldots, \tag{26}$$

of (9). We will prove that Condition (1) holds.

We see from (9) that

$$p = \frac{(a+b)\,p}{c+dp^2}, \qquad q = \frac{(a+b)\,q}{c+dq^2}. \tag{27}$$

Then,

$$c + dp^2 = a + b, \tag{28}$$

$$c + dq^2 = a + b. \tag{29}$$

Subtracting (28) from (29) gives

$$d\left(p^2 - q^2\right) = 0. \tag{30}$$

Since $p \neq q$, it follows that

$$p = -q. \tag{31}$$

Again, from (28) and (29)

$$p^2 = q^2 = \frac{a+b-c}{d}, \tag{32}$$

and so

$$a + b - c > 0. \tag{33}$$

Therefore, inequality (1) holds.

Second, suppose that inequality (1) is true. We will show that (9) has a prime period two solution.

Assume that

$$p = +\sqrt{\frac{a+b-c}{d}}, \qquad q = -\sqrt{\frac{a+b-c}{d}}. \tag{34}$$

We see from inequality (1) that

$$a + b - c > 0. \tag{35}$$

Therefore, p and q are distinct real numbers.

Set

$$x_{-l} = x_{-k} = p, \qquad x_{-2} = q, \qquad x_{-1} = p, \qquad x_0 = q. \tag{36}$$

We wish to show that

$$x_1 = x_{-1} = p, \qquad x_2 = x_0 = q. \tag{37}$$

It follows from (9) that

$$x_1 = \frac{(a+b)\,p}{c+dp^2} = \frac{(a+b)\sqrt{(a+b-c)/d}}{c+d\left((a+b-c)/d\right)} = \sqrt{\frac{a+b-c}{d}} = p. \tag{38}$$

Similarly, we see that

$$\begin{aligned} x_2 &= \frac{(a+b)\,q}{c+dq^2} = \frac{-(a+b)\sqrt{(a+b-c)/d}}{c+d\left((a+b-c)/d\right)} \\ &= -\sqrt{\frac{a+b-c}{d}} = q. \end{aligned} \tag{39}$$

Then, it follows by induction that

$$x_{2n} = q, \quad x_{2n+1} = p, \quad \forall n \geq -1. \tag{40}$$

Thus, (9) has the prime period two solution

$$\ldots, p, q, p, q, \ldots, \tag{41}$$

where p and q are distinct roots of a quadratic equation, and the proof is complete. □

5. Global Attractor of the Equilibrium Point of (9)

In this section, we investigate the global asymptotic stability of (9). If we take the function $f(u,v)$ defined by (16), then we have four cases of the monotonicity behavior in its arguments (all of these cases we suppose that $a+b>c$).

Theorem 4. *If the function $f(u,v)$ defined by* (16) *is nondecreasing (or nonincreasing) in u, v, then the positive equilibrium point $\overline{x} = \sqrt{(a+b-c)/d}$ is a global attractor of* (9).

Proof. Let $\{x_n\}_{n=-t}^{\infty}$ be a solution of (9) and again let f be a function defined by (16).

We will prove the theorem when $f(u,v)$ is nondecreasing and the proof of the other cases is similar, and so we will omit it.

Suppose that (m, M) is a solution of the systems $M = f(M,M)$ and $m = g(m,m)$. Then, from (9), we see that

$$M = \frac{aM + bM}{c + dM^2}, \qquad m = \frac{am + bm}{c + dm^2}, \tag{42}$$

or

$$c + dM^2 = a + b, \qquad c + dm^2 = a + b. \tag{43}$$

Subtracting these two equations, we obtain

$$d(M-m)(M+m)=0. \tag{44}$$

Under the condition $d>0$, we see that

$$M=m. \tag{45}$$

It follows by Theorem 2 that $\bar{x}$ is a global attractor of (9), and then the proof is complete. □

Theorem 5. *If the function $f(u,v)$ defined by (16) is nondecreasing in u and nonincreasing in v, then the positive equilibrium point $\bar{x}=\sqrt{(a+b-c)/d}$ is a global attractor of (9) if $c+b>a$.*

Proof. Let $\{x_n\}_{n=-t}^{\infty}$ be a solution of (9) and again let f be a function defined by (16).

Suppose that (m,M) is a solution of the systems $M=f(M,m)$ and $m=g(m,M)$. Then, from (9), we see that

$$M=\frac{aM+bm}{c+dmM}, \qquad m=\frac{am+bM}{c+dmM}, \tag{46}$$

or

$$\begin{gathered} cM+dmM^2=aM+bm, \\ cm+dMm^2=am+bM. \end{gathered} \tag{47}$$

Subtracting these two equations, we obtain

$$\begin{gathered} c(M-m)+dMm(M-m)=(a-b)(M-m), \\ (M-m)\{c+b-a+dMm\}=0. \end{gathered} \tag{48}$$

Under the condition $c+b>a$, we see that

$$M=m. \tag{49}$$

It follows by Theorem 2 that $\bar{x}$ is a global attractor of (9), and then the proof is complete. □

Theorem 6. *If the function $f(u,v)$ defined by (16) is nondecreasing in v, nonincreasing in u. Then the positive equilibrium point $\bar{x}=\sqrt{(a+b-c)/d}$ is a global attractor of (9) if $c+a>b$.*

Proof. The proof is similar to the previous Theorem and so we will be omit it. □

Lemma 7. *When $c\geq a+b$ then the equilibrium point $\bar{x}=0$ of (9) is global attractor.*

Proof. If $c\geq a+b$, then the proof follows by Theorem 2. □

6. Numerical Examples

For confirming the results of this paper, we consider numerical examples which represent different types of solutions to (9).

Example 1. We assume that $l=1$, $k=2$, $x_{-2}=3$, $x_{-1}=2$, $x_0=6$, $a=2$, $b=5$, $c=8$, and $d=6$. See Figure 1.

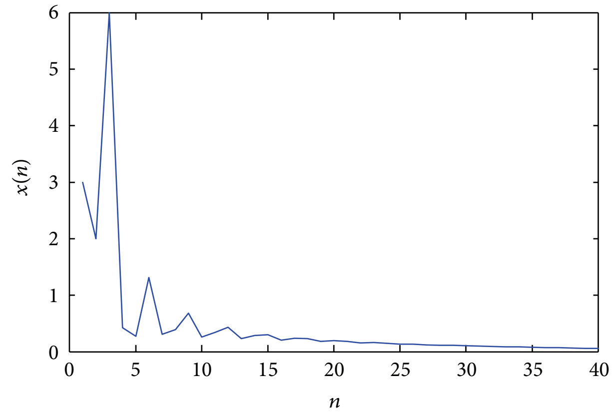

Figure 1: It shows the solution of (9) with $l=1$, $k=2$, $x_{-2}=3$, $x_{-1}=2$, $x_0=6$, $a=2$, $b=5$, $c=8$, and $d=6$.

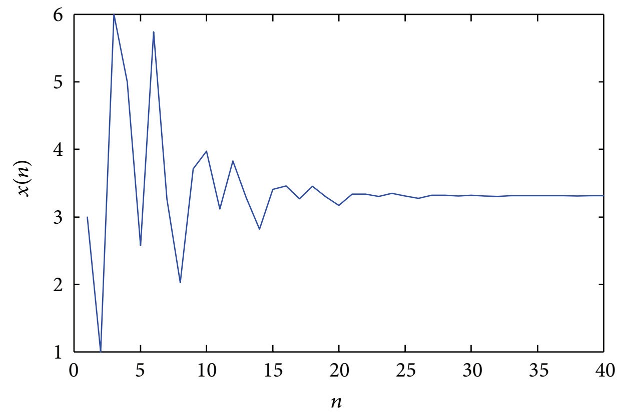

Figure 2: It shows the behavior of the solution of (9) with $l=1$, $k=3$, $x_{-3}=3$, $x_{-2}=1$, $x_{-1}=6$, $x_0=5$, $a=9$, $b=13$, $c=0.1$, and $d=2$.

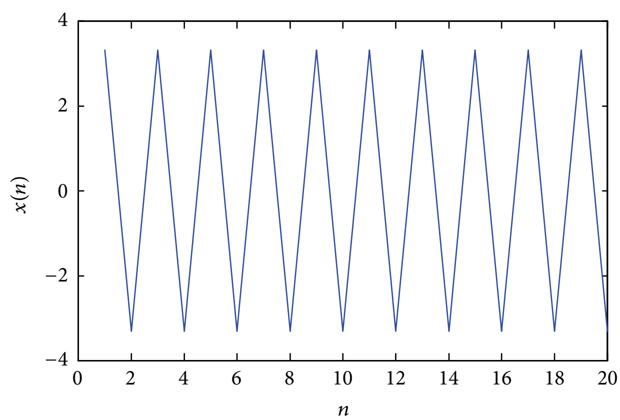

Figure 3: It shows the periodicity of the solution of (9) when $l=3$, $k=1$, $x_{-3}=x_{-1}=p$, $x_{-2}=x_0=q$, $a=9$, $b=13$, $c=0.1$, and $d=2$.

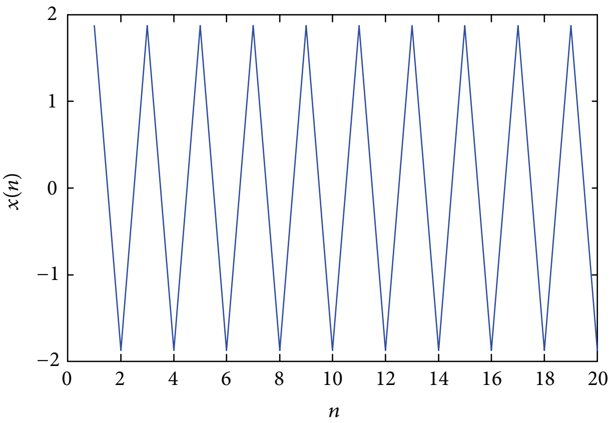

FIGURE 4: It shows the periodicity of the solution of (9) when $l = 4$, $k = 3$, $x_{-3} = x_{-1} = q$, $x_{-4} = x_{-2} = x_0 = p$, $a = 9$, $b = 5$, $c = 3$, and $d = 2$.

Example 2. See Figure 2, since $l = 1$, $k = 3$, $x_{-3} = 3$, $x_{-2} = 1$, $x_{-1} = 6$, $x_0 = 5$, $a = 9$, $b = 13$, $c = 0.1$, $d = 2$.

Example 3. Figure 3 shows the solutions when $l = 3$, $k = 1$, $x_{-3} = x_{-1} = p$, $x_{-2} = x_0 = q$, $a = 9$, $b = 13$, $c = 0.1$, and $d = 2$. (Since $p, q = \pm\sqrt{(a+b-c)/d}$).

Example 4. Figure 4 shows the solutions when $l = 4$, $k = 3$, $x_{-3} = x_{-1} = q$, $x_{-4} = x_{-2} = x_0 = p$, $a = 9$, $b = 5$, $c = 3$, and $d = 2$. (Since $p, q = \pm\sqrt{(a+b-c)/d}$).

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