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Research Article

Permanence, Extinction, and Almost Periodic Solution of a Nicholson's Blowflies Model with Feedback Control and Time Delay

Haihui Wu and Shengbin Yu

Sunshine College, Fuzhou University, Fuzhou, Fujian 350015, China

Correspondence should be addressed to Haihui Wu; whh3346@sina.com

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A Nicholson's blowflies model with feedback control and time delay is studied. By applying the comparison theorem of the differential equation and fluctuation lemma and constructing a suitable Lyapunov functional, sufficient conditions which guarantee the permanence, extinction, and existence of a unique globally attractive positive almost periodic solution of the system are obtained. It is proved that the feedback control variable and time delay have no influence on the permanence and extinction of the system.

1. Introduction

Let $f(t)$ be any continuous bounded function defined on $[0, +\infty)$; we set

$$f^l = \inf_{t\geq 0} f(t), \qquad f^u = \sup_{t\geq 0} f(t). \tag{1}$$

In order to describe the dynamics of Nicholson's blowflies, Gurney et al. [1] proposed the following mathematical model in 1980:

$$\dot{N}(t) = -\delta N(t) + PN(t-\tau)e^{-aN(t-\tau)}, \tag{2}$$

where $N(t)$ is the size of the population at time t, P is the maximum per capita daily egg production rate, $(1/a)$ is the size at which the population reproduces at its maximum rate, δ is the per capita daily adult death rate, and τ is the generation time. Kulenović and Ladas [2], Győri and Ladas [3], and Győri and Trofimchuk [4] investigated the oscillatory behaviors of the solutions of (2). For the attractivity, Kulenović et al. [5] and So and Yu [6] have shown that, when $P > \delta$, every positive solution $N(t)$ of (2) tends to a positive equilibrium $N^* = (1/a)\ln(P/\delta)$ as $t \to \infty$ if

$$\left(e^{\delta\tau} - 1\right)\left(\frac{P}{\delta} - 1\right) < 1. \tag{3}$$

Reference [5] further showed that, for $P \leq \delta$, every nonnegative solution of (2) tends to zero as $t \to \infty$, and for $P > \delta$, (2) is uniformly persistent. Furthermore, Li and Fan [7] considered the following nonautonomous equation:

$$\dot{x}(t) = x(t)\left(-\delta(t) + p(t)\exp\{-\alpha(t)x(t)\}\right), \tag{4}$$

where $\alpha(t)$, $\delta(t)$, and $p(t)$ are all positive ω-periodic functions. The authors show that (4) has a unique globally attractive ω-periodic positive solution if

$$p(t) > \delta(t) \quad \text{for } t \in [0, \omega]. \tag{5}$$

Their results improved the results of Saker and Agarwal [8] who considered system (4) with $\alpha(t) = a$ (a is a constant).

Recently, Wang and Fan [9] proposed the following discrete Nicholson's blowflies model with feedback control:

$$\begin{aligned} x(n+1) &= x(n)\exp\{-\delta(n) + p(n)\exp\{-\alpha(n)x(n)\} \\ &\qquad - c(n)\mu(n)\}, \\ \Delta\mu(n) &= -a(n)\mu(t) + b(n)x(n-m). \end{aligned} \tag{6}$$

Sufficient conditions are established for the permanence and the extinction of the system (6). They show that the bounded feedback terms do not have any influence on the permanence or extinction of (6). The authors in [9] also proposed the following continuous model:

$$\dot{x}(t) = x(t)\left(-\delta(t) + p(t)\exp\{-\alpha(t)x(t)\} - c(t)\mu(t)\right),$$
$$\dot{\mu}(t) = -a(t)\mu(t) + b(t)x(t-\tau); \tag{7}$$

however, they did not discuss the dynamic behaviors of the system (7). Considering that continuous models can excellently show the dynamic behaviors of those populations who have a long life cycle, overlapping generations, and large quantity, sufficient conditions for the permanence, global attractivity, and the existence of a unique, globally attractive, strictly positive almost periodic solution of the system (7) with $\tau = 0$ are obtained by Yu [10]. As pointed out by Nindjin et al. [11], time delay plays an important role in many biological dynamical systems, being particularly relevant in ecology and a model with time delay is a more realistic approach to the understanding of dynamics. Hence, it is necessary to study the model (7) which contains time delay.

In the following discussion, we always assume that $\delta(t)$, $p(t)$, $\alpha(t)$, $c(t)$, $a(t)$, $b(t)$ are all continuous, positive almost periodic functions. Also, from the viewpoint of mathematical biology, we consider (7) together with the following initial conditions:

$$\begin{aligned} x(\theta) &= \varphi(\theta) \geq 0, \quad \theta \in [-\tau, 0] \quad \varphi(0) > 0, \\ \mu(\theta) &= \psi(\theta) \geq 0, \quad \theta \in [-\tau, 0] \quad \psi(0) > 0, \end{aligned} \tag{8}$$

where $\varphi(s)$ and $\psi(s)$ are continuous on $[-\tau, 0]$. It is not difficult to see that solutions of (7) and (8) are well defined for all $t \geq 0$ and satisfy

$$x(t) > 0, \quad \mu(t) > 0, \quad \text{for } t \geq 0. \tag{9}$$

The aim of this paper is, by constructing a suitable Lyapunov functional and applying the analysis technique of Feng et al. [12], to obtain sufficient conditions for the existence of a unique globally attractive positive almost periodic solution of the system (7) with initial condition (8).

This paper is organized as follows. In Section 2, by applying the analysis technique of [13, 14] and Fluctuation lemma [15, 16], we present the permanence and the extinction of model (7) and (8). In Section 3, by constructing a suitable Lyapunov functional, a sufficient conditions for the existence of a unique globally attractive positive almost periodic solution of the system (7) and (8). Examples together with their numeric simulations are stated in Section 4. For more works on almost periodic solutions of the ecosystem with feedback control, one could refer to [17–23] and the references cited therein.

2. Permanence and Extinction

Now let us state several lemmas which will be useful in proving the main result of this section.

Lemma 1 (see [13]). *Assume that $a > 0$, $b(t) > 0$ is a boundedness continuous function and $x(0) > 0$. Further suppose that*

(i)

$$\dot{x}(t) \leq -ax(t) + b(t) \tag{10}$$

then for all $t \geq s$,

$$x(t) \leq x(t-s)\exp\{-as\} + \int_{t-s}^{t} b(\tau)\exp\{a(\tau - t)\}\,d\tau. \tag{11}$$

Particularly, if $b(t)$ is bounded above with respect to M, then

$$\limsup_{t\to+\infty} x(t) \leq \frac{M}{a}. \tag{12}$$

(ii) *Also suppose that*

$$\dot{x}(t) \geq -ax(t) + b(t); \tag{13}$$

then for all $t \geq s$,

$$x(t) \geq x(t-s)\exp\{-as\} + \int_{t-s}^{t} b(\tau)\exp\{a(\tau - t)\}\,d\tau. \tag{14}$$

Particularly, if $b(t)$ is bounded above with respect to m, then

$$\liminf_{t\to+\infty} x(t) \geq \frac{m}{a}. \tag{15}$$

Lemma 2 (see [20]). *If $a > 0$, $b > 0$ and $\dot{x} \geq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, one has*

$$\liminf_{t\to+\infty} x(t) \geq \frac{b}{a}. \tag{16}$$

If $a > 0$, $b > 0$, and $\dot{x} \leq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, one has

$$\limsup_{t\to+\infty} x(t) \leq \frac{b}{a}. \tag{17}$$

Lemma 3. *Let $(x(t), \mu(t))^T$ be any solution of system (7) with initial condition (8); there exists positive numbers M_1 and M_2, which are independent of the solution of the system, such that*

$$\limsup_{t\to+\infty} x(t) \leq M_1, \qquad \limsup_{t\to+\infty} \mu(t) \leq M_2. \tag{18}$$

Proof. Let $(x(t), \mu(t))^T$ be any solution of system (7) satisfying initial condition (8). Since $x\exp\{-\alpha^l x(t)\} \leq (1/\alpha^l e)$ for $x > 0$, according to the positivity of solution and the first equation of system (7), for $t \geq 0$,

$$\begin{aligned} \dot{x}(t) &\leq x(t)\left(-\delta^l + p^u \exp\left\{-\alpha^l x(t)\right\}\right) \\ &\leq -\delta^l x(t) + \frac{p^u}{\alpha^l e}, \end{aligned} \tag{19}$$

where e is the mathematical constant.

By applying Lemma 1(i) to (19), we have

$$\limsup_{t\to+\infty} x(t) \le \frac{p^u}{\delta^l \alpha^l e} \triangleq M_1. \tag{20}$$

Hence, there exists $T_1 > 0$ such that

$$x(t) \le 2M_1, \quad \forall t \ge T_1. \tag{21}$$

Equation (21) together with the second equation of (7) leads to

$$\dot{\mu}(t) \le -a^l \mu(t) + 2b^u M_1, \quad \forall t \ge T_1 + \tau. \tag{22}$$

Using Lemma 1(i) again, one has

$$\limsup_{t\to+\infty} \mu(t) \le \frac{2b^u M_1}{a^l} \triangleq M_2. \tag{23}$$

Obviously, $M_i (i = 1, 2)$ are independent of the solution of system (7). Equations (20) and (23) show that the conclusion of Lemma 3 holds. The proof is completed. □

Lemma 4. *Assume that*

(H_1)

$$p(t) > \delta(t), \quad t \ge 0, \tag{24}$$

holds. Then there exists positive constants m_1 and m_2, which are independent of the solution of system (7)*, such that*

$$\liminf_{t\to+\infty} x(t) \ge m_1, \qquad \liminf_{t\to+\infty} \mu(t) \ge m_2. \tag{25}$$

Proof. Let $(x(t), \mu(t))^T$ be any solution of system (7) satisfying initial condition (8). From Lemma 3, there exists a $T_2 > T_1 + \tau$ such that for all $t \ge T_2$, $x(t) \le M$, $\mu(t) \le M$, where $M = 2\max\{M_1, M_2\}$. According to the first equation of system (7) and the positivity of solution, for $t \ge T_2$,

$$\begin{aligned}\dot{x}(t) &= x(t)\left(-\delta(t) + p(t)\exp\{-\alpha(t)x(t)\} - c(t)\mu(t)\right) \\ &\ge x(t)\left(-\delta(t) + p(t)\exp\{-\alpha(t)M\} - c(t)M\right) \\ &\triangleq -Q(t)x(t),\end{aligned} \tag{26}$$

where $Q(t) = \delta(t) - p(t)\exp\{-\alpha(t)M\} + c(t)M$.

Integrating both sides of (26) from η $(\eta \le t)$ to t leads to

$$\frac{x(t)}{x(\eta)} \ge \exp\left\{-\int_\eta^t Q(s)\,ds\right\}, \tag{27}$$

or

$$x(\eta) \le x(t)\exp\left\{\int_\eta^t Q(s)\,ds\right\}. \tag{28}$$

Particularly, taking $\eta = t - \tau$, one can get

$$x(t-\tau) \le x(t)\exp\left\{\int_{t-\tau}^t Q(s)\,ds\right\}. \tag{29}$$

Substituting (29) into the second equation of system (7) leads to

$$\dot{\mu}(t) \le -a^l\mu(t) + b^u x(t)\exp\left\{\int_{t-\tau}^t Q(s)\,ds\right\}. \tag{30}$$

Applying Lemma 1(i) to the above differential inequality, for $0 \le s \le t$, one has

$$\begin{aligned}\mu(t) &\le \mu(t-s)\exp\left\{-a^l s\right\} \\ &\quad + \int_{t-s}^t b^u x(\eta)\exp\left\{\int_{\eta-\tau}^\eta Q(u)\,du\right\}\exp\left\{a^l(\eta-t)\right\}d\eta \\ &\overset{\text{from (28)}}{\le} \mu(t-s)\exp\left\{-a^l s\right\} \\ &\quad + \int_{t-s}^t b^u x(t)\exp\left\{\int_\eta^t Q(u)\,du\right\}\exp\left\{\int_{\eta-\tau}^\eta Q(u)\,du\right\} \\ &\quad \times \exp\left\{a^l(\eta-t)\right\}d\eta \\ &\le \mu(t-s)\exp\left\{-a^l s\right\} + b^u x(t)\int_{t-s}^t \exp\left\{\int_\eta^t Q(u)\,du\right\} \\ &\quad \times \exp\left\{\int_{\eta-\tau}^\eta Q(u)\,du\right\}d\eta,\end{aligned} \tag{31}$$

where we used the fact $\max_{\eta\in[t-s,t]}\exp\{a^l(\eta-t)\} = \exp\{0\} = 1$.

Note that there exists a K, such that $2c^u M\exp\{-a^l s\} < (\beta/2)$, as $s \ge K$, where $\beta = \inf_{t\ge0}(p(t) - \delta(t))$. In fact, we can choose $K > (1/a^l)\ln(4c^u M/\beta)$. And so, fixing K, combined with (31), we can obtain

$$\begin{aligned}\mu(t) &\le M\exp\left\{-a^l K\right\} + b^u x(t)\int_{t-K}^t \exp\left\{\int_\eta^t Q(u)\,du\right\} \\ &\quad \times \exp\left\{\int_{\eta-\tau}^\eta Q(u)\,du\right\}d\eta \\ &\le M\exp\left\{-a^l K\right\} + Dx(t),\end{aligned} \tag{32}$$

for all $t > T_2 + K$, where $D = \sup_{t\ge T_3}(b^u \int_{t-K}^t \exp\{\int_\eta^t Q(u)du\}\exp\{\int_{\eta-\tau}^\eta Q(u)du\}d\eta) > 0$.

Considering that $e^{-x} \ge 1 - x$, for $x > 0$, from the first equation of system (7) and the positivity of the solution, for $t > T_2 + K$, we can get

$$\begin{aligned}\dot{x}(t) &= x(t)\left(-\delta(t) + p(t)\exp\{-\alpha(t)x(t)\} - c(t)\mu(t)\right) \\ &\ge x(t)\Big(-\delta(t) + p(t) - p(t)\alpha(t)x(t) \\ &\qquad -2c^u M\exp\left\{-a^l K\right\} - c^u Dx(t)\Big) \\ &\ge x(t)\left(\frac{\beta}{2} - \left(p^u\alpha^u + c^u D\right)x(t)\right).\end{aligned} \tag{33}$$

By Lemma 2, we have

$$\liminf_{t \to +\infty} x(t) \geq \frac{\beta}{2(p^u \alpha^u + c^u D)} \triangleq m_1. \tag{34}$$

Thus, there exists $T_3 > T_2 + K$ such that for all $t > T_3$,

$$x(t) \geq \frac{m_1}{2}. \tag{35}$$

Equation (35) together with the second equation of (7) leads to

$$\dot{\mu}(t) \geq -a^u \mu(t) + b^l \frac{m_1}{2}, \quad \forall t > T_3. \tag{36}$$

By applying Lemma 1(ii) to the above differential inequality, we have

$$\liminf_{t \to +\infty} \mu(t) \geq \frac{b^l m_1}{2a^u} \triangleq m_2. \tag{37}$$

Obviously, m_i $(i = 1, 2)$ are independent of the solution of system (7). Equations (34) and (37) show that the conclusion of Lemma 4 holds. The proof is completed. □

From Lemmas 3–4 and the definition of permanence, we can obtain the following conclusion.

Theorem 5. *Assume that* (H_1) *holds; then system* (7) *with initial condition* (8) *is permanent.*

As a direct corollary of Theorem 2 in [24], from Theorem 5, we have the following.

Corollary 6. *Suppose that* (H_1) *holds; then system* (7) *admits at least one positive* ω*-periodic solution if* $\delta(t)$, $p(t)$, $\alpha(t)$, $c(t)$, $a(t)$, $b(t)$ *are all continuous positive* ω*-periodic functions.*

Theorem 7. *Let* $(x(t), \mu(t))^T$ *be any positive solution of the system* (7) *with initial condition* (8). *Assume that*

$$\int_0^\infty (p(t) - \delta(t))\, dt = -\infty \quad \text{or} \quad p(t) \leq \delta(t), \quad t \geq 0 \tag{38}$$

holds; then

$$\lim_{t \to +\infty} x(t) = 0, \qquad \lim_{t \to +\infty} \mu(t) = 0. \tag{39}$$

Proof. Firstly, from the the first equation of (7),

$$\dot{x}(t) \leq x(t)(p(t) - \delta(t)). \tag{40}$$

If the former case of (38) holds, then

$$0 < x(t) \leq x(0) \exp\left[\int_0^t (p(t) - \delta(t))\, dt\right] \longrightarrow 0, \quad \text{as } t \longrightarrow \infty, \tag{41}$$

which shows that $\lim_{t \to +\infty} x(t) = 0$.

If the latter case of (38) holds, from (40) we have $\dot{x}(t) < 0$ or $x(t)$ is decreasing; therefore, $\lim_{t \to +\infty} x(t) = q \in [0, +\infty)$. Hence $\limsup_{t \to +\infty} = \liminf_{t \to +\infty} x(t) = q$. We only need to show that $q = 0$. Otherwise, if $q > 0$, then there exists a $T_4 > 0$, such that $x(t) > (q/2)$ for $t \geq T_4$. According to the Fluctuation lemma, there exists a sequence $\xi_n \to \infty$ as $n \to \infty$ such that $\dot{x}(\xi_n) \to 0$, $x(\xi_n) \to \limsup_{t \to \infty} = q$, as $n \to \infty$. We can choose a large enough number N such that $\xi_n > T_4$ for $n > N$; hence, $x(\xi_n) > (q/2)$ for all $n > N$.

For $n > N$, $p(t) \leq \delta(t)$ together with the first equation of (7) leads to

$$\begin{aligned}
\dot{x}(\xi_n) &\leq x(\xi_n)(-\delta(\xi_n) + p(\xi_n)\exp\{-\alpha(\xi_n)x(\xi_n)\}) \\
&\leq x(\xi_n)\left(-\delta(\xi_n) + \delta(\xi_n)\exp\left\{-\alpha(\xi_n)\frac{q}{2}\right\}\right) \\
&\leq x(\xi_n)\left(-1 + \exp\left\{-\alpha^l \frac{q}{2}\right\}\right)\delta^l.
\end{aligned} \tag{42}$$

Let $n \to \infty$; we obtain that $0 \leq q(-1 + \exp\{-\alpha^l(q/2)\})\delta^l$ or $\exp\{-\alpha^l(q/2)\} > 1$ which is impossible. Hence, $q = 0$ or

$$\lim_{t \to +\infty} x(t) = 0. \tag{43}$$

Now, we come to prove that

$$\lim_{t \to +\infty} \mu(t) = 0. \tag{44}$$

For any $\epsilon > 0$, according to (43), there exists a $T_5 > 0$, such that

$$x(t) < \epsilon \quad \forall t > T_5. \tag{45}$$

Then, for $t > T_5 + \tau$,

$$\dot{\mu}(t) \leq -a^l \mu(t) + b^u \epsilon. \tag{46}$$

Thus, by applying Lemma 1(i) to the above differential inequality, we have

$$0 < \mu(t) \leq \frac{b^u \epsilon}{a^l} \tag{47}$$

which implies that

$$\lim_{t \to +\infty} \mu(t) = 0. \tag{48}$$

The proof is complete. □

3. Existence of a Unique Almost Periodic Solution

Now, we give the definition of the almost periodic function.

Definition 8 (see [25, 26]). A function $f(t, x)$, where f is an m-vector, t is a real scalar, and x is an n-vector, is said to be almost periodic in t uniformly with respect to $x \in X \subset R^n$, if $f(t, x)$ is continuous in $t \in R$ and $x \in X$, and if for any $\varepsilon > 0$, there is a constant $l(\varepsilon) > 0$, such that in any interval of length $l(\varepsilon)$, there exists τ such that the inequality

$$\|f(t+\tau) - f(t)\| = \sum_{i=1}^{m} \left|f_i(t+\tau, x) - f_i(t, x)\right| < \varepsilon \quad (49)$$

is satisfied for all $t \in (-\infty, +\infty)$, $x \in X$. The number τ is called an ε-translation number of $f(t, x)$.

Definition 9 (see [25, 26]). A function $f : R \to R$ is said to be an asymptotically almost periodic function if there exists an almost periodic function $q(t)$ and a continuous function $r(t)$ such that

$$f(t) = q(t) + r(t), \quad t \in R,\ r(t) \longrightarrow 0 \text{ as } t \longrightarrow \infty. \quad (50)$$

We denote by $S(E)$ the set of all solutions $z(t) = (x(t), \mu(t))^T$ of system (7) satisfying $m_1 \le x(t) \le M_1$, $m_2 \le \mu(t) \le M_2$ for all $t \in R$.

Lemma 10. *One has* $S(E) \ne \emptyset$.

Proof. Since $\delta(t)$, $p(t)$, $\alpha(t)$, $c(t)$, $a(t)$, $b(t)$ are almost periodic functions, there exists a sequence $\{t_n\}$, $t_n \to \infty$ as $n \to \infty$ such that

$$\begin{aligned} &\delta(t+t_n) \longrightarrow \delta(t), \qquad p(t+t_n) \longrightarrow \delta(t), \\ &\alpha(t+t_n) \longrightarrow \delta(t), \qquad c(t+t_n) \longrightarrow \delta(t), \\ &a(t+t_n) \longrightarrow \delta(t), \qquad b(t+t_n) \longrightarrow \delta(t), \end{aligned} \quad (51)$$

as $n \to \infty$ uniformly on R. Suppose $z(t) = (x(t), \mu(t))^T$ is a solution of (7) satisfying $m_1 \le x(t) \le M_1$, $m_2 \le \mu(t) \le M_2$ for $t > T$. Obviously, the sequence $(z(t + t_n))$ is uniformly bounded and equicontinuous on each bounded subset of R. Therefore, by the Ascoli-Arzela theorem, there exists a subsequence of $\{t_n\}$, which we still denote by $\{t_n\}$, such that $x(t + t_n) \to m(t)$, $\mu(t + t_n) \to n(t)$, as $n \to \infty$ uniformly on each bounded subset of R. For any $T_1 \in R$, we may assume that $t_n + T_1 \ge T$ for all n. For $t \ge 0$, we have

$$\begin{aligned} &x(t+t_n+T_1) - x(t_n+T_1) \\ &= \int_{T_1}^{t+T_1} x(s+t_n)\big(-\delta(s+t_n) + p(s+t_n) \\ &\qquad \times \exp\{-\alpha(s+t_n)x(s+t_n)\} \\ &\qquad -c(s+t_n)\mu(s)\big)\,ds, \\ &\mu(t+t_n+T_1) - \mu(t_n+T_1) \\ &= \int_{T_1}^{t+T_1} \big(-a(s+t_n)\mu(s+t_n) + b(s+t_n)x(s+t_n-\tau)\big)\,ds. \end{aligned} \quad (52)$$

Applying Lebesgue's dominated convergence theorem and letting $n \to \infty$ in to previous equations, we obtain

$$\begin{aligned} &m(t+T_1) - m(T_1) \\ &\quad = \int_{T_1}^{t+T_1} m(s)\big(-\delta(s) + p(s) \\ &\qquad \times \exp\{-\alpha(s)m(s)\} - c(s)n(s)\big)\,ds, \\ &n(t+T_1) - n(T_1) = \int_{T_1}^{t+T_1} \big(-a(s)n(s) + b(s)m(s-\tau)\big)\,ds, \end{aligned} \quad (53)$$

for all $t \ge 0$. Since $T_1 \in R$ is arbitrarily given, $(m(t), n(t))^T$ is a solution of system (7) on R. It is clear that $m_1 \le m(t) \le M_1$, $m_2 \le n(t) \le M_2$ for $t \in R$. That is to say, $(m(t), n(t))^T \in S(E)$. This completes the proof. □

Lemma 11 (see [27]). *Let f be a nonnegative function defined on $[0, +\infty)$ such that f is integrable on $[0, +\infty)$ and is uniformly continuous on $[0, +\infty)$. Then,* $\lim_{t\to+\infty} f(t) = 0$.

Theorem 12. *In addition to* (H_1)*, further suppose that*

(H_2) *there exists a* $h > 0$*, such that*

$$p^l\alpha^l \exp(\alpha^u M_1) - b^u > h, \qquad a^l - c^u > h, \quad (54)$$

where M_1 is defined in (23)*; then system* (7) *with initial conditions* (8) *is globally attractive. That is to say, for any two positive solutions, one has*

$$\lim_{t\to+\infty} \left|x(t) - x^*(t)\right| = 0, \qquad \lim_{t\to+\infty} \left|\mu(t) - \mu^*(t)\right| = 0. \quad (55)$$

Proof. Let $(x^*(t), \mu^*(t))^T$ and $(x(t), \mu(t))^T$ be any two positive solutions of system (7)-(8). Theorem 5 implies there exist positive constants T, m_i, and M_i $(i = 1, 2)$ such that for $t \ge T$

$$\begin{aligned} &m_1 \le x(t) \le M_1, \qquad m_1 \le x^*(t) \le M_1, \\ &m_2 \le \mu(t) \le M_2, \qquad m_2 \le \mu^*(t) \le M_2, \end{aligned} \quad (56)$$

where m_i and M_i $(i = 1, 2)$ are defined in Lemma 3 and Lemma 4. Set $V(t) = V_1(t) + V_2(t)$, where

$$\begin{aligned} &V_1(t) = \left|\ln x(t) - \ln x^*(t)\right|, \\ &V_2(t) = \left|\mu(t) - \mu^*(t)\right| + b^u \int_{t-\tau}^{t} \left|x(u) - x^*(u)\right| du. \end{aligned} \quad (57)$$

Calculating the upper right derivatives of $V_1(t)$ along the solution of (7) leads to

$$\begin{aligned} D^+V_1(t) &= \operatorname{sgn}[x(t)-x^*(t)] \\ &\quad\times\left(p(t)\left(\exp\{-\alpha(t)x(t)\}-\exp\{-\alpha(t)x^*(t)\}\right)\right. \\ &\quad\quad\left.+c(t)\left(\mu^*(t)-\mu(t)\right)\right) \\ &= \operatorname{sgn}[x(t)-x^*(t)] \\ &\quad\times\left(p(t)\left(-\alpha(t)\exp\{-\xi(t)\}\left(x(t)-x^*(t)\right)\right)\right. \\ &\quad\quad\left.+c(t)\left(\mu^*(t)-\mu(t)\right)\right) \\ &\le -p(t)\alpha(t)\left(\exp\{-\xi(t)\}\left|x(t)-x^*(t)\right|\right) \\ &\quad+c(t)\left|\mu(t)-\mu^*(t)\right|, \end{aligned} \tag{58}$$

where we used the elementary mean value theorem of differential calculus and $\xi(t)$ lies between $\alpha(t)x(t)$ and $\alpha(t)x^*(t)$. Then, for $t \ge T$, we have

$$\alpha^l m_1 \le \xi(t) \le \alpha^u M_1. \tag{59}$$

Hence, by (58), we can have

$$\begin{aligned} D^+V_1(t) &\le -p^l\alpha^l\exp\left(\alpha^u M_1\right)\left|x(t)-x^*(t)\right| \\ &\quad+c^u\left|\mu(t)-\mu^*(t)\right|. \end{aligned} \tag{60}$$

Calculating the upper right derivatives of $V_2(t)$ along the solution of (7), one has

$$\begin{aligned} D^+V_2(t) &= \operatorname{sgn}[\mu(t)-\mu^*(t)] \\ &\quad\times\left(-a(t)\left(\mu(t)-\mu^*(t)\right)\right. \\ &\quad\quad\left.+b(t)\left(x(t-\tau)-x^*(t-\tau)\right)\right) \\ &\quad+b^u\left(\left|x(t)-x^*(t)\right|-\left|x(t-\tau)-x^*(t-\tau)\right|\right) \\ &\le -a^l\left|\mu(t)-\mu^*(t)\right|+b^u\left|x(t)-x^*(t)\right|. \end{aligned} \tag{61}$$

According to (58), (66), and condition (H_2), we can obtain

$$\begin{aligned} D^+V(t) &\le \left(b^u-p^l\alpha^l\exp\left(\alpha^u M_1\right)\right)\left|x(t)-x^*(t)\right| \\ &\quad+\left(c^u-a^l\right)\left|\mu(t)-\mu^*(t)\right| \\ &< h\left[\left|\mu(t)-\mu^*(t)\right|+\left|x(t)-x^*(t)\right|\right]. \end{aligned} \tag{62}$$

Integrating both sides of (62) from T to t leads to

$$\begin{aligned} V(t)+h\int_T^t\left[\left|\mu(s)-\mu^*(s)\right|+\left|x(s)-x^*(s)\right|\right]ds \\ < V(T) < +\infty, \quad t \ge T. \end{aligned} \tag{63}$$

Then,

$$\int_T^t\left[\left|\mu(s)-\mu^*(s)\right|+\left|x(s)-x^*(s)\right|\right]ds < \frac{V(T)}{h} < +\infty, \quad t \ge T. \tag{64}$$

Hence, $|\mu(t)-\mu^*(t)|+|x(t)-x^*(t)| \in L^1([T,+\infty))$. By system (7) and Theorem 5, we get $\mu(t)$, $\mu^*(t)$, $x(t)$, $x^*(t)$, and their derivatives are bounded on $[T,+\infty)$, which implies that $|\mu(t)-\mu^*(t)|+|x(t)-x^*(t)|$ is uniformly continuous on $[T,+\infty)$. By Lemma 11, we obtain

$$\lim_{t\to+\infty}\left|x(t)-x^*(t)\right| = 0, \qquad \lim_{t\to+\infty}\left|\mu(t)-\mu^*(t)\right| = 0. \tag{65}$$

The proof of Theorem 12 is complete. □

Theorem 13. *Suppose all conditions of Theorem 12 hold; then there exists a unique almost periodic solution of systems* (7) *and* (8).

Proof. According to Lemma 10, there exists a bounded positive solution $u(t) = (u_1(t), u_2(t))^T$ of (7) with initial condition (8). Then there exists a sequence $\{t_k'\}$, $\{t_k'\} \to \infty$ as $k \to \infty$, such that $(u_1(t+t_k'), u_2(t+t_k'))^T$ is a solution of the following system:

$$\begin{aligned} \dot{x}(t) &= x(t)\left(-\delta\left(t+t_k'\right)+p\left(t+t_k'\right)\exp\left\{-\alpha\left(t+t_k'\right)x(t)\right\}\right. \\ &\quad\left.-c\left(t+t_k'\right)\mu(t)\right), \\ \dot{\mu}(t) &= -a\left(t+t_k'\right)\mu(t)+b\left(t+t_k'\right)x(t-\tau). \end{aligned} \tag{66}$$

According to Theorem 5 and the fact that $\delta(t)$, $p(t)$, $\alpha(t)$, $c(t)$, $a(t)$, $b(t)$ are all continuous, positive almost periodic functions, we know that both $\{u_i(t+t_k')\}$ $(i=1,2)$ and its derivative function $\{\dot{u}(t+t_k')\}$ $(i=1,2)$ are uniformly bounded; thus, $\{u_i(t+t_k')\}$ $(i=1,2)$ are uniformly bounded and equi-continuous. By Ascoli's theorem, there exists a uniformly convergent subsequence $\{u_i(t+t_k)\} \subseteq \{u_i(t+t_k')\}$ such that for any $\varepsilon > 0$, there exists a $K(\varepsilon) > 0$ with the property that if $m, k \ge K(\varepsilon)$, then

$$\left|u_i(t+t_m)-u_i(t+t_k)\right| < \varepsilon, \quad i = 1,2. \tag{67}$$

That is to say, $u_i(t)$ $(i=1,2)$ are asymptotically almost periodic functions. Hence there exists two almost periodic functions $r_i(t+t_k)$ $(i=1,2)$ and two continuous functions $s_i(t+t_k)$ $(i=1,2)$ such that

$$u_i(t+t_k) = r_i(t+t_k)+s_i(t+t_k), \quad i = 1,2, \tag{68}$$

where

$$\lim_{k\to+\infty} r_i\left(t+t_k\right)=r_i\left(t\right),\qquad \lim_{k\to+\infty} s_i\left(t+t_k\right)=0,\qquad i=1,2, \tag{69}$$

$r_i(t)$ $(i=1,2)$ are also almost periodic functions.

Therefore,

$$\lim_{k\to+\infty} u_i\left(t+t_k\right)=r_i\left(t\right),\qquad i=1,2. \tag{70}$$

On the other hand,

$$\begin{aligned}\lim_{k\to+\infty}\dot{u}_i\left(t+t_k\right)&=\lim_{k\to+\infty}\lim_{h\to 0}\frac{u_i\left(t+t_k+h\right)-u_i\left(t+t_k\right)}{h}\\&=\lim_{h\to 0}\lim_{k\to+\infty}\frac{u_i\left(t+t_k+h\right)-u_i\left(t+t_k\right)}{h}\\&=\lim_{h\to 0}\frac{r_i\left(t+h\right)-r_i\left(t\right)}{h}.\end{aligned} \tag{71}$$

So $\dot{r}_i(t)$ $(i=1,2)$ exist. Moreover,

$$\begin{aligned}\dot{r}_1\left(t\right)&=\lim_{k\to+\infty}\dot{u}_1\left(t+t_k\right)\\&=\lim_{k\to+\infty}\{u_1\left(t+t_k\right)\\&\quad\times\left(-\delta\left(t+t_k\right)-c\left(t+t_k\right)u_2\left(t+t_k\right)+p\left(t+t_k\right)\right.\\&\quad\left.\times\exp\left\{-\alpha\left(t+t_k\right)u_1\left(t+t_k\right)\right\}\right)\}\\&=r_1\left(t\right)\left(-\delta\left(t\right)+p\left(t\right)\exp\left\{-\alpha\left(t\right)r_1\left(t\right)\right\}-c\left(t\right)r_2\left(t\right)\right),\\\dot{r}_2\left(t\right)&=\lim_{k\to+\infty}\dot{u}_2\left(t+t_k\right)\\&=\lim_{k\to+\infty}\{-a\left(t+t_k\right)r_2\left(t+t_k\right)\\&\qquad+b\left(t+t_k\right)r_1\left(t+t_k-\tau\right)\}\\&=-a\left(t\right)r_2\left(t\right)+b\left(t\right)r_1\left(t-\tau\right).\end{aligned} \tag{72}$$

These show that $(r_1(t),r_2(t))^T$ satisfied system (7). Hence, $(r_1(t),r_2(t))^T$ is a positive almost periodic solution of (7). Then, it follows from Theorem 12 that system (7) has a unique positive almost periodic solution. The proof is completed. □

Without the feedback terms, that is $a(t)=0$, $b(t)=0$, $c(t)=0$, and (7) becomes the following equation:

$$\dot{x}\left(t\right)=x\left(t\right)\left(-\delta\left(t\right)+p\left(t\right)\exp\left\{-\alpha\left(t\right)x\left(t\right)\right\}\right). \tag{73}$$

Equation (73) with periodic coefficients has been studied by Li and Fan [7] and Saker and Agarwal [8] with $\alpha(t)=a$. Since the periodic case is a special case of almost periodic, hence, as a direct corollary of Theorem 13, we have the following.

Corollary 14. *Suppose $\delta(t)$, $p(t)$, $\alpha(t)$, $c(t)$, $a(t)$, $b(t)$ are all continuous positive ω-periodic functions and $p(t)>\delta(t)$ for $t\in[0,\omega]$; then* (73) *has a unique globally attractive ω-periodic positive solution.*

Remark 15. Li and Fan in [7] show that (73) has a unique globally attractive ω-periodic positive solution if $p(t)>\delta(t)$ for $t\in[0,\omega]$, which is the same as Corollary 14. Thus, Theorem 13 supplements and generalizes results in [7, 8].

4. Examples and Numeric Simulations

Now we give several examples together with their numeric simulations to show the feasibility of our main results.

Example 16. Consider the following example:

$$\begin{aligned}\dot{x}\left(t\right)&=x\left(t\right)\left(-5-\sin\left(\sqrt{5}t\right)\right.\\&\qquad+10\exp\left\{-\left(30+\cos\left(\sqrt{11}t\right)\right)x\left(t\right)\right\}\\&\qquad\left.-\left(1+0.5\sin\left(\sqrt{7}t\right)\right)\mu\left(t\right)\right),\\\dot{\mu}\left(t\right)&=-\left(2.5+0.5\cos\left(\sqrt{7}t\right)\right)\mu\left(t\right)\\&\qquad+\left(5.8+0.2\sin\left(\sqrt{3}t\right)\right)x\left(t-2\right).\end{aligned} \tag{74}$$

In this case, corresponding to system (7), we have $\delta(t)=5+\sin(\sqrt{5}t)$, $p(t)=10$, $\alpha(t)=30+\cos(\sqrt{11}t)$, $a(t)=2.5+0.5\cos(\sqrt{7}t)$, $b(t)=5.8+0.2\sin(\sqrt{3}t)$, $c(t)=1+0.5\sin(\sqrt{7}t)$, $\tau=2$. According to the proof of Lemmas 3 and 4, one has

$$\begin{aligned}M_1&=\frac{3p^u}{2\delta^l\alpha^l e}=0.04757,\qquad M_2=\frac{3b^uM_1}{2a^l}=0.214066,\\m_1&=\frac{p^l-\delta^u-c^uM_2}{2p^l\alpha^u}=0.005934,\\m_2&=\frac{b^lm_1}{2a^u}=0.0055384.\end{aligned} \tag{75}$$

Hence,

$$\begin{gathered}p\left(t\right)>\delta\left(t\right),\qquad p^l\alpha^l\exp\left(\alpha^uM_1\right)-b^u\cong 1261.193896>0,\\a^l-c^u=0.5>0.\end{gathered} \tag{76}$$

Thus, all the conditions of Theorem 13 are satisfied, and so, there exists a unique almost periodic solution of systems (74). Figure 1 shows this property.

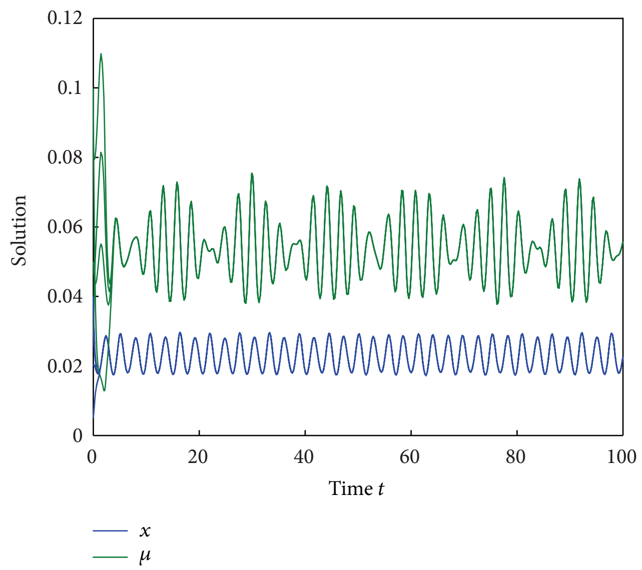

FIGURE 1: Dynamics of the $x(t)$) and $\mu(t)$ of system (74) with the initial values $(x(0), \mu(0))^T = (0.02, 0.05)^T, (0.03, 0.03)^T, (0.04, 0.08)^T, (0.005, 0.1)^T$; here $t \in [0, 100]$.

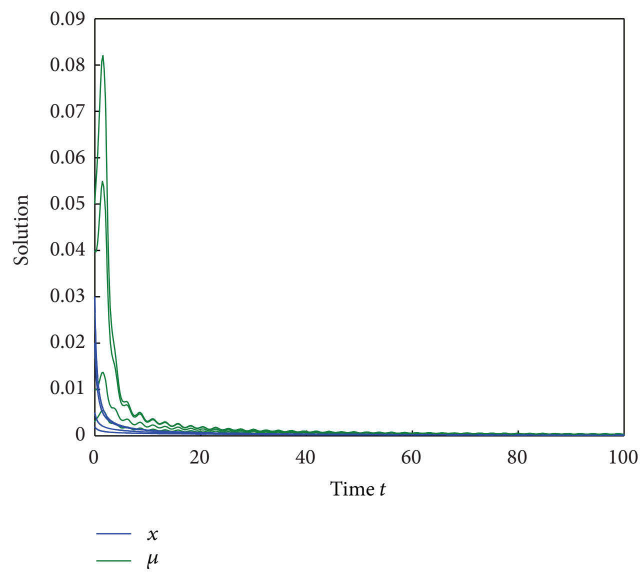

FIGURE 2: The dynamic behavior of system (77) with initial condition $(x(0), \mu(0))^T = (0.03, 0.05)^T, (0.02, 0.04)^T, (0.005, 0.01)^T, (0.002, 0.003)^T$; here $t \in [0, 100]$.

Example 17. Consider the following example:

$$\begin{aligned} \dot{x}(t) = x(t) \Bigg(&-2 - \frac{1}{(t+1)^2} \\ &+ 2\exp\left\{-\left(30 + \cos\left(\sqrt{11}t\right)\right) x(t)\right\} \\ &- \left(1 + 0.5\sin\left(\sqrt{7}t\right)\right)\mu(t) \Bigg), \\ \dot{\mu}(t) = &-\left(2.5 + 0.5\cos\left(\sqrt{7}t\right)\right)\mu(t) \\ &+ \left(5.8 + 0.2\sin\left(\sqrt{3}t\right)\right) x(t-2). \end{aligned} \tag{77}$$

In this case, we have

$$p(t) < \delta(t). \tag{78}$$

Hence, By Theorem 7, we know that any positive solution of system (77) satisfies $\lim_{t\to+\infty} x(t) = 0$, $\lim_{t\to+\infty} \mu(t) = 0$. Numerical simulation also confirms our result (see Figure 2).

5. Conclusion

In this paper, we consider a Nicholson's blowflies model with feedback control and time delay. It is shown that feedback control variable and time delay have no influence on the permanence and extinction of the system. Also, by constructing a suitable Lyapunov functional, a set of sufficient conditions which ensure the existence of a unique globally attractive positive almost periodic solution of the system is established. Moreover, compared with the main result of the relative discrete model (see [9]), we can see that the continuous and discrete models have similar results on permanence and the extinction of the Nicholson's blowflies model with feedback control and time delay. At the end of this paper, two examples together with their numerical simulations show the verification of our main results. Our results supplement and generalize the results in [7, 8].

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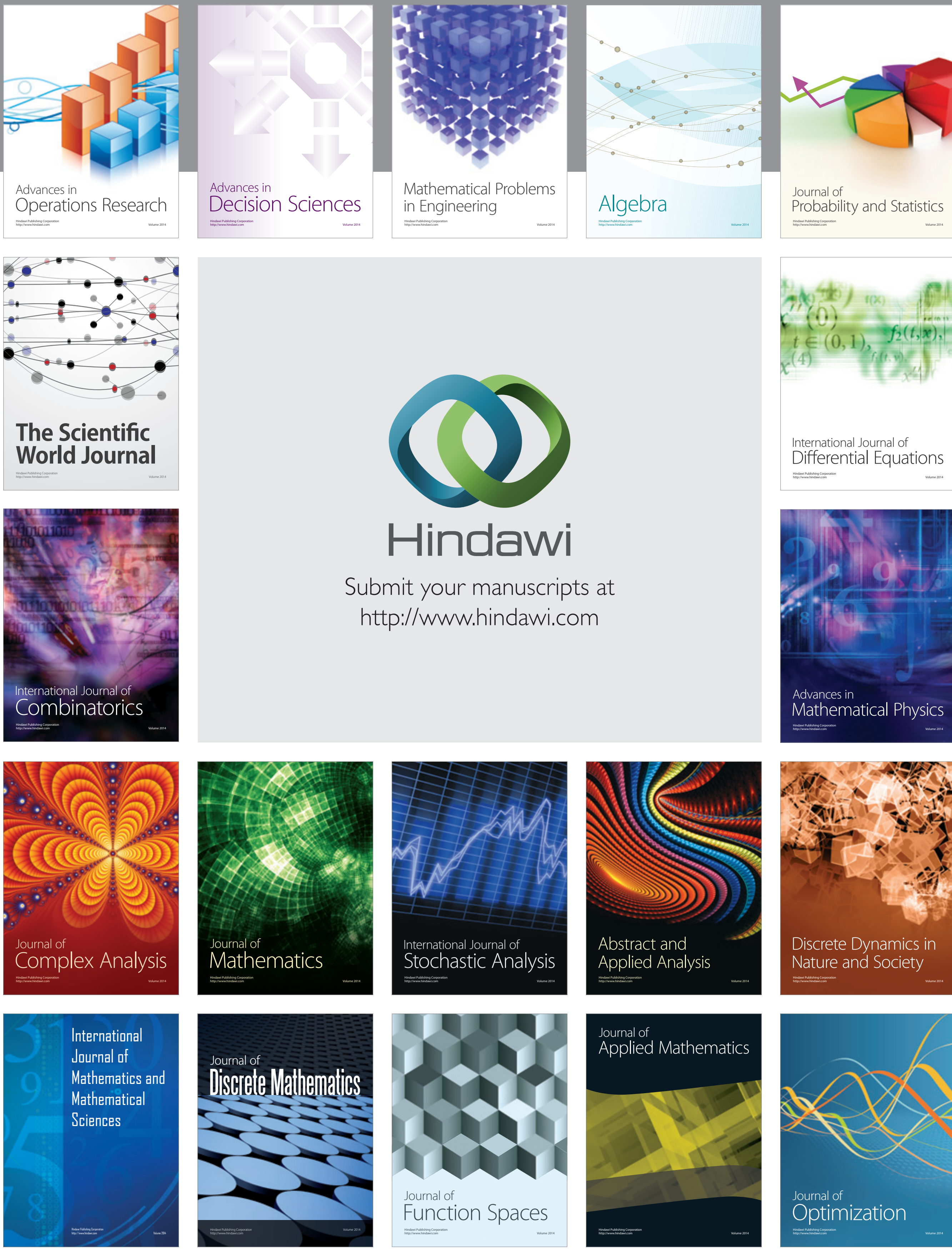
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