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Research Article

Pricing Decisions of a Two-Echelon Supply Chain in Fuzzy Environment

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Pricing decisions of a two-echelon supply chain with one manufacturer and duopolistic retailers in fuzzy environment are considered in this paper. The manufacturer produces a product and sells it to the two retailers, who in turn retail it to end customers. The fuzziness is associated with the customers' demand and the manufacturing cost. The purpose of this paper is to analyze the effect of two retailers' different pricing strategies on the optimal pricing decisions of the manufacturer and the two retailers themselves in MS Game scenario. As a reference model, the centralized decision scenario is also considered. The closed-form optimal pricing decisions of the manufacturer and the two retailers are derived in the above decision scenarios. Some insights into how pricing decisions vary with decision scenarios and the two retailers' pricing strategies in fuzzy environment are also investigated, which can serve as the basis for empirical study in the future.

1. Introduction

There is abundant literature on the pricing/ordering policies for two-echelon supply chain management. Most of them focused on the two-echelon supply chain with one manufacturer/supplier and one retailer/buyer and adopted the following assumption on the channel structure: the manufacturer/supplier wholesales a product to the retailer/buyer who in turn retails it to end consumers. The retail market demand varies with the retail price according to a deterministic/stochastic demand function that is assumed to be known to both the manufacturer and the retailer, and the costs incurred in the manufacturing and inventory process are positive constant numbers. Moreover, a most common gaming assumption on the pricing/ordering decision process is that the manufacturer is a Stackelberg leader and the retailer is a Stackelberg follower (hereafter "MS Game") in the existing two-echelon supply chain literature. For example, A. H. L. Lau and H. S. Lau [1] studied the effects of different demand curves on the optimal solution of a two-echelon system in the manufacturer-Stackelberg process. They found out that under a downward-sloping price-versus-demand relationship the manufacturer's profit is the double of the retailer's. Subsequently, Lau et al. [2] considered a two-echelon system with one manufacturer and one retailer. They presented a procedure for the dominant manufacturer to design a profit maximizing volume-discount scheme with stochastic and asymmetric demand information by modeling this supply chain as a manufacturer-Stackelberg game. Zhao et al. [3] studied the pricing problem of substitutable products in a supply chain with one manufacturer and two competitive retailers.

In fact, in order to make effective supply chain management, uncertainties that happen in the real world cannot be ignored. Those uncertainties are usually associated with product supply, manufacturing cost, customer demand, and so on. The quantitative demand forecasts based on manager's judgements, intuitions, and experience seem to be more appropriate, and the fuzzy theory rather than probability theory should be applied to model this kind of uncertainties [4]. Zadeh [5] initialized the concept of a fuzzy set via membership function. From then on, many researchers such as Nahmias [6], Kaufmann and Gupta [7], Liu [8], and B. Liu and Y. K. Liu [9] made great contributions to this field. Fuzzy theory provides a reasonable way to deal with possibility and linguistic expressions (i.e., decision maker's judgements; e.g., manufacturing cost may be expressed as "low cost" or "high cost" to make rough estimates, and market base can be expressed as "large market base" or "small market base" to make rough estimates, etc.).

Many researchers have already adopted fuzzy theory to depict uncertainties in the supply chain model. Zhao et al. [3] considered the pricing problem of substitutable products in a fuzzy supply chain by using game theory in this paper. Xie et al. [10] developed a new two-level coordination strategy that aims to improve the overall supply chain performance through hierarchical inventories control and by introducing a coordination function. They supposed that the supply chain operates under uncertainty in customer demand, which is described by imprecise terms and modelled by fuzzy sets.

This paper extends the current model related to two-echelon supply chain pricing issue from two aspects: one is considering fuzziness associated with customer's demands and the manufacturing cost; the other is analyzing the effect of the two retailers' different pricing strategies (e.g., Bertrand, Cooperation and Stackelberg) on the optimal pricing decisions of the manufacturer and the duopolistic retailers in MS Game scenario. First, as a benchmark model, one centralized pricing model (namely, assume that the manufacturer and the duopolistic retailers behave as part of a unified system) is established. Second, based on the two retailers' different pricing strategies, three decentralized pricing models are constructed in fuzzy environment (e.g., the MSB model where the two retailers implement the Bertrand competition, the MSC model where the two retailers implement the cooperation strategy, and the MSS model where the two retailers implement the Stackelberg competition) and the effect of the two retailers' different pricing strategies on the pricing decisions of the manufacturer and the two retailers is considered. Third, the closed-form solutions for these models are provided. Finally, we provide numerical examples to show the difference among each firm's optimal pricing decisions, the difference among each firm's maximum expected profits, and the variation of each firm's optimal pricing strategy and maximum expected profit with the two retailers' pricing strategies and these decision scenarios in fuzzy environment.

The rest of the paper is organized as follows. Section 2 presents preliminaries of fuzzy theory for this paper. Section 3 gives the problem description and notations, and Section 4 details our key analytical results. Numerical studies are given in Section 5. Finally, some concluding comments are presented in Section 6.

2. Preliminaries

A possibility space is defined as a triplet $(\Theta, \mathscr{P}(\Theta), \mathrm{Pos})$, where Θ is a nonempty set, $\mathscr{P}(\Theta)$ is the power set of Θ, and Pos is a possibility measure. Each element in $\mathscr{P}(\Theta)$ is called an event. For each event A, $\mathrm{Pos}\{A\}$ indicates the possibility that A will occur. Nahmias [6] and Liu [11] gave the following four axioms.

Axiom 1. $\mathrm{Pos}\{\Theta\} = 1$.

Axiom 2. $\mathrm{Pos}\{\phi\} = 0$, where ϕ denotes the empty set.

Axiom 3. $\mathrm{Pos}\{\bigcup_{i=1}^{m} A_i\} = \sup_{1\le i\le m} \mathrm{Pos}\{A_i\}$ for any collection A_i in $\mathscr{P}(\Theta)$.

Axiom 4. Let Θ_i be a nonempty set, on which Pos_i is the possibility measure satisfying the above three axioms, $i = 1, 2, \ldots, n$, and $\Theta = \prod_{i=1}^{n} \Theta_i$, then

$$\mathrm{Pos}(A) = \sup_{(\theta_1,\theta_2,\ldots,\theta_n)\in A} \mathrm{Pos}_1(\theta_1) \wedge \mathrm{Pos}_2(\theta_2) \wedge \cdots \wedge \mathrm{Pos}_n(\theta_n), \quad (1)$$

for each $A \in \mathscr{P}(\Theta)$. In that case we write $\mathrm{Pos} = \wedge_{i=1}^{n} \mathrm{Pos}_i$.

Lemma 1 (see [12]). *Suppose that* $(\Theta_i, \mathscr{P}(\Theta_i), \mathrm{Pos}_i), i = 1, 2, \cdots, n$ *is a possibility space. By Axiom 4,* $(\prod_{i=1}^{n}\Theta_i, \mathscr{P}(\prod_{i=1}^{n}\Theta_i), \wedge_{i=1}^{n} \mathrm{Pos}_i)$ *is also a possibility space, which is called the product possibility space.*

Definition 2 (see [6]). A fuzzy variable is defined as a function from the possibility space $(\Theta, \mathscr{P}(\Theta), \mathrm{Pos})$ to the set of real numbers.

Definition 3 (see [12]). A fuzzy variable ξ is said to be nonnegative (or positive) if $\mathrm{Pos}\{\xi < 0\} = 0$ (or $\mathrm{Pos}\{\xi \le 0\} = 0$).

Definition 4 (see [12]). Let $f : R^n \to R$ be a function and let ξ_i be a fuzzy variable defined on the possibility space $(\Theta_i, \mathscr{P}(\Theta_i), \mathrm{Pos}_i), i = 1, 2, \ldots, n$, respectively. Then $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ is a fuzzy variable defined on the product possibility space $(\prod_{i=1}^{n}\Theta_i, \mathscr{P}(\prod_{i=1}^{n}\Theta_i), \wedge_{i=1}^{n} \mathrm{Pos}_i)$ as $\xi(\theta_1, \theta_2, \ldots, \theta_n) = f(\xi_1(\theta_1), \xi_2(\theta_2), \ldots, \xi_n(\theta_n))$ for any $(\theta_1, \theta_2, \ldots, \theta_n) \in \prod_{i=1}^{n}\Theta_i$.

The independence of fuzzy variables was discussed by several researchers, such as Zadeh [13] and Nahmias [6].

Definition 5. The fuzzy variables $\xi_1, \xi_2, \ldots, \xi_n$ are independent if for any sets $\mathscr{B}_1, \mathscr{B}_2, \ldots, \mathscr{B}_n$ of R,

$$\mathrm{Pos}\{\xi_i \in \mathscr{B}_i, i = 1, 2, \ldots, n\} = \min_{1\le i\le n} \mathrm{Pos}\{\xi_i \in \mathscr{B}_i\}. \quad (2)$$

Lemma 6 (see [11]). *Let* ξ_i *be independent fuzzy variable, and* $f_i : R \to R$ *function,* $i = 1, 2, \ldots, m$, *then* $f_1(\xi_1), f_2(\xi_2), \ldots, f_m(\xi_m)$ *are independent fuzzy variables.*

Definition 7 (see [12]). Let ξ be a fuzzy variable on the possibility space $(\Theta, \mathscr{P}(\Theta), \mathrm{Pos})$, and $\alpha \in (0, 1]$, then

$$\xi_\alpha^L = \inf\{r \mid \mathrm{Pos}\{\xi \le r\} \ge \alpha\},$$
$$\xi_\alpha^U = \sup\{r \mid \mathrm{Pos}\{\xi \ge r\} \ge \alpha\} \quad (3)$$

are called the α-pessimistic value and the α-optimistic value of ξ, respectively.

Example 8. The triangular fuzzy variable $\xi = (a_1, a_2, a_3)$ has its α-pessimistic value and α-optimistic value as

$$\xi_\alpha^L = a_2\alpha + a_1(1-\alpha), \qquad \xi_\alpha^U = a_2\alpha + a_3(1-\alpha). \quad (4)$$

Lemma 9 (see [14]). *Let ξ_i be an independent fuzzy variable defined on the possibility space $(\Theta_i, \mathscr{P}(\Theta_i), \text{Pos}_i)$ with continuous membership function, $i = 1, 2, \ldots, n$, and $f : X \subset \mathscr{R}^n \to \mathscr{R}$ a measurable function. If $f(x_1, x_2, \ldots, x_n)$ is monotonic with respect to x_i, respectively, then*

(a) $f^U_\alpha(\xi) = f(\xi^V_{1\alpha}, \xi^V_{2\alpha}, \ldots, \xi^V_{n\alpha})$, *where* $\xi^V_{i\alpha} = \xi^U_{i\alpha}$, *if* $f(x_1, x_2, \ldots, x_n)$ *is nondecreasing with respect to* x_i; $\xi^V_{i\alpha} = \xi^L_{i\alpha}$, *otherwise;*

(b) $f^L_\alpha(\xi) = f(\xi^{\overline{V}}_{1\alpha}, \xi^{\overline{V}}_{2\alpha}, \ldots, \xi^{\overline{V}}_{n\alpha})$, *where* $\xi^{\overline{V}}_{i\alpha} = \xi^L_{i\alpha}$, if $f(x_1, x_2, \ldots, x_n)$ *is nondecreasing with respect to* x_i; $\xi^{\overline{V}}_{i\alpha} = \xi^U_{i\alpha}$, *otherwise,*

where $f^U_\alpha(\xi)$ *and* $f^L_\alpha(\xi)$ *denote the* α*-optimistic value and the* α*-pessimistic value of fuzzy variable* $f(\xi)$*, respectively.*

Definition 10 (see [9]). Let $(\Theta, \mathscr{P}(\Theta), \text{Pos})$ be a possibility space and A a set in $\mathscr{P}(\Theta)$. The credibility measure of A is defined as

$$\text{Cr}\{A\} = \frac{1}{2}\left(1 + \text{Pos}\{A\} - \text{Pos}\{A^c\}\right), \tag{5}$$

where A^c denotes the complement of A.

Definition 11 (see [9]). Let ξ be a fuzzy variable; the expected value of ξ is defined as

$$E[\xi] = \int_0^{+\infty} \text{Cr}\{\xi \geq x\}\,dx - \int_{-\infty}^0 \text{Cr}\{\xi \leq x\}\,dx, \tag{6}$$

provided that at least one of the two integrals is finite.

Example 12. The triangular fuzzy variable $\xi = (a_1, a_2, a_3)$ has an expected value

$$E[\xi] = \frac{a_1 + 2a_2 + a_3}{4}. \tag{7}$$

Definition 13 (see [9]). Let f be a function on $R \to R$ and let ξ be a fuzzy variable. Then the expected value $E[f(\xi)]$ is defined as

$$E[f(\xi)] = \int_0^{+\infty} \text{Cr}\{f(\xi) \geq x\}\,dx - \int_{-\infty}^0 \text{Cr}\{f(\xi) \leq x\}\,dx, \tag{8}$$

provided that at least one of the two integrals is finite.

Lemma 14 (see [15]). *Let ξ be a fuzzy variable with finite expected value. Then*

$$E[\xi] = \frac{1}{2}\int_0^1 \left(\xi^L_\alpha + \xi^U_\alpha\right) d\alpha. \tag{9}$$

Lemma 15 (see [15]). *Let ξ and η be independent fuzzy variables with finite expected values. Then for any numbers a and b, we have*

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta]. \tag{10}$$

3. Problem Description

Consider a two-echelon supply chain with one monopolistic manufacturer and two duopolistic retailers (retailer 1 and retailer 2) in fuzzy environment. The monopolistic manufacturer manufactures products and sells them to the duopolistic retailers, who in turn retail them to end customers. The manufacturer produces products with unit manufacturing cost $\tilde{c}$, which is a fuzzy variable, and wholesales them to the retailers with unit wholesale price w, respectively. The retailer i then sells products to end consumers with unit retail price $p_i, i = 1, 2$. The manufacturer and retailers must make their pricing decisions in order to achieve the maximum expected profits and behave as if they have perfect information of the demand and the cost structures of other channel members.

Similar to McGuire and Staelin [16], we assume that the demand for each retailer's product is sensitive to the retail prices of the duopolistic retailers, which uses a set of basic characteristics of the type of demand of each product, for example, downward sloping in its own price, and increases with respect to the competitor's price. And we assume that all activities occur within a single period. Specifically, the demand faced by the retailer i can be expressed as

$$D_i\left(p_i, p_j\right) = \tilde{a}_i - \tilde{\beta} p_i + \tilde{\gamma} p_j, \quad i = 1, 2,\ j = 3 - i, \tag{11}$$

where $\tilde{a}_i$, $\tilde{\beta}$, and $\tilde{\gamma}$ are nonnegative fuzzy variables. $\tilde{a}_i$ denotes the primary demand faced by the retailer i $(i = 1, 2)$, $\tilde{\beta}$ denotes the measure of the responsiveness of each retailer's market demand to the price charged by herself, and $\tilde{\gamma}$ denotes the measure of the responsiveness of each retailer's market demand to her competitor's price. Here we assume that the fuzzy variables $\tilde{\beta}$ and $\tilde{\gamma}$ satisfy $E[\tilde{\beta}] > E[\tilde{\gamma}]$, which means that the expected demand for a retailer's product is more sensitive to the change in its own price than to the change in the price of the other competitor's product. This assumption is reasonable in reality.

In our model, the manufacturer can influence the market demand by setting his wholesale price, and the retailers can also influence the market demands by making their retail prices, respectively. We assume that the chain members are independent, risk neutral, and profit maximizing. The chain members choose their decisions sequentially in a manufacturer-Stackelberg game (namely, the manufacturer acts as the Stackelberg leader and the duopolistic retailers act as the followers), and they have complete information about the other members. Moreover, the logistic cost components of the chain members (i.e., carrying cost and inventory cost, etc.) are not considered in our paper for analytical convenience. As explained in A. H. L. Lau and H. S. Lau [1], by removing the confounding effect of the logistic cost components, their profit functions are more effective to reveal the effects of different game procedures.

From the above problem description, the manufacturer's objective is to maximize his expected profit $E[\pi_m(w)]$

(For convenience, it is $E[\pi_m]$ for short sometimes in this paper), which can be described as

$$\max_{w} E\left[\pi_m(w)\right] = \max_{w} E\left[\sum_{i=1}^{2}(w-\tilde{c})\left(\tilde{a}_i - \tilde{\beta}p_i + \tilde{\gamma}p_j\right)\right]. \quad (12)$$

The objectives of the retailers are to maximize their respective expected profits $E[\pi_{r1}(p_1, p_2)]$ and $E[\pi_{r2}(p_1, p_2)]$ (For convenience, abbreviated to $E[\pi_{r1}]$ and $E[\pi_{r2}]$ for short sometimes in this paper), which can be described as

$$\max_{p_1} E\left[\pi_{r1}(p_1, p_2)\right] = \max_{p_1} E\left[(p_1 - w)\left(\tilde{a}_1 - \tilde{\beta}p_1 + \tilde{\gamma}p_2\right)\right], \quad (13)$$

$$\max_{p_2} E\left[\pi_{r2}(p_1, p_2)\right] = \max_{p_2} E\left[(p_2 - w)\left(\tilde{a}_2 - \tilde{\beta}p_2 + \tilde{\gamma}p_1\right)\right]. \quad (14)$$

4. Analytical Results

4.1. Centralized Pricing Model (CD Model). As a benchmark to evaluate channel decisions under different decision cases, we first give the centralized pricing model; namely, there is one entity that aims to optimize the whole supply chain system performance, so both the duopolistic retailers' and the manufacturer's decisions are fully coordinated in the centralized decision case. The wholesale price charged by the manufacturer is seen as inner transfer price and thus will be neglected. The total profit is determined by the production cost and retail prices.

Let π_c be the total profit of the centralized supply chain; we have

$$\pi_c = \sum_{i=1}^{2}(p_i - \tilde{c})\left(\tilde{a}_i - \tilde{\beta}p_i + \tilde{\gamma}p_j\right), \quad j = 3 - i. \quad (15)$$

To maximize the system expected profit $E[\pi_c(p_1, p_2)]$, the objective is

$$\max_{p_1,p_2} E\left[\pi_c\right] = \max_{p_1,p_2} E\left[\sum_{i=1}^{2}(p_i - \tilde{c})\left(\tilde{a}_i - \tilde{\beta}p_i + \tilde{\gamma}p_j\right)\right]. \quad (16)$$

Proposition 16. *In the CD model, the optimal retail prices p_{c1}^* and p_{c2}^* are given as*

$$p_{c1}^* = \frac{A_1 E[\tilde{\beta}] + A_2 E[\tilde{\gamma}]}{2\left(E^2[\tilde{\beta}] - E^2[\tilde{\gamma}]\right)},$$
$$p_{c2}^* = \frac{A_2 E[\tilde{\beta}] + A_1 E[\tilde{\gamma}]}{2\left(E^2[\tilde{\beta}] - E^2[\tilde{\gamma}]\right)}, \quad (17)$$

where

$$A_1 = E[\tilde{a}_1] + E[\tilde{c}\tilde{\beta}] - \frac{1}{2}\int_0^1\left(\tilde{c}_\alpha^U\tilde{\gamma}_\alpha^L + \tilde{c}_\alpha^L\tilde{\gamma}_\alpha^U\right)d\alpha,$$
$$A_2 = E[\tilde{a}_2] + E[\tilde{c}\tilde{\beta}] - \frac{1}{2}\int_0^1\left(\tilde{c}_\alpha^U\tilde{\gamma}_\alpha^L + \tilde{c}_\alpha^L\tilde{\gamma}_\alpha^U\right)d\alpha. \quad (18)$$

Proof. From (15), the expected profit $E[\pi_c]$ can be expressed as

$$\begin{aligned} E[\pi_c] = & -E[\tilde{\beta}]\left(p_1^2 + p_2^2\right) + 2E[\tilde{\gamma}]p_1p_2 \\ & + \left(E[\tilde{a}_1] + E[\tilde{c}\tilde{\beta}] - \frac{1}{2}\int_0^1\left(\tilde{c}_\alpha^U\tilde{\gamma}_\alpha^L + \tilde{c}_\alpha^L\tilde{\gamma}_\alpha^U\right)d\alpha\right)p_1 \\ & + \left(E[\tilde{a}_2] + E[\tilde{c}\tilde{\beta}] - \frac{1}{2}\int_0^1\left(\tilde{c}_\alpha^U\tilde{\gamma}_\alpha^L + \tilde{c}_\alpha^L\tilde{\gamma}_\alpha^U\right)d\alpha\right)p_2 \\ & - \frac{1}{2}\int_0^1\left(\tilde{a}_{1\alpha}^U\tilde{c}_\alpha^L + \tilde{a}_{1\alpha}^L\tilde{c}_\alpha^U\right)d\alpha \\ & - \frac{1}{2}\int_0^1\left(\tilde{a}_{2\alpha}^U\tilde{c}_\alpha^L + \tilde{a}_{2\alpha}^L\tilde{c}_\alpha^U\right)d\alpha. \end{aligned} \quad (19)$$

Then, the first-order derivatives of $E[\pi_c]$ to p_1 and p_2 are

$$\begin{aligned} \frac{\partial E[\pi_c]}{\partial p_1} = & -2E[\tilde{\beta}]p_1 + E[\tilde{a}_1] + 2E[\tilde{\gamma}]p_2 + E[\tilde{c}\tilde{\beta}] \\ & - \frac{1}{2}\int_0^1\left(\tilde{c}_\alpha^U\tilde{\gamma}_\alpha^L + \tilde{c}_\alpha^L\tilde{\gamma}_\alpha^U\right)d\alpha, \\ \frac{\partial E[\pi_c]}{\partial p_2} = & -2E[\tilde{\beta}]p_2 + E[\tilde{a}_2] + 2E[\tilde{\gamma}]p_1 \\ & + E[\tilde{c}\tilde{\beta}] - \frac{1}{2}\int_0^1\left(\tilde{c}_\alpha^U\tilde{\gamma}_\alpha^L + \tilde{c}_\alpha^L\tilde{\gamma}_\alpha^U\right)d\alpha, \end{aligned} \quad (20)$$

and the second-order derivatives are

$$\begin{aligned} \frac{\partial^2 E[\pi_c]}{\partial p_1^2} = \frac{\partial^2 E[\pi_c]}{\partial p_2^2} = -2E[\tilde{\beta}] < 0, \\ \frac{\partial^2 E[\pi_c]}{\partial p_1 \partial p_2} = \frac{\partial^2 E[\pi_c]}{\partial p_2 \partial p_1} = 2E[\tilde{\gamma}]. \end{aligned} \quad (21)$$

By (21), together with the assumption $E[\tilde{\beta}] > E[\tilde{\gamma}]$, we can get a negative definite Hessian Matrix, so the expected profit $E[\pi_c]$ is jointly concave in p_1 and p_2. Let (20) be equal to zeros; we derive the first order conditions as

$$\begin{aligned} & -2E[\tilde{\beta}]p_1 + E[\tilde{a}_1] + 2E[\tilde{\gamma}]p_2 + E[\tilde{c}\tilde{\beta}] \\ & - \frac{1}{2}\int_0^1\left(\tilde{c}_\alpha^U\tilde{\gamma}_\alpha^L + \tilde{c}_\alpha^L\tilde{\gamma}_\alpha^U\right)d\alpha = 0, \\ & -2E[\tilde{\beta}]p_2 + E[\tilde{a}_2] + 2E[\tilde{\gamma}]p_1 + E[\tilde{c}\tilde{\beta}] \\ & - \frac{1}{2}\int_0^1\left(\tilde{c}_\alpha^U\tilde{\gamma}_\alpha^L + \tilde{c}_\alpha^L\tilde{\gamma}_\alpha^U\right)d\alpha = 0. \end{aligned} \quad (22)$$

So, solving (22) simultaneously, we get the solutions (17), and then the proposition is proved. □

4.2. Pricing Models in MS Game. In this section, we assume that the manufacturer acts as the Stackelberg leader and the duopolistic retailers act as the followers. The game-theoretical approach is used to analyze the models established in the following. For this case, the manufacturer chooses the wholesale prices of the product using the response functions of both the retailers. Then, given the wholesale prices made by the manufacturer, the duopolistic retailers determine their retail prices.

4.2.1. The MSB Model. When the two retailers pursue the Bertrand solution, the manufacturer first announces the wholesale price and the two retailers observe the wholesale price and then decide the retail prices simultaneously. Then the MSB model is formulated as

$$\begin{cases} \max_{w} E\left[\pi_m\left(w, p_1^*(w), p_2^*(w)\right)\right] \\ p_1^*(w), p_2^*(w) \text{ are derived from solving} \\ \text{the following problem} \\ \begin{cases} \max_{p_1} E\left[\pi_{r_1}\left(p_1, p_2^*(w)\right)\right] \\ \max_{p_2} E\left[\pi_{r_2}\left(p_2, p_1^*(w)\right)\right]. \end{cases} \end{cases} \tag{23}$$

We first derive the retailers' Bertrand decisions as follows.

Proposition 17. *When the duopolistic retailers pursue the Bertrand solution, the optimal retail prices (denoted by* p_{msb1} *and* p_{msb2}*, resp.), given earlier decision made by the manufacturer* w*, are*

$$\begin{aligned} p_{\text{msb1}} &= \frac{2E[\tilde{a}_1]E[\tilde{\beta}] + E[\tilde{a}_2]E[\tilde{\gamma}]}{4E^2[\beta] - E^2[\gamma]} + \frac{\left(E[\tilde{\beta}]E[\tilde{\gamma}] + 2E^2[\tilde{\beta}]\right)w}{4E^2[\beta] - E^2[\gamma]}, \\ p_{\text{msb2}} &= \frac{2E[\tilde{a}_2]E[\tilde{\beta}] + E[\tilde{a}_1]E[\tilde{\gamma}]}{4E^2[\beta] - E^2[\gamma]} + \frac{\left(E[\tilde{\beta}]E[\tilde{\gamma}] + 2E^2[\tilde{\beta}]\right)w}{4E^2[\beta] - E^2[\gamma]}. \end{aligned} \tag{24}$$

Proof. Using (13) and (14), we get the expected value of π_{r1} and π_{r2} as

$$E[\pi_{r1}] = (p_1 - w)\left(E[\tilde{a}_1] - E[\tilde{\beta}]p_1 + E[\tilde{\gamma}]p_2\right), \tag{25}$$

$$E[\pi_{r2}] = (p_2 - w)\left(E[\tilde{a}_2] - E[\tilde{\beta}]p_2 + E[\tilde{\gamma}]p_1\right). \tag{26}$$

By (25), the first order derivative of $E[\pi_{r1}]$ to p_1 is

$$\frac{\partial E[\pi_{r1}]}{\partial p_1} = E[\tilde{a}_1] - 2E[\tilde{\beta}]p_1 + E[\tilde{\gamma}]p_2 + E[\tilde{\beta}]w, \tag{27}$$

and the second order derivative is given below to check for the optimality:

$$\frac{\partial^2 E[\pi_{r1}]}{\partial p_1^2} = -2E[\tilde{\beta}] < 0. \tag{28}$$

From (28), the expected profit $E[\pi_{r1}]$ is concave in p_1. Let (27) be equal to zero; we get

$$E[\tilde{a}_1] - 2E[\tilde{\beta}]p_1 + E[\tilde{\gamma}]p_2 + E[\tilde{\beta}]w = 0. \tag{29}$$

Similarly, from (26), we get

$$E[\tilde{a}_2] - 2E[\tilde{\beta}]p_2 + E[\tilde{\gamma}]p_1 + E[\tilde{\beta}]w = 0. \tag{30}$$

Therefore, solving (29) and (30) simultaneously, we get the solutions (24), and thus Proposition 17 is proved. □

Having the information about the decisions of the two retailers, the manufacturer would then use them to maximize his expected profit $E[\pi_m]$. So, we get the following result.

Proposition 18. *When the duopolistic retailers pursue the Bertrand solution, the manufacturer's optimal wholesale price (denoted by* w^*_{msb} *) is*

$$w^*_{\text{msb}} = \frac{E[\tilde{a}_1] + E[\tilde{a}_2] + 2E[\tilde{c}\tilde{\beta}] - \int_0^1 \left(\tilde{c}_\alpha^U\tilde{\gamma}_\alpha^L + \tilde{c}_\alpha^L\tilde{\gamma}_\alpha^U\right)d\alpha}{4\left(E[\tilde{\beta}] - E[\tilde{\gamma}]\right)}. \tag{31}$$

Proof. By (13), together with some manipulations, we get

$$\begin{aligned} E[\pi_m] = {} & \left(E[\tilde{a}_1] + E[\tilde{a}_2] + \left(E[\tilde{\gamma}] - E[\tilde{\beta}]\right)(p_1 + p_2)\right)w \\ & + \left(E[\tilde{c}\tilde{\beta}] - \frac{1}{2}\int_0^1 \left(\tilde{c}_\alpha^U\tilde{\gamma}_\alpha^L + \tilde{c}_\alpha^L\tilde{\gamma}_\alpha^U\right)d\alpha\right)(p_1 + p_2) \\ & - \frac{1}{2}\int_0^1 \left(\tilde{a}_{1\alpha}^U\tilde{c}_\alpha^L + \tilde{a}_{1\alpha}^L\tilde{c}_\alpha^U\right)d\alpha \\ & - \frac{1}{2}\int_0^1 \left(\tilde{a}_{2\alpha}^U\tilde{c}_\alpha^L + \tilde{a}_{2\alpha}^L\tilde{c}_\alpha^U\right)d\alpha. \end{aligned} \tag{32}$$

Then, from (24) and (32), the first order derivative of $E[\pi_m]$ to w is

$$\begin{aligned} \frac{\partial E[\pi_m]}{\partial w} = {} & E[\tilde{a}_1] + E[\tilde{a}_2] \\ & + \left(E[\tilde{c}\tilde{\beta}] - \frac{1}{2}\int_0^1 \left(\tilde{c}_\alpha^U\tilde{\gamma}_\alpha^L + \tilde{c}_\alpha^L\tilde{\gamma}_\alpha^U\right)d\alpha\right) \\ & \times \frac{2E[\tilde{\beta}]}{2E[\tilde{\beta}] - E[\tilde{\gamma}]} \\ & + \frac{\left(E[\tilde{\gamma}] - E[\tilde{\beta}]\right)\left(E[\tilde{a}_1] + E[\tilde{a}_2]\right)}{2E[\tilde{\beta}] - E[\tilde{\gamma}]} \\ & + \frac{4E[\tilde{\beta}]\left(E[\tilde{\gamma}] - E[\tilde{\beta}]\right)}{2E[\tilde{\beta}] - E[\tilde{\gamma}]}w. \end{aligned} \tag{33}$$

Furthermore, its second order derivative satisfies

$$\frac{\partial^2 E[\pi_m]}{\partial w^2} = \frac{4E[\tilde{\beta}]\left(E[\tilde{\gamma}] - E[\tilde{\beta}]\right)}{2E[\tilde{\beta}] - E[\tilde{\gamma}]} < 0. \tag{34}$$

So, $E[\pi_m]$ is concave in w. Therefore, let (33) be equal to zero; the proposition is proved. □

Proposition 19. *When the duopolistic retailers pursue the Bertrand solution, their optimal retail prices (denoted by p^*_{msb1} and p^*_{msb2}, resp.) are given as*

$$p^*_{\text{msb1}} = \frac{2E[\tilde{a}_1]E[\tilde{\beta}] + E[\tilde{a}_2]E[\tilde{\gamma}]}{4E^2[\beta] - E^2[\gamma]} + \frac{\left(E[\tilde{\beta}]E[\tilde{\gamma}] + 2E^2[\tilde{\beta}]\right)w^*_{\text{msb}}}{4E^2[\beta] - E^2[\gamma]},$$
$$p^*_{\text{msb2}} = \frac{2E[\tilde{a}_2]E[\tilde{\beta}] + E[\tilde{a}_1]E[\tilde{\gamma}]}{4E^2[\beta] - E^2[\gamma]} + \frac{\left(E[\tilde{\beta}]E[\tilde{\gamma}] + 2E^2[\tilde{\beta}]\right)w^*_{\text{msb}}}{4E^2[\beta] - E^2[\gamma]}, \tag{35}$$

where w^*_{msb} is given in Proposition 18.

Proof. By Propositions 17 and 18, we can easily see that Proposition 19 holds. □

4.2.2. The MSC Model. In this decision case where the two retailers adopt the cooperation strategy, we assume that the retailers recognize their interdependence and agree to act in union in order to maximize the total expected profit of the downstream retail market. So, the manufacturer first announces the wholesale price and the retailers observe the wholesale price and then decide their retail prices with the objective to maximize the total expected profit of the downstream retail market. Thus, the MSC model is formulated as

$$\begin{cases} \max\limits_{w} E\left[\pi_m\left(w, p_1^*(w), p_2^*(w)\right)\right] \\ p_1^*(w), p_2^*(w) \text{ are derived from solving} \\ \text{the following problem} \\ \quad \max\limits_{p_1, p_2} E\left[\pi_{r_1}(p_1, p_2) + \pi_{r_2}(p_1, p_2)\right]. \end{cases} \tag{36}$$

We first derive the retailers' decisions.

Proposition 20. *When the two retailers adopt the cooperation strategy, their optimal retail prices p_{msc1} and p_{msc2}, given earlier decision made by the manufacturer w, are*

$$p_{\text{msc1}} = \frac{E[\tilde{a}_1]E[\tilde{\beta}] + E[\tilde{a}_2]E[\tilde{\gamma}] + \left(E^2[\tilde{\beta}] - E^2[\tilde{\gamma}]\right)w}{2\left(E^2[\tilde{\beta}] - E^2[\tilde{\gamma}]\right)},$$
$$p_{\text{msc2}} = \frac{E[\tilde{a}_2]E[\tilde{\beta}] + E[\tilde{a}_1]E[\tilde{\gamma}] + \left(E^2[\tilde{\beta}] - E^2[\tilde{\gamma}]\right)w}{2\left(E^2[\tilde{\beta}] - E^2[\tilde{\gamma}]\right)}. \tag{37}$$

Proof. By (13) and (14), we have

$$E[\pi_{r1} + \pi_{r2}] = (p_1 - w)\left(E[\tilde{a}_1] - E[\tilde{\beta}]p_1 + E[\tilde{\gamma}]p_2\right) + (p_2 - w)\left(E[\tilde{a}_2] - E[\tilde{\beta}]p_2 + E[\tilde{\gamma}]p_1\right). \tag{38}$$

Then

$$\frac{\partial E[\pi_{r1} + \pi_{r2}]}{\partial p_1} = E[\tilde{a}_1] + \left(E[\tilde{\beta}] - E[\tilde{\gamma}]\right)w - 2E[\tilde{\beta}]p_1 + 2E[\tilde{\gamma}]p_2,$$
$$\frac{\partial E[\pi_{r1} + \pi_{r2}]}{\partial p_2} = E[\tilde{a}_2] + \left(E[\tilde{\beta}] - E[\tilde{\gamma}]\right)w - 2E[\tilde{\beta}]p_2 + 2E[\tilde{\gamma}]p_1, \tag{39}$$

$$\frac{\partial^2 E[\pi_{r1} + \pi_{r2}]}{\partial p_1^2} = \frac{\partial^2 E[\pi_{r1} + \pi_{r2}]}{\partial p_2^2} = -2E[\tilde{\beta}],$$
$$\frac{\partial^2 E[\pi_{r1} + \pi_{r2}]}{\partial p_1 \partial p_2} = \frac{\partial^2 E[\pi_{r1} + \pi_{r2}]}{\partial p_2 \partial p_1} = 2E[\tilde{\gamma}]. \tag{40}$$

From (40) and the assumption $E[\tilde{\beta}] > E[\tilde{\gamma}]$, its Hessian Matrix is negative definite, so the expected profit $E[\pi_{r1} + \pi_{r2}]$ is jointly concave in p_1 and p_2. Let (39) be equal to 0, respectively; we get

$$E[\tilde{a}_1] + \left(E[\tilde{\beta}] - E[\tilde{\gamma}]\right)w - 2E[\tilde{\beta}]p_1 + 2E[\tilde{\gamma}]p_2 = 0,$$
$$E[\tilde{a}_2] + \left(E[\tilde{\beta}] - E[\tilde{\gamma}]\right)w - 2E[\tilde{\beta}]p_2 + 2E[\tilde{\gamma}]p_1 = 0. \tag{41}$$

Thus, solving (41) simultaneously, we get (37), so the proposition is proved. □

Having the information about the decisions of the retailers, the manufacturer would then use them to maximize his expected profit $E[\pi_m]$. So, we get the following result.

Proposition 21. *When the two retailers adopt the cooperation strategy, the manufacturer's optimal wholesale price (denoted as w^*_{msc}) is*

$$w^*_{\text{msc}} = \frac{E[\tilde{a}_1] + E[\tilde{a}_2] + 2E[\tilde{c}\tilde{\beta}] - \int_0^1 \left(\tilde{c}^U_\alpha \tilde{\gamma}^L_\alpha + \tilde{c}^L_\alpha \tilde{\gamma}^U_\alpha\right) d\alpha}{4\left(E[\tilde{\beta}] - E[\tilde{\gamma}]\right)}. \tag{42}$$

Proof. By (32) and (37), we get

$$\frac{\partial E[\pi_m]}{\partial w} = \frac{E[\tilde{a}_1] + E[\tilde{a}_2]}{2} + E[\tilde{c}\tilde{\beta}] - \frac{1}{2}\int_0^1 \left(\tilde{c}_\alpha^U \tilde{\gamma}_\alpha^L + \tilde{c}_\alpha^L \tilde{\gamma}_\alpha^U\right) d\alpha + 2\left(E[\tilde{\gamma}] - E[\tilde{\beta}]\right) w, \quad (43)$$

$$\frac{\partial^2 E[\pi_m]}{\partial w^2} = 2\left(E[\tilde{\gamma}] - E[\tilde{\beta}]\right) < 0. \quad (44)$$

By (44), $E[\pi_m]$ is concave in w. Then, let (43) be equal to zero; we can easily get the proposition. □

Proposition 22. *When the two retailers adopt the cooperation strategy, their optimal retail prices p^*_{msc1} and p^*_{msc2} are given as follows:*

$$p^*_{\text{msc1}} = \frac{E[\tilde{a}_1] E[\tilde{\beta}] + E[\tilde{a}_2] E[\tilde{\gamma}] + \left(E^2[\tilde{\beta}] - E^2[\tilde{\gamma}]\right) w^*_{\text{col}}}{2\left(E^2[\tilde{\beta}] - E^2[\tilde{\gamma}]\right)},$$

$$p^*_{\text{msc2}} = \frac{E[\tilde{a}_2] E[\tilde{\beta}] + E[\tilde{a}_1] E[\tilde{\gamma}] + \left(E^2[\tilde{\beta}] - E^2[\tilde{\gamma}]\right) w^*_{\text{col}}}{2\left(E^2[\tilde{\beta}] - E^2[\tilde{\gamma}]\right)}, \quad (45)$$

where w^*_{msc} is given in Proposition 20.

Proof. By Propositions 20 and 21, we can easily see that Proposition 22 holds. □

4.2.3. The MSS Model. In this decision case when the duopolistic retailers play Stackelberg Game, we assume that one of the duopolistic retailers (e.g., retailer 1) acts as a Stackelberg leader and the other (i.e., retailer 2) acts as a Stackelberg follower. The manufacturer first announces the wholesale price of the product, and retailer 1 then decides the retail price to maximize her expected profit and retailer 2 finally decides the retail price when knowing both the manufacturer and retailer 1 decisions. So, we first need to derive retailer 2's decision (as the Stackelberg game's follower). The MSS model is formulated as follows:

$$\begin{cases} \max\limits_{w} E\left[\pi_m\left(w, p_1^*(w), p_2^*(w, p_1^*(w))\right)\right] \\ p_1^*(w), p_2^*(w, p_1^*(w)) \text{ are derived from solving} \\ \text{the following problem} \\ \begin{cases} \max\limits_{p_1} E\left[\pi_{r_1}\left(p_1, p_2^*(w, p_1)\right)\right] \\ p_2^*(w, p_1) \text{ is derived from solving} \\ \text{the following problem} \\ \max\limits_{p_2} E\left[\pi_{r_2}(p_1, p_2)\right]. \end{cases} \end{cases} \quad (46)$$

We first derive retailer 2's decision as follows.

Proposition 23. *When the duopolistic retailers play Stackelberg Game, retailer 2's optimal decision (denoted as p_{mss2}), given earlier decisions made by the manufacturer and retailer 1 which are w and p_1, respectively, is*

$$p_{\text{mss2}} = \frac{E[\tilde{a}_2] + E[\tilde{\gamma}] p_1 + E[\tilde{\beta}] w}{2E[\tilde{\beta}]}. \quad (47)$$

Proof. Using (26), given earlier decisions made by the manufacturer and retailer 1 which are w and p_1, respectively, we can have the first-and second order derivatives of $E[\pi_{r2}]$ to p_2 as follows:

$$\frac{\partial E[\pi_{r2}]}{\partial p_2} = E[\tilde{a}_2] - 2E[\tilde{\beta}] p_2 + E[\tilde{\gamma}] p_1 + E[\tilde{\beta}] w, \quad (48)$$

$$\frac{\partial^2 E[\pi_{r2}]}{\partial p_2^2} = -2E[\tilde{\beta}] < 0. \quad (49)$$

By (49), we know that $E[\pi_{r2}]$ is concave in p_2 for given earlier decisions made by the manufacturer and retailer 1 which are w and p_1, respectively. Therefore, equating (48) to zero and solving it, we can easily have Proposition 23. □

Proposition 24. *When the duopolistic retailers play Stackelberg Game, retailer 1's optimal decision (denoted as p_{mss1}), given earlier decision made by the manufacturer which is w, is*

$$p_{\text{mss1}} = B_2 + B_1 w, \quad (50)$$

where

$$B_1 = \frac{E[\tilde{\beta}] E[\tilde{\gamma}] - E^2[\tilde{\gamma}] + E^2[\tilde{\beta}]}{3E^2[\tilde{\beta}] - 2E^2[\tilde{\gamma}]},$$

$$B_2 = \frac{2E[\tilde{a}_1] E[\tilde{\beta}] + E[\tilde{a}_2] E[\tilde{\gamma}]}{3E^2[\tilde{\beta}] - 2E^2[\tilde{\gamma}]}. \quad (51)$$

Proof. Using (25) and (47), given earlier decision made by the manufacturer which is w, we can have the first-and second order derivatives of $E[\pi_{r1}]$ to p_1 as follows:

$$\frac{\partial E[\pi_{r1}]}{\partial p_1} = E[\tilde{a}_1] + \frac{E[\tilde{a}_2] E[\tilde{\gamma}] + \left(E[\tilde{\beta}] E[\tilde{\gamma}] - E^2[\tilde{\gamma}] + E^2[\tilde{\beta}]\right) w}{2E[\tilde{\beta}]} + \frac{2E^2[\tilde{\gamma}] - 3E^2[\tilde{\beta}]}{2E[\tilde{\beta}]} p_1, \quad (52)$$

$$\frac{\partial^2 E[\pi_{r1}]}{\partial p_1^2} = \frac{2E^2[\tilde{\gamma}] - 3E^2[\tilde{\beta}]}{2E[\tilde{\beta}]} < 0. \quad (53)$$

By (53), we know that $E[\pi_{r1}]$ is concave in p_1 for given earlier decision made by the manufacturer which is w. Therefore, equating (52) to zero and solving it, we can easily have Proposition 24. □

Proposition 25. *When the duopolistic retailers play Stackelberg Game, retailer 2's optimal decision (denoted as p_{mss2}), given earlier decisions made by the manufacturer which is w, is*

$$\begin{aligned} p_{\text{mss2}} = {} & \frac{E[\tilde{a}_2]}{2E[\beta]} + \frac{w}{2} \\ & + E[\tilde{\gamma}]\left(2E[\tilde{a}_1]E[\tilde{\beta}] + E[\tilde{a}_2]E[\tilde{\gamma}]\right. \\ & \left. + \left(E[\tilde{\beta}]E[\tilde{\gamma}] - E^2[\tilde{\gamma}] + E^2[\tilde{\beta}]\right)w\right) \\ & \times \left(2E[\beta]\left(3E^2[\beta] - 2E^2[\tilde{\gamma}]\right)\right)^{-1}. \end{aligned} \tag{54}$$

Proof. Using Propositions 23 and 24, one can easily have Proposition 25. □

Proposition 26. *When the duopolistic retailers play Stackelberg Game, the manufacturer's optimal decision is (denoted by w^*_{mss}) given as follows:*

$$\begin{aligned} w^*_{\text{mss}} = {} & \frac{E[\tilde{a}_1] + E[\tilde{a}_2] + B_3(B_4 + B_1)}{2\left(E[\tilde{\beta}] - E[\tilde{\gamma}]\right)(B_4 + B_1)} \\ & + \frac{\left(E[\tilde{\gamma}] - E[\tilde{\beta}]\right)(B_2 + B_5)}{2\left(E[\tilde{\beta}] - E[\tilde{\gamma}]\right)(B_4 + B_1)}. \end{aligned} \tag{55}$$

Proof. Using (32), (50), and (54), one can have the first-and second order derivatives of $E[\pi_m]$ to w as follows:

$$\begin{aligned} \frac{\partial E[\pi_m]}{\partial w} = {} & E[\tilde{a}_1] + E[\tilde{a}_2] + B_3(B_4 + B_1) \\ & + \left(E[\tilde{\gamma}] - E[\tilde{\beta}]\right)(B_2 + B_5) \\ & + 2\left(E[\tilde{\gamma}] - E[\tilde{\beta}]\right)(B_4 + B_1)w, \end{aligned} \tag{56}$$

where

$$\begin{aligned} B_1 &= E[\tilde{c}\tilde{\beta}] - \frac{1}{2}\int_0^1 \left(\tilde{c}^U_\alpha \tilde{\gamma}^L_\alpha + \tilde{c}^L_\alpha \tilde{\gamma}^U_\alpha\right) d\alpha, \\ B_4 &= \frac{1}{2} + \frac{E[\tilde{\gamma}]\left(E[\tilde{\beta}]E[\tilde{\gamma}] - E^2[\tilde{\gamma}] + E^2[\tilde{\beta}]\right)}{2E[\tilde{\beta}]\left(3E^2[\tilde{\beta}] - 2E^2[\tilde{\gamma}]\right)}, \\ B_5 &= \frac{E[\tilde{a}_2]}{2E[\tilde{\beta}]} + \frac{E[\tilde{\gamma}]\left(2E[\tilde{a}_1]E[\tilde{\beta}] + E[\tilde{a}_2]E[\tilde{\gamma}]\right)}{2E[\tilde{\beta}]\left(3E^2[\tilde{\beta}] - 2E^2[\tilde{\gamma}]\right)}, \\ \frac{\partial^2 E[\pi_m]}{\partial w^2} &= 2\left(E[\tilde{\gamma}] - E[\tilde{\beta}]\right)(B_4 + B_1)w. \end{aligned} \tag{57}$$

By (57), we know that $E[\pi_m]$ is concave in w. Therefore, equating (56) to zero and solving it, we can easily have Proposition 26. □

Proposition 27. *When the duopolistic retailers play Stackelberg Game, their optimal retail prices (denoted by p^*_{mss1} and p^*_{mss2}, resp.) are given as follows:*

$$\begin{aligned} p^*_{\text{mss1}} &= B_2 + B_1 w^*_{\text{msta}}, \\ p^*_{\text{mss2}} &= B_5 + B_4 w^*_{\text{msta}}, \end{aligned} \tag{58}$$

where w^*_{mss} is given as in Proposition 26.

Proof. By Propositions 24, 25, and 26, we can easily see that Proposition 27 holds. □

5. Numerical Studies

In this section, we compare analytical results obtained from the above different decision scenarios using numerical approach and study the behavior of firms facing changing fuzzy environment. Here, we assume that the fuzzy variables used in this paper are all triangular fuzzy variables which take values as follows: the manufacturing cost $\tilde{c}$ is high ($\tilde{c}$ is about 9), the market bases $\tilde{a}_1$ and $\tilde{a}_2$ are large ($\tilde{a}_1$ is about 1400 and $\tilde{a}_2$ is about 1200), and price elasticities $\tilde{\beta}$ and $\tilde{\gamma}$ are sensitive ($\tilde{\beta}$ is about 180 and $\tilde{\gamma}$ is about 130). More specifically, $\tilde{c} = (6, 9, 12)$, $\tilde{a}_1 = (1300, 1400, 1600)$, $\tilde{a}_2 = (1000, 1200, 1300)$, $\tilde{\beta} = (170, 180, 200)$, and $\tilde{\gamma} = (120, 130, 140)$. Similar to Example 8 in Preliminaries (see Section 2), the expected values of the above triangular fuzzy variables can be obtained as follows: $E[\tilde{a}_1] = (1300 + 2\times1400 + 1600)/4 = 5700/4$, $E[\tilde{a}_2] = (1000 + 2\times1200 + 1300)/4 = 4700/4$, and $E[\tilde{\beta}] = (170 + 2\times180 + 200)/4 = 730/4$, $E[\tilde{\gamma}] = (120 + 2\times130 + 140)/4 = 520/4$, $E[\tilde{c}] = (6 + 2\times9 + 12)/4 = 9$. Similar to Example 12 in Preliminaries (see Section 2), the α-pessimistic values and α-optimistic values of triangular fuzzy variables $\tilde{c}$, $\tilde{a}_1, \tilde{a}_2, \tilde{\beta}$, and $\tilde{\gamma}$ are $\tilde{c}^L_\alpha = 6 + 3\alpha$, $\tilde{c}^U_\alpha = 12 - 3\alpha$, $\tilde{a}^L_{1\alpha} = 1300 + 100\alpha$, $\tilde{a}^U_{1\alpha} = 1600 - 200\alpha$, $\tilde{a}^L_{2\alpha} = 1000 + 200\alpha$, $\tilde{a}^U_{2\alpha} = 1300 - 100\alpha$, $\tilde{\beta}^L_\alpha = 170 + 10\alpha$, $\tilde{\beta}^U_\alpha = 200 - 20\alpha$, $\tilde{\gamma}^L_\alpha = 120 + 10\alpha$, $\tilde{\gamma}^U_\alpha = 140 - 10\alpha$, respectively. Using the analytical results obtained in this paper, we can easily have the following numerical results expressed in Tables 1 and 2 when the parameters take the values described above.

Remark 28. From Tables 1 and 2, we derive the following results when the two retailers have different market bases.

(1.1) The expected profit of the total supply chain in the centralized decision case is higher than that in all decentralized decision cases.

(1.2) One can observe directly from Table 1 that different pricing strategies of the two retailers affect the maximum expected profits of the manufacturer and the two retailers. The manufacturer achieves his largest expected profit in the MSB model, retailer 1 achieves her largest expected profit in MSC model, and retailer 2 achieves her largest expected profit in MSS model.

(1.3) From Table 1, we can see that the two retailers' Bertrand action benefits the manufacturer as well

Table 1: Maximum expect profit of total system and every firm under different pricing models.

Pricing model	$E[\pi_c]$	$E[\pi_m]$	$E[\pi_{r_1}]$	$E[\pi_{r_2}]$
CD model	7696.5			
MSB model	7388.7	6301.3	701.1	386.3
MSC model	6163.1	4604.8	1058.1	500.2
MSS model	6778.0	5492.1	546.5	739.4

Table 2: Optimal retail prices and wholesale price under different pricing models.

Pricing model	p_1^*	$p_1^* - w^*$	p_2^*	$p_2^* - w^*$	w^*
CD model	17.3190		16.9190		
MSB model	19.0790	1.9600	18.5740	1.4549	17.1190
MSC model	21.1405	4.0214	20.7405	3.6214	17.1190
MSS model	20.5076	3.4869	19.0337	2.0129	17.0208

as the total supply chain. On the other hand, the two retailers' cooperation action will always make the manufacturer and the total supply chain obtain the lowest expected profits, which implies that the manufacturer who acts as the leader does not always have the superiority of gaining expected profit in a two-echelon supply chain with two retailers. This is counterintuitive. Therefore, the manufacturer, as a Stackelberg leader, should find a way to induce the two retailers to implement Bertrand policy.

(1.4) From Table 1, we can also see that the cooperation action does not always benefit every retailer; for example, retailer 2's expected profit in the MSC model is lower than that in the MSS model. This insight is helpful to a retailer who is the follower, which tells the retailer that a suitable profit-split should be negotiated with his rival before agreeing to act in union.

(1.5) From Table 2 we find that the two retailers' cooperation behavior will result in the highest unit margins for themselves.

(1.6) From Table 2, we can see that the wholesale price in the MSB model is equal to that in the MSC model, which is consistent with the results expressed in both Propositions 18 and 21, and the wholesale price in the MSS model is the lowest one among the three decentralized decision models.

(1.7) We observe from Table 1 that the manufacturer's expected profit is bigger than the sum of the two retailers' expected profits in the above all Game cases. Moreover, retailer 2's expected profit is bigger than that of retailer 1 in the MSS model. However, retailer 1's expected profit is bigger than that of retailer 2 in other Game cases.

In order to see how the two retailers' different competitive behaviors affect the optimal pricing policy and the total expected profits of the manufacturer and the two retailers, we further assume that the retailers have the same market bases (here we set $\tilde{a}_1 = \tilde{a}_2 = (1000, 1200, 1300)$), which can be intuitively explained as the duopolistic retailers facing the similar market demand. Tables 3 and 4 present the optimal solutions when the two retailers have the same market bases.

Remark 29. From Tables 3 and 4, we can have the following results.

(2.1) The expected profit of the whole supply chain system in the centralized decision is higher than that in all decentralized decisions. This is consistent with the general case when the two retailers have different market bases.

(2.2) One can observe directly from Table 3 that the two retailers' different pricing strategies affect the total maximum expected profit of the manufacturer and the two retailers. First, retailer 1 achieves her highest expected profit in the RSC model while both the manufacturer and the whole supply chain achieve the lowest expected profits in this case. Secondly, the manufacturer achieves his highest expected profit in the MSB model while retailer 2 achieves her lowest expected profit in this case. This is consistent with the general case when the two retailers have different market bases. Thirdly, from Table 3, one can easily see that both retailer 1 and retailer 2 will achieve the same expected profit in the MSB model. Similar results occur in the MSC model. This is against to the general case when the two retailers have different market bases.

(2.3) From Table 3, we can see that the two retailers' Bertrand action benefits the manufacturer as well as the total supply chain, and the duopolistic retailers' cooperation action will always make the manufacturer and the total supply chain obtain the lowest expected profits. This is consistent with the general case when the two retailers have different market bases.

(2.4) From Table 3, we can also see that action in cooperation does not always benefit every duopolistic retailer; for example, retailer 2's expected profit in the MSC model is lower than that in the MSS model. This is also

Table 3: Maximum profit of total system and every firm under different pricing models.

Pricing model	$E[\pi_c]$	$E[\pi_m]$	$E[\pi_{r_1}]$	$E[\pi_{r_2}]$
CD model	5790.5			
MSB model	5572.3	4813.9	379.2	379.2
MSC model	4697.6	3604.8	546.4	546.4
MSS model	5212.3	4311.1	289.1	612.1

Table 4: Optimal retail prices and wholesale price under different pricing models.

Pricing model	p_1^*	$p_1^* - w^*$	p_2^*	$p_2^* - w^*$	w^*
CD model	15.9286		15.9286		
MSB model	17.3701	1.4415	17.3701	1.4415	15.9286
MSC model	19.1548	3.2262	19.1548	3.2262	15.9286
MSS model	18.4646	2.5361	17.7599	1.8313	15.9286

consistent with the general case when the two retailers have different market bases.

(2.5) One can observe directly from Table 4 that the duopolistic retailers' cooperation action will result in the highest unit margins for themselves. Moreover, we can have the following insights: firstly, the two retailers achieve the lowest unit margin in the MSB model, followed by the MSS model, then the MSC model; secondly, the two retailers will achieve equal unit margins in the MSB/MSC models. Thirdly, the retail prices charged by the two retailers achieve the highest value the retail prices charged by the two retailers achieve the highest value in the MSC model and achieve the lowest value in the CD model. Finally, the two retailers will charge the same retail price in the MSB and MSC models.

(2.6) From Table 4, we can see that the wholesale price charged by the manufacturer in three models does not vary with the two retailers' pricing strategies. This is against the general case when the two retailers have different market bases.

6. Conclusions

We have analyzed the duopolistic retailers' and the manufacturer's pricing decisions by considering the duopolistic retailers' three kinds of pricing strategies: Bertrand, Cooperation, and Stackelberg in fuzzy environment. As a benchmark to evaluate channel decision in different decision case, we first developed the pricing model in centralized decision case and derived the optimal retail prices. We then established the pricing models in decentralized decision cases by considering the duopolistic retailers' three kinds of pricing strategies and obtained the analytic equilibrium decisions. Finally, we provided comparison of the expected profits and optimal pricing decisions of the whole supply chain and every supply chain members in both the general case (namely, the two retailers have different market bases) and the special case (viz., the two retailers have the same market bases). The analytical and numerical results revealed some insights into the economic behavior of firms.

Our results, however, are based upon some assumptions about the two-echelon supply chain models. Thus, several extensions to the analysis in this paper are possible by considering the duopolistic retailers' three kinds of pricing strategies. First, as opposed to the risk neutral two-echelon supply chain members considered in this paper, one could study the case where the supply chain members with different attitudes toward risk and could also examine the influence of their attitudes toward risk on individual expected profits and the expected profit of the whole supply chain. This would add complications to the analysis of the two-echelon supply chain members' decisions. Second, we assumed that both the duopolistic retailers and the manufacturer have symmetric information about costs and demands. So, an extension would be to consider the two-echelon supply chain models in information asymmetry, such as asymmetry in cost information and demand information. Finally, we can also consider the coordination of the two-echelon supply chain under linear or isoelastic demand with symmetric and asymmetric information.

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