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Research Article

Robust $l_2 - l_\infty$ Filtering for Takagi-Sugeno Fuzzy Systems with Norm-Bounded Uncertainties

Wenbai Li, Yu Xu, and Huaizhong Li

College of Physics and Electronic Information Engineering, Wenzhou University, Wenzhou, Zhejiang 325035, China

Correspondence should be addressed to Wenbai Li; wenbaili12@gmail.com

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We study the filter design problem for Takagi-Sugeno fuzzy systems which are subject to norm-bounded uncertainties in each subsystem. As we know that the Takagi-Sugeno fuzzy linear systems can be used to represent smooth nonlinear systems, the studied plants can also be uncertain complex systems. We suppose to design a filter with the order of the original system which is also dependent on the normalized fuzzy-weighting function; that is, the filter is also a Takagi-Sugeno fuzzy filter. With the augmentation technique, an uncertain filtering error system can be obtained and the system matrices in the filtering error system are reorganized into two categories (without uncertainties and with uncertainties). For the filtering error system, we have two objectives. (1) The first one is that the filtering error system should be robust stable; that is, the filtering error system is stable though there are uncertainties in the original system. (2) The second one is that the robust energy-to-peak performance should be guaranteed. With the well-known Finsler's lemma, we provide the conditions for the robust energy-to-peak performance of the filtering error system in which three slack matrices are introduced. Finally, a numerical example is used to show the effectiveness of the proposed design methodology.

1. Introduction

It is well known that the Takagi-Sugeno (T-S) linear systems can be used to approximate smooth complex systems. Therefore, recently, fuzzy systems based on Takagi-Sugeno (T-S) [1–8] model have attracted a lot of attention [9–16]. In [17], the parameterized linear matrix inequality techniques were used in fuzzy control system design. In [18], the stability analysis and synthesis of fuzzy singularly perturbed systems were investigated. In [19], by assuming that there are some uncertainties in the controller, the robust nonfragile H_∞ filtering of fuzzy systems with linear fractional parametric uncertainties was studied. In [20], the fuzzy guaranteed cost control for nonlinear systems with time-varying delay was exploited. In [21], the variable structure system approach was used in the stabilization of nonlinear systems. The tracking control problem was studied in [22]. The authors in [23] studied the robust quadratic-optimal control for TS-fuzzy-model-based dynamic systems by considering both elemental parametric uncertainties and norm-bounded approximation error simultaneously. In the filtering aspect for T-S fuzzy systems or nonlinear systems, we can see that robust filter design approaches were proposed in [24] and filtering problem for discrete-time T-S fuzzy systems with time-varying delays was studied in [25].

The filtering problem is quite important since this technique can be used to reduce the cost and improve the measuring performance. Therefore, it is unsurprising that a lot of effort has been made during the past few decades. Among all the filtering work, the Kalman filtering [26] has been utilized for many industrial and astronautics projects since it was proposed in the 1960s. To design the Kalman filter, it requires the precise linear system model and the covariance of the measurement noise at each sampling time. If the system is a nonlinear one, the traditional Kalman cannot be directly applied. Moreover in many practical applications, the above requirements cannot be satisfied at the same time and the online computation cannot be guaranteed [27, 28].

If the external disturbance is energy-bounded, we have three additional strategies: energy-to-energy filtering, energy-to-peak filtering, and peak-to-peak filtering in the new filtering aspects [29–31]. In the energy-to-peak filtering [32–37], the infinity norm of the final filtering error aimes to be minimized when the energy of the external noise is bounded. When the peak value of the final filtering error is required to be within a certain range or required to be minimized, the energy-to-peak filtering is one of the best strategies.

In this paper, we study the robust filtering problem for T-S fuzzy discrete-time systems. We proposed to design a full-order T-S fuzzy filter such that the filtering error can satisfy the energy-to-peak performance. By using the augmentation method, we derive a filtering error system. For the filtering error system, we set two objectives. (1) The first one is that the filtering error system should be robust stable. (2) The second one is that the robust energy-to-peak performance should be guaranteed. To achieve the objective, with two well-known lemmas, we offer the sufficient conditions which can guarantee the above two objectives. With the proposed new conditions in which there are several slack matrices to reduce the conservativeness, we provide the design method of the filter parameters of the T-S fuzzy filter. Finally, an illustrative example is used to show the efficacy of the proposed design methodology. The main contributions of this paper can be organized as follows. (1) We introduce a lot of slack matrices in the robust energy-to-peak performance for T-S fuzzy systems. These matrices can be used to reduce the conservativeness of the results in robust control and filtering. (2) Compared with the work in the existing paper, the norm-bounded uncertainties are considered in the T-S fuzzy systems, which have be great potential to applied in filtering problem for complex systems.

2. Problem Formulation

In this paper, we consider a class of uncertain discrete-time T-S fuzzy systems as follows.

Plant Rule i. If $\eta_1(k)$ is $\mathcal{N}_{i1}$ and $\eta_2(k)$ is $\mathcal{N}_{i2} \ldots$ and $\eta_s(k)$ is $\mathcal{N}_{is}$, then

$$\begin{aligned} x_{k+1} &= (A_i + \Delta A_i) x_k + (B_i + \Delta B_i)\omega_k, \\ y_k &= C_i x_k + D_i \omega_k, \\ z_k &= G_i x_k, \end{aligned} \tag{1}$$

where $\mathcal{N}_{ij}$ for $i = 1, \ldots s_1$, $j = 1, \ldots, s$ are the fuzzy sets; s denotes the number of the premise variables; s_1 denotes the number of fuzzy rules; $\eta_k = [\eta_{1k}, \eta_{2k}, \ldots, \eta_{sk}]$ denotes the premise variable vector; x_k is the system state; ω_k is the external input which are bounded. ΔA_i and ΔB_i denote the norm-bounded uncertainties satisfying

$$[\Delta A_i, \Delta B_i] = Q_i H(k)[X_i, Y_i], \tag{2}$$

where Q_i, X_i, Y_i are given matrices and the time-varying $H(k)$ is bounded as $H^T(k)H(k)$. In addition, A_i, B_i, C_i, D_i, and G_i are matrices with appropriate dimensions and fixed values.

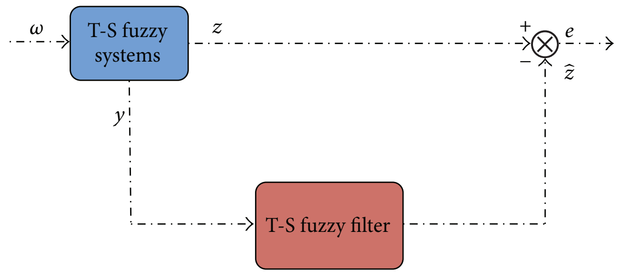

Figure 1: Filtering problem for uncertain T-S fuzzy systems.

In order to deal with the fuzzy rules, similar with the existing work on the T-S fuzzy systems, we can choose the normalized fuzzy-membership functions as follows:

$$\mu_i(\eta(k)) = \frac{\prod_{j=1}^{s}\theta_{ij}(\eta_j(k))}{\sum_{i=1}^{s_1}\prod_{j=1}^{s}\theta_{ij}(\eta_j(k))}, \tag{3}$$

where $\theta_{ij}(\eta_j(k))$ is the grade of the membership of $\eta_j(k)$ in the ith fuzzy rule. After the definition, we can see that the normalized fuzzy-membership functions have the following properties:

$$\begin{gathered} 0 \le \mu_i(\eta(k)) \le 1, \\ \sum_{i=1}^{s_1}\mu_i(\eta(k)) = 1. \end{gathered} \tag{4}$$

The principle of the filter design problem in this paper can be shown in Figure 1. The uncertain T-S fuzzy system is driven by the external input ω_k. The output of the system is measured and denoted by y_k which is used to drive the filter. $\hat{z}_k$ is the output of the filter and $\hat{z}_k$ is the estimation of z_k.

In order to utilize the on-line information of the T-S fuzzy system, we propose to design a full-order T-S fuzzy filter which has the following structure: *Filter rule i:* if $\theta_1(k)$ is $\mathcal{N}_{i1}$, and $\theta_2(k)$ is $\mathcal{N}_{i2}, \ldots$ and $\theta_s(k)$ is $\mathcal{N}_{is}$, then

$$\begin{aligned} \hat{x}_{k+1} &= A_{fi}\hat{x}_k + B_{fi} y_k, \\ \hat{z}_k &= G_{fi}\hat{x}_k, \\ i &= 1, \ldots, s_1, \end{aligned} \tag{5}$$

where $\hat{x}_k$ denotes the state in the filter; A_{fi}, B_{fi}, and G_{fi} are filter parameters to be determined in the filter design.

In the filter design, we wish the estimate $\hat{z}_k$ can track the exact value z_k which is the most interesting signal. To fulfill this wish, we can define and study the filtering error as $e_k := z_k - \hat{z}_k$. With the definition of the filtering error, we can have the following filtering error system as

$$\begin{aligned} \xi_{k+1} &= \overline{A}(\mu_k)\xi_k + \overline{B}(\mu_k)\omega_k, \\ e_k &= \overline{G}(\mu_k)\xi_k. \end{aligned} \tag{6}$$

Here, we have the following notations:

$$\xi_k = \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix},$$
$$\overline{A}(\mu_k) = \begin{bmatrix} A(\mu_k) + \Delta A(\mu_k) & 0 \\ B_f(\mu_k) C(\mu_k) & A_f(\mu_k) \end{bmatrix},$$
$$\overline{B}(\mu_k) = \begin{bmatrix} B(\mu_k) + \Delta B(\mu_k) \\ B_f(\mu_k) D(\mu_k) \end{bmatrix}, \quad (7)$$
$$\overline{G}(\mu_k) = \begin{bmatrix} G(\mu_k) & -G_f(\mu_k) \end{bmatrix}.$$

For the simplicity, we use the following notations to represent these parameters:

$$A(\mu(k)) = \sum_{i=1}^{s_1} \mu_i(\eta(k)) A_i,$$
$$\Delta A(\mu(k)) = \sum_{i=1}^{s_1} \mu_i(\eta(k)) \Delta A_i,$$
$$B(\mu(k)) = \sum_{i=1}^{s_1} \mu_i(\eta(k)) B_i,$$
$$\Delta B(\mu(k)) = \sum_{i=1}^{s_1} \mu_i(\eta(k)) \Delta B_i,$$
$$C(\mu(k)) = \sum_{i=1}^{s_1} \mu_i(\eta(k)) C_i, \quad (8)$$
$$D(\mu(k)) = \sum_{i=1}^{s_1} \mu_i(\eta(k)) D_i,$$
$$G(\mu(k)) = \sum_{i=1}^{s_1} \mu_i(\eta(k)) G_i,$$
$$A_f(\mu(k)) = \sum_{i=1}^{s_1} \mu_i(\eta(k)) A_{fi},$$
$$B_f(\mu(k)) = \sum_{i=1}^{s_1} \mu_i(\eta(k)) B_{fi},$$
$$G_f(\mu(k)) = \sum_{i=1}^{s_1} \mu_i(\eta(k)) G_{fi}. \quad (9)$$

As the norm-bounded uncertainties are involved in the filtering error system, we reorganize the system as

$$\xi_{k+1} = \left(\widehat{A}(\mu_k) + \Delta\widehat{A}(\mu_k)\right)\xi_k + \left(\widehat{B}(\mu_k) + \Delta\widehat{B}(\mu_k)\right)\omega_k, \quad (10)$$
$$e_k = \overline{G}(\mu_k)\xi_k,$$

where

$$\widehat{A}(\mu_k) = \begin{bmatrix} A(\mu_k) & 0 \\ B_f(\mu_k) C(\mu_k) & A_f(\mu_k) \end{bmatrix},$$
$$\Delta\widehat{A}(\mu_k) = \begin{bmatrix} \Delta A(\mu_k) & 0 \\ 0 & 0 \end{bmatrix} = \widehat{Q}(\mu_k) H(k) \widehat{X}(\mu_k),$$
$$\widehat{Q}(\mu_k) = \begin{bmatrix} Q(\mu_k) \\ 0 \end{bmatrix}, \qquad \widehat{X}(\mu_k) = \begin{bmatrix} X(\mu_k) & 0 \end{bmatrix}, \quad (11)$$
$$\widehat{B}(\mu_k) = \begin{bmatrix} B(\mu_k) \\ B_f(\mu_k) D(\mu_k) \end{bmatrix},$$
$$\widehat{B}(\mu_k) = \begin{bmatrix} \Delta B(\mu_k) \\ 0 \end{bmatrix} = \widehat{Q}(\mu_k) H(k) Y(\mu_k).$$

In the filtering work, the main objective is to obtain a smallest filtering error. The question becomes how to evaluate the external input ω_k and the filtering error e_k? Inspired by the filtering work in [35], we use the energy-to-peak gain γ which has the following meaning: if the initial conditions for the system and the filter are all zeros, the energy-to-peak gain from the external disturbance ω_k to the defined filtering error e_k should be smaller than the preset energy-to-peak index γ; that is,

$$\|e\|_\infty < \gamma \|\omega\|_2. \quad (12)$$

To make the design objective more clear, the objectives of the paper are summarized in the following two points.

(1) The uncertain filtering error system in (6) is robustly stable for zero disturbance input ω_k.

(2) For zero-initial conditions for the original system and the filter, the energy-to-peak gain from the external input to the filtering error should be no greater than a prescribed value γ; that is,

$$\|e\|_\infty < \gamma \|\omega\|_2. \quad (13)$$

In order to have a better result, we also include the following lemmas.

Lemma 1 (see [38]). *Given $x \in \mathbb{R}^{\overline{n}}$, $\Theta = \Theta^T \in \mathbb{R}^{\overline{n}\times\overline{n}}$, and $\mathscr{B} \in \mathbb{R}^{\overline{m}\times\overline{n}}$, if* $\operatorname{rank}(\mathscr{B}) < \overline{n}$, *the following conditions are equivalent:*

(i) $x^T \Theta x < 0$, $\forall \mathscr{B} x = 0$, $x \neq 0$,

(ii) $\exists \mathscr{X} \in \mathbb{R}^{\overline{n}\times\overline{m}}$ *such that* $\Theta + \mathscr{X}\mathscr{B} + \mathscr{B}^T \mathscr{X}^T < 0$.

Lemma 2. *Suppose that $\Theta = \Theta^T$, $\mathscr{Q}$ and $\mathscr{X}$ are real matrices with compatible dimensions, and $H(k)$ is a time-varying matrix satisfying $H^T(k)H(k) < I$. The following condition*

$$\Theta + \mathscr{Q} H(k) \mathscr{X} + \mathscr{Q}^T H^T(k) \mathscr{X}^T < 0 \quad (14)$$

holds if and only if there exists a positive scaler ε such that

$$\begin{bmatrix} \Theta & \mathscr{Q} & \varepsilon \mathscr{X}^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0 \quad (15)$$

is satisfied. Here, we use an asterisk $(*)$ *as an ellipsis for the terms that are introduced by symmetry.*

Remark 3. Though the T-S fuzzy system is used to approximate the nonlinear systems, we still introduce the norm-bounded uncertainties in the system models. The main reason is due to the approximate error and the uncertain values for each linear model. This uncertain T-S fuzzy model is more generalized and can be used to model more complex systems. The norm-bounded uncertainties have been widely used in the linear systems to cover more applications; see [39, 40] and the references therein.

3. Main Results

3.1. Robust Stability and Robust Energy-to-Peak Performance Analysis. We firstly propose a new theorem which can guarantee the robust stability and the robust energy-to-peak performance of the filtering error system.

Theorem 4. *Consider an uncertain T-S fuzzy system in* (1) *and suppose that the filter parameters* $A_f(\mu_k)$, $B_f(\mu_k)$, $G_f(\mu_k)$, *and a positive* γ *are given. The filtering error system in* (6) *is robustly stable with a prescribed robust energy-to-peak performance index* γ, *if there exist matrices* $P(\mu_k) = P^T(\mu_k) > 0$, $P(\mu_{k+1}) = P^T(\mu_{k+1}) > 0$, $L(\mu_k)$, $M(\mu_k)$, $N(\mu_k)$, *and* ε *such that*

$$\mathcal{J}_1 = \begin{bmatrix} P(\mu_{k+1}) + \operatorname{sym}(L(\mu_k)) & -L(\mu_k)\overline{A}(\mu_k) + M^T(\mu_k) & -L(\mu_k)\overline{B}(\mu_k) + N^T(\mu_k) & L(\mu_k)\widehat{Q}(\mu_k) & 0 \\ * & -P(\mu_k) - \operatorname{sym}\left(M(\mu_k)\overline{A}(\mu_k)\right) & -M(\mu_k)\overline{B}(\mu_k) - \left(N(\mu_k)\overline{A}(\mu_k)\right)^T & M(\mu_k)\widehat{Q}(\mu_k) & -\varepsilon\widehat{X}^T(\mu_k) \\ * & * & -I - \operatorname{sym}\left(N(\mu_k)\overline{B}(\mu_k)\right) & N(\mu_k)\widehat{Q}(\mu_k) & -\varepsilon Y^T(\mu_k) \\ * & * & * & -\varepsilon I & 0 \\ * & * & * & * & -\varepsilon I \end{bmatrix} < 0, \quad (16)$$

$$\begin{bmatrix} -P(\mu_k) & \overline{G}^T(\mu_k) \\ * & -\gamma^2 I \end{bmatrix}. \quad (17)$$

Proof. By using Lemma 2, the condition in (16) holds if and only if the following condition is satisfied:

$$\begin{bmatrix} P(\mu_{k+1}) + \operatorname{sym}(L(\mu_k)) & -L(\mu_k)\overline{A}(\mu_k) + M^T(\mu_k) & -L(\mu_k)\overline{B}(\mu_k) + N^T(\mu_k) \\ * & -P(\mu_k) - \operatorname{sym}\left(M(\mu_k)\overline{A}(\mu_k)\right) & -M(\mu_k)\overline{B}(\mu_k) - \left(N(\mu_k)\overline{A}(\mu_k)\right)^T \\ * & * & -I - \operatorname{sym}\left(N(\mu_k)\overline{B}(\mu_k)\right) \end{bmatrix} < 0. \quad (18)$$

Choose a Lyapunov function candidate for the filtering error system in (6) as follows:

$$V(\xi_k, \mu_k, k) = \xi_k^T P(\mu_k)\xi_k, \quad (19)$$

where $P(\mu_k) = P^T(\mu_k) > 0$ is a positive-definite matrix which is also dependent on the normalized fuzzy-weighting functions.

It can be seen that the matrix inequality (18) is rewritten as follows:

$$\Theta + \mathcal{X}\mathcal{B} + \mathcal{B}^T\mathcal{X}^T < 0, \quad (20)$$

where

$$\Theta = \begin{bmatrix} P(\mu_{k+1}) & 0 & 0 \\ * & -P(\mu_k) & 0 \\ * & * & -I \end{bmatrix},$$
$$\mathcal{X} = \begin{bmatrix} L(\mu_k) \\ M(\mu_k) \\ N(\mu_k) \end{bmatrix}, \quad (21)$$
$$\mathcal{B}^T(\mu_k) = \begin{bmatrix} I \\ -\overline{A}^T(\mu_k) \\ -\overline{B}^T(\mu_k) \end{bmatrix},$$

and $L(\mu_k)$, $M(\mu_k)$, and $N(\mu_k)$ are matrices with appropriate dimensions.

Recalling the filtering error system (6), we can see that $\mathcal{B}\overline{x}_k$ is equal to zero for all $\overline{x}_k$ which is defined as

$\overline{x}_k^T := [\xi_{k+1}^T \ \ \xi_k^T \ \ \omega_k^T]$. According to Lemma 1, the condition (18) has an equivalent one: $\overline{x}_k^T \Theta \overline{x}_k < 0$, $\forall \mathscr{B}\overline{x}_k = 0$, $\overline{x}_k \neq 0$.

Substituting the representative of Θ into $\overline{x}_k^T \Theta \overline{x}_k$, we have the following inequality:

$$\xi_{k+1}^T P(\mu_{k+1}) \xi_{k+1} - \xi_k^T P(\mu_k) \xi_k - \omega_k^T \omega_k < 0. \tag{22}$$

When the external disturbance is zero, the inequality (22) becomes

$$\xi_{k+1}^T P(\mu_{k+1}) \xi_{k+1} - \xi_k^T P(\mu_k) \xi_k < 0, \tag{23}$$

which implies that

$$\Delta V = \xi_{k+1}^T P(\mu_{k+1}) \xi_{k+1} - \xi_k^T P(\mu_k) \xi_k. \tag{24}$$

According to the Lyapunov theory, the uncertain filtering error system is robustly stable.

In the sequel, we are going to establish the $l_2 - l_\infty$ performance attenuation level γ. Suppose that the nonzero external disturbance is no-zero. It is infered from (23) that the difference of the Lyapunov function satisfies

$$\Delta V < \omega_k^T \omega_k. \tag{25}$$

Under the assumption of the zero initial values for the filter and the original system, the Lyapunov function satisfies

$$V(\xi_k, \mu_k, k) = \sum_{i=0}^{k-1} \Delta V < \sum_{i=0}^{k-1} \omega_i^T \omega_i. \tag{26}$$

In terms of the Schur complement, the inequality (17) implies that

$$\overline{G}^T(\mu_k) \overline{G}(\mu_k) < \gamma^2 P(\mu_k). \tag{27}$$

Recalling the filtering error system (6), (19), and (26), we can conclude that

$$\begin{aligned} e_k^T e_k &= [\xi_k]^T \overline{G}^T(\mu_k) \overline{G}(\mu_k) [\xi_k] \\ &< \gamma^2 [\xi_k]^T P(\mu_k) [\xi_k] \\ &= \gamma^2 \left(V(\xi_k, \mu_k, k) + \omega_k^T \omega_k \right) \\ &< \gamma^2 \sum_{i=0}^{\infty} \omega_i^T \omega_i. \end{aligned} \tag{28}$$

Taking the supremum over time $k > 0$ to the above inequality can result in the following condition $\|e\|_\infty < \gamma \|\omega\|_2$ for all nonzero $\omega_k \in l_2[0, \infty)$. Thus, the robust energy-to-peak performance can be guaranteed. This completes the proof. □

3.2. T-S Fuzzy Filter Design Approach. In the previous subsection, we assumed the parameters of the filter are given and obtained two conditions which can guarantee the robust stability and the robust energy-to-peak performance. In this subsection, we will propose the design method of the filter parameters based on the main results in the previous subsection.

Theorem 5. *Consider a discrete-time robust T-S fuzzy system in* (1) *and give a positive scalar* γ. *Then the filtering error system in* (6) *is robustly stable with a filter in the form of* (5) *if there exist matrices* $P_{11,i} = P_{11,i}^T > 0$, $P_{12,i}$, $P_{22,i} = P_{22,i}^T > 0$, $L_{11,i}$, L_{12}, $L_{21,i}$, $M_{11,i}$, $M_{21,i}$, $N_{1,i}$, AF_i, BF_i, *and* GF_i, $\forall i = 1, \ldots, s_1$, $j = i, \ldots, s_1$, $r = 1, \ldots, s_1$, *such that the following conditions hold:*

$$\begin{bmatrix} -P_{11,i} & -P_{12,i} & G_i^T \\ * & -P_{22,i} & -GF_i^T \\ * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{29}$$

$$\mathscr{H}_{i,j,r} + \mathscr{H}_{j,i,r} < 0, \tag{30}$$

where

$$\mathscr{H}_{i,j,r} = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} & 0 \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} & \Omega_{26} & 0 \\ * & * & \Omega_{33} & \Omega_{34} & \Omega_{35} & \Omega_{36} & \Omega_{37} \\ * & * & * & \Omega_{44} & \Omega_{45} & \Omega_{46} & 0 \\ * & * & * & * & \Omega_{55} & \Omega_{56} & \Omega_{57} \\ * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & -\varepsilon I \end{bmatrix},$$

$$\Omega_{11} = P_{11,r} + \mathrm{sym}(L_{11,i}),$$

$$\Omega_{12} = P_{12,r} + L_{12} + L_{21,i}^T,$$

$$\Omega_{13} = M_{11,i}^T - L_{11,i} A_j - BF_j C_i,$$

$$\Omega_{14} = M_{21,i}^T - AF_i,$$

$$\Omega_{15} = -L_{11,i} B_j - BF_j D_i + N_{1,i}^T,$$

$$\Omega_{16} = L_{11,i} Q_j,$$

$$\Omega_{22} = P_{22,r} + \mathrm{sym}(L_{12}),$$

$$\Omega_{23} = -L_{21,i} A_j - BF_j C_i,$$

$$\Omega_{24} = -AF_i,$$

$$\Omega_{25} = -L_{21,i} B_j - BF_j D_i,$$

$$\Omega_{26} = L_{21,i} Q_j,$$

$$\Omega_{33} = -P_{11,i} - \mathrm{sym}(M_{11,i} A_j),$$

$$\Omega_{34} = -P_{12,i} - A_i^T M_{21,j}^T,$$

$$\Omega_{35} = -M_{11,i} B_j - (N_{1,i} A_j)^T,$$

$$\Omega_{36} = M_{11,i} Q_j,$$

$$\Omega_{37} = -\varepsilon X_i^T,$$

$$\Omega_{44} = -P_{22,i},$$

$$\Omega_{45} = -M_{21,i}B_j,$$
$$\Omega_{46} = M_{21,i}Q_j,$$
$$\Omega_{55} = -I - \text{sym}\, N_{1,i}B_j,$$
$$\Omega_{56} = N_{1,i}Q_j,$$
$$\Omega_{57} = -\varepsilon Y_i^T. \quad (31)$$

Moreover, the parameters for each subfilter can be determined by the following equations:

$$A_{fi} = L_{12}^{-1}AF_i, \qquad B_{fi} = L_{12}^{-1}BF_i, \qquad G_{fi} = GF_i. \quad (32)$$

Proof. The proof of the theorem can be done by partitioning these matrices in Theorem 4 as

$$P_i = \begin{bmatrix} P_{11,i} & P_{12,i} \\ * & P_{22,i} \end{bmatrix}, \qquad L_i = \begin{bmatrix} L_{11,i} & L_{12} \\ L_{21,i} & L_{12} \end{bmatrix},$$
$$M_i = \begin{bmatrix} M_{11,i} & 0 \\ M_{21,i} & 0 \end{bmatrix}, \qquad N_i = \begin{bmatrix} N_{1,i} & 0 \end{bmatrix}. \quad (33)$$

This completes the proof. □

The minimal value for the robust energy-to-peak performance γ can be obtained in the following corollary.

Corollary 6. *The minimum robust energy-to-peak performance index γ for the filtering error system in (6) can be found by solving the following minimization problem:*

$$\min \quad \gamma^2,$$
$$\text{s.t.} \quad (29), (30) \quad \forall i \leq j, i, r, \quad j = 1, \ldots, s_1. \quad (34)$$

4. Illustrative Example

In this section, we consider a discrete-time T-S fuzzy system as follows.

Plant Rule 1. If x_{1k} is $\mathcal{N}_{11}$, then

$$x_{k+1} = \begin{bmatrix} 0.60 & 0.15 \\ -0.25 & 0.05 \end{bmatrix} x_k + \begin{bmatrix} -0.20 \\ -0.05 \end{bmatrix} \omega_k,$$
$$y_k = \begin{bmatrix} 1.0 & 0.5 \end{bmatrix} x_k + 0.01\omega_k, \quad (35)$$
$$z_k = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix} x_k.$$

Plant Rule 2. If x_{1k} is $\mathcal{N}_{21}$, then

$$x_{k+1} = \begin{bmatrix} 0.40 & -0.20 \\ -0.20 & 0.20 \end{bmatrix} x_k + \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix} \omega_k, \quad (36)$$
$$y_k = \begin{bmatrix} -1.0 & 0.3 \end{bmatrix} x_k + 0.03\omega_k, \quad (37)$$
$$z_k = \begin{bmatrix} 0.3 & 1.5 \end{bmatrix} x_k, \quad (38)$$

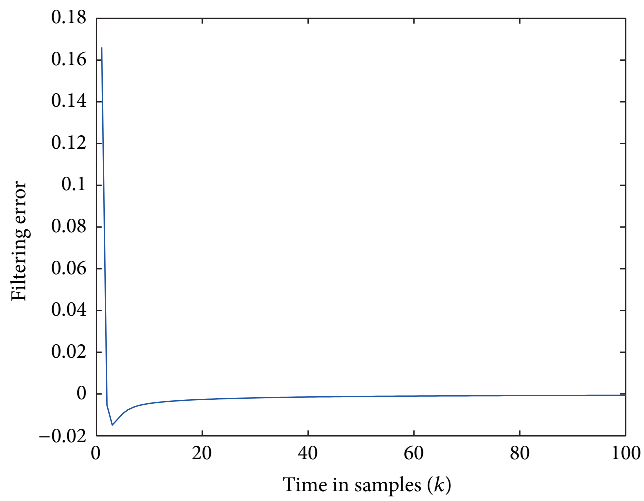

Figure 2: The filtering error in this example.

where x_{1k} is the first state of the system

$$\mathcal{N}_{11} = \begin{cases} 0.5 + 0.3 \left| \dfrac{\sin(x_{1k})}{x_{1k}} \right|, & \text{for } x_{1k} \neq 0, \\ 1, & \text{for } x_{1k} = 0, \end{cases} \quad (39)$$

$$\mathcal{N}_{21} = \begin{cases} 0.5 - 0.3 \left| \dfrac{\sin(x_{1k})}{x_{1k}} \right|, & \text{for } x_{1k} \neq 0, \\ 0, & \text{for } x_{1k} = 0. \end{cases} \quad (40)$$

For the previous discrete-time T-S fuzzy system, using Corollary 6, the obtained minimal γ is 0.1337 and the corresponding parameters of the filters are

$$A_{f1} = \begin{bmatrix} -0.0810 & -1.3402 \\ 0.0183 & 0.2738 \end{bmatrix},$$
$$B_{f1} = \begin{bmatrix} -3.3804 \\ 0.5396 \end{bmatrix},$$
$$G_{f1} = \begin{bmatrix} -0.0550 & -0.3265 \end{bmatrix},$$
$$A_{f2} = \begin{bmatrix} -0.0165 & -0.1819 \\ 0.0120 & 0.1310 \end{bmatrix}, \quad (41)$$
$$B_{f2} = \begin{bmatrix} 0.4062 \\ -0.2234 \end{bmatrix},$$
$$G_{f2} = \begin{bmatrix} -0.1315 & -1.1957 \end{bmatrix}.$$

In order to verify the performance of the designed filter, it is assumed that the initial state of the system is $[0.5, 0.1]^T$. Figure 2 shows the filtering error. It can be seen that the filtering error converges to zero quickly.

5. Conclusions

In this paper, we have investigated the robust energy-to-peak filtering problem for uncertain T-S fuzzy systems. The full-order filter is dependent on the normalized fuzzy-weighting functions. By using two novel lemmas, the conditions for the robust stability and the robust energy-to-peak performance of the filtering error systems were provided. The filter design method was proposed by using the partition method. The parameters of the T-S fuzzy filters can be obtained by solving LMIs.

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