ON A SIMILARITY SOLUTION OF MHD BOUNDARY LAYER FLOW OVER A MOVING VERTICAL CYLINDER

MARYEM AMKADNI AND ADNANE AZZOUZI

The steady flow of an incompressible electrically conducting fluid over a semi-infinite moving vertical cylinder in the presence of a uniform transverse magnetic field is analyzed. The partial differential equations governing the flow are reduced to an ordinary differential equation, using the self-similarity transformation. The analysis deals with the existence of an exact solution to the boundary value problem by a shooting method.

1. Introduction

The study of laminar flow over a continuously moving surface in a viscous incompressible fluid is of considerable interest in many industrial applications and a large number of papers investigating different aspects of this problem have been published. Boundary layer flow behavior on a cylinder moving in a Newtonian fluid was initially studied by Sakiadis [19], and obtained a numerical solution using a similarity transformation. Later, this problem has received the attention of certain researchers (see [9, 14, 18]).

More recently the problem of MHD flow over infinite surfaces has become more important due to the possibility of applications in areas like nuclear fusion, chemical engineering, medicine, and high-speed, noiseless printing. Problem of MHD flow in the vicinity of infinite plate has been studied intensively by a number of investigators (see, e.g., [12, 16, 17, 20–23] and the references therein). But only very few authors studied the flow past semi-infinite vertical cylinder (see, e.g., [1, 2, 5, 10, 13] and the references therein). It may be remarked that most exact solutions in fluid mechanics and MHD are similarity solutions in the sense that the number of independent variable is reduced by one or more.

Most of previous investigations were concerned with numerical studies and there are only few papers in the literature that deal with a theoretical analysis of problem of MHD flow along a vertical cylinder, however, an important number of theoretical investigations

are concerned with flow past vertical and flat plates without magnetic field (see, e.g., [3, 4, 7, 8, 11, 15] and the references therein). The subject of the present note is to give an analytic investigation to the problem of boundary layer in a laminar flow of a viscous incompressible and electrically conducting fluid past a permeable moving vertical semi-infinite cylinder under the action of a uniform magnetic field in the case of a linear external velocity. The governing boundary layer equations with initial and boundary conditions are reduced to an ordinary differential equation which is solved using a shooting method, and favorable conditions for the existence of solutions are established.

2. Mathematical formulation

We consider a steady laminar and incompressible viscous MHD flow past a moving permeable semi-infinite vertical cylinder of radius R. The applied transverse magnetic field B_0 is assumed to be uniform. All fluid properties are assumed to be constant and the magnetic Reynolds number is assumed to be small so that the magnetic field can be neglected. No electric field is assumed to exist. Axial coordinate x is measured along the axis of the cylinder. The radial coordinate r is measured normal to the axis of cylinder. We denote by $u_e(x) = u_\infty x$ the external velocity with $u_\infty > 0$. Under these assumptions and with the boundary layer approximation, the governing equations describing the problem are

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0,$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial x} = \frac{v}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + u_e\frac{du_e}{dx} + \frac{\sigma B^2}{\rho}(u_e - u),$$

(2.1)

with initial and boundary conditions

$$u(R,x) = u_w x, \qquad v(R,x) = -v_w, \qquad u(\infty,x) = u_e(x),$$

(2.2)

we denote by u and v the velocity components in the x and r directions, respectively. v is the kinematic viscosity, ρ is the fluid density, and σ is the electric conductivity of the fluid. v_ω is the suction/injection parameter, with $v_\omega > 0$ corresponding to the wall suction, $v_\omega < 0$ corresponding to the wall blowing, and the case $v_\omega = 0$ characterizing the impermeable wall. In our following analysis we assume that $v_\omega > 0$ and $u_\omega > 0$.

The stream function ψ is defined by $ru = \partial\psi/\partial r$ and $rv = -\partial\psi/\partial x$, substituting these expressions in (2.1)-(2.2), the continuity equation is automatically satisfied and we obtain the boundary value problem

$$\frac{1}{r^2}\frac{\partial\psi}{\partial r}\frac{\partial^2\psi}{\partial x\partial r} + \frac{1}{r^3}\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial r} - \frac{1}{r^2}\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial r^2}$$

$$= \frac{v}{r^3}\frac{\partial\psi}{\partial r} - \frac{v}{r^2}\frac{\partial^2\psi}{\partial r^2} + \frac{v}{r}\frac{\partial^3\psi}{\partial r^3} + u_e\frac{du_e}{dx} + \frac{\sigma B_0^2}{\rho}\left(u_e - \frac{1}{r}\frac{\partial\psi}{\partial r}\right),$$

(2.3)

$$\frac{\partial\psi}{\partial r}(R,x) = Ru_\omega x, \qquad \frac{\partial\psi}{\partial x}(R,x) = Rv_\omega, \qquad \lim_{r\to\infty}\left(\frac{1}{r}\frac{\partial\psi}{\partial r}(r,x)\right) = u_\infty x.$$

We look for self-similar solutions under the form

$$\psi(r,x) = \sqrt{\frac{\nu u_\infty R}{2}} x f(t), \tag{2.4}$$

where f is the dimensionless stream function and $t = \sqrt{u_\infty R/2\nu}((r^2 - R^2)/R)$ is the similarity variable.

In terms of this variable, the governing equation and boundary conditions (2.3) are transformed into

$$(Kt + 2R)f'''(t) + Kf''(t) + f(t)f''(t) - f'^2(t) - M(f'(t) - 1) + 1 = 0, \tag{2.5}$$

$$f(0) = a, \qquad f'(0) = b, \qquad f'(\infty) = 1, \tag{2.6}$$

with

$$K = 2\sqrt{\frac{2\nu}{u_\infty R}}, \qquad a = \frac{v_\omega R}{\sqrt{\nu u_\infty R}}, \qquad b = \frac{u_\omega}{2u_\infty}, \qquad M = \frac{\sigma B_0^2}{\rho u_\infty}. \tag{2.7}$$

Note that $M > 0$ is the magnetic parameter and $a > 0$ plays the role of suction parameter.

3. Main result

The objective in this section is to establish a sufficient condition for the existence of exact solutions of the problem (2.5)-(2.6) with respect to the three parameters a, b, and M. For this we will study the related initial value problem

$$(Kt + 2R)f'''(t) + Kf''(t) + f(t)f''(t) - f'^2(t) - M(f'(t) - 1) + 1 = 0,$$
$$f(0) = a, \qquad f'(0) = b, \qquad f''(0) = c, \tag{3.1}$$

where the real c is the shooting parameter. Problem (3.1) has a unique local solution f_c defined on its maximal interval of existence $[0, T_c)$, $T_c \leq \infty$. This solution is of class C^∞ on $[0, T_c)$. Let us note that if $T_c < \infty$, then

$$\lim_{t \to T_c} |f_c(t)| + |f'_c(t)| + |f''_c(t)| = +\infty. \tag{3.2}$$

THEOREM 3.1. *For $a > 0$ and $b > (3/2)(\sqrt{M^2/4 + (4/3)(M+1)} - M/2)$, if*

$$-\sqrt{\frac{1}{R}\left(\frac{b^3}{3} + \frac{M}{2}b^2 - (M+1)b\right)} < c < 0, \tag{3.3}$$

the problem (2.5)-(2.6) has at most one solution.

To prove this theorem, we use the following lemmas.

LEMMA 3.2. *If a solution f_c of problem (3.1) is defined on $[0, T_c)$ with $T_c < \infty$, then f_c, f'_c, and f''_c are unbounded for $t \to T_c$.*

Proof. We use the same idea in [6] for the Falkner-skan equation. If f_c'' were bounded for $t \to T_c$ then f_c' and f_c would also be bounded. But we have

$$\lim_{t \to T_c} |f_c(t)| + |f_c'(t)| + |f_c''(t)| = +\infty; \tag{3.4}$$

a contradiction. If f_c' were bounded for $t \to T_c$, f_c would also be bounded, on the other hand integrating $(3.1)_1$ between the limits 0 and t we get

$$Kt f_c''(t) + 2R(f_c''(t) - c) = -f_c(t)f_c'(t) + ab + 2\int_0^t f_c'^2(s)ds + M(f_c(t) - a) - (M+1)t, \tag{3.5}$$

this implies that f_c'' is bounded and we have seen that this is impossible.

Now suppose f_c were bounded for $t \to T_c$, by integration of (3.5) between 0 and t we obtain

$$(Kt + 2R)f_c'(t) + \frac{1}{2}f_c^2(t) - K f_c(t) - (2cR + ab - Ma)t + (M+1)\frac{t^2}{2} - M\int_0^t f_c(s)ds \tag{3.6}$$

$$= 2\int_0^t \int_0^\eta f_c'^2(\eta)d\eta\, dt,$$

since f_c' is unbounded for $t \to T_c$, then $\int_0^t \int_0^\eta f_c'^2(\eta)d\eta\, dt$ is also unbounded. Whereas $\int_0^t \int_0^\eta f_c'^2(\eta)d\eta\, dt$ is a monotonic function of t, therefore it tends to infinity and also does $f_c'(t)$.

If we put $\omega = \int_0^t \int_0^\eta f_c'^2(\eta)d\eta\, dt$, then

$$\omega'' = f_c'^2 \sim \frac{4}{(KT_c + 2R)^2}\omega^2 \quad \text{for } t \longrightarrow T_c. \tag{3.7}$$

Multiplying by ω', integrating and using the fact that ω tends to infinity for $t \to T_c$, we obtain

$$\frac{1}{2}\omega'^2 \sim \frac{4}{3(KT_c + 2R)^2}\omega^3, \tag{3.8}$$

$$\omega' \sim \frac{\sqrt{8}}{\sqrt{3}(KT_c + 2R)}\omega^{3/2}.$$

By the theory of indeterminate forms it follows that

$$\omega^{-1/2} \sim c_1(T_c - t) \quad \text{for } t \longrightarrow T_c, \tag{3.9}$$

where c_1 is a positive constant. Hence

$$f_c' \sim \frac{2}{KT_c + 2R}\omega \sim c_2(T_c - t)^{-2}, \tag{3.10}$$

where $c_2 \neq 0$. Integrating, we get

$$f_c \sim c_2 (T_c - t)^{-1}, \tag{3.11}$$

this contradicts the fact that f_c is bounded for $t \to T_c$. □

LEMMA 3.3. *Let* $a > 0$, $b > (3/2)(\sqrt{M^2/4 + (4/3)(M+1)} - M/2)$, *and*

$$-\sqrt{\left(\frac{1}{R}\right)\left(\frac{b^3}{3} + \left(\frac{M}{2}\right)b^2 - (M+1)b\right)} < c < 0, \tag{3.12}$$

then $f_c' > 1$ *and* $f_c > 0$ *on* $[0, T_c)$.

Proof. Since $f_c'(0) = b > 1$, then $f_c' > 1$ on some $[0, t_0)$, $0 < t_0 < T_c$. Since $f_c(0) = a > 0$, we obtain that $f_c > 0$ on $[0, t_0)$. Suppose that there exists $t_0 \leq t_1 < T_c$ such that $f_c'(t_1) = 1$ and $f_c' > 1$ on $[0, t_1)$. We introduce the function E defined by

$$E(t) = \left(\frac{Kt}{2} + R\right)f_c''^2(t) - \frac{f_c'^3(t)}{3} - \frac{M}{2}f_c'^2(t) + (M+1)f_c'(t)$$
$$+ \frac{K}{2}\int_0^t f_c''^2(s)ds \quad \forall t \in [0, T_c), \tag{3.13}$$

then by $(3.1)_1$ we get

$$E'(t) = -f_c(t)f_c''^2(t) \leq 0 \quad \text{on } [0, t_1). \tag{3.14}$$

In addition we have $E(0) = Rc^2 - b^3/3 - (M/2)b^2 + (M+1)b \leq 0$, it follows that $E(t_1) = E(0) = 0$. Thus $E(t) = E'(t) = 0$ for all $t \in [0, t_1)$. Consequently $f_c''(t) = 0$ for all $t \in [0, t_1)$, which implies $c = 0$; a contradiction. Hence $f_c' > 1$ and so $f_c > 0$. □

LEMMA 3.4. *Let* $-\sqrt{(1/R)(b^3/3 + (M/2)b^2 - (M+1)b)} < c < 0$ *such that* f_c' *is bounded, then* $f_c'' < 0$ *on* $[0, T_c)$.

Proof. Since $f_c''(0) < 0$ then f_c'' is negative on a neighborhood of 0. Suppose that there exists a number $t_0 > 0$ such that $f_c''(t_0) = 0$ and $f_c'' < 0$ on $(0, t_0)$. We will show that $f_c''' > 0$ for $t \geq t_0$. In fact, because $f_c'' < 0$ on $(0, t_0)$ we have $f_c'''(t_0) \geq 0$. Suppose that $f_c'''(t_0) = 0$, then it follows from (2.5) that

$$(1 - f_c'(t_0))(1 + f_c'(t_0) + M) = 0, \tag{3.15}$$

which implies that $f_c'(t_0) = 1$ or $f_c'(t_0) = -(1+M)$, this contradicts the fact that $f_c' > 1$ and then $f_c'''(t_0) > 0$. Now we suppose that there exists a number $t_1 > t_0$ such that $f'''(t_1) = 0$, let us observe that $f_c^{(4)}(t_1) \leq 0$ and $f_c''(t_1) \geq 0$. Differentiation of (2.5) yields

$$(Kt + 2R)f_c^{(4)}(t) + 2Kf_c'''(t) + f_c(t)f_c'''(t) - f_c'(t)f_c''(t) - Mf_c''(t) = 0, \tag{3.16}$$

and hence at $t = t_1$, we obtain

$$(Kt_1 + 2R) f_c^{(4)}(t_1) - M f_c''(t_1) = f_c'(t_1) f_c''(t_1), \qquad (3.17)$$

thus we deduce that $f_c'(t_1) \leq 0$; a contradiction. Consequently $f_c''' > 0$ and then $f_c'' > 0$ on (t_0, ∞), we get that f_c' is increasing and becomes unbounded as t tends to infinity, this contradicts the fact that f_c' is bounded and then $f_c'' < 0$ on $[0, T_c)$. □

Proof of Theorem 3.1. First let us distinguish two cases.
  (1) For any $-\sqrt{(1/R)(b^3/3 + (M/2)b^2 - (M+1)b)} < c < 0$, f_c' is unbounded.
  (2) There exists a real c_1 satisfying $-\sqrt{(1/R)(b^3/3 + (M/2)b^2 - (M+1)b)} < c_1 < 0$ such that f_{c_1}' is bounded.
In the case (1) it is clear that f_c' is not a solution of (2.5)-(2.6) since the boundary condition at infinity could not be satisfied. Then to investigate solutions of problem (2.5)-(2.6), it remains to consider the case (2). Since f_{c_1}' is bounded, then from function E we have f_{c_1}'' is also bounded on $[0, T_{c_1})$. Assume that T_{c_1} is finite, since f_{c_1}' is bounded, then f_{c_1} is also bounded. But this contradicts Lemma 3.2, consequently $T_{c_1} = +\infty$. Hence from Lemma 3.3, there exists $L \in (0, +\infty]$ such that $\lim_{t \to +\infty} f_{c_1}(t) = L$, assume that $L < +\infty$, this implies in particular the existence of a sequence (t_n) tending to $+\infty$ with n such that $\lim_{n \to +\infty} f_{c_1}'(t_n) = 0$, by using the function E, we get $0 \leq E(+\infty) \leq E(0) \leq 0$ and then $E(t) = E'(t) = 0$ for all $t \in [0, +\infty)$. Hence $f_{c_1}''(t) = 0$ for all $t \in [0, +\infty)$, which implies $c = 0$, this is impossible. We use again the function E to find the limit of f_{c_1}'', since $E(t) \leq E(0)$ we have

$$\left(\frac{Kt}{2} + R \right) f_{c_1}''^2 + \frac{K}{2} \int_0^t f_{c_1}''^2(s)\,ds \leq B, \qquad (3.18)$$

where B is a constant. There exists $t_2 \in (0, t)$ such that

$$R f_{c_1}''^2(t) + \frac{Kt}{2} f_{c_1}''^2(t) + \frac{K}{2} f_{c_1}''^2(t_2) t \leq B, \qquad (3.19)$$

then

$$f_{c_1}''^2(t) \leq \frac{1}{t} \left(\frac{2B}{K} \right), \qquad (3.20)$$

thus we get $\lim_{t \to +\infty} f_{c_1}''(t) = 0$. Finally we prove that f_{c_1}' tends to 1 as t approaches infinity, for this suppose that f_{c_1}' is oscillating indefinitely, this implies the existence of two sequences:

$(t_n)_n$ the sequence of points where the local maximums of f_{c_1}' are reached,
$(\tau_n)_n$ the sequence of points where the local minimums of f_{c_1}' are reached, $\qquad (3.21)$

then $(t_n)_n$ and $(\tau_n)_n$ are tending to $+\infty$ with n, and satisfying

$$f_{c_1}''(t_n) = f_{c_1}''(\tau_n) = 0, \quad f_{c_1}'''(t_n) < 0, \quad f_{c_1}'''(\tau_n) > 0 \quad \forall n \in \mathbb{N}. \qquad (3.22)$$

By using the polynomial $p(x) = x^2 + Mx - (M+1)$, $x \in \mathbb{R}^+$, and $(3.1)_1$ we get

$$
\begin{aligned}
(Kt_n + 2R) f_{c_1}'''(t_n) &= f_{c_1}'^2(t_n) + M f_{c_1}'(t_n) - (M+1) = p(f_{c_1}'(t_n)) < 0, \\
(K\tau_n + 2R) f_{c_1}'''(\tau_n) &= f_{c_1}'^2(\tau_n) + M f_{c_1}'(\tau_n) - (M+1) = p(f_{c_1}'(\tau_n)) > 0,
\end{aligned}
\tag{3.23}
$$

and we deduce that $0 < f_{c_1}'(t_n) < 1$ and $f_{c_1}'(\tau_n) > 1$ $\forall n \in \mathbb{N}$; a contradiction.

Then it follows that f_{c_1}' is monotone on $(t_1, +\infty)$, where t_1 is large enough. Since f_{c_1}' is bounded, hence there exists $l \in \mathbb{R}^+$ such that

$$
\lim_{t \to +\infty} f_{c_1}'(t) = l,
\tag{3.24}
$$

then

$$
f_{c_1} \sim lt \quad \text{for } t \longrightarrow +\infty,
\tag{3.25}
$$

and from identity (3.5) we have

$$
(Kt + 2R) f_{c_1}'' \sim l^2 t + Mlt - (M+1)t \quad \text{for } t \longrightarrow +\infty.
\tag{3.26}
$$

We deduce that for $t \to +\infty$

$$
\begin{aligned}
f_{c_1}'' &\sim \frac{1}{Kt} [l^2 t + Mlt - (M+1)t] \\
&\sim \frac{1}{K} p(l).
\end{aligned}
\tag{3.27}
$$

Thus, since $\lim_{t \to +\infty} f_{c_1}''(t) = 0$, we get $p(l) = 0$ and then $l = 1$. Thus f_{c_1} is a solution of (2.5)-(2.6).

Suppose that there exists another real c_2 satisfying $-\sqrt{(1/R)(b^3/3 + (M/2)b^2 - (M+1)b)}$ $< c_2 < 0$ such that f_{c_2}' is bounded, then f_{c_2} is also a solution of (2.5)-(2.6).

Assume that $c_1 > c_2$ and consider the function $g = f_{c_1} - f_{c_2}$, we have $g'(0) = g'(\infty) = 0$ and $g''(0) > 0$, then g' is a positive maximum at some point $t_0 > 0$ such that $g' > 0$ on $(0, t_0]$, therefore we have

$$
g(t_0) > 0, \qquad g'(t_0) > 0, \qquad g''(t_0) = 0, \qquad g'''(t_0) \le 0.
\tag{3.28}
$$

From (2.5) we obtain

$$
(Kt_0 + 2R) f_{c_1}'''(t_0) + K f_{c_1}''(t_0) + f_{c_1}(t_0) f_{c_1}''(t_0) + 1 - f_{c_1}'(t_0)^2 + M(1 - f_{c_1}'(t_0)) = 0,
$$

$$
(Kt_0 + 2R) f_{c_2}'''(t_0) + K f_{c_2}''(t_0) + f_{c_2}(t_0) f_{c_2}''(t_0) + 1 - f_{c_2}'(t_0)^2 + M(1 - f_{c_2}'(t_0)) = 0.
\tag{3.29}
$$

Using (3.29) we obtain

$$
(Kt_0 + 2R) g'''(t_0) = -f_{c_2}''(t_0) g(t_0) + g'(t_0) (f_{c_2}'(t_0) + f_{c_1}'(t_0) + M).
\tag{3.30}
$$

From Lemma 3.4 we have $f''_{c_2}(t_0) < 0$ and then the right-hand side is positive but $g'''(t_0) \leq 0$; a contradiction. Therefore $c_1 = c_2$ and we conclude that if there exists a real c in the interval $]-\sqrt{(1/R)(b^3/3 + (M/2)b^2 - (M+1)b)}, 0[$ such that f_c is a solution of (2.5)-(2.6), then c is unique. This ends the proof of Theorem 3.1. □

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Maryem Amkadni: Faculté de Mathématiques et d'Informatique, Université de Picardie Jules Verne, 33 rue Saint-Leu, Amiens 80039, France
E-mail address: maryem.amkadni@u-picardie.fr

Adnane Azzouzi: Faculté de Mathématiques et d'Informatique, Université de Picardie Jules Verne, 33 rue Saint-Leu, Amiens 80039, France
E-mail address: adnane.azzouzi@u-picardie.fr

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Thinking about nonlinearity in engineering areas, up to the 70s, was focused on intentionally built nonlinear parts in order to improve the operational characteristics of a device or system. Keying, saturation, hysteretic phenomena, and dead zones were added to existing devices increasing their behavior diversity and precision. In this context, an intrinsic nonlinearity was treated just as a linear approximation, around equilibrium points.

Inspired on the rediscovering of the richness of nonlinear and chaotic phenomena, engineers started using analytical tools from "Qualitative Theory of Differential Equations," allowing more precise analysis and synthesis, in order to produce new vital products and services. Bifurcation theory, dynamical systems and chaos started to be part of the mandatory set of tools for design engineers.

This proposed special edition of the *Mathematical Problems in Engineering* aims to provide a picture of the importance of the bifurcation theory, relating it with nonlinear and chaotic dynamics for natural and engineered systems. Ideas of how this dynamics can be captured through precisely tailored real and numerical experiments and understanding by the combination of specific tools that associate dynamical system theory and geometric tools in a very clever, sophisticated, and at the same time simple and unique analytical environment are the subject of this issue, allowing new methods to design high-precision devices and equipment.

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Guest Editors

José Roberto Castilho Piqueira, Telecommunication and Control Engineering Department, Polytechnic School, The University of São Paulo, 05508-970 São Paulo, Brazil; piqueira@lac.usp.br

Elbert E. Neher Macau, Laboratório Associado de Matemática Aplicada e Computação (LAC), Instituto Nacional de Pesquisas Espaciais (INPE), São Josè dos Campos, 12227-010 São Paulo, Brazil ; elbert@lac.inpe.br

Celso Grebogi, Center for Applied Dynamics Research, King's College, University of Aberdeen, Aberdeen AB24 3UE, UK; grebogi@abdn.ac.uk