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Research Article

Global Existence of the Higher-Dimensional Linear System of Thermoviscoelasticity

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We obtain a global existence result for the higher-dimensional thermoviscoelastic equations. Using semigroup approach, we will establish the global existence of homogeneous, nonhomogeneous, linear, semilinear, and nonlinear, thermoviscoelastic systems.

1. Introduction

In this paper, we consider global existence of the following thermoviscoelastic model:

$$\begin{aligned} &u_{tt} - \mu\Delta u - (\lambda+\mu)\nabla \operatorname{div} u \\ &\quad + \mu g * \Delta u + (\lambda+\mu) g * \nabla \operatorname{div} u + \alpha \nabla\theta_t = f, \quad (x,t) \in \Omega \times (0,\infty), \\ &\theta_{tt} - \Delta\theta_t - \Delta\theta + \beta \operatorname{div} u_t = h, \quad (x,t) \in \Omega \times (0,\infty), \end{aligned} \tag{1.1}$$

where the sign "$*$" denotes the convolution product in time, which is defined by

$$g * v(t) = \int_{-\infty}^{t} g(t-s) v(x,s)\, ds \tag{1.2}$$

with the initial data

$$\begin{aligned} &u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad \theta(x,0) = \theta_0(x), \quad x \in \Omega, \\ &\theta_t(x,0) = \theta_1(x), \quad u(x,0) - u(x,-s) = w_0(x,s), \quad (x,s) \in \Omega \times (0,\infty) \end{aligned} \tag{1.3}$$

and boundary condition

$$u = 0, \qquad \theta = 0, \qquad (x,t) \in \Gamma \times (0,\infty). \tag{1.4}$$

The body Ω is a bounded domain in R^n with smooth boundary $\Gamma = \partial\Omega$ (say C^2) and is assumed to be linear, homogeneous, and isotropic. $u(x,t) = (u_1(x,t), u_2(x,t), \ldots, u_n(x,t))$, and $\theta(x,t)$ represent displacement vector and temperature derivations, respectively, from the natural state of the reference configuration at position x and time t. $\lambda, \mu > 0$ are Lamé's constants and $\alpha, \beta > 0$ the coupling parameters; $g(t)$ denotes the relaxation function, $w_0(x,s)$ is a specified "history," and $u_0(x), u_1(x), \theta_0(x)$ are initial data. Δ, ∇, div denote the Laplace, gradient, and divergence operators in the space variables, respectively.

We refer to the work by Dafermos [1–3]. The following basic conditions on the relaxation function $g(t)$ are

(H_1) $g \in C^1[0,\infty) \cap L^1(0,\infty)$;

(H_2) $g(t) \geq 0$, $g'(t) \leq 0, t > 0$;

(H_3) $\kappa = 1 - \int_0^\infty g(t)dt > 0$.

In what follows, we denote by $\|\cdot\|$ the norm of $L^2(\Omega)$, and we use the notation

$$\|v\|^2 = \sum_{i=1}^{n} \|v_i\|^2, \quad \text{for } v = (v_1, v_2, \ldots, v_n). \tag{1.5}$$

When $f = g = h = 0$, system (1.1)–(1.4) is reduced to the thermoelastic system:

$$\begin{gathered}
u_{tt} - \mu\Delta u - (\lambda + \mu)\nabla \operatorname{div} u + \alpha\nabla\theta_t = 0, \quad (x,t) \in \Omega \times (0,\infty), \\
\theta_{tt} - \Delta\theta_t - \Delta\theta + \beta \operatorname{div} u_t = 0, \quad (x,t) \in \Omega \times (0,\infty), \\
u = 0, \quad \theta = 0, \quad (x,t) \in \Gamma \times (0,\infty), \\
u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x), \quad \theta(x,0) = \theta_0(x), \qquad x \in \Omega.
\end{gathered} \tag{1.6}$$

In the one-dimensional space case, there are many works (see e.g., [4–8]) on the global existence and uniqueness. Liu and Zheng [9] succeeded in deriving in energy decay under the boundary condition (1.4) or

$$u|_{x=0} = 0, \qquad \sigma|_{x=l} = 0, \qquad \theta_x|_{x=0,l} = 0, \tag{1.7}$$

$$u|_{x=0} = 0, \qquad \sigma|_{x=l} = 0, \qquad \theta|_{x=0} = 0, \qquad \theta_x|_{x=l} = 0, \tag{1.8}$$

or

$$u|_{x=0} = 0, \qquad \sigma|_{x=l} = 0, \qquad \theta|_{x=0,\, l} = 0, \tag{1.9}$$

and Hansen [10] used the method of combining the Fourier series expansion with decoupling technique to solve the exponential stability under the following boundary condition:

$$u|_{x=0} = 0, \qquad \sigma|_{x=l} = 0, \qquad \theta_x|_{x=0} = 0, \qquad \theta|_{x=l} = 0, \tag{1.10}$$

where $\sigma = u_x - \gamma\theta$ is the stress. Zhang and Zuazua [11] studied the decay of energy for the problem of the linear thermoelastic system of type III by using the classical energy method and the spectral method, and they obtained the exponential stability in one space dimension, and in two or three space dimensions for radially symmetric situations while the energy decays polynomially for most domains in two space dimensions.

When $\alpha = \beta = 0$, $f = h = 0$, system (1.1)-(1.4) is decoupled into the following viscoelastic system:

$$
\begin{gathered}
u_{tt} - \mu\Delta u - (\lambda+\mu)\nabla \operatorname{div} u + \mu g * \Delta u + (\lambda+\mu) g * \nabla \operatorname{div} u = 0, \quad (x,t) \in \Omega \times (0,\infty), \\
u = 0, \quad (x,t) \in \Gamma \times (0,\infty), \\
u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad (x,t) \in \Omega, \\
u(x,0) - u(x,-s) = w_0(x), \quad (x,t) \in \Omega \times (0,\infty),
\end{gathered}
\tag{1.11}
$$

and the wave equation.

There are many works (see, e.g., [9, 12–15]) on exponential stability of energy and asymptotic stability of solution under different assumptions. The notation in this paper will be as follows. $L^p, 1 \le p \le +\infty, W^{m,p}, m \in N, H^1 = W^{1,2}, H_0^1 = W_0^{1,2}$ denote the usual (Sobolev) spaces on Ω. In addition, $\|\cdot\|_B$ denotes the norm in the space B; we also put $\|\cdot\| = \|\cdot\|_{L^2(\Omega)}$. We denote by $C^k(I,B)$, $k \in \mathbb{N}_0$, the space of k-times continuously differentiable functions from $J \subseteq I$ into a Banach space B, and likewise by $L^p(I,B), 1 \le p \le +\infty$, the corresponding Lebesgue spaces. $C^\beta([0,T],B)$ denotes the Hölder space of B-valued continuous functions with exponent $\beta \in (0,1]$ in variable t.

2. Main Results

Let the "history space" $L^2(g,(0,\infty),(H_0^1(\Omega))^n)$ consist of $((H_0^1(\Omega))^n)$-valued functions w on $(0,\infty)$ for which

$$
\|w\|^2_{L^2(g,\,(0,\infty),(H_0^1(\Omega))^n)} = \int_0^\infty g(s)\|w(s)\|^2_{(H_0^1(\Omega))^n} ds < \infty. \tag{2.1}
$$

Put

$$
\mathscr{H} = \left(H_0^1(\Omega)\right)^n \times \left(L^2(\Omega)\right)^n \times H_0^1(\Omega) \times L^2(\Omega) \times L^2\left(g,(0,\infty),\left(H_0^1(\Omega)\right)^n\right) \tag{2.2}
$$

with the energy norm

$$
\begin{aligned}
\|(u,v,\theta,\theta_t,w)\|_{\mathscr{H}} = \Bigg\{ &\kappa\|u\|^2_{(H_0^1(\Omega))^n} + \frac{1}{2}\left(\|v\|^2 + \frac{\alpha}{\beta}\|\theta_t\|^2 + \|\nabla\theta\|\right) \\
&+ \int_0^\infty g(s)\|w(s)\|^2_{(H_0^1(\Omega))^n}\, ds \Bigg\}^{1/2},
\end{aligned}
\tag{2.3}
$$

where κ denotes the positive constant in (H_3), that is,

$$
\kappa = 1 - \int_0^\infty g(t)dt > 0. \tag{2.4}
$$

Thus we consider the following thermoviscoelastic system:

$$
\begin{gathered}
u_{tt} - \mu\Delta u - (\lambda+\mu)\nabla \operatorname{div} u + \mu g * \Delta u + (\lambda+\mu) g * \nabla \operatorname{div} u + \alpha\nabla\theta_t = 0, \quad (x,t) \in \Omega\times(0,\infty), \\
\theta_{tt} - \Delta\theta_t - \Delta\theta + \beta \operatorname{div} u_t = 0, \quad (x,t)\in\Omega\times(0,\infty), \\
u = 0, \quad \theta = 0, \quad (x,t)\in\Gamma\times(0,\infty), \\
u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad \theta(x,0) = \theta_0(x), \quad \theta_t(x,0) = \theta_1(x), \quad x\in\Omega, \\
u(x,0) - u(x,-s) = w_0(x,s), \quad (x,t)\in\Omega\times(0,\infty).
\end{gathered}
\tag{2.5}
$$

Let

$$
v(x,t) = u_t(x,t), \qquad w(x,t,s) = u(x,t) - u(x,t-s). \tag{2.6}
$$

Since

$$
\begin{aligned}
\frac{\partial}{\partial\nu}\int_{-\infty}^{t} g(t-s)u(s)ds &= \frac{\partial}{\partial\nu}\int_0^\infty g(s)u(t-s)ds \\
&= \int_0^\infty g(s)\frac{\partial}{\partial\nu}(u(t)-w(t,s))ds \\
&= (1-\kappa)\frac{\partial u(x,t)}{\partial\nu} - \int_0^\infty g(s)\frac{\partial w(t,s)}{\partial\nu}ds,
\end{aligned}
\tag{2.7}
$$

System (2.5) can be written as follows:

$$
\begin{aligned}
& u_{tt} - \kappa\mu\Delta u - \kappa(\lambda+\mu)\nabla\operatorname{div} u \\
&\quad + \alpha\nabla\theta_t - \mu\int_0^\infty g(s)\Delta w(t,s)ds \\
&\quad - (\lambda+\mu)\int_0^\infty g(s)\nabla\operatorname{div} w(t,s)ds = 0, \quad (x,t)\in\Omega\times(0,\infty), \\
& \theta_{tt} - \Delta\theta_t - \Delta\theta + \beta\operatorname{div} u_t = 0, \quad (x,t)\in\Omega\times(0,\infty), \\
& w(x,t,s) = u(x,t) - u(x,t-s), \quad (x,t,s)\in\Omega\times(0,\infty)\times(0,\infty), \\
& u = 0, \quad \theta = 0, \quad (x,t)\in\Gamma\times(0,\infty), \\
& u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad \theta(x,0) = \theta_0(x), \quad \theta_t(x,0) = \theta_1(x), \quad x\in\Omega, \\
& w(0,s) = w_0(s), \quad (x,t)\in\Omega\times(0,\infty).
\end{aligned}
\tag{2.8}
$$

We define a linear unbounded operator A on $\mathscr{H}$ by

$$
A(u,v,\theta,\theta_t,w) = \left(v, B(u,w) - \alpha\nabla\theta_t, \theta_t, \Delta\theta_t + \Delta\theta - \beta\operatorname{div} v, v - w_s\right), \tag{2.9}
$$

where $w_s = \partial w / \partial s$ and

$$B(u,w) = \kappa\mu\Delta u + \kappa(\lambda+\mu)\nabla \operatorname{div} u + \mu\int_0^\infty g(s)\Delta w(s)ds + (\lambda+\mu)\int_0^\infty g(s)\nabla \operatorname{div} w(s)ds. \tag{2.10}$$

Set

$$v(x,t) = u_t(x,t), \qquad w(x,t,s) = u(x,t) - u(x,t-s), \\ \Phi = (u,v,\theta,\theta_t,w), \qquad K = (0,f,0,h,0). \tag{2.11}$$

Then problem (2.8) can be formulated as an abstract Cauchy problem

$$\frac{d\Phi}{dt} = A\Phi + K, \tag{2.12}$$

on the Hilbert space $\mathscr{H}$ for an initial condition $\Phi(0) = (u_0, u_1, \theta_0, \theta_1, w_0)$. The domain of A is given by

$$\begin{aligned} D(A) = \Big\{ &(u,v,\theta,w) \in \mathscr{H} : \theta \in H_0^1(\Omega), \theta_t \in H_0^1(\Omega),\ \theta+\theta_t \in H^2(\Omega)\cap H_0^1(\Omega), \\ &v \in \left(H_0^1(\Omega)\right)^n,\ \kappa u + \int_0^\infty g(s)w(s)ds \in \left(H^2(\Omega)\cap H_0^1(\Omega)\right)^n, \\ &w(s) \in H^1\left(g,(0,\infty),\left(H_0^1(\Omega)\right)^n\right), w(0) = 0 \Big\}, \end{aligned} \tag{2.13}$$

where

$$H^1\left(g,(0,\infty),\left(H_0^1(\Omega)\right)^n\right) = \left\{ w : w, w_s \in L^2\left(g,(0,\infty),\left(H_0^1(\Omega)\right)^n\right)\right\}. \tag{2.14}$$

It is clear that $D(A)$ is dense in $\mathscr{H}$.

Our hypotheses on f, h can be stated as follows, which will be used in different theorems:

(A_1) $f = h = 0$;

(A_2) $f = f(x,t) \in C^1([0,\infty),(L^2(\Omega))^n)$, $h = h(x,t) \in C^1([0,\infty),L^2(\Omega))$;

(A_3) $f(x,t) \in C([0,\infty),(H_0^1(\Omega))^n), h(x,t) \in C([0,\infty),H^2(\Omega))$;

(A_4) $f(x,t) \in C([0,\infty),(L^2(\Omega))^n)$, $h(x,t) \in C([0,\infty),L^2(\Omega))$, and for any $T > 0$, $f_t \in L^1((0,T),(L^2(\Omega))^n)$, $h_t \in L^1((0,T),L^2(\Omega))$.

We are now in a position to state our main theorems.

Theorem 2.1. *Suppose that condition (A_1) holds. Relaxation function g satisfies (H_1)–(H_3). Then for any $\Phi(0) = (u_0,u_1,\theta_0,\theta_1,w_0) \in D(A)$, there exists a unique global classical solution $\Phi = (u,v,\theta,\theta_t,w)$ to system* (2.8) *satisfying $\Phi = (u,v,\theta,\theta_t,w) \in C^1([0,\infty),\mathscr{H}) \cap C([0,\infty),D(A))$.*

Theorem 2.2. *Suppose that condition (A_2) holds. Relaxation function g satisfies (H_1)–(H_3). Then for any $\Phi(0) = (u_0,u_1,\theta_0,\theta_1,w_0)$, there exists a unique global classical solution $\Phi = (u,v,\theta,\theta_t,w)$*

to system (2.8) *satisfying* $\Phi = (u, v, \theta, \theta_t, w) \in C^1([0,\infty), \mathscr{H}) \cap C([0,\infty), D(A))$*, that is,*

$$
\begin{aligned}
&u \in C^1\Big([0,\infty), \Big(H_0^1(\Omega)\Big)^n\Big) \cap C\Big([0,\infty), \Big(H^2(\Omega) \cap H_0^1(\Omega)\Big)^n\Big),\\
&v \in C^1\Big([0,\infty), \Big(L^2(\Omega)\Big)^n\Big) \cap C\Big([0,\infty), \Big(H_0^1(\Omega)\Big)^n\Big),\\
&\theta \in C^1\Big([0,\infty), H_0^1(\Omega)\Big) \cap C\Big([0,\infty), H^2(\Omega) \cap H_0^1(\Omega)\Big),\\
&\theta_t \in C^1\Big([0,\infty), L^2(\Omega)\Big) \cap C\Big([0,\infty), H_0^1(\Omega)\Big),\\
&w \in C^1\Big([0,\infty), L^2\Big(g, (0,\infty), \Big(H_0^1(\Omega)\Big)^n\Big)\Big) \cap C\Big([0,\infty), H^1\Big(g, (0,\infty), \Big(H_0^1(\Omega)\Big)^n\Big)\Big).
\end{aligned}
\tag{2.15}
$$

Corollary 2.3. *Suppose that condition* (A_3) *or* (A_4) *holds. Relaxation function g satisfies* (H_1)*–*(H_3)*. Then for any* $\Phi(0) = (u_0, u_1, \theta_0, \theta_1, w_0) \in D(A)$*, there exists a unique global classical solution* $\Phi = (u, v, \theta, \theta_t, w) \in C^1([0,\infty), \mathscr{H}) \cap C([0,\infty), D(A))$ *to system* (2.8)*.*

Corollary 2.4. *If* $f(x,t)$ *and* $h(x,t)$ *are Lipschitz continuous functions from* $[0,T]$ *into* $(L^2(\Omega))^n$ *and* $L^2(\Omega)$*, respectively, then for any* $\Phi = (u, v, \theta, \theta_t, w) \in D(A)$*, there exists a unique global classical solution* $\Phi = (u, v, \theta, \theta_t, w) \in C^1([0,\infty), \mathscr{H}) \cap C([0,\infty), D(A))$ *to system* (2.8)*.*

Theorem 2.5. *Suppose relaxation function g satisfies* (H_1)*–*(H_3)*,* $f = f(\Phi)$*, and* $h = h(\Phi), \Phi = (u, v, \theta, \theta_t, w)$*, and* $K = (0, f, 0, h, 0)$ *satisfies the global Lipschitz condition on* $\mathscr{H}$*; that is, there is a positive constant L such that for all* $\Phi_1, \Phi_2 \in \mathscr{H}$*,*

$$
\|K(\Phi_1) - K(\Phi_2)\|_{\mathscr{H}} \le L\|\Phi_1 - \Phi_2\|_{\mathscr{H}}. \tag{2.16}
$$

Then for any $\Phi(0) = (u_0, u_1, \theta_0, \theta_1, w_0) \in \mathscr{H}$*, there exists a global mild solution Φ to system* (2.8) *such that* $\Phi \in C([0,\infty), \mathscr{H})$*, that is,*

$$
\begin{gathered}
u \in C\Big([0,\infty), \Big(H_{\Gamma_1}^1(\Omega)\Big)^n\Big), \quad \theta \in C\Big([0,\infty), H_0^1(\Omega)\Big), \quad \theta_t \in C\Big([0,\infty), L^2(\Omega)\Big),\\
v \in C\Big([0,\infty), \Big(L^2(\Omega)\Big)^n\Big), \quad w \in C([0,\infty), L^2\Big(g, (0,\infty), \Big(H_0^1(\Omega)\Big)^n\Big).
\end{gathered}
\tag{2.17}
$$

Theorem 2.6. *Suppose* $f = f(\Phi)$ *and* $h = h(\Phi), \Phi = (u, v, \theta, \theta_t, w)$*, and* $K = (0, f, 0, h, 0)$ *is a nonlinear operator from* $D(A)$ *into* $D(A)$ *and satisfies the global Lipschitz condition on* $D(A)$*; that is, there is a positive constant L such that for all* $\Phi_1, \Phi_2 \in D(A)$*,*

$$
\|K(\Phi_1) - K(\Phi_2)\|_{D(A)} \le L\|\Phi_1 - \Phi_2\|_{D(A)}. \tag{2.18}
$$

Then for any $\Phi(0) = (u_0, u_1, \theta_0, \theta_1, w_0) \in D(A)$*, there exists a unique global classical solution* $\Phi = (u, v, \theta, \theta_t, w) \in C^1([0,\infty), \mathscr{H}) \cap C([0,\infty), D(A))$ *to system* (2.8)*.*

3. Some Lemmas

In this section in order to complete proofs of Theorems 2.1–2.6, we need first Lemmas 3.1–3.5. For the abstract initial value problem,

$$\frac{du}{dt} + Bu = K,$$
$$u(0) = u_0, \tag{3.1}$$

where B is a maximal accretive operator defined in a dense subset $D(B)$ of a Banach space H. We have the following.

Lemma 3.1. *Let B be a linear operator defined in a Hilbert space H,$B : D(B) \subset H \mapsto H$. Then the necessary and sufficient conditions for B being maximal accretive are*

(i) $\mathrm{Re}(Bx, x) \geq 0$, *for all* $x \in D(B)$,

(ii) $R(I + B) = H$.

Proof. We first prove the necessity. B is an accretive operator, so we have

$$(x, x) = \|x\|^2 \leq \|x + \lambda Bx\|^2 = (x, x) + 2\lambda \,\mathrm{Re}(Bx, x) + \lambda^2 \|Bx\|^2. \tag{3.2}$$

Thus, for all $\lambda > 0$,

$$\mathrm{Re}(Bx, x) \geq -\frac{\lambda}{2} \|Bx\|^2. \tag{3.3}$$

Letting $\lambda \to 0$, we get (i). Furthermore, (ii) immediately follows from the fact that B is m-accretive.

We now prove the sufficiency. It follows from (i) that for all $\lambda > 0$,

$$\begin{aligned} \|x - y\|^2 &\leq \mathrm{Re}(x - y, x - y + \lambda B(x - y)) \\ &\leq \|x - y\| \|x - y + \lambda(Bx - By)\|. \end{aligned} \tag{3.4}$$

Now it remains to prove that B is densely defined. We use a contradiction argument. Suppose that it is not true. Then there is a nontrivial element x_0 belonging to orthogonal supplement of $\overline{D(B)}$ such that for all $x \in D(B)$,

$$(x, x_0) = 0. \tag{3.5}$$

It follows from (ii) that there is $x^* \in D(B)$ such that

$$x^* + Bx^* = x_0. \tag{3.6}$$

Taking the inner product of (3.5) with x^*, we deduce that

$$(x^* + Bx^*, x^*) = 0. \tag{3.7}$$

Taking the real part of (3.7), we deduce that $x^* = 0$, and by (3.6), $x_0 = 0$, which is a contradiction. Thus the proof is complete. □

Lemma 3.2. *Suppose that B is m-accretive in a Banach space H, and $u_0 \in D(B)$. Then problem* (3.1) *has a unique classical solution u such that*

$$u \in C^1([0,\infty), H) \cap C([0,\infty), D(B)). \tag{3.8}$$

Lemma 3.3. *Suppose that $K = K(t)$, and*

$$K(t) \in C^1([0,\infty), H), \qquad u_0 \in D(B). \tag{3.9}$$

Then problem (3.1) *admits a unique global classical solution u such that*

$$u \in C^1([0,\infty), H) \cap C([0,\infty), D(B)) \tag{3.10}$$

which can be expressed as

$$u(t) = S(t)u_0 + \int_0^t S(t-\tau)K(\tau)d\tau. \tag{3.11}$$

Proof. Since $S(t)u_0$ satisfies the homogeneous equation and nonhomogeneous initial condition, it suffices to verify that $w(t)$ given by

$$w(t) = \int_0^t S(t-\tau)K(\tau)d\tau \tag{3.12}$$

belongs to $C^1([0,\infty), H) \cap C([0,\infty), D(B))$ and satisfies the nonhomogeneous equation. Consider the following quotient of difference

$$\begin{aligned}
\frac{w(t+h)-w(t)}{h} &= \frac{1}{h}\left(\int_0^{t+h} S(t+h-\tau)K(\tau)d\tau - \int_0^t S(t-\tau)K(\tau)d\tau\right) \\
&= \frac{1}{h}\int_t^{t+h} S(t+h-\tau)K(\tau)d\tau + \frac{1}{h}\int_0^t (S(t+h-\tau)-S(t-\tau))K(\tau)d\tau \\
&= \frac{1}{h}\int_t^{t+h} S(z)K(t+h-z)dz + \frac{1}{h}\int_0^t S(z)(K(t+h-z)-K(t-z))dz.
\end{aligned} \tag{3.13}$$

When $h \to 0$, the terms in the last line of (3.13) have limits:

$$S(t)K(0) + \int_0^t S(z)K'(t-z)dz \in C([0,\infty), H). \tag{3.14}$$

It turns out that $w \in C^1([0,\infty), H)$ and the terms in the third line of (3.13) have limits too, which should be

$$S(0)K(t) - Bw(t) = K(t) - Bw(t). \tag{3.15}$$

Thus the proof is complete. □

Lemma 3.4. *Suppose that* $K = K(t)$, *and*

$$K(t) \in C([0,\infty), D(B)), \qquad u_0 \in D(B). \tag{3.16}$$

Then problem (3.1) *admits a unique global classical solution.*

Proof. From the proof of Lemma 3.2, we can obtain

$$\begin{aligned}\frac{w(t+h)-w(t)}{h} &= \frac{1}{h}\int_t^{t+h} S(t+h-\tau)K(\tau)d\tau + \frac{1}{h}\int_0^t (S(t+h-\tau)-S(t-\tau))K(\tau)d\tau \\ &= \frac{1}{h}\int_t^{t+h} S(t+h-\tau)K(\tau)d\tau + \frac{1}{h}\int_0^t S(t-\tau)\left(\frac{S(h)-I}{h}\right)K(\tau)d\tau.\end{aligned} \tag{3.17}$$

When $h \to 0$, the last terms in the line of (3.17) have limits:

$$\begin{aligned}&S(0)K(t) - \int_0^t S(t-\tau)BK(\tau)d\tau \\ &\quad = S(0)K(t) - B\int_0^t S(t-\tau)K(\tau)d\tau = K(t) - Bw(t).\end{aligned} \tag{3.18}$$

Combining the results of Lemma 3.3 proves the lemma. □

Lemma 3.5. *Suppose that* $K = K(t)$, *and*

$$K(t) \in C([0,\infty), H), \qquad u_0 \in D(B), \tag{3.19}$$

and for any $T > 0$,

$$K_t \in L^1([0,T], H). \tag{3.20}$$

Then problem (3.1) *admits a unique global classical solution.*

Proof. We first prove that for any $K_1 \in L^1([0,T], H)$, the function w given by the following integral:

$$w(t) = \int_0^t S(t-\tau)K_1 d\tau \tag{3.21}$$

belongs to $C([0,T],H)$. Indeed, we infer from the difference

$$\begin{aligned} w(t+h)-w(t) &= \int_0^{t+h} S(t+h-\tau)K_1(\tau)d\tau - \int_0^t S(t-\tau)K_1(\tau)d\tau \\ &= (S(h)-I)w(t) + \int_t^{t+h} S(t+h-\tau)K_1(\tau)d\tau \end{aligned} \tag{3.22}$$

that as $h \longrightarrow 0$,

$$\|w(t+h)-w(t)\| \le \|(S(h)-I)w(t)\| + \int_t^{t+h} \|K_1(\tau)\|d\tau \longrightarrow 0, \tag{3.23}$$

where we have used the strong continuity of $S(t)$ and the absolute continuity of integral for $\|K_1\| \in L^1[0,t]$.

Now it can be seen from the last line of (3.13) that for almost every $t \in [0,T], dw/dt$ exists, and it equals

$$\begin{aligned} &S(t)K(0) + \int_0^t S(z)K'(t-z)dz \\ &\quad = S(t)K(0) + \int_0^t S(t-\tau)K'(\tau)d\tau \in C([0,T],H). \end{aligned} \tag{3.24}$$

Thus, for almost every t,

$$\frac{dw}{dt} = -Bw + K. \tag{3.25}$$

Since w and K both belong to $C([0,T],H)$, it follows from (3.25) that for almost every t, Bw equals a function belonging to $C([0,T],H)$. Since B is a closed operator, we conclude that

$$w \in C([0,T],D(B)) \cap C^1([0,T],H) \tag{3.26}$$

and (3.25) holds for every t. Thus the proof is complete. □

To prove that the operator A defined by (2.14) is dissipative, we need the following lemma.

Lemma 3.6. *If the function $f : [0,\infty) \longrightarrow R$ is uniformly continuous and is in $L^1(0,\infty)$, then*

$$\lim_{t\to\infty} f(t) = 0. \tag{3.27}$$

Lemma 3.7. *Suppose that the relaxation function g satisfies (H_1) and (H_2). If $w \in H^1(g,(0,\infty),$ $(H_0^1(\Omega))^n)$ and $w(0)=0$, then*

$$\begin{aligned} &g'(s)\|w(s)\|^2_{(H_0^1(\Omega))^n} \in L^1(0,\infty), \\ &\lim_{s\to\infty} g(s)\|w(s)\|^2_{(H_0^1(\Omega))^n} = 0. \end{aligned} \tag{3.28}$$

Proof. See, for example, the work by Liu in [16]. □

Lemma 3.8. *Suppose relaxation function g satisfies (H_1)–(H_3). The operator A defined by* (2.13) *is dissipative; furthermore, $0 \in \rho(A)$, where $\rho(A)$ is the resolvent of the operator A.*

Proof. By a straightforward calculation, it follows from Lemma 3.7 that

$$\begin{aligned}
&\langle A(u,v,\theta,\theta_t,w),(u,v,\theta,\theta_t,w)\rangle_{\mathscr{H}} \\
&= \kappa(v,u)_{(H_0^1(\Omega))^n} + \frac{1}{2}(B(u,w)-\alpha\nabla\theta_t,v) + \frac{\alpha}{2\beta}(\nabla\theta_t,\nabla\theta) \\
&\quad + \frac{\alpha}{2\beta}(\Delta\theta_t+\Delta\theta-\operatorname{div} v,\theta_t) + (v-w_s,w)_{L^2(g,(0,\infty),(H_0^1(\Omega))^n)} \\
&= -\frac{\alpha}{2\beta}\|\nabla\theta_t\|^2 + \int_0^\infty g'(s)\|w(s)\|^2_{(H_0^1(\Omega))^n}ds \\
&\le 0.
\end{aligned} \tag{3.29}$$

Thus, A is dissipative.

To prove that $0 \in \rho(A)$, for any $G = (g_1, g_2, g_3, g_4, g_5) \in \mathscr{H}$, consider

$$A\Phi = G, \tag{3.30}$$

that is,

$$v = g_1, \quad \text{in } \left(H_0^1(\Omega)\right)^n, \tag{3.31}$$

$$B(u,w) - \alpha\nabla\theta_t = g_2, \quad \text{in } \left(L^2(\Omega)\right)^n, \tag{3.32}$$

$$\theta_t = g_3, \quad \text{in } L^2(\Omega), \tag{3.33}$$

$$\Delta\theta_t + \Delta\theta - \beta \operatorname{div} v = g_4, \quad \text{in } L^2(\Omega), \tag{3.34}$$

$$v - w_s = g_5, \quad \text{in } L^2\left(g,(0,\infty),\left(H_0^1(\Omega)\right)^n\right). \tag{3.35}$$

Inserting $v = g_1$ and $\theta_t = g_3$ obtained from (3.31), (3.33) into (3.34), we obtain

$$\Delta\theta = g_4 + \beta \operatorname{div} g_1 - \Delta g_3 \in L^2(\Omega). \tag{3.36}$$

By the standard theory for the linear elliptic equations, we have a unique $\theta \in H^2(\Omega) \cap H_0^1(\Omega)$ satisfying (3.36).

We plug $v = g_1$ obtained from (3.31) into (3.35) to get

$$w_s = g_1 - g_5 \in L^2\left(g,(0,\infty),\left(H_{\Gamma_1}^1(\Omega)\right)^n\right). \tag{3.37}$$

Applying the standard theory for the linear elliptic equations again, we have a unique $w \in H^1(g,(0,\infty),(H_0^1(\Omega))^n)$ satisfying (3.37). Then plugging θ and w just obtained from solving (3.36), (3.37), respectively, into (3.32) and applying the standard theory for the linear elliptic equations again yield the unique solvability of $u \in D(A)$ for (3.32) and such that $\kappa u + \int_0^\infty g(s)w(s)ds \in (H^2(\Omega) \cap H_0^1(\Omega))^n$. Thus the unique solvability of (3.30) follows. It is clear from the regularity theory for the linear elliptic equations that $\|\Phi\|_{\mathscr{H}} \leq K\|G\|_{\mathscr{H}}$ with K being a positive constant independent of Φ. Thus the proof is completed. □

Lemma 3.9. *The operator A defined by* (2.13) *is closed.*

Proof. To prove that A is closed, let $(u_n, v_n, \theta_n, \theta_{nt}, w_n) \in D(A)$ be such that

$$\begin{aligned} &(u_n, v_n, \theta_n, \theta_{nt}, w_n) \longrightarrow (u, v, \theta, \theta_t, w) \quad \text{in } \mathscr{H}, \\ &A(u_n, v_n, \theta_n, \theta_{tn}, w_n) \longrightarrow (a, b, c, d, e) \quad \text{in } \mathscr{H}. \end{aligned} \tag{3.38}$$

Then we have

$$u_n \longrightarrow u \quad \text{in } \left(H_0^1(\Omega)\right)^n, \tag{3.39}$$

$$v_n \longrightarrow v \quad \text{in } \left(L^2(\Omega)\right)^n, \tag{3.40}$$

$$\theta_n \longrightarrow \theta \quad \text{in } H_0^1(\Omega), \tag{3.41}$$

$$\theta_{nt} \longrightarrow \theta_t \quad \text{in } L^2(\Omega), \tag{3.42}$$

$$w_n \longrightarrow w \quad \text{in } L^2\left(g, (0,\infty), \left(H_0^1(\Omega)\right)^n\right), \tag{3.43}$$

$$v_n \longrightarrow a \quad \text{in} \left(H_0^1(\Omega)\right)^n, \tag{3.44}$$

$$B(u_n, w_n) - \alpha \nabla \theta_{nt} \longrightarrow b \quad \text{in } \left(L^2(\Omega)\right)^n, \tag{3.45}$$

$$\theta_{nt} \longrightarrow c \quad \text{in } H_0^1(\Omega), \tag{3.46}$$

$$\Delta\theta_{nt} + \Delta\theta_n - \beta \operatorname{div} v_n \longrightarrow d \quad \text{in } L^2(\Omega), \tag{3.47}$$

$$v_n - w_{ns} \longrightarrow e \quad \text{in } L^2\left(g, (0,\infty), \left(H_0^1(\Omega)\right)^n\right). \tag{3.48}$$

By (3.40) and (3.44), we deduce

$$v_n \longrightarrow v \quad \text{in } \left(H_0^1(\Omega)\right)^n, \tag{3.49}$$

$$v = a \in \left(H_0^1(\Omega)\right)^n. \tag{3.50}$$

By (3.42) and (3.46), we deduce

$$\theta_{nt} \longrightarrow \theta_t \quad \text{in } H_0^1(\Omega), \tag{3.51}$$

$$\theta_t = c \in H_0^1(\Omega). \tag{3.52}$$

By (3.47) and (3.49), we deduce

$$\Delta\theta_{nt} + \Delta\theta_n \longrightarrow d + \beta \operatorname{div} v \quad \text{in } L^2(\Omega), \tag{3.53}$$

and consequently, it follows from (3.41), that

$$\theta_{nt} + \theta_n \longrightarrow \theta_t + \theta \quad \text{in } H^2(\Omega) \cap H_0^1(\Omega), \tag{3.54}$$

since Δ is an isomorphism from $H^2(\Omega) \cap H_0^1(\Omega)$ onto $L^2(\Omega)$. It therefore follows from (3.47) and (3.54) that

$$d = \Delta\theta_{nt} + \Delta\theta_n - \beta \operatorname{div} v, \qquad \theta_t + \theta \in H^2(\Omega) \cap H_0^1(\Omega). \tag{3.55}$$

By (3.43), (3.48), and (3.49), we deduce

$$w_n \longrightarrow w \quad \text{in } H^1\left(g, (0,\infty), \left(H_{\Gamma_1}^1(\Omega)\right)^n\right), \tag{3.56}$$

$$e = v - w_s, \quad w \in H^1\left(g, (0,\infty), \left(H_0^1(\Omega)\right)^n\right), \quad w(0) = 0. \tag{3.57}$$

In addition, it follows from (3.39), (3.43), (3.51) that

$$B(u_n, w_n) - \alpha\nabla\theta_{nt} \longrightarrow B(u, w) - \alpha\nabla\theta_t \tag{3.58}$$

in the distribution. It therefore follows from (3.45) and (3.58) that

$$b = B(u, w) - \alpha\nabla\theta_t, \qquad B(u, w) \in \left(L^2(\Omega)\right)^n, \tag{3.59}$$

and consequently,

$$\kappa u + \int_0^\infty g(s) w(s) ds \in \left(H^2(\Omega) \cap H_0^1(\Omega)\right)^n, \tag{3.60}$$

since $\mu\Delta + (\lambda + \mu)\nabla \operatorname{div}$ is an isomorphism from $H^2(\Omega) \cap H_0^1(\Omega)$ onto $L^2(\Omega)$. Thus, by (3.50), (3.52), (3.55), (3.57), (3.59), (3.60), we deduce

$$A(u, v, \theta, \theta_t, w) = (a, b, c, d, e), \qquad (u, v, \theta, \theta_t, w) \in D(A). \tag{3.61}$$

Hence, A is closed. □

Lemma 3.10. *Let A be a linear operator with dense domain $D(A)$ in a Hilbert space H. If A is dissipative and $0 \in \rho(A)$, the resolvent set of A, then A is the infinitesimal generator of a C_0-semigroup of contractions on H.*

Proof. See, for example, the work by Liu and Zheng in [17] and by Pazy in [18]. □

Lemma 3.11. *Let A be a densely defined linear operator on a Hilbert space H. Then A generates a C_0-semigroup of contractions on H if and only if A is dissipative and $R(I-A)=H$.*

Proof. See, for example, the work by Zheng in [19]. □

4. Proofs of Theorems 2.1–2.5

Proof of Theorem 2.1. By (2.2), it is clear that $\mathscr{H}$ is a Hilbert space. By Lemmas 3.8–3.10, we can deduce that the operator A is the infinitesimal generator of a C_0-semigroup of contractions on Hilbert space $\mathscr{H}$. Applying the result and Lemma 3.2, we can obtain our result. □

Proof of Theorem 2.2. we have known that the operator A is the infinitesimal generator of a C_0-semigroup of contractions on Hilbert space $\mathscr{H}$. Applying the result and Lemma 3.11, we can conclude that $R(I-A)=H$. If we choose operator $B=-A$, we can obtain $D(B)=D(A)$ and $D(B)$ is dense in $\mathscr{H}$. Noting that by (A_2), we know that $K=(0,f,0,h,0)\in C^1([0,\infty),\mathscr{H})$; therefore, applying Lemma 3.1, we can conclude the operator B is the maximal accretive operator. Then we can complete the proof of Theorem 2.2 in term of Lemma 3.3. □

Proof of Corollary 2.3. By (A_3) or (A_4), we derive that $K=(0,f,0,h,0)\in C([0,\infty),D(A))$ or $K\in C([0,\infty),\mathscr{H})$, and for any $T>0, K_t\in L^1((0,T),\mathscr{H})$. Noting that $B=-A$ is the maximal accretive operator, we use Lemmas 3.4 and 3.5 to prove the corollary. □

Proof of Corollary 2.4. We know that $K(x,t)=(0,f,0,h,0)$ are Lipschitz continuous functions from $[0,T]$ into $\mathscr{H}$. Moreover, by (2.2), it is clear that $\mathscr{H}$ is a reflexive Banach space. Therefore, $K_t\in L^1([0,T],H)$. Hence applying Lemma 3.5, we may complete the proof of the corollary. □

Proof of Theorem 2.5. By virtue of the proof of Theorem 2.2, we know that $B=-A$ is the maximal accretive operator of a C_0 semigroup $S(t)$. On the other hand, $K=(0,f,0,h,0)$ satisfies the global Lipschitz condition on $\mathscr{H}$. Therefore, we use the contraction mapping theorem to prove the present theorem. Two key steps for using the contraction mapping theorem are to figure out a closed set of the considered Banach space and an auxiliary problem so that the nonlinear operator defined by the auxiliary problem maps from this closed set into itself and turns out to be a contraction. In the following we proceed along this line.

Let

$$\phi(\Phi)=S(t)\Phi_0+\int_0^t S(t-\tau)K(\Phi(\tau))d\tau, \tag{4.1}$$

$$\Omega=\left\{\Phi\in C([0,+\infty),H)\mid \sup_{t\geq 0}\left(\|\Phi(t)\|e^{-kt}\right)<\infty\right\}, \tag{4.2}$$

where k is a positive constant such that $k>L$. In Ω, we introduce the following norm:

$$\|\Phi\|_\Omega=\sup_{t\geq 0}\left(\|\Phi(t)\|e^{-kt}\right). \tag{4.3}$$

Clearly, Ω is a Banach space. We now show that the nonlinear operator ϕ defined by (4.1) maps Ω into itself, and the mapping is a contraction. Indeed, for $\Phi \in \Omega$, we have

$$\begin{aligned}
\|\phi(\Phi)\| &\leq \|S(t)\Phi_0\| + \int_0^t \|S(t-\tau)\| \|K(\Phi)\| d\tau \\
&\leq \|\Phi_0\| + \int_0^t \|K(\Phi)\| d\tau \leq \|\Phi_0\| + \int_0^t (L\|\Phi(\tau)\| + \|K(0)\|) d\tau \\
&\leq \|\Phi_0\| + C_0 t + L \sup_{t \geq 0} \|\Phi(t)\| e^{-kt} \int_0^t e^{k\tau} d\tau \\
&\leq \|\Phi_0\| + C_0 t + \frac{L}{k} e^{kt} \|\Phi\|_\Omega,
\end{aligned} \tag{4.4}$$

where $C_0 = \|K(0)\|$. Thus,

$$\|\phi(\Phi)\|_\Omega \leq \sup_{t \geq 0} \left[(\|\Phi_0\| + C_0 t) e^{-kt} \right] + \frac{L}{k} \|\Phi\|_\Omega < \infty. \tag{4.5}$$

that is, $\phi(\Phi) \in \Omega$.

For $\Phi_1, \Phi_2 \in \Omega$, we have

$$\begin{aligned}
\|\phi(\Phi_1) - \phi(\Phi_2)\|_\Omega &= \sup_{t \geq 0} e^{-kt} \left\| \int_0^t S(t-\tau)(K(\Phi_1(\tau)) - K(\Phi_2(\tau))) d\tau \right\| \\
&\leq \sup_{t \geq 0} e^{-kt} L \int_0^t \|\Phi_1 - \Phi_2\| d\tau \leq \sup_{t \geq 0} \left(e^{-kt} \cdot \frac{L}{k} \cdot \left(e^{kt} - 1 \right) \right) \|\Phi_1 - \Phi_2\|_\Omega \\
&\leq \frac{L}{k} \|\Phi_1 - \Phi_2\|_\Omega.
\end{aligned} \tag{4.6}$$

Therefore, by the contraction mapping theorem, the problem has a unique solution in Ω. To show that the uniqueness also holds in $C([0,\infty), H)$, let $\Phi_1, \Phi_2 \in C([0,\infty), H)$ be two solutions of the problem and let $\Phi = \Phi_1 - \Phi_2$. Then

$$\begin{aligned}
\Phi(t) &= \int_0^t S(t-\tau)(K(\Phi_1) - K(\Phi_2)) d\tau, \\
\|\Phi(t)\| &\leq L \int_0^t \|\Phi(\tau)\| d\tau.
\end{aligned} \tag{4.7}$$

By the Gronwall inequality, we immediately conclude that $\Phi(t) = 0$; that is, the uniqueness in $C([0,\infty), H)$ follows. Thus the proof is complete. □

Proof of Theorem 2.6. Since B is the maximal accretive operator, $K = (0, f, 0, h, 0)$ satisfies the global Lipschitz condition on $D(A)$. Let

$$A_1 = D(B), \ B_1 = B^2 : D(B_1) = D\left(B^2\right) \longmapsto A_1. \tag{4.8}$$

Then A_1 is a Banach space, and $B_1 = B^2$ is a densely defined operator from $D(B^2)$ into A_1. In what follows we prove that B_1 is m-accretive in $A_1 = D(\mathrm{B})$.

Indeed, for any $x, \ y \in D(B^2)$, since B is accretive in H, we have

$$\begin{aligned}\|x - y + \lambda(Bx - By)\|_{D(B)} &= \left(\|x - y + \lambda(Bx - By)\|^2 + \left\|Bx - By + \lambda\left(B^2x - B^2y\right)\right\|^2\right)^{1/2} \\ &\geq \left(\|x - y\|^2 + \|Bx - By\|^2\right)^{1/2} = \|x - y\|_{D(B)}.\end{aligned} \tag{4.9}$$

that is, B_1 is accretive in A_1. Furthermore, since B is m-accretive in H, for any $y \in H$, there is a unique $x \in D(B)$ such that

$$x + Bx = y. \tag{4.10}$$

Now for any $y \in A_1 = D(B)$, (4.10) admits a unique solution $x \in D(B)$. It turns out that

$$Bx = y - x \in D(B). \tag{4.11}$$

Thus $x \in D(B^2)$; that is, B_1 is m-accretive in A_1. Let $S_1(t)$ be the semigroup generated by B_1. If $\Phi_0 \in D(B^2) = D(B_1)$, then

$$\Phi(t) = S_1(t)\Phi_0 \in C\left([0, +\infty), D\left(B^2\right)\right) \cap C^1([0, +\infty), D(B)) \tag{4.12}$$

is unique classical solution of the problem. On the other hand, $\Phi(t) = S_1(t)\Phi_0$ is also a classical solution in

$$C([0, +\infty), D(B)) \cap C^1([0, +\infty), H). \tag{4.13}$$

This implies that $S_1(t)$ is a restriction of $S(t)$ on A_1. By virtue of the proof of Theorem 2.5, there exists a unique mild solution $\Phi \in C([0, +\infty), A_1)$. Since $S_1(t)$ is a restriction of $S(t)$ on $D(B)$, and moreover, we infer from $K(\Phi)$ being an operator from $D(B)$ to $D(B)$ and Lemma 3.4 that Φ is a classical solution to the problem. Thus the proof is complete. □

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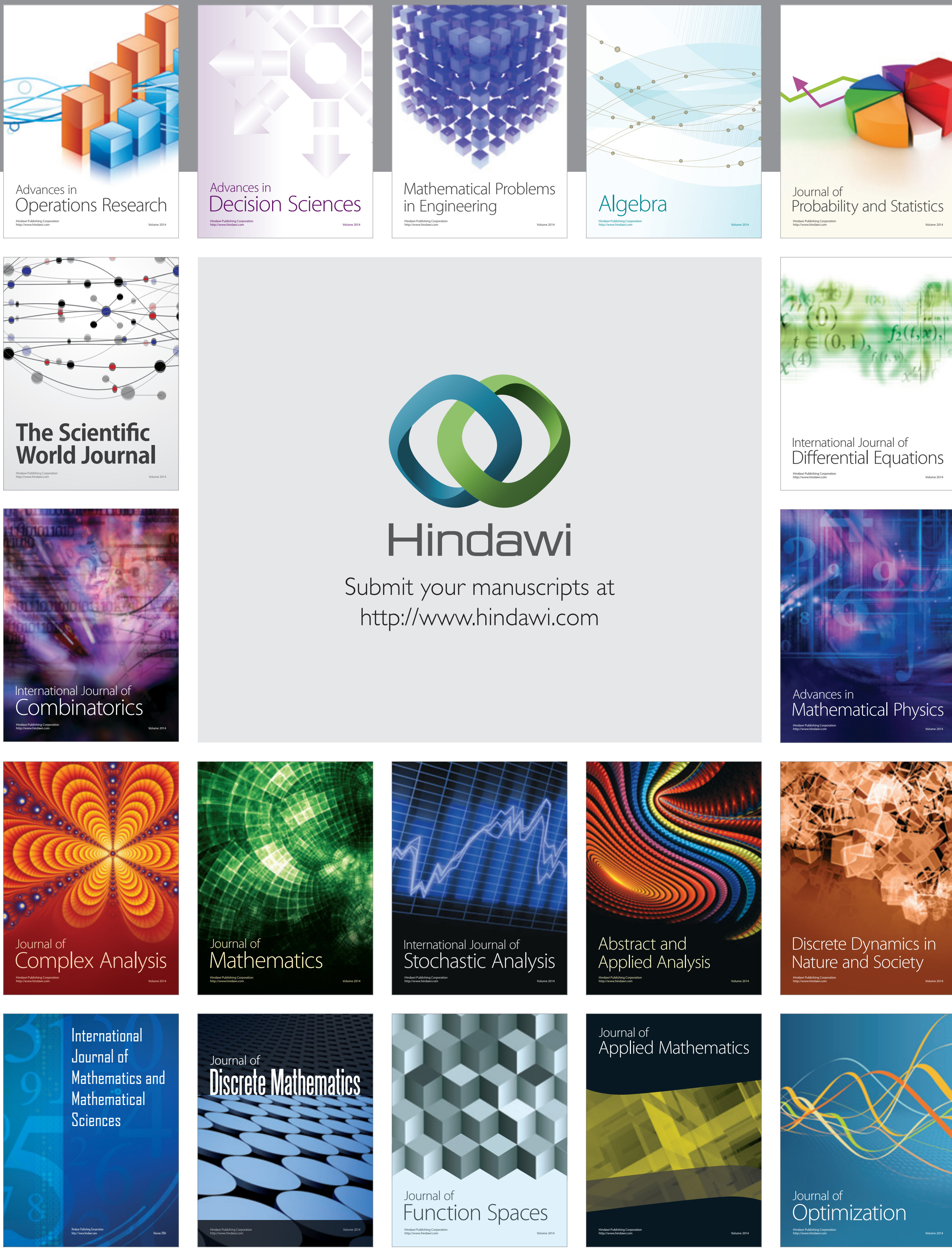

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