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Research Article

Positive Periodic Solutions of Cooperative Systems with Delays and Feedback Controls

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This paper studies a class of periodic n species cooperative Lotka-Volterra systems with continuous time delays and feedback controls. Based on the continuation theorem of the coincidence degree theory developed by Gaines and Mawhin, some new sufficient conditions on the existence of positive periodic solutions are established.

1. Introduction

Mathematical ecological system has become one of the most important topics in the study of modern applied mathematics. Its dynamical behavior includes persistence, permanence and extinction of species, global stability of systems, the existence of positive periodic solutions, positive almost periodic solutions, and strictly positive solutions. The existence of positive periodic solutions has already become one of the most interesting subjects for scholars. In the recent years, the application of fixed-point theorems to the existence of positive periodic solutions in mathematical ecology has been studied extensively, for example, Brouwer's fixed point theorem [1–4], Schauder's fixed-point theorem [5–8], Krasnoselskii's fixed-point theorem [9–14], Horn's fixed-point theorem [15, 16], and Mawhins continuation theorem [17–36], and so forth. In particular, Mawhins continuation theorem is a powerful tool for studying the existence of periodic solutions of periodic high-dimensional time-delayed problems. When dealing with a time-delayed problem, it is very convenient and the result is relatively simple [30]. Recently, a considerable number of mathematical models with delays have been proposed in the study of population dynamics. One of the most celebrated models for population is the Lotka-Volterra system. Subsequently, a lot of the literature related to the study of the existence of positive periodic solutions for various Lotka-Volterra-type population dynamical systems with delays by using the method of continuation theorem was published and extensive research results were obtained [17–21, 24–34].

On the other hand, in some situations, people may wish to change the position of the existing periodic solution but to keep its stability. This is of significance in the control of ecology balance. One of the methods for the realization of it is to alter the system structurally by introducing some feedback control variables so as to get a population stabilizing at another periodic solution. The realization of the feedback control mechanism might be implemented by means of some biological control scheme or by harvesting procedure [21]. In fact, during the last decade, the existence of positive periodic solutions for the population dynamics with feedback control has been studied extensively [8, 14, 17–21, 24, 29]. To the best of our knowledge, studies on the existence of positive periodic solutions for cooperative systems with delays and feedback controls are fairly rare.

In [21], the authors studied the following neutral Lotka-Volterra system with feedback controls:

$$\dot{y}_i(t) = y_i(t)\Bigg[r_i(t) - \sum_{j=1}^{n} a_{ij} y_j(t) - \sum_{j=1}^{n} b_{ij} y_j\left(t - \tau_{ij}(t)\right) - \sum_{j=1}^{n} c_{ij} \dot{y}_j\left(t - \gamma_{ij}(t)\right)$$

$$-f_i(t)u_i(t) - e_i(t)u_i(t-\sigma_i(t))\Big]$$

$$\dot{u}_i(t) = -\alpha_i(t)u_i(t) + \beta_i(t)y_i(t) + \gamma_i(t)y_i(t)(t-\delta_i(t)), \quad i = 1,2,\ldots,n. \tag{1}$$

By using Mawhin's continuation theorem, the sufficient conditions on the existence of positive periodic solutions are established. In [24], the authors considered the following delay differential system with feedback control:

$$\frac{dx}{dt} = F(t, x(t-\tau_1(t)), \ldots, x(t-\tau_n(t)), u(t-\delta(t)))$$
$$\frac{du}{dt} = -\eta(t) + a(t)x(t-\sigma(t)). \tag{2}$$

A set of natural and easily verifiable sufficient conditions of the existence of positive periodic solutions are established, by using Mawhin's continuation theorem. In [29], the authors considered the following single-species periodic logistic systems with feedback regulation and infinite distributed delay:

$$\dot{N}(t) = r(t)N(t) \times \left(1 - \frac{1}{K(t)}\int_0^\infty H(s)N(t-s)\,ds - c(t)u(t)\right)$$
$$\dot{u}(t) = -a(t)u(t) + b(t)\int_0^\infty H(s)N^2(t-s)\,ds. \tag{3}$$

The sufficient conditions for the existence of positive periodic solutions are established, based on Mawhin's continuation theorem.

Motivated by the above works, in this paper, we investigate the following n species periodic Lotka-Volterra-type cooperative systems with continuous time delays and feedback controls:

$$\dot{x}_i(t) = x_i(t)\Big[r_i(t) - \sum_{l=1}^{m} a_{iil}(t)x_i(t-\tau_{iil}(t)) + \sum_{j\neq i}^{n}\sum_{l=1}^{m} a_{ijl}(t)x_j(t-\tau_{ijl}(t))\Big]$$
$$- d_i(t)u_i(t) - e_i(t)u_i(t-\varepsilon_i(t))$$
$$\dot{u}_i(t) = -b_i(t)u_i(t) + \beta_i(t)x_i(t) + \gamma_i(t)x_i(t-\sigma_i(t)), \quad i = 1,2,\ldots,n. \tag{4}$$

By using the technique of coincidence degree developed by Gaines and Mawhin in [36], we will establish some new sufficient conditions which guarantee that the system has at least one positive periodic solution.

2. Preliminaries

In system (4), we have that $x_i(t)$ $(i = 1,2,\ldots n)$ represent the density of n cooperative species x_i $(i = 1,2,\ldots n)$ at time t, respectively; $r_i(t)$ $(i = 1,2,\ldots n)$ represent the intrinsic growth rate of species x_i $(i = 1,2,\ldots n)$ at time t, respectively; $a_{iil}(t)$ $(i = 1,2,\ldots,n,\ l = 1,2,\ldots,m)$ represent the intrapatch restriction density of species x_i $(i = 1,2,\ldots n)$ at time t, respectively; $a_{ijl}(t)$ $(l = 1,2,\ldots,m,\ i \neq j,\ i,j = 1,2,\ldots,n)$ represent the cooperative coefficients between n species x_i $(i = 1,2,\ldots n)$ at time t, respectively. $u_i(t)$ $(i = 1,2,\ldots n)$ represent the indirect feedback control variables [21] at time t, respectively. $\beta_i(t), e_i(t), b_i(t), d_i(t)$, and $\gamma_i(t)$ $(i = 1,2,\ldots n)$ represent the feedback control coefficients at time t, respectively. In this paper, we always assume that

(H1) $\tau_{ijl}(t)$ $(l = 1,2,\ldots,m,\ i,j = 1,2,\ldots,n)$, $\sigma_i(t), \varepsilon_i(t), r_i(t)$ $(i = 1,2,\ldots n)$ are continuous ω-periodic functions with $\tau'_{ijl}(t) < 1$ and $\int_0^\omega r_i(t)dt > 0$. $a_{ijl}(t)$ $(i,j = 1,2,\ l = 1,2,\ldots,m)$, $\beta_i(t)$, $e_i(t)$, $b_i(t)$, $d_i(t)$, and $\gamma_i(t)$ $(i = 1,2,\ldots n)$ are continuous, positive ω-periodic functions.

From the viewpoint of mathematical biology, in this paper for system (4) we only consider the solution with the following initial conditions:

$$x_i(t) = \phi_i(t), \quad \forall t \in [-\sigma, 0],\ i = 1,2,\ldots n,$$
$$u_i(t) = \psi_i(t), \quad \forall t \in [-\sigma, 0],\ i = 1,2,\ldots n, \tag{5}$$

where $\phi_i(t), \psi_i(t)$ $(i = 1,2,\ldots n)$ are nonnegative continuous functions defined on $[-\sigma, 0]$ satisfying $\phi_i(0) > 0$, $\psi_i(0) > 0$ $(i = 1,2,\ldots n)$ with $\sigma = \max_{t\in[0,\omega]}\{\tau_{ijl}(t), \sigma_i(t), \varepsilon_i(t)\ (i,j = 1,2,\ l = 1,2,\ldots,m)\}$.

In this paper, for any ω-periodic continuous function $f(t)$ we denote

$$f^L = \min_{t\in[0,\omega]} f(t), \qquad f^M = \max_{t\in[0,\omega]} f(t),$$
$$\overline{f} = \frac{1}{\omega}\int_0^\omega f(t)\,dt. \tag{6}$$

In order to obtain the existence of positive ω-periodic solutions of system (4), we will use the continuation theorem developed by Gaines and Mawhin in [36]. For the reader's convenience, we will introduce the continuation theorem in the following.

Let X and Z be two normed vector spaces. Let $L : \operatorname{Dom} L \subset X \to Z$ be a linear operator and let $N : X \to Z$ be a continuous operator. The operator L is called a Fredholm operator of index zero, if $\dim\operatorname{Ker} L = \operatorname{codim}\operatorname{Im} L < \infty$ and $\operatorname{Im} L$ is a closed set in Z. If L is a Fredholm operator of index zero, then there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $\operatorname{Im} P = \operatorname{Ker} L$ and $\operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I - Q)$. It follows that $L \mid \operatorname{Dom} L \cap \operatorname{Ker} P : \operatorname{Dom} L \cap \operatorname{Ker} P \to \operatorname{Im} L$ is invertible and its inverse is denoted by K_P; denote by $J : \operatorname{Im} Q \to \operatorname{Ker} L$ an isomorphism of $\operatorname{Im} Q$ onto $\operatorname{Ker} L$. Let Ω be a bounded open subset of X; we say that the operator N is L-compact on $\overline{\Omega}$, where $\overline{\Omega}$ denotes the closure of Ω in X, if $QN(\overline{\Omega})$ is bounded and $K_P(I-Q)N : \overline{\Omega} \to X$ is compact.

Lemma 1 (see [35]). *Suppose* $\tau \in C^1(R, R)$ *with* $\tau(t+\omega) \equiv \tau(t)$ *and* $\tau'(t) < 1$, $\forall t \in [0, \omega]$. *Then the function* $t - \tau(t)$ *has a unique inverse function* $\mu(t)$ *satisfying* $\mu \in C(R, R)$, $\mu(u+\omega) = \mu(u) + \omega$, $\forall u \in R$.

Lemma 2 (see [36]). *Let* L *be a Fredholm operator of index zero and let* N *be L-compact on* $\overline{\Omega}$. *If*

(a) *for each* $\lambda \in (0,1)$ *and* $x \in \partial\Omega \cap \operatorname{Dom} L$, $Lx \neq \lambda Nx$;

(b) *for each* $x \in \partial\Omega \cap \operatorname{Ker} L$, $QNx \neq 0$;

(c) $\deg\{JQN, \Omega \cap \operatorname{Ker} L, 0\} \neq 0$,

then the operator equation $Lx = Nx$ *has at least one solution lying in* $\operatorname{Dom} L \cap \overline{\Omega}$.

3. Main Results

In order to obtain the existence of positive periodic solutions of system (4), firstly, we introduce the following lemma.

Lemma 3. *Suppose that* $(x_1^*(t), x_2^*(t), \ldots, x_n^*(t), u_1^*(t), u_2^*(t), \ldots, u_n^*(t))$ *is an* ω*-periodic solution of* (4) *and* (5); *then* $(x_1^*(t), x_2^*(t), \ldots, x_n^*(t), u_1^*(t), u_2^*(t), \ldots, u_n^*(t))$ *satisfies the system*

$$\begin{aligned} \dot{x}_i(t) &= x_i(t)\left(r_i(t) - \sum_{j=1}^{2}(-1)^{i+j}\sum_{l=1}^{m} a_{ijl}(t)\, x_j\left(t - \tau_{ijl}(t)\right)\right) \\ &\quad - d_i(t)\, u_i(t) - e_i(t)\, u_i\left(t - \varepsilon_i(t)\right), \\ u_i(t) &= \int_t^{t+\omega} \left[\beta_i(s)\, x_i(s) + \gamma_i(s)\, x_i\left(s - \sigma_i(s)\right)\right] G_i(t,s)\, ds, \\ &\qquad i = 1, 2, \ldots, n, \end{aligned} \tag{7}$$

where

$$G_i(t,s) = \frac{\exp\left\{\int_t^s b_i(\theta)\, d\theta\right\}}{\exp\left\{\int_0^{\omega} b_i(\theta)\, d\theta\right\} - 1}, \quad i = 1, 2, \ldots, n. \tag{8}$$

The converse is also true.

Proof. By (4), (5), and the variation constants formula in ordinary differential equations, we have

$$\begin{aligned} u_i(t) = &\left(\int_0^t \left(\beta_i(s)\, x_i(s) + \gamma_i(s)\, x_i\left(s - \sigma_i(s)\right)\right)\right. \\ &\left.\times \exp\left\{\int_0^s b_i(\theta)\, d\theta\right\} ds + \psi_i(0)\right) \\ &\times \exp\left\{-\int_0^t b_i(\theta)\, d\theta\right\}, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{9}$$

From (9), we obtain

$$\begin{aligned} u_i(t+\omega) = &\left(\int_0^{t+\omega} \left(\beta_i(s)\, x_i^*(s) + \gamma_i(s)\, x_i^*\left(s - \sigma_i(s)\right)\right)\right. \\ &\left.\times \exp\left\{\int_0^s b_i(\theta)\, d\theta\right\} ds + \psi_i(0)\right) \\ &\times \exp\left\{-\int_0^{t+\omega} b_i(\theta)\, d\theta\right\}, \quad i = 1, 2, \ldots, n, \\ u_i^*(t) = &\left(\int_0^t \left(\beta_i(s)\, x_i^*(s) + \gamma_i(s)\, x_i^*\left(s - \sigma_i(s)\right)\right)\right. \\ &\left.\times \exp\left\{\int_0^s b_i(\theta)\, d\theta\right\} ds + \psi_i(0)\right) \\ &\times \exp\left\{-\int_0^t b_i(\theta)\, d\theta\right\}, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{10}$$

Considering that $(x_1^*(t), x_2^*(t), \ldots, x_n^*(t), u_1^*(t), u_2^*(t), \ldots, u_n^*(t))$ is an ω-periodic solution of system (4) and (5), we obtain

$$\begin{aligned} &\left(\int_0^t \left(\beta_i(s)\, x_i^*(s) + \gamma_i(s)\, x_i^*\left(s - \sigma_i(s)\right)\right)\right. \\ &\quad\left.\times \exp\left\{\int_0^s b_i(\theta)\, d\theta\right\} ds + \psi_i(0)\right) \exp\left\{-\int_0^t b_i(\theta)\, d\theta\right\} \\ &= \left(\int_0^{t+\omega} \left(\beta_i(s)\, x_i^*(s) + \gamma_i(s)\, x_i^*\left(s - \sigma_i(s)\right)\right)\right. \\ &\quad\left.\times \exp\left\{\int_0^s b_i(\theta)\, d\theta\right\} ds + \psi_i(0)\right) \\ &\quad\times \exp\left\{-\int_0^{t+\omega} b_i(\theta)\, d\theta\right\}, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{11}$$

Then

$$\begin{aligned} &\left(\int_0^t \left(\beta_i(s)\, x_i^*(s) + \gamma_i(s)\, x_i^*\left(s - \sigma_i(s)\right)\right)\right. \\ &\quad\left.\times \exp\left\{\int_0^s b_i(\theta)\, d\theta\right\} ds + \psi_i(0)\right) \exp\left\{\int_0^{\omega} b_i(\theta)\, d\theta\right\} \\ &= \left(\int_0^{t+\omega} \left(\beta_i(s)\, x_i^*(s) + \gamma_i(s)\, x_i^*\left(s - \sigma_i(s)\right)\right)\right. \\ &\quad\left.\times \exp\left\{\int_0^s b_i(\theta)\, d\theta\right\} ds + \psi_i(0)\right) \\ &= \int_0^t \left(\beta_i(s)\, x_i^*(s) + \gamma_i(s)\, x_i^*\left(s - \sigma_i(s)\right)\right) \\ &\quad\times \exp\left\{\int_0^s b_i(\theta)\, d\theta\right\} ds + \psi_i(0) \end{aligned}$$

$$
\begin{aligned}
&+\int_t^{t+\omega}\left(\beta_i(s)\,x_i^*(s)+\gamma_i(s)\,x_i^*\left(s-\sigma_i(s)\right)\right)\\
&\quad\times\exp\left\{\int_0^s b_i(\theta)\,d\theta\right\}ds,\quad i=1,2,\ldots,n,
\end{aligned}
\tag{12}
$$

which implies

$$
\begin{aligned}
&\left(\int_0^t\left(\beta_i(s)\,x_i^*(s)+\gamma_i(s)\,x_i^*\left(s-\sigma_i(s)\right)\right)\right.\\
&\quad\left.\times\exp\left\{\int_0^s b_i(\theta)\,d\theta\right\}ds+\psi_i(0)\right)\left(e^{\bar{b}\omega}-1\right)\\
&=\int_t^{t+\omega}\left(\beta_i(s)\,x_i^*(s)+\gamma_i(s)\,x_i^*\left(s-\sigma_i(s)\right)\right)\\
&\quad\times\exp\left\{\int_0^s b_i(\theta)\,d\theta\right\}ds,\quad i=1,2,\ldots,n.
\end{aligned}
\tag{13}
$$

That is,

$$
\begin{aligned}
&\int_0^t\left(\beta_i(s)\,x_i^*(s)+\gamma_i(i)\,x_i^*\left(s-\sigma_i(s)\right)\right)\\
&\quad\times\exp\left\{\int_0^s b_i(\theta)\,d\theta\right\}ds+\psi_i(0)\\
&=u_i^*(t)\exp\left\{\int_0^t b_i(\theta)\,d\theta\right\}\\
&=\frac{1}{e^{\bar{b}\omega}-1}\int_t^{t+\omega}\left(\beta_i(s)\,x_i^*(s)+\gamma_i(s)\,x_i^*\left(s-\sigma_i(s)\right)\right)\\
&\quad\times\exp\left\{\int_0^s b_i(\theta)\,d\theta\right\}ds,\quad i=1,2,\ldots,n.
\end{aligned}
\tag{14}
$$

Hence

$$
\begin{aligned}
u_i^*(t)&=\int_t^{t+\omega}\left(\beta_i(s)\,x_i^*(s)+\gamma_i(s)\,x_i^*\left(s-\sigma_i(s)\right)\right)\\
&\quad\times\frac{\exp\left\{\int_t^s b_i(\theta)\,d\theta\right\}}{\exp\left\{\int_0^\omega b_i(\theta)\,d\theta\right\}-1}ds,\quad i=1,2,\ldots,n.
\end{aligned}
\tag{15}
$$

On the other hand, assume that $(x_1^*(t), x_2^*(t), \ldots, x_n^*(t), u_1^*(t), u_2^*(t), \ldots, u_n^*(t))$ is an ω-periodic solution of system (7), then

$$
\begin{aligned}
u_i^*(t)&=\int_t^{t+\omega}\left(\beta_i(s)\,x_i^*(s)+\gamma_i(s)\,x_i^*\left(s-\sigma_i(s)\right)\right)\\
&\quad\times\frac{\exp\left\{\int_t^s b_i(\theta)\,d\theta\right\}}{\exp\left\{\int_0^\omega b_i(\theta)\,d\theta\right\}-1}ds,\quad i=1,2,\ldots,n.
\end{aligned}
\tag{16}
$$

By a direct calculation, we have

$$
\begin{aligned}
&\frac{du^*(t)}{dt}\\
&=-b_i(t)\int_t^{t+\omega}\left(\beta_i(s)\,x_i^*(s)+\gamma_i(s)\,x_i^*\left(s-\sigma_i(s)\right)\right)\\
&\quad\times\frac{\exp\left\{\int_t^s b_i(\theta)\,d\theta\right\}}{\exp\left\{\int_0^\omega b_i(\theta)\,d\theta\right\}-1}ds\\
&\quad+\left(\beta_i(t+\omega)\,x_i^*(t+\omega)\right.\\
&\qquad\left.+\gamma_i(t+\omega)\,x_i^*\left(t+\omega-\sigma_i(t+\omega)\right)\right)\\
&\quad\times\frac{\exp\left\{\int_t^{t+\omega} b_i(\theta)\,d\theta\right\}}{\exp\left\{\int_0^\omega b_i(\theta)\,d\theta\right\}-1}\\
&\quad-\left(\beta_i(t)\,x_i^*(t)+\gamma_i(t)\,x_i^*\left(t-\sigma_i(t)\right)\right)\\
&\quad\times\frac{1}{\exp\left\{\int_0^\omega b_i(\theta)\,d\theta\right\}-1}\\
&=-b_i(t)\,u_i(t)+\beta_i(t)\,x_i^*(t)+\gamma_i(t)\,x_i^*\left(t-\sigma_i(t)\right),\\
&\qquad i=1,2,\ldots,n.
\end{aligned}
\tag{17}
$$

This completes the proof. □

It is easy to see that system (7) is equivalent to the following system:

$$
\begin{aligned}
\dot{x}_i(t)&=x_i(t)\left(r_i(t)-\sum_{j=1}^{2}(-1)^{i+j}\sum_{l=1}^{m}a_{ijl}(t)\,x_j\left(t-\tau_{ijl}(t)\right)\right)\\
&\quad-d_i(t)\,u_i(t)-e_i(t)\,u_i\left(t-\varepsilon_i(t)\right),\\
u_i(t)&=\int_t^{t+\omega}K\left(x_i\right)G_i(t,s)\,ds,\quad i=1,2,\ldots,n,
\end{aligned}
\tag{18}
$$

where

$$
\begin{aligned}
G_i(t,s)&=\frac{\exp\left\{\int_t^s b_i(\theta)\,d\theta\right\}}{\exp\left\{\int_0^\omega b_i(\theta)\,d\theta\right\}-1},\\
K\left(x_i\right)&=\beta_i(s)\,x_i(s)+\gamma_i(s)\,x_i\left(s-\sigma_i(s)\right),\\
&\qquad i=1,2,\ldots,n.
\end{aligned}
\tag{19}
$$

It is clear that in order to prove that systems (4) and (5) have at least one ω-periodic solution, we only need to prove that system (18) has at least one ω-periodic solution.

Now, for convenience of statements we denote the functions

$$
a_{ij}(t)=\sum_{l=1}^{m}a_{ijl}(t),\quad i,j=1,2,\ldots,n.
\tag{20}
$$

The following theorem is about the existence of positive periodic solutions of system (4).

Theorem 4. *Suppose that assumption (H1) holds and there exists a constant $\theta_i > 0$, $\zeta_i > 0$, $i = 1, 2, \ldots, n$, such that*

$$\min_{t\in[0,\omega]}\left\{\sum_{l=1}^{m}\left[\delta_{iil}(t)\theta_i - \sum_{j\neq i}^{n}\delta_{jil}(t)\theta_j\right]\right\} =: \zeta_i > 0, \quad i = 1, 2, \ldots, n, \tag{21}$$

where

$$\delta_{ijl}(t) = \frac{a_{ijl}(\varphi_{ijl}(t))}{1 - \tau'_{ijl}(\varphi_{ijl}(t))}, \quad i, j = 1, 2, \ldots, n, \; l = 1, 2, \ldots, m, \tag{22}$$

and the algebraic equation

$$\overline{r}_i - \overline{H}_i - \overline{a}_{ii}v_i + \sum_{j\neq i}^{n}\overline{a}_{ij}v_j = 0, \quad i = 1, 2, \ldots, n, \tag{23}$$

where

$$H_i = \int_0^\omega (\beta_i(s) + \gamma_i(s)) \times (d_i(t)G_i(t,s) + e_i(t)G_i(t-\varepsilon_i(t), s))\,ds, \quad i = 1, 2, \ldots, n, \tag{24}$$

has a unique positive solution. Then system (4) *has at least one positive ω-periodic solution.*

Proof. For system (18) we introduce new variables $y_i(t)$ ($i = 1, 2, \ldots, n$) such that

$$x_i(t) = \exp\{y_i(t)\}, \quad i = 1, 2, \ldots, n. \tag{25}$$

Then system (18) is rewritten in the following form:

$$\begin{aligned}\dot{y}_i(t) &= r_i(t) - \sum_{j=1}^{2}(-1)^{i+j}\sum_{l=1}^{m}a_{ijl}(t)\exp\{y_j(t-\tau_{ijl}(t))\} \\ &\quad - \exp\{-y_i(t)\}d_i(t)U_i(t), \\ &\quad - \exp\{-y_i(t)\}e_i(t)U_i(t-\varepsilon_i(t))\end{aligned} \tag{26}$$

$$U_i(t) = \int_t^{t+\omega} K(e^{y_i})G_i(t,s)\,ds, \quad i = 1, 2,$$

where

$$K(e^{y_i}) = \beta_i(s)\exp\{y_i(s)\} + \gamma_i(s)\exp\{y_i(s-\sigma_i(s))\}, \quad i = 1, 2. \tag{27}$$

In order to apply Lemma 2 to system (26), we introduce the normed vector spaces X and Z as follows. Let $C(R, R^n)$ denote the space of all continuous functions $y(t) = (y_1(t), y_2(t), \ldots, y_n(t)) : R \to R^n$. We take

$$X = Z = \{y(t) \in C(R, R^n) : y(t) \text{ is an } \omega\text{-periodic function}\}, \tag{28}$$

with norm

$$\|y\| = \sum_{i=1}^{n}\max_{t\in[0,\omega]}|y_i(t)|. \tag{29}$$

It is obvious that X and Z are the Banach spaces. We define a linear operator $L : \operatorname{Dom} L \subset X \to Z$ and a continuous operator $N : X \to Z$ as follows:

$$\begin{aligned}Ly(t) &= \dot{y}(t), \\ Ny(t) &= (Ny_1(t), Ny_2(t), \ldots, Ny_n(t)),\end{aligned} \tag{30}$$

where

$$\begin{aligned}Ny_i(t) &= r_i(t) - \sum_{l=1}^{m}a_{iil}(t)\exp\{y_i(t-\tau_{iil}(t))\} \\ &\quad + \sum_{j\neq i}^{n}\sum_{l=1}^{m}a_{ijl}(t)\exp\{y_j(t-\tau_{ijl}(t))\} \\ &\quad - \exp\{-y_i(t)\}d_i(t)u_i(t) - \exp\{-y_i(t)\} \\ &\quad \times e_i(t)u_i(t-\varepsilon_i(t)).\end{aligned} \tag{31}$$

Further, we define continuous projectors $P : X \to X$ and $Q : Z \to Z$ as follows:

$$Py(t) = \frac{1}{\omega}\int_0^\omega y(t)\,dt, \qquad Qv(t) = \frac{1}{\omega}\int_0^\omega v(t)\,dt. \tag{32}$$

We easily see $\operatorname{Im} L = \{v \in Z : \int_0^\omega v(t)dt = 0\}$ and $\operatorname{Ker} L = R^n$. It is obvious that $\operatorname{Im} L$ is closed in Z and $\dim\operatorname{Ker} L = n$. Since for any $v \in Z$ there are unique $v_1 \in R^n$ and $v_2 \in \operatorname{Im} L$ with

$$v_1 = \frac{1}{\omega}\int_0^\omega v(t)\,dt, \qquad v_2(t) = v(t) - v_1, \tag{33}$$

such that $v(t) = v_1 + v_2(t)$, we have $\operatorname{codim}\operatorname{Im} L = n$. Therefore, L is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L) $K_p : \operatorname{Im} L \to \operatorname{Ker} P \cap \operatorname{Dom} L$ is given in the following form:

$$K_p v(t) = \int_0^t v(s)\,ds - \frac{1}{\omega}\int_0^\omega\int_0^t v(s)\,ds\,dt. \tag{34}$$

For convenience, we denote $F(t) = (F_1(t), F_2(t), \ldots, F_n(t))$ as follows:

$$\begin{aligned}F_i(t) &= r_i(t) - \sum_{l=1}^{m}a_{iil}(t)\exp\{y_i(t-\tau_{iil}(t))\} \\ &\quad + \sum_{j\neq i}^{n}\sum_{l=1}^{m}a_{ijl}(t)\exp\{y_j(t-\tau_{ijl}(t))\} \\ &\quad - \exp\{-y_i(t)\}d_i(t)u_i(t) - \exp\{-y_i(t)\} \\ &\quad \times e_i(t)u_i(t-\varepsilon_i(t)).\end{aligned} \tag{35}$$

Thus, we have

$$QNy(t) = \frac{1}{\omega}\int_0^\omega F(t)\,dt,$$
$$K_p(I-Q)Nu(t) = K_pINu(t) - K_pQNu(t) = \int_0^t F(s)\,ds - \frac{1}{\omega}\int_0^\omega\int_0^t F(s)\,ds\,dt + \left(\frac{1}{2}-\frac{t}{\omega}\right)\int_0^\omega F(s)\,ds. \tag{36}$$

From formulas (36), we easily see that QN and $K_p(I-Q)N$ are continuous operators. Furthermore, it can be verified that $\overline{K_p(I-Q)N(\overline{\Omega})}$ is compact for any open bounded set $\Omega\subset X$ by using the Arzela-Ascoli theorem and $QN(\overline{\Omega})$ is bounded. Therefore, N is L-compact on $\overline{\Omega}$ for any open bounded subset $\Omega\subset X$.

Now, we reach the position to search for an appropriate open bounded subset Ω for the application of the continuation theorem (Lemma 2) to system (26).

Corresponding to the operator equation $Ly(t)=\lambda Ny(t)$ with parameter $\lambda\in(0,1)$, we have

$$\dot{y}_i(t) = \lambda F_i(t), \quad i=1,2,\ldots,n, \tag{37}$$

where $F_i(t)$ $(i=1,2,\ldots,n)$ are given in (35).

Assume that $y(t)=(y_1(t),y_2(t),\ldots,y_n(t))\in X$ is a solution of system (37) for some parameter $\lambda\in(0,1)$. By integrating system (37) over the interval $[0,\omega]$, we obtain

$$\int_0^\omega\Bigg[r_i(t) - \sum_{l=1}^m a_{iil}(t)\exp\{y_i(t-\tau_{iil}(t))\} + \sum_{j\neq i}^n\sum_{l=1}^m a_{ijl}(t)\exp\{y_j(t-\tau_{ijl}(t))\} - e_i(t)\exp\{-y_i(t)\}\int_{t-\varepsilon_i(t)}^{t-\varepsilon_i(t)+\omega}K(e^{y_i})G_i(t-\varepsilon_i(t),s)\,ds - d_i(t)\exp\{-y_i(t)\}\int_t^{t+\omega}K(e^{y_i})G_i(t,s)\,ds\Bigg]dt = 0,$$
$$i=1,2,\ldots,n. \tag{38}$$

Consequently,

$$\bar{r}_i\omega + \int_0^\omega\left[\sum_{j\neq i}^n\sum_{l=1}^m a_{ijl}(t)\exp\{y_j(t-\tau_{ijl}(t))\}\right]dt = \int_0^\omega\left[\sum_{l=1}^m a_{iil}(t)\exp\{y_i(t-\tau_{iil}(t))\}\right]dt + \int_0^\omega e_i(t)\exp\{-y_i(t)\}\times\int_{t-\varepsilon_i(t)}^{t-\varepsilon_i(t)+\omega}K(e^{y_i})G_i(t-\varepsilon_i(t),s)\,ds\,dt + \int_0^\omega d_i(t)\exp\{-y_i(t)\}\int_t^{t+\omega}K(e^{y_i})G_i(t,s)\,ds\,dt,$$
$$i=1,2,\ldots,n. \tag{39}$$

From the continuity of $y(t)=(y_1(t),y_2(t),\ldots,y_n(t))$, there exist constants $\xi_i,\eta_i\in[0,\omega]$ $(i=1,2,\ldots,n)$ such that

$$y_i(\xi_i) = \max_{t\in[0,\omega]} y_i(t), \qquad y_i(\eta_i) = \min_{t\in[0,\omega]} y_i(t), \quad i=1,2,\ldots,n. \tag{40}$$

By (39) and (40) we obtain

$$\bar{r}_i\omega \le \int_0^\omega a_{ii}(t)\exp\{y_i(\xi_i)\}\,dt + \int_0^\omega A_i(t)\exp\{y_i(\xi_i)\}\,dt, \quad i=1,2,\ldots,n, \tag{41}$$

where

$$A_i(t) = e_i(t)\int_{t-\varepsilon_i(t)}^{t-\varepsilon_i(t)+\omega}G_i(t-\varepsilon_i(t),s)(\beta_i(s)+\gamma_i(s))\,ds + d_i(t)\int_t^{t+\omega}G_i(t,s)(\beta_i(s)+\gamma_i(s))\,ds,$$
$$i=1,2,\ldots,n. \tag{42}$$

Therefore, we further have

$$y_i(\xi_i) \ge \ln\left(\frac{\bar{r}_i}{\bar{a}_{ii}+\bar{A}_i}\right), \quad i=1,2,\ldots,n. \tag{43}$$

Let $s_{ijl}(t) = t-\tau_{ijl}(t)$ $(i,j=1,2,\ldots,n,\ l=1,2,\ldots,m)$; then from Lemma 1 and (H1) we get that function $s_{ijl}(t)$ has a unique ω periodic inverse function $\varphi_{ijl}(t)$; then, for every $i,j=1,2,\ldots,n,\ l=1,2,\ldots,m$, we have

$$\int_0^\omega a_{ijl}(t)\exp\{y_i(t-\tau_{ijl}(t))\}\,dt = \int_{-\tau_{ijl}(0)}^{\omega-\tau_{ijl}(\omega)}\frac{a_{ijl}(\varphi_{ijl}(t))}{1-\tau'_{ijl}(\varphi_{ijl}(t))}\exp\{y_i(t)\}\,dt. \tag{44}$$

One can see that

$$\frac{a_{ijl}(\varphi_{ijl}(t))}{1-\tau'_{ijl}(\varphi_{ijl}(t))} =: \delta_{ijl}(t), \quad i,j=1,2,\ldots,n,\ l=1,2,\ldots,m, \tag{45}$$

are ω periodic functions. Then, for every $i,j=1,2,\ldots,n$, $l=1,2,\ldots,m$, we have

$$\int_0^\omega a_{ijl}(t)\exp\{y_i(t-\tau_{ijl}(t))\}\,dt = \int_0^\omega \delta_{ijl}(t)\exp\{y_i(t)\}\,dt. \tag{46}$$

From (39) and (46) we further obtain

$$\begin{aligned}\sum_{i=1}^{n}\Bigg(\int_0^{\omega}\Bigg(&\sum_{l=1}^{m}\delta_{iil}(t)\exp\{y_i(t)\}\\&+d_i(t)\exp\{-y_i(t)\}\int_t^{t+\omega}K(e^{y_i})G_i(t,s)\,ds\\&+e_i(t)\exp\{-y_i(t)\}\\&\times\int_{t-\varepsilon_i(t)}^{t-\varepsilon_i(t)+\omega}K(e^{y_i})G_i(t-\varepsilon_i(t),s)\,ds\Bigg)dt\Bigg)\\&=\sum_{i=1}^{n}\left(\bar{r}_i\omega+\int_0^{\omega}\left[\sum_{j\neq i}^{n}\sum_{l=1}^{m}\delta_{ijl}(t)\exp\{y_j(t)\}\right]dt\right).\end{aligned}\tag{47}$$

From the above equality we have

$$\begin{aligned}&\sum_{i=1}^{n}\left(\int_0^{\omega}\left[\sum_{l=1}^{m}\delta_{iil}(t)-\sum_{j\neq i}^{n}\sum_{l=1}^{m}\delta_{jil}(t)\right]\exp\{y_i(t)\}\,dt\right)\\&\quad\le\sum_{i=1}^{n}\bar{r}_i\omega,\\&\int_0^{\omega}\left[\sum_{l=1}^{m}\delta_{iil}(t)-\sum_{j\neq i}^{n}\sum_{l=1}^{m}\delta_{jil}(t)\right]\exp\{y_i(t)\}\,dt\\&\quad\le\sum_{i=1}^{n}\bar{r}_i\omega,\quad i=1,2,\ldots,n.\end{aligned}\tag{48}$$

From the assumptions of Theorem 4 and (48) we can obtain

$$\zeta_i\int_0^{\omega}\exp\{y_i(t)\}\,dt\le\sum_{i=1}^{n}\bar{r}_i\omega,\quad i=1,2,\ldots,n.\tag{49}$$

Consequently,

$$\int_0^{\omega}\exp\{y_i(t)\}\,dt\le\frac{\sum_{i=1}^{n}\bar{r}_i\omega}{\zeta_i},\quad i=1,2,\ldots,n.\tag{50}$$

From (40) and (50), we further obtain

$$y_i(\eta_i)\le\ln\left(\frac{\sum_{i=1}^{n}\bar{r}_i}{\zeta_i}\right),\qquad i=1,2,\ldots,n.\tag{51}$$

On the other hand, directly from system (26) we have

$$\begin{aligned}&\int_0^{\omega}|\dot{y}_i(t)|\,dt\\&\quad\le\int_0^{\omega}|r_i(t)|\,dt\\&\qquad+\int_0^{\omega}\Bigg(\sum_{l=1}^{m}a_{iil}(t)\exp\{y_i(t-\tau_{iil}(t))\}\\&\qquad\qquad+\sum_{j\neq i}^{n}\sum_{l=1}^{m}a_{ijl}(t)\exp\{y_j(t-\tau_{ijl}(t))\}\Bigg)dt\\&\quad\le\int_0^{\omega}|r_i(t)|\,dt\\&\qquad+\int_0^{\omega}\sum_{l=1}^{m}\left(\delta_{iil}(t)+\sum_{j\neq i}^{n}\delta_{jil}(t)\right)\exp\{y_i(t)\}\,dt\\&\quad\le\int_0^{\omega}|r_i(t)|\,dt+D_i^M\int_0^{\omega}\exp\{y_i(t)\}\,dt\\&\quad\le|\bar{r}_i|\,\omega+\frac{D_i^M\sum_{i=1}^{n}\bar{r}_i\omega}{\zeta_i},\quad i=1,2,\ldots,n,\end{aligned}\tag{52}$$

where

$$D_i(t)=\sum_{l=1}^{m}\left(\delta_{iil}(t)+\sum_{j\neq i}^{n}\delta_{jil}(t)\right),\quad i=1,2,\ldots,n.\tag{53}$$

From (43), (51), and (52), we have for any $t\in[0,\omega]$

$$\begin{aligned}y_i(t)&\le y_i(\eta_i)+\int_0^{\omega}|\dot{y}_i(t)|\,dt\\&\le\ln\left(\frac{\sum_{i=1}^{n}\bar{r}_i}{\zeta_i}\right)+|\bar{r}_i|\,\omega+\frac{D_i^M\sum_{i=1}^{n}\bar{r}_i\omega}{\zeta_i}=:M_i,\\&\qquad i=1,2,\ldots,n,\end{aligned}\tag{54}$$

$$\begin{aligned}y_i(t)&\ge y_i(\xi_i)-\int_0^{\omega}|\dot{y}_i(t)|\,dt\\&\ge\ln\left(\frac{\bar{r}_i}{\bar{a}_{ii}+\bar{A}_i}\right)-|\bar{r}_i|\,\omega-\frac{D_i^M\sum_{i=1}^{n}\bar{r}_i\omega}{\zeta_i}=:N_i,\\&\qquad i=1,2,\ldots,n.\end{aligned}\tag{55}$$

Therefore, from (54) and (55), we have

$$\max_{t\in[0,\omega]}|y_i(t)|\le\max\{|M_i|,|N_i|\}=:B_i,\quad i=1,2,\ldots,n.\tag{56}$$

It can be seen that the constants B_i $(i = 1, 2, \ldots, n)$ are independent of parameter $\lambda \in (0, 1)$. For any $y = (y_1, y_2, \ldots, y_n) \in R^n$, from (31) we can obtain

$$QNy = (QNy_1, QNy_2, \ldots, QNy_n)$$

$$QNy_i = \overline{r}_i - \overline{H}_i - \overline{a}_{ii} \exp\{y_i\} + \sum_{j \neq i}^{n} \overline{a}_{ij} \exp\{y_j\}, \quad i = 1, 2, \ldots, n, \tag{57}$$

$$H_i(t) = \int_0^{\omega} (\beta_i(s) + \gamma_i(s)) (d_i(t) G_i(t, s) + e_i(t) G_i(t - \varepsilon_i(t), s) ds, \quad i = 1, 2, \ldots, n.$$

We consider the following algebraic equation:

$$\overline{r}_i - \overline{H}_i - \overline{a}_{ii} v_i + \sum_{j \neq i}^{n} \overline{a}_{ij} v_j = 0, \quad i = 1, 2, \ldots, n. \tag{58}$$

From the assumption of Theorem 4, the equation has a unique positive solution $v^* = (v_1^*, v_2^*, \ldots, v_n^*)$. Hence, the equation $QNy = 0$ has a unique solution $y^* = (y_1^*, y_2^*, \ldots, y_n^*) = (\ln v_1^*, \ln v_2^*, \ldots, \ln v_n^*) \in R^n$.

Choosing constant $B > 0$ large enough such that $|y_1^*| + |y_2^*| + \cdots + |y_n^*| < B$ and $B > B_1 + B_2 + \cdots + B_n$, we define a bounded open set $\Omega \subset X$ as follows:

$$\Omega = \{y \in X : \|y\| < B\}. \tag{59}$$

It is clear that Ω satisfies conditions (a) and (b) of Lemma 2. On the other hand, by directly calculating we can obtain

$$\deg\{JQN, \Omega \cap \operatorname{Ker} L, (0, 0, \ldots, 0)\} = \operatorname{sgn} \begin{vmatrix} f_{y_1}^1 & f_{y_2}^1 & \cdots & f_{y_n}^1 \\ f_{y_1}^2 & f_{y_2}^2 & \cdots & f_{y_n}^2 \\ \cdots & \cdots & \cdots & \cdots \\ f_{y_1}^n & f_{y_2}^n & \cdots & f_{y_n}^n \end{vmatrix}, \tag{60}$$

where

$$f_{y_j}^i = -\overline{a}_{ij} \exp\{y_j^*\}, \quad i = j$$
$$f_{y_j}^i = \overline{a}_{ij} \exp\{y_j^*\}, \quad i \neq j \; i, j = 1, 2, \ldots, n. \tag{61}$$

From the assumption of Theorem 4, we have

$$\begin{vmatrix} f_{y_1}^1 & f_{y_2}^1 & \cdots & f_{y_n}^1 \\ f_{y_1}^2 & f_{y_2}^2 & \cdots & f_{y_n}^2 \\ \cdots & \cdots & \cdots & \cdots \\ f_{y_1}^n & f_{y_2}^n & \cdots & f_{y_n}^n \end{vmatrix} \neq 0. \tag{62}$$

From this, we finally have

$$\deg\{JQN, \Omega \cap \operatorname{Ker} L, (0, 0, \ldots, 0)\} \neq 0. \tag{63}$$

This shows that Ω satisfies condition (c) of Lemma 2. Therefore, system (26) has an ω-periodic solution $y^*(t) = (y_1^*(t), y_2^*(t), \ldots, y_n^*(t)) \in \overline{\Omega}$. Finally, we have system that (4) has a positive ω-periodic solution. This completes the proof. □

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