Hindawi Publishing Corporation
Fixed Point Theory and Applications
Volume 2007, Article ID 48174, 9 pages
doi:10.1155/2007/48174

Research Article

Strong Convergence of Modified Implicit Iteration Processes for Common Fixed Points of Nonexpansive Mappings

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Received 21 December 2006; Accepted 19 March 2007

Recommended by William Art Kirk

Strong convergence theorems are obtained by hybrid method for modified composite implicit iteration process of nonexpansive mappings in Hilbert spaces. The results presented in this paper generalize and improve the corresponding results of Nakajo and Takahashi (2003) and others.

1. Introduction and preliminaries

Throughout this paper, let H be a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$. Let C be a nonempty closed convex subset of H, we denote by $P_C(\cdot)$ the metric projection from H onto C. It is known that $z = P_C(x)$ is equivalent to $\langle z - y, x - z \rangle \geq 0$ for every $y \in C$. Recall that $T : C \to C$ is nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$. A point $x \in C$ is a fixed point of T provided that $Tx = x$. Denote by $F(T)$ the set of fixed points of T, that is, $F(T) = \{x \in C : Tx = x\}$. It is known that $F(T)$ is closed and convex.

Construction of fixed points of nonexpansive mappings (and asymptotically nonexpansive mappings) is an important subject in the theory of nonexpansive mappings and finds application in a number of applied areas, in particular, in image recovery and signal processing (see, e.g., [1–5]). However, the sequence $\{T^n x\}_{n=0}^{\infty}$ of iterates of the mapping T at a point $x \in C$ may not converge even in the weak topology. Thus averaged iterations prevail. Indeed, Mann's iterations do have weak convergence. More precisely, Mann's iteration procedure is a sequence $\{x_n\}$ which is generated in the following recursive way:

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n) T x_n, \quad n \geq 0, \tag{1.1}$$

where the initial value $x_0 \in C$ is chosen arbitrarily. For example, Reich [6] proved that if X is a uniformly convex Banach space with a Fréchet differentiable norm and if $\{\alpha_n\}$ is chosen such that $\sum_{n=1}^{\infty} \alpha_n(1-\alpha_n) = \infty$, then the sequence $\{x_n\}$ defined by (1.1) converges weakly to a fixed point of T. However we note that Mann's iterations have only weak convergence even in a Hilbert space [7].

Attempts to modify the Mann iteration method (1.1) so that strong convergence is guaranteed have recently been made. Nakajo and Takahashi [8] proposed the following modification of Mann iteration method (1.1) for a single nonexpansive mapping T in a Hilbert space H:

$$\begin{gathered} x_0 \in C \text{ chosen arbitrarily}, \\ y_n = \alpha_n x_n + (1-\alpha_n) T x_n, \\ C_n = \{z \in C : \|y_n - z\| \leq \|x_n - z\|\}, \\ Q_n = \{z \in C : \langle x_n - z, x_0 - x_n \rangle \geq 0\}, \\ x_{n+1} = P_{C_n \cap Q_n}(x_0). \end{gathered} \tag{1.2}$$

They proved that if the sequence $\{\alpha_n\}$ is bounded above from one, then the sequence $\{x_n\}$ generated by (1.2) converges strongly to $P_{F(T)}(x_0)$.

In recent years, the implicit iteration scheme for approximating fixed points of nonlinear mappings has been introduced and studied by several authors.

In 2001, Xu and Ori [9] introduced the following implicit iteration scheme for common fixed points of a finite family of nonexpansive mappings $\{T_i\}_{i=1}^N$ in Hilbert spaces:

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n) T_n x_n, \quad n \geq 1, \tag{1.3}$$

where $T_n = T_{n \bmod N}$, and they proved weak convergence theorem.

In 2004, Osilike [10] extended results of Xu and Ori from nonexpansive mappings to strictly pseudocontractive mappings. By this implicit iteration scheme (1.3) he proved some convergence theorems in Hilbert spaces and Banach spaces.

We note that it is the same as Mann's iterations that have only weak convergence theorems with implicit iteration scheme (1.3). In this paper, we introduce the following two general composite implicit iteration schemes and modify them by hybrid method, so strong convergence theorems are obtained:

$$\begin{aligned} x_n &= \alpha_n x_{n-1} + (1-\alpha_n) T_n y_n, \\ y_n &= \beta_n x_n + (1-\beta_n) T_n x_n, \end{aligned} \tag{1.4}$$

$$\begin{aligned} x_n &= \alpha_n x_{n-1} + (1-\alpha_n) T_n y_n, \\ y_n &= \beta_n x_{n-1} + (1-\beta_n) T_n x_n, \end{aligned} \tag{1.5}$$

where $T_n = T_{n \bmod N}$.

Observe that if K is a nonempty closed convex subset of a real Banach space E and $T : K \to K$ is a nonexpansive mapping, then for every $u \in K$, $\alpha,\beta \in [0,1]$, and positive integer n, the operator $S = S_{(\alpha,\beta)} : K \to K$ defined by

$$Sx = \alpha u + (1-\alpha)T(\beta x + (1-\beta)Tx) \tag{1.6}$$

satisfies

$$\begin{aligned}\|Sx - Sy\| &= (1-\alpha)\|T(\beta x + (1-\beta)Tx) - T(\beta y + (1-\beta)Ty)\| \\ &\le (1-\alpha)\|(\beta x + (1-\beta)Tx) - (\beta y + (1-\beta)Ty)\| \\ &\le (1-\alpha)[\beta\|x-y\| + (1-\beta)\|Tx - Ty\|] \\ &\le (1-\alpha)[\beta\|x-y\| + (1-\beta)\|x-y\|] \le (1-\alpha)\|x-y\|,\end{aligned} \tag{1.7}$$

for all $x, y \in K$. Thus, if $\alpha > 0$, then S is a contraction and so has a unique fixed point $x^* \in K$. Thus there exists a unique $x^* \in K$ such that

$$x^* = \alpha u + (1-\alpha)T(\beta x^* + (1-\beta)Tx^*). \tag{1.8}$$

This implies that if $\alpha_n > 0$, the general composite implicit iteration scheme (1.4) can be employed for the approximation of common fixed points of a finite family of nonexpansive mappings.

For the same reason, the operator $S = S_{(\alpha,\beta)} : K \to K$ defined by

$$Sx = \alpha u + (1-\alpha)T(\beta u + (1-\beta)Tx) \tag{1.9}$$

satisfies

$$\begin{aligned}\|Sx - Sy\| &= (1-\alpha)\|T(\beta u + (1-\beta)Tx) - T(\beta u + (1-\beta)Ty)\| \\ &\le (1-\alpha)\|(\beta u + (1-\beta)Tx) - (\beta u + (1-\beta)Ty)\| \\ &\le (1-\alpha)(1-\beta)\|Tx - Ty\| \le (1-\alpha)(1-\beta)\|x-y\|,\end{aligned} \tag{1.10}$$

for all $x, y \in K$. Thus, if $(1-\alpha)(1-\beta) < 1$, the S is a contractive mapping, then S has a unique fixed point $x^* \in K$. Thus there exists a unique $x^* \in K$ such that

$$x^* = \alpha u + (1-\alpha)T(\beta u + (1-\beta)Tx^*). \tag{1.11}$$

This implies that if $(1-\alpha_n)(1-\beta_n) < 1$, the general composite implicit iteration scheme (1.5) can be employed for the approximation of common fixed points of a finite family of nonexpansive mappings.

It is the purpose of this paper to modify iteration processes (1.4) and (1.5) by hybrid method as follows:

$$\begin{aligned}
&x_0 \in C \text{ chosen arbitrarily},\\
&y_n = \alpha_n x_n + (1-\alpha_n) T_n z_n,\\
&z_n = \beta_n y_n + (1-\beta_n) T_n y_n,\\
&C_n = \{z \in C : ||y_n - z|| \leq ||x_n - z||\},\\
&Q_n = \{z \in C : \langle x_n - z, x_0 - x_n\rangle \geq 0\},\\
&x_{n+1} = P_{C_n \cap Q_n}(x_0).
\end{aligned} \tag{1.12}$$

$$\begin{aligned}
&x_0 \in C \text{ chosen arbitrarily},\\
&y_n = \alpha_n x_n + (1-\alpha_n) T_n z_n,\\
&z_n = \beta_n x_n + (1-\beta_n) T_n y_n,\\
&C_n = \{z \in C : ||y_n - z|| \leq ||x_n - z||\},\\
&Q_n = \{z \in C : \langle x_n - z, x_0 - x_n\rangle \geq 0\},\\
&x_{n+1} = P_{C_n \cap Q_n}(x_0),
\end{aligned} \tag{1.13}$$

where $T_n = T_{n \bmod N}$, for common fixed points of a finite family of nonexpansive mappings $\{T_i\}_{i=1}^N$ in Hilbert spaces and to prove strong convergence theorems.

We will use the notation (1) $\rightharpoonup$ for weak convergence and $\to$ for strong convergence. (2) $w_w(x_n) = \{x : \exists x_{n_j} \rightharpoonup x\}$ denotes the weak w-limit set of $\{x_n\}$. We need some facts and tools in a real Hilbert space H which are listed as lemmas below.

Lemma 1.1 (see Martinez-Yanes and Xu [11]). *Let H be a real Hilbert space, C a closed convex subset of H. Given points $x, y \in H$, the set*

$$D = \{v \in C : \|y - v\| \leqslant \|x - v\|\} \tag{1.14}$$

is closed and convex.

Lemma 1.2 (see Goebel and Kirk [12]). *Let C be a closed convex subset of a real Hilbert space H and let $T : C \to C$ be a nonexpansive mapping such that $Fix(T) \neq \emptyset$. If a sequence $\{x_n\}$ in C is such that $x_n \rightharpoonup z$ and $x_n - Tx_n \to 0$, then $z = Tz$.*

Lemma 1.3 (see Martinez-Yanes and Xu[11]). *Let K be a closed convex subset of H. Let $\{x_n\}$ be a sequence in H and $u \in H$. Let $q = P_k u$. If $\{x_n\}$ is such that $w_w(x_n) \subset K$ and satisfies the condition*

$$||x_n - u|| \leqslant \|u - q\|, \quad \forall n, \tag{1.15}$$

then $x_n \to q$.

2. Main results

Let C be a nonempty closed convex subset of H, let $\{T_i\}_{i=1}^N : C \to C$ be N nonexpansive mappings with nonempty common fixed points set F. Assume that $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in $[0,1]$. We consider the sequence $\{x_n\}$ generated by (1.12). We assume that $\alpha_n > 0$ (for all $n \in N$) in Lemmas 2.1, 2.2, and 2.3.

LEMMA 2.1. *$\{x_n\}$ is well defined and $F \subset C_n \cap Q_n$ for every $n \in N \cup \{0\}$.*

Proof. First observe that C_n is convex by Lemma 1.1. Next, we show that $F \subset C_n$ for all n. Indeed, we have, for all $p \in F$,

$$\begin{aligned}
||y_n - p|| = ||\alpha_n x_n + (1-\alpha_n)T_n z_n - p|| &\leqslant \alpha_n||x_n - p|| + (1-\alpha_n)||T_n z_n - p|| \\
&\leqslant \alpha_n||x_n - p|| + (1-\alpha_n)||z_n - p|| \\
&\leqslant \alpha_n||x_n - p|| + (1-\alpha_n)||\beta_n y_n + (1-\beta_n)T_n y_n - p|| \\
&\leqslant \alpha_n||x_n - p|| + (1-\alpha_n)[\beta_n||y_n - p|| + (1-\beta_n)||T_n y_n - p||] \\
&\leqslant \alpha_n||x_n - p|| + (1-\alpha_n)||y_n - p||.
\end{aligned} \tag{2.1}$$

It follows that

$$||y_n - p|| \leq ||x_n - p||. \tag{2.2}$$

So $p \in C_n$ for every $n \geq 0$, therefore $F \subset C_n$ for every $n \geq 0$.

Next, we show that $F \subset C_n \cap Q_n$ for all $n \geq 0$. It suffices to show that $F \subset Q_n$, for all $n \geq 0$. We prove this by mathematical induction. For $n = 0$, we have $F \subset C = Q_0$. Assume that $F \subset Q_n$. Since x_{n+1} is the projection of x_0 onto $C_n \cap Q_n$, we have

$$\langle x_{n+1} - z, x_0 - x_{n+1}\rangle \geq 0, \quad \forall z \in Q_n \cap C_n, \tag{2.3}$$

as $F \subset C_n \cap Q_n$, the last inequality holds, in particular, for all $z \in F$. This together with the definition of Q_{n+1}, implies that $F \subset Q_{n+1}$. Hence the $F \subset C_n \cap Q_n$ holds for all $n \geq 0$. This completes the proof. □

LEMMA 2.2. *$\{x_n\}$ is bounded.*

Proof. Since F is a nonempty closed convex subset of C, there exists a unique element $z_0 \in F$ such that $z_0 = P_F(x_0)$. From $x_{n+1} = P_{C_n \bigcap Q_n}(x_0)$, we have

$$||x_{n+1} - x_0|| \leq ||z - x_0||, \tag{2.4}$$

for every $z \in C_n \cap Q_n$. As $z_0 \in F \subset C_n \cap Q_n$, we get

$$||x_{n+1} - x_0|| \leq ||z_0 - x_0||, \tag{2.5}$$

for each $n \geq 0$. This implies that $\{x_n\}$ is bounded, so the proof is complete. □

LEMMA 2.3. $\|x_{n+1} - x_n\| \to 0$.

Proof. Indeed, by the definition of Q_n, we have that $x_n = P_{Q_n}(x_0)$ which together with the fact that $x_{n+1} \in C_n \cap Q_n$ implies that

$$||x_0 - x_n|| \leq ||x_0 - x_{n+1}||. \tag{2.6}$$

This shows that the sequence $\{\|x_n - x_0\|\}$ is increasing, from Lemma 2.2, we know that $\lim_{n\to\infty} \|x_n - x_0\|$ exists. Noticing again that $x_n = P_{Q_n}(x_0)$ and $x_{n+1} \in Q_n$ which implies that $\langle x_{n+1} - x_n, x_n - x_0\rangle \geq 0$, and noticing the identity

$$\|u - v\|^2 = \|u\|^2 - \|v\|^2 - 2\langle u - v, v\rangle, \quad \forall u, v \in H, \tag{2.7}$$

we have

$$\begin{aligned} ||x_{n+1} - x_n||^2 &= ||(x_{n+1} - x_0) - (x_n - x_0)||^2 \\ &\leq ||x_{n+1} - x_0||^2 - ||x_n - x_0||^2 - 2\langle x_{n+1} - x_n, x_n - x_0\rangle \\ &\leq ||x_{n+1} - x_0||^2 - ||x_n - x_0||^2 \longrightarrow 0, \ n \longrightarrow \infty. \end{aligned} \tag{2.8}$$

□

THEOREM 2.4. *If* $\{\alpha_n\} \subset (0, a]$ *for some* $a \in (0,1)$ *and* $\{\beta_n\} \subset [b, 1]$ *for some* $b \in (0,1]$, *then* $x_n \to z_0$, *where* $z_0 = P_F(x_0)$.

Proof. We first prove that $\|T_n z_n - x_n\| \to 0$, indeed,

$$||T_n z_n - x_n|| = \frac{1}{1-\alpha_n}||y_n - x_n|| \leq \frac{1}{1-\alpha_n}(||y_n - x_{n+1}|| + ||x_{n+1} - x_n||). \tag{2.9}$$

Since $x_{n+1} \in C_n$, then

$$||y_n - x_{n+1}|| \leq ||x_n - x_{n+1}||, \tag{2.10}$$

by Lemma 2.3 $\|x_{n+1} - x_n\| \to 0$, so that $\|y_n - x_{n+1}\| \to 0$, which leads to

$$||T_n z_n - x_n|| \longrightarrow 0. \tag{2.11}$$

On the other hand, we have

$$\begin{aligned} ||T_n x_n - x_n|| &\leq ||T_n x_n - T_n z_n|| + ||T_n z_n - x_n|| \leq ||z_n - x_n|| + ||T_n z_n - x_n|| \\ &\leq \beta_n ||y_n - x_n|| + (1-\beta_n)||T_n y_n - x_n|| + ||T_n z_n - x_n|| \\ &\leq \beta_n ||y_n - x_n|| + (1-\beta_n)[||T_n y_n - T_n x_n|| + ||T_n x_n - x_n||] + ||T_n z_n - x_n|| \\ &\leq \beta_n ||y_n - x_n|| + (1-\beta_n)[||y_n - x_n|| + ||T_n x_n - x_n||] + ||T_n z_n - x_n|| \\ &\leq ||y_n - x_n|| + (1-\beta_n)||T_n x_n - x_n|| + ||T_n z_n - x_n||, \end{aligned} \tag{2.12}$$

which implies that

$$||T_nx_n - x_n|| \le \frac{1}{\beta_n}||y_n - x_n|| + \frac{1}{\beta_n}||T_nz_n - x_n||. \tag{2.13}$$

By the condition $0 < b \le \beta_n$ and (2.11), we obtain that

$$||T_nx_n - x_n|| \longrightarrow 0, \quad \text{as } n \longrightarrow \infty, \tag{2.14}$$

from Lemma 2.3, we know that $\|x_{n+1} - x_n\| \to 0$, so that for all $j = 1,2,\dots,N$,

$$||x_n - x_{n+j}|| \longrightarrow 0, \quad \text{as } n \longrightarrow \infty. \tag{2.15}$$

So, for any $i = 1,2,\dots,N$, we also have

$$\begin{aligned}||x_n - T_{n+i}x_n|| &\le ||x_n - x_{n+i}|| + ||x_{n+i} - T_{n+i}x_{n+i}|| + ||T_{n+i}x_{n+i} - T_{n+i}x_n|| \\ &\le ||x_n - x_{n+i}|| + ||x_{n+i} - T_{n+i}x_{n+i}|| + ||x_{n+i} - x_n|| \\ &\le 2||x_n - x_{n+i}|| + ||x_{n+i} - T_{n+i}x_{n+i}||.\end{aligned} \tag{2.16}$$

Thus, it follows from (2.15) and (2.14) that

$$\lim_{n\to+\infty} ||T_{n+i}x_n - x_n|| = 0, \quad i = 1,2,3,\dots,N. \tag{2.17}$$

Because $T_n = T_{n \bmod N}$, it is easy to see, for any $l = 1,2,3,\dots,N$, that

$$\lim_{n\to+\infty} ||T_lx_n - x_n|| = 0. \tag{2.18}$$

By Lemma 1.2 and (2.18), we obtain that $w_w(x_n) \subset F(T_l)$. So, $w_w(x_n) \subset F = \bigcap_{l=1}^N F(T_l)$, this, together with $\|x_n - x_0\| \leqslant \|P_F(x_0) - x_0\|$ (for all $n \in N$) and Lemma 1.3, guarantees the strong convergence of $\{x_n\}$ to $P_F(x_0)$. □

Remark 2.5. If we set $\beta_n = 1$ for all n, then $z_n = y_n$ and $y_n = \alpha_nx_n + (1-\alpha_n)T_ny_n$, the iteration scheme (1.12) becomes modified implicit iteration scheme, so we, from Theorem 2.4, obtain the convergence theorem of composite modified implicit iteration scheme.

THEOREM 2.6. *Let C be a nonempty closed convex subset of H, let $\{T_i\}_{i=1}^N : C \to C$ be N nonexpansive mappings with nonempty common fixed points set F. Assume that $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in $[0,1]$ and $\{\alpha_n\} \subset [0,a]$ for some $a \in [0,1)$ and $\{\beta_n\} \subset [b,1]$ for some $b \in (0,1]$, then the sequence $\{x_n\}$ generated by (1.13) has $x_n \to z_0$, where $z_0 = P_F(x_0)$.*

Proof. First, we prove that $\{x_n\}$ is well defined and $F \subset C_n \cap Q_n$ for every $n \in N \cup \{0\}$. Observe that C_n is convex by Lemma 1.1. Next, we show that $F \subset C_n$ for all n. Indeed, we have, for all $p \in F$,

$$\begin{aligned}
\|y_n - p\| &= \|\alpha_n x_n + (1-\alpha_n) T_n z_n - p\| \\
&\leqslant \alpha_n \|x_n - p\| + (1-\alpha_n)\|T_n z_n - p\| \\
&\leqslant \alpha_n \|x_n - p\| + (1-\alpha_n)\|z_n - p\| \\
&\leqslant \alpha_n \|x_n - p\| + (1-\alpha_n)\|[\beta_n x_n + (1-\beta_n) T_n y_n] - p\| \\
&\leqslant \alpha_n \|x_n - p\| + (1-\alpha_n)[\beta_n \|x_n - p\| + (1-\beta_n)\|T_n y_n - p\|] \\
&\leqslant (\alpha_n + \beta_n - \alpha_n\beta_n)\|x_n - p\| + (1-\alpha_n)(1-\beta_n)\|y_n - p\|.
\end{aligned} \tag{2.19}$$

It follows that

$$\|y_n - p\| \le \|x_n - p\|. \tag{2.20}$$

So $p \in C_n$ for every $n \ge 0$, therefore $F \subset C_n$ for every $n \ge 0$.

Next, we show that $F \subset C_n \cap Q_n$ for all $n \ge 0$. It suffices to show that $F \subset Q_n$, for all $n \ge 0$. We prove this by mathematical induction. For $n = 0$, we have $F \subset C = Q_0$. Assume that $F \subset Q_n$. Since x_{n+1} is the projection of x_0 onto $C_n \cap Q_n$, we have

$$\langle x_{n+1} - z, x_0 - x_{n+1}\rangle \ge 0, \quad \forall z \in Q_n \cap C_n, \tag{2.21}$$

as $F \subset C_n \cap Q_n$, the last inequality holds, in particular, for all $z \in F$. This together with the definition of Q_{n+1} implies that $F \subset Q_{n+1}$. Hence $F \subset C_n \cap Q_n$ holds for all $n \ge 0$. This completes the proof. □

By Lemma 2.2 $\{x_n\}$ is bounded and by Lemma 2.3 $\|x_{n+1} - x_n\| \to 0$, so that $\|y_n - x_n\| \to 0$, which leads to

$$\|T_n z_n - x_n\| = \frac{1}{1-\alpha_n}\|y_n - x_n\| \longrightarrow 0. \tag{2.22}$$

On the other hand, we have

$$\begin{aligned}
\|T_n x_n - x_n\| &\le \|T_n x_n - T_n z_n\| + \|T_n z_n - x_n\| \\
&\le \|z_n - x_n\| + \|T_n z_n - x_n\| \\
&\le (1-\beta_n)\|T_n y_n - x_n\| + \|T_n z_n - x_n\| \\
&\le (1-\beta_n)[\|T_n y_n - T_n x_n\| + \|T_n x_n - x_n\|] + \|T_n z_n - x_n\| \\
&\le (1-\beta_n)[\|y_n - x_n\| + \|T_n x_n - x_n\|] + \|T_n z_n - x_n\|,
\end{aligned} \tag{2.23}$$

which implies that

$$\|T_n x_n - x_n\| \le \frac{1-\beta_n}{\beta_n}\|y_n - x_n\| + \frac{1}{\beta_n}\|T_n z_n - x_n\|. \tag{2.24}$$

By the condition $0 < b \leq \beta_n$ and (2.22), we obtain that

$$||T_n x_n - x_n|| \longrightarrow 0, \quad \text{as } n \longrightarrow \infty. \tag{2.25}$$

As in the proof of Theorem 2.4 we have for any $l = 1, 2, 3, \ldots, N$ that

$$\lim_{n \to +\infty} ||T_l x_n - x_n|| = 0. \tag{2.26}$$

By Lemma 1.2 and (2.26), we obtain that $w_w(x_n) \subset F(T_l)$. So, $w_w(x_n) \subset F = \bigcap_{l=1}^{N} F(T_l)$, this together with $\|x_n - x_0\| \leqslant \|P_F(x_0) - x_0\|$ (for all $n \in N$) and Lemma 1.3 guarantees the strong convergence of $\{x_n\}$ to $P_F(x_0)$.

Remark 2.7. If we set $\beta_n = 1$ for all n, then $z_n = x_n$ and $y_n = \alpha_n x_n + (1 - \alpha_n) T_n x_n$, so the iteration scheme (1.13) becomes modified Mann iteration, and if there is only one nonexpansive mapping, we can obtain the theorem of Nakajo and Takahashi [8].

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